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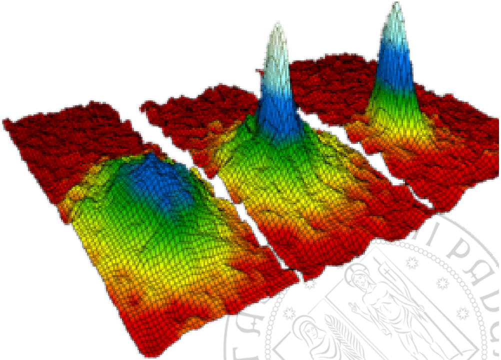
December 2, 2014

Plan of the talk

- Ultracold Fermi gases: condensate fraction for an unbalanced Fermi gas. Role of the fluctuations throughout the BCS-BEC crossover. (supervisor: prof. Luca Salasnich)
- A gauge approach to superfluid density in cuprates (prof. Pieralberto Marchetti)
- Other activities/future plans.

Ultracold Fermi gases (1/3)

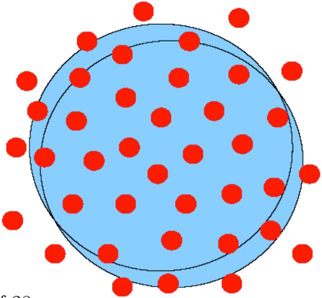
- Ultracold gases: experimental observation of quantum properties of matter. Vortices in a superfluid, BEC.
- Bose-Einstein condensation (1995), degenerate Fermi gas and fermionic condensate (2003).
- Very clean experimental environment: control over the temperature, the number of particles, the interaction.



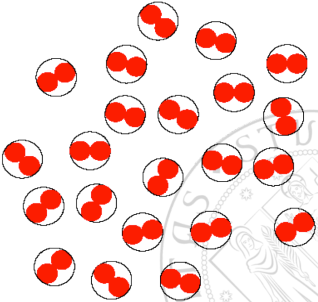
Ultracold Fermi gases (2/3)

Why are ultracold Fermi gases interesting? The fermion-fermion interaction can be tuned (using a Feshbach resonance), from weakly to strongly interacting: the **BCS-BEC crossover**.

BCS regime: coherence in momentum space.

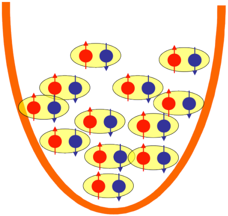


BEC regime: coherence in coordinate space.

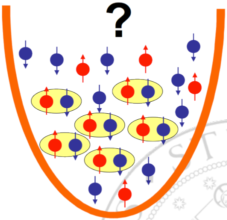


Ultracold Fermi gases (3/3)

Balanced Fermi gas



Polarized Fermi gas



Path integral and the partition function

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S[\psi,\bar{\psi}]}$$

$$S\left[\psi,\bar{\psi}\right] = \int_0^\beta \mathrm{d}\tau \sum_{\mathbf{k}} \sum_{\sigma=\uparrow,\downarrow} \bar{\psi}_{\mathbf{k}\sigma}(\partial_\tau + \xi_{\mathbf{k}})\psi_{\mathbf{k}\sigma} + \frac{g}{\Omega} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \bar{\psi}_{\mathbf{k}+\mathbf{q}\uparrow}\bar{\psi}_{-\mathbf{k}\downarrow}\psi_{-\mathbf{k}'+\mathbf{q}\downarrow}\psi_{\mathbf{k}'\uparrow}$$

- Hubbard-Stratonovich transformation, introducing the pairing field $\Delta(\mathbf{r},\tau) \sim \bar{\psi}\psi$
- The fermionic variables are integrated out, the pairing field integration is replaced by the mean field approximation:

$$\Delta(\mathbf{r},\tau) = \Delta_0 + \cancel{\delta(\mathbf{r},\tau)}$$

- Gap equation/number equation:

$$\frac{\partial\Omega}{\partial\Delta_0} = 0 \qquad n = -\left.\frac{\partial\Omega}{\partial\mu}\right|_{T,\zeta} \qquad \Omega = -\frac{1}{\beta}\ln\mathcal{Z}$$

Condensate fraction (1/2)

- Why? The condensate fraction is *the* fundamental signature of Bose-Einstein condensation: a finite fraction of particles occupying the ground state.

$$N_0 = 2\sum_{\mathbf{k}} u_{\mathbf{k}}^2 v_{\mathbf{k}}^2 = \sum_{\mathbf{k}} \frac{\Delta_0^2}{4E_{\mathbf{k}}^2} \qquad E_{\mathbf{k}} = \sqrt{\left(\hbar^2 \frac{k^2}{2m} - \mu\right)^2 + \Delta_0^2}$$

Condensate fraction (2/2)

Paper (published in J. Phys. B): a study of the condensate fraction across the BCS-BEC crossover for an unbalanced Fermi gas, in the uniform and trapped cases.

Journal of Physics B: Atomic, Molecular and Optical Physics

Journal of Physics B: Atomic, Molecular and Optical Physics > Volume 47 >
Number 19

Pair condensation of polarized fermions in the BCS–BEC crossover

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Show affiliations

G Bighin *et al* 2014 *J. Phys. B: At. Mol. Opt. Phys.* **47** 195302. doi:10.1088/0953-4075/47/19/195302
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I will briefly review the main results from this paper.

Condensate fraction for an unbalanced Fermi gas: theory

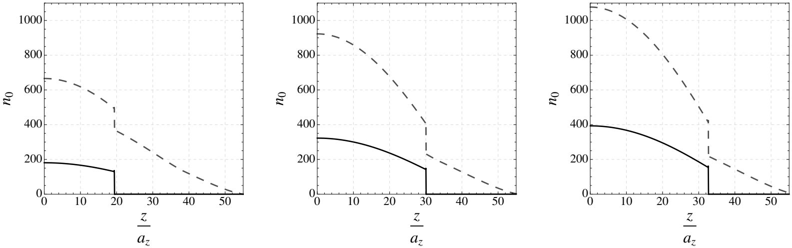


Figure: Condensate density profile $n_0(z)$ (solid line) and total density profile $n(z)$ (dashed line) in the axial direction z for three different scattering lengths. From left to right: $y = -0.44$, $y = 0.0$, $y = 0.11$, where $y = (k_F a_s)^{-1}$ with $k_F = (3\pi^2 n(\mathbf{0}))^{\frac{1}{3}}$ and $n(\mathbf{0})$ the total density at the center of the trap. Number of atoms $N = 2.3 \times 10^7$ and polarization $P = (N_\uparrow - N_\downarrow)/N = 0.2$. Here $a_z = \frac{1}{\sqrt{m\omega_z}}$ is the characteristic length of the axial harmonic confinement.

Condensate fraction for an unbalanced Fermi gas: theory vs. experiments

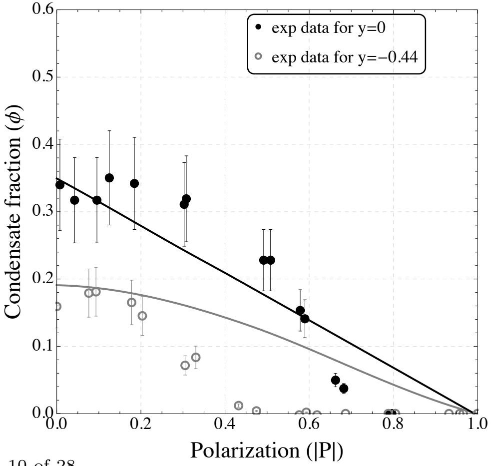


Figure: Condensate fraction ϕ as a function of the absolute value of the polarization $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$ for two values of the dimensionless interaction parameter $y = (k_F a_s)^{-1}$: $y = -0.44$ (open circles) and $y = 0.0$ (filled circles), T=0. y is a measure of the interaction strength ($y \gg 1$ corresponding to the deep-BEC regime, and $y \ll -1$ corresponding to the deep-BCS regime). Circles with error bars are experimental data of ^6Li atoms taken from MIT experiment. Solid lines are our theoretical calculations for the trapped system.

The role of the fluctuations in the BCS-BEC crossover (1/2)

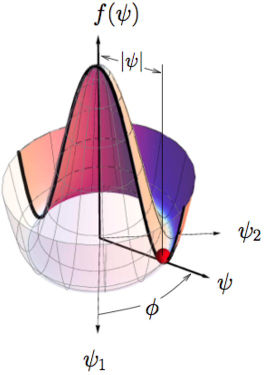
A natural extension consists of improving the mean-field theory with Gaussian fluctuations for the pairing field. (supervisor: Luca Salasnich, in collaboration with Flavio Toigo, Pieralberto Marchetti, Luca Dell'Anna)

We go back to the path integral formulation; now we retain the order parameter fluctuations up to Gaussian order:

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\Delta \mathcal{D}\bar{\Delta} e^{-S[\psi,\bar{\psi},\Delta,\bar{\Delta}]} = \int \mathcal{D}\Delta \mathcal{D}\bar{\Delta} e^{-\beta F[\Delta,\bar{\Delta}]}$$

The dynamics of the pairing field Δ are determined by the effective potential $F[\Delta, \bar{\Delta}]$

The role of the fluctuations in the BCS-BEC crossover (2/2)



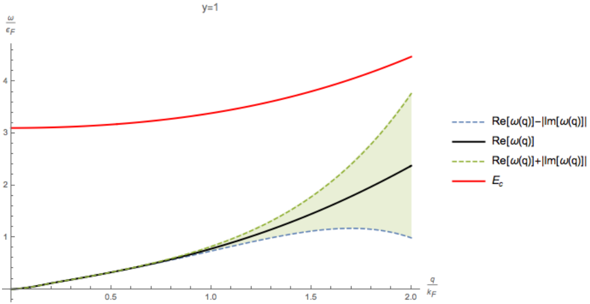
The effective potential F reads (modulo a constant factor, note that $g < 0$):

$$F = -\frac{1}{\beta} \sum_n \sum_{\mathbf{k}} \log\left(1 + \frac{\Delta\bar{\Delta}}{\omega_n^2 + \xi_{\mathbf{k}}^2}\right) + \frac{V}{g} \Delta\bar{\Delta}$$

$$F^{(4)} \approx \frac{V}{g} \Delta\bar{\Delta} + \frac{7}{16} \zeta(3) \frac{\rho V}{(\pi T)^2} \left(\Delta\bar{\Delta}\right)^2$$

In the broken symmetry phase ($T < T_c$) we expect a phase (Goldstone) mode and an amplitude (Higgs) mode.

The Goldstone mode



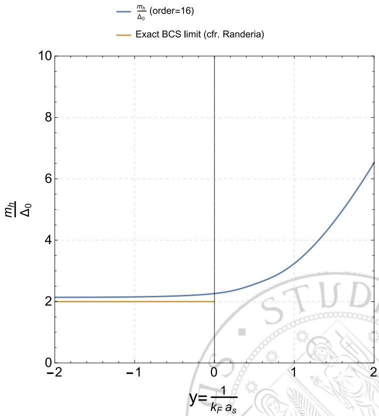
- It is the usual Bogoliubov excitation spectrum:

$$\hbar\omega_{\mathbf{q}} = \sqrt{\frac{\hbar^2 q^2}{2m}\left(\frac{\hbar^2 q^2}{2m} + 2gn\right)}$$

- Beliaev decay: does it define a natural cutoff scale?

The Higgs mode

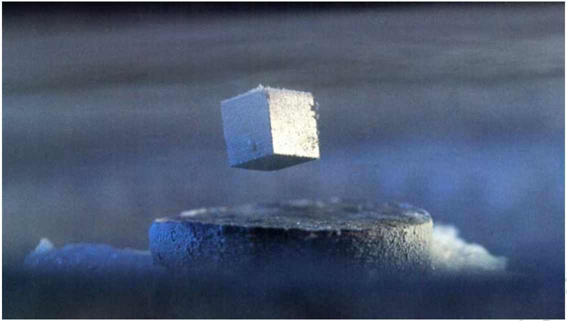
Higgs mode mass as a function of the interaction parameter $y = (k_F a_s)^{-1}$.



Open questions:

- Stability of the Higgs mode. Calculation of the decay width.
- Full spectrum.

High-T_c superconductivity in cuprates



Main reference: P. A. Marchetti, F. Ye, Z. B. Su, and L. Yu Phys. Rev. B **84**, 214525

Cuprates: an overview (1/2)

- **Superconducting cuprates**: a class of superconducting materials with very high critical temperatures (up to 135 °K).
- Discovered in 1986 by J. G. Bednorz e K. A. Müller; Nobel prize awarded in 1987, the fastest in history.
- Very active research field: more than 100,000 research articles in ~ 25 years.
- Up to date, the microscopical mechanism behind SC in cuprates is not completely understood.

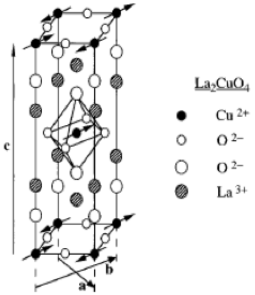
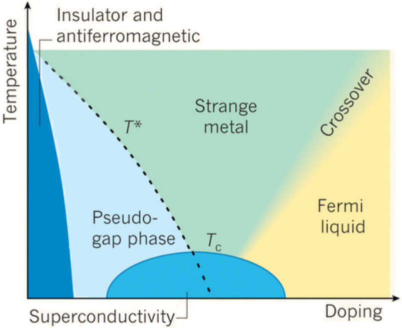


Figure: Unitary cell for La_2CuO_4.

Cuprates: an overview (2/2)

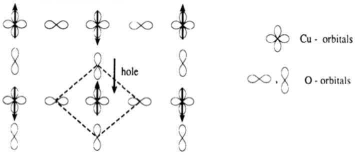
- Different chemical compositions (YBCO, LSCO, BSSCO) the only common chemical features being the CuO_2 planes. $\Longrightarrow$ The CuO_2 planes are believed to be the main seat of superconductivity.



- Dependence on doping and universality for the **phase diagram**.
- BCS theory can not account for SC in cuprates.

From the CuO_2 planes to the t/J model

CuO_2 planes in terms of Zhang-Rice singlets:



ZR: Doping-induced hole reside (primarily) on combinations of four oxygen p orbitals centered around a copper site.

From ZR singlets to the t/J model:

- Strong on-site repulsion (P_G)
- Nearest neighbour hopping ($t \approx 0.3\,\mathrm{eV}$)
- Anti-ferromagnetic Heisenberg term ($J \approx 0.1\,\mathrm{eV}$)

$$H_{\mathrm{t/J}} = \sum_{\langle i,j \rangle} P_G \left[-t \sum_{\alpha} c_{i\alpha}^{\dagger} c_{j\alpha} + h.c. + J\mathbf{S}_i \cdot \mathbf{S}_j \right] P_G$$

"Doping a Mott insulator", P.A. Lee, N. Nagaosa, X.-G. Wen, Rev. Mod. Phys. **78**, 17

Spin-charge separation

The electron creation/annihilation operators are decomposed as follows:

$$\hat{c}_{i,\alpha} = \hat{s}_{i,\alpha}\hat{h}_i^\dagger$$

where:

- $\hat{h}_i$ is a spinless fermion (holon): the P_G constraint is automatically satisfied due to Pauli exlusion principle.
- $\hat{s}_{i,\alpha}$ is a spin $\frac{1}{2}$ boson (spinon).

Local invariance introduced by this process:

$$U\left(1\right)_{h/s} \qquad \begin{cases} \hat{s}_{i,\alpha} \longrightarrow \hat{s}_{i,\alpha} e^{\mathrm{i}\phi(x)} \\ \hat{h}_i \longrightarrow \hat{h}_i e^{\mathrm{i}\phi(x)} \end{cases}$$

Emergent $U\left(1\right)$ gauge field: $A_\mu \approx s^*_\alpha \partial_\mu s_\alpha + \cdots$

Effective action

Effective action for the model:

- Holons are (formally) relativistic Dirac fermions.
- Spinons are described by a non-linear massive σ model, with $m \sim |\delta \ln \delta|$.
- The gauge field A_μ is minimally coupled to holons and spinons: it corresponds to the h/s symmetry: it provides a "gauge glue" between the two electron components.

$\longrightarrow$ we have an effective description of the t/J model in terms of holons and spinons.

Towards superconductivity

The electron has a composite structure:
spinon + holon

$\longmapsto$

Superconductivity is achieved in three stages:

- Holon pairing
- Spinon pairing
- Phase coherence

The pairing process

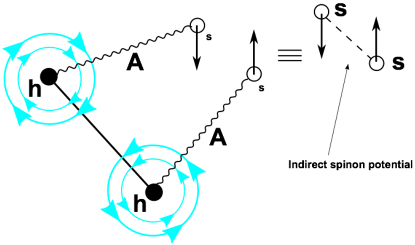


Figure: The attractive potential between the spinons, essential for the SC, is mediated by a gauge field "binding" holon and spinons, and by the holon attraction.

The superconductivity is achieved in **three steps**: holon pairing (T_{ph}), spinon pairing (T_{ps}), phase coherence (T_c):

$$\Delta_c \sim \frac{|\Delta^s|}{|\Delta^h|} e^{\mathrm{i}\overbrace{(\phi_s - \phi_h)}^{\equiv \phi}}$$

$$SC \Longleftrightarrow \langle \Delta_c \rangle \neq 0$$

The direct spinon-spinon interaction is repulsive, the gauge fluctuations play a key role.

Superfluid density

I studied the superfluid density within this model.

- Superfluid density (ρ_s). $S_{\mathrm{EFF}} = \frac{\rho_s}{2} \int \mathrm{d}\tau \mathrm{d}^d r \, (\nabla\theta)^2 + \cdots$
- Importance:
  - Lots of experimental data, $\rho_s \propto \lambda^{-2}$.
  - Very different from BCS.
  - Empirical relations (Uemura relation: $\rho_s(T=0) \propto T_c$).

Results:

- Summation formula ($\sim$ Ioffe-Larkin)

$$\rho_s = \frac{\rho_s^s \rho_s^h}{\rho_s^s + \rho_s^h}$$

- Very good agreement with experimental data. The critical exponent is exactly reproduced: $\rho_{\mathrm{s}} \sim \left|\frac{T-T_c}{T}\right|^{\frac{2}{3}}$ for $T \longrightarrow T_c$

“A gauge approach to superfluid density in cuprates”

These results led to a paper (recently submitted); we also compared our theoretical model with experimental data.

Gauge approach to superfluid density in cuprates

P.A. Marchetti[1,2] and G. Bighin[1,2]
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[2] *Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Via Marzolo 8, 35131 Padova, Italy*
(Dated: November 9, 2014)

We prove that a gauge approach based on a composite structure of the hole in hole-doped cuprates is able to capture analytically many features of the experimental data on superfluid density in the moderate-underdoping to optimal doping region, including critical exponent, Uemura relation and almost universality of the normalized superfluid density.

PACS numbers: 71.10.Hf, 71.27.+a, 11.15.-q, 74.25.-q, 74.72.Gh

Comparison with experimental data (1/3)

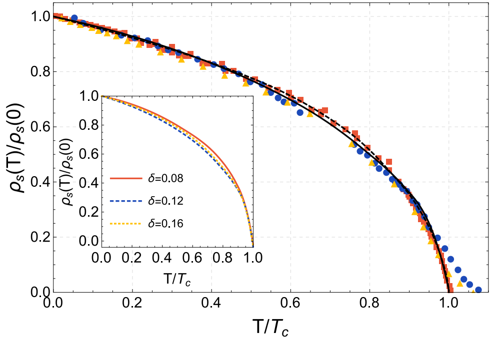
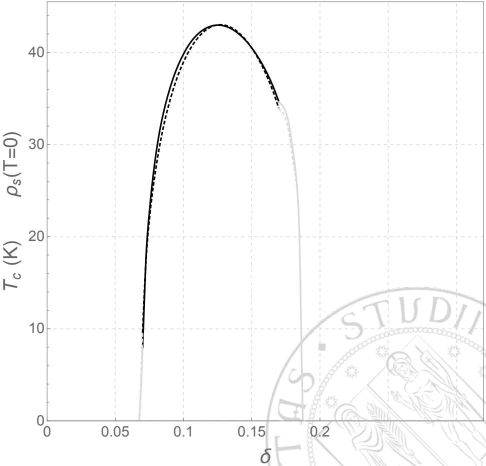


Figure: The normalized superfluid density vs. $\frac{T}{T_c}$. Our theoretical calculation (solid line, $\delta = 0.12$) is compared with a pure 3D XY model (dashed line), squares corresponding to the near-universal YBCO behavior for superfluid density, circles for $\delta = 0.075$ LSCO, triangles for near-optimal-doping BSCCO. The near-universal behavior of our ρ_s is shown in the inset.

Comparison with experimental data (2/3)

Figure: Uemura et al. (Phys. Rev. Lett. **62**, 2317 (1989)) observed strong linearity between $\rho_s(T=0)$ and T_c in underdoped cuprates ($0 \leq \delta \leq 0.15$). Theoretically calculated $T = 0$ superfluid density (solid line, arbitrary units) and critical temperature (dashed line) vs. δ exhibits an approximate Uemura.



Activities of the second year

- I spent a two months research period at University of Antwerp, Belgium, working with the Theory of quantum systems and complex systems (TQC) group. I gave a talk there.
- Conference: *"Multi-Condensate Superconductivity and Superfluidity in Solids and Ultracold Gases"*, June 24-27, 2014, University of Camerino
- International School of Physics "Enrico Fermi": *"Quantum Matter at Ultracold Temperatures"*, June 30-July 14, 2014, Varenna (LC)
- Last exam: "Processi stocastici e dinamica dei mercati" (A. Stella, F. Baldovin), due before the end of the year.

Thanks for your attention.