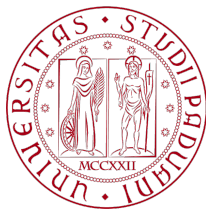


# Scuola di Dottorato di Ricerca in Fisica, XVIII ciclo



Giacomo Bighin

Università degli Studi di Padova

November 25, 2013

# Contents

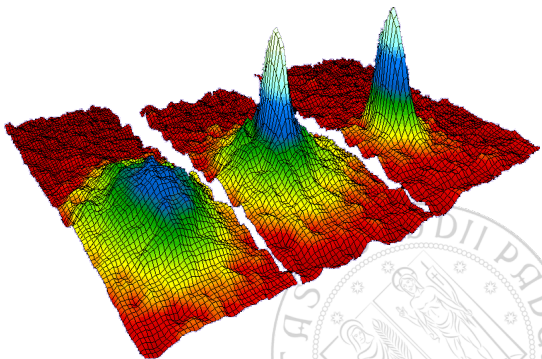
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- Ultracold Fermi gases
- High- $T_c$  superconducting cuprates
- Summer schools
- Exams
- Future work



# Ultracold Fermi gases (1/4)

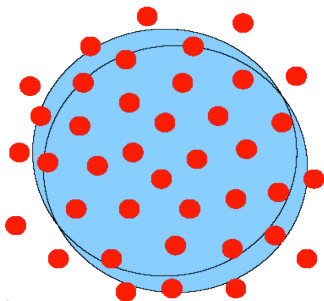
- My main research topic (supervisor: Prof. Luca Salasnich)
- Ultracold gases: experimental observation of quantum properties of matter. Vortices in a superfluid, BEC.
- Bose-Einstein condensation (1995), degenerate Fermi gas (2003)



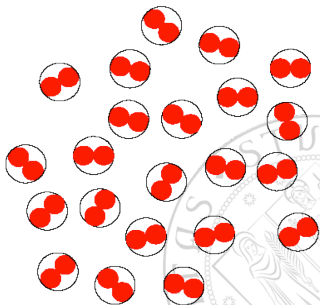
## Ultracold Fermi gases (2/4)

Why are ultracold Fermi gases interesting? The fermion-fermion attractive interaction can be tuned (using a Feshbach resonance), from weakly to strongly interacting: the **BCS-BEC crossover**.

**BCS regime:** coherence in momentum space.

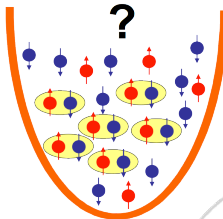
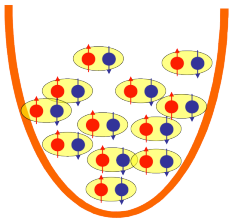


**BEC regime:** coherence in coordinate space.



## Ultracold Fermi gases (3/4)

### Polarized Fermi gases



# Ultracold Fermi gases (4/4)

My work on ultracold Fermi gases:

- Starting point: BCS-Leggett theory. The BCS trial wavefunction is valid throughout the crossover up to the strong-coupling regime at  $T = 0$ .
- Extension to the unbalanced case.
- Phase diagram: QPT.
- Condensate fraction, preliminary results, as a function of  $y = \frac{1}{k_F a_s}$  and

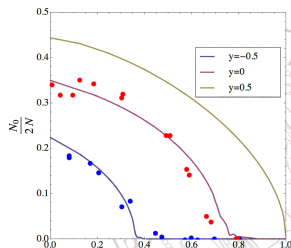
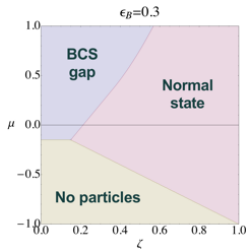
$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$



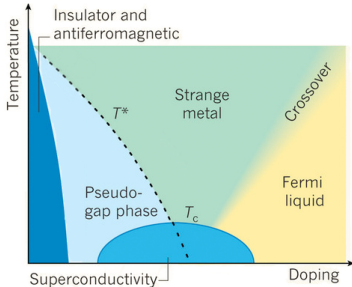
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Condensate fraction for an unbalanced 3D Fermi gas

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(Dated: November 18, 2013)



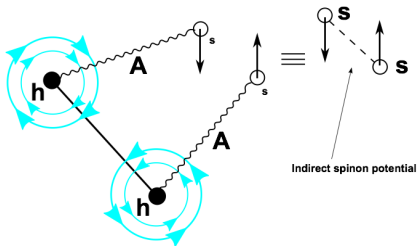
# High- $T_c$ superconducting cuprates (1/3)



- Very complicated phenomenology; superconducting up to 135 °K; no widely-accepted microscopical theoretical model.
- Discovered in 1985 (Bednorz, Müller), very active research field, over 100k scientific papers in  $\sim 25$  years.
- I am continuing the work I began with my Master's thesis, supervisor: Prof. P.A. Marchetti.

## High- $T_c$ superconducting cuprates (2/3)

In the formalism I am working with the electron has a **composite structure**: spinon + holon.



As a consequence, the superconductivity is achieved in **three steps**: holon pairing ( $T_{ph}$ ), spinon pairing ( $T_{ps}$ ), phase coherence ( $T_c$ ):

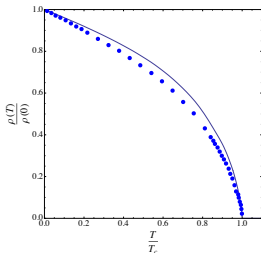
**Figure :** The attractive potential between the spinons, essential for the SC, is mediated by a gauge field “binding” holon and spinons, and by the holon attraction.

$$\Delta_c \sim \frac{|\Delta^s|}{|\Delta^h|} e^{i(\overbrace{\phi_s - \phi_h}^{\equiv \phi})}$$

$$SC \iff \langle \Delta_c \rangle \neq 0$$



## High- $T_c$ superconducting cuprates (3/3)



**Figure :** Superfluid density as a function of the temperature: our model (solid line) vs. experimental data (blue points).

- My focus: superfluid density ( $\rho_s$ ).  
 $S_{\text{EFF}} = \frac{\rho_s}{2} \int d\tau d^d r (\nabla\theta)^2 + \dots$
- Importance: lot of experimental data,  $\rho_s \propto \lambda^{-2}$ , very different from BCS.
- Summation formula ( $\sim$  Ioffe-Larkin)

$$\rho_s = \frac{\rho_s^s \rho_s^h}{\rho_s^s + \rho_s^h}$$

- Our results are in fairly good agreement with experimental data. The critical exponent is exactly reproduced:

$$\rho_s \sim \left| \frac{T - T_c}{T} \right|^{\frac{2}{3}} \quad \text{for } T \rightarrow T_c$$

# Summer school: Quantum Matter — Foundations and Applications

## CONDENSATE FRACTION FOR AN UNBALANCED FERMIGAS

G. BIGHINI, L. SALASNIH, G. MAZZARELLA  
Università degli Studi di Padova

### PHYSICAL SYSTEM

Starting point: a homogeneous unbalanced Fermi gas (UFG) with contact + range attractive interactions, described by the following Lagrangian density:

$$\mathcal{L} = \sum_{\sigma} \bar{\psi}_{\sigma}(\mathbf{r}) \left( \nabla_{\mathbf{r}}^2 - \mu_{\sigma} \right) \psi_{\sigma}(\mathbf{r}) + g \bar{\psi}_{\sigma}(\mathbf{r}) \bar{\psi}_{\sigma}(\mathbf{r}) \psi_{\sigma}(\mathbf{r}) \psi_{\sigma}(\mathbf{r}) + \bar{\psi}_{\sigma}(\mathbf{r}) \bar{\psi}_{\sigma}(\mathbf{r}) \psi_{\sigma}(\mathbf{r}) \psi_{\sigma}(\mathbf{r})$$

The spin-dependent chemical potential,  $\mu_{\sigma}$ , allows for a different number of  $\uparrow$  and  $\downarrow$  fermions; each system has been experimentally realized in 3000 (see [1]), and the interaction strength  $g$  can be finely tuned by means of Feshbach resonances. Through standard mean-field (Hartree-Fock) + MFA we derive the following mean-field effective action:

$$S_{\text{MF}} = - \sum_{\sigma} \int d\mathbf{r} \left[ 2 \text{mash}(\mathcal{E}_{\sigma}) + 2 \text{mash}(\mathcal{E}_{\sigma}^{\dagger}) \right] + \delta \sum_{\sigma} \bar{\psi}_{\sigma} \psi_{\sigma} \frac{\partial \mu_{\sigma}}{\partial \rho_{\sigma}}$$

where  $\mathcal{E}_{\sigma} = \frac{p^2}{2m} - \mu_{\sigma}$ ,  $\mathcal{E}_{\sigma}^{\dagger} = \sqrt{\frac{p^2}{2m} - \mu_{\sigma}^{\dagger}}$ ,  $\mu_{\sigma}^{\dagger} = \mu_{\sigma} + \frac{g}{2} \rho_{\sigma}$ ,  $\mu_{\sigma} = \mu_{\sigma}^{\dagger}$  and  $\rho_{\sigma} = \langle \bar{\psi}_{\sigma} \psi_{\sigma} \rangle = \langle \bar{\psi}_{\sigma}^{\dagger} \psi_{\sigma}^{\dagger} \rangle$ .

### THE CONDENSATE FRACTION

Given the Green function in the Nambu-Gorkov basis, we can calculate the condensate number as follows [2]:

$$N_0 = \frac{1}{2} \sum_{\sigma} \sum_{\mathbf{p}} \sum_{\mathbf{q}} G_{\sigma}(\mathbf{p}, \mathbf{q}) G_{\sigma}(\mathbf{q}, \mathbf{p})$$

after carrying out the summation of Matsubara fermionic frequencies the formula above is seen to be (as an extension to the balanced case formula in [3]):

$$N_0 = \sum_{\sigma} \frac{1}{2\pi i} \oint_{\mathcal{C}} \left( \frac{1}{2} \text{mash} \left( \frac{p^2}{2m} + \mu_{\sigma} \right) + \frac{1}{2} \text{mash} \left( \frac{p^2}{2m} - \mu_{\sigma} \right) \right)$$

### 2D PHASE DIAGRAM: QUANTUM PHASE TRANSITION

We take into account the 2D phase diagram at  $T = 0$ . The grandpotential ( $\Omega$ ), derived from the effective action shows two minima, for  $\Delta_{\sigma} = 0$  and for  $\Delta_{\sigma} \neq 0$  (the values  $\Delta_{\sigma}$  assumes at these minima are determined by  $\mathcal{C}$ ). For a certain range of values of the unbalanced chemical potential  $\mathcal{C}$ , the normal state is energetically favored with respect to the BCS gapped state. In other words at  $T = 0$  the system undergoes a phase transition [4] by tuning an external parameter: a quantum phase transition [4].

In the figure above the phase diagram, as a function of  $\mu$  and  $\mathcal{C}$ , at fixed  $\epsilon_F$ , the pair binding energy, the 2D analogues of the 3D scattering length introduced with regularization.

### CONDENSATE FRACTION IN A 3D UFG

In 3D we can compare our approach with the experimental data in [5], having defined  $\rho = \frac{N_0}{N}$  and the polarization  $P = \frac{N_{\uparrow} - N_{\downarrow}}{N}$ .

- Finite temperature effects: we assume that  $T \ll T_c$ . For all data experimental, however, we report they had had control on system's temperature.
- Need to include order parameter fluctuations.

### THE EBCS EQUATIONS

We write the extended BCS (EBCS) equations for an unbalanced Fermi gas:

$$\frac{1}{g} = \frac{1}{2} \sum_{\sigma} \frac{1}{\mathcal{E}_{\sigma}} \frac{\text{mash}(\mathcal{E}_{\sigma})}{2 \text{mash}(\mathcal{E}_{\sigma}) + 2 \text{mash}(\mathcal{E}_{\sigma}^{\dagger})}$$

$$\delta = \sum_{\sigma} \left( 1 - \frac{1}{2} \frac{\text{mash}(\mathcal{E}_{\sigma})}{\mathcal{E}_{\sigma}} \frac{\text{mash}(\mathcal{E}_{\sigma}^{\dagger})}{2 \text{mash}(\mathcal{E}_{\sigma}) + 2 \text{mash}(\mathcal{E}_{\sigma}^{\dagger})} \right)$$

$$\mathcal{E}_{\sigma} = \sum_{\mathbf{p}} \frac{\text{mash}(\mathcal{E}_{\sigma})}{2 \text{mash}(\mathcal{E}_{\sigma}) + 2 \text{mash}(\mathcal{E}_{\sigma}^{\dagger})}$$

The gap equation is found imposing the saddle-point condition for the effective action  $\Omega = \mathcal{E}_{\sigma}^{\dagger} N_0$ . Regularization for the 3D case [6]:

$$\frac{1}{g} = \frac{1}{2} \sum_{\sigma} \frac{1}{\mathcal{E}_{\sigma}} \frac{\text{mash}(\mathcal{E}_{\sigma})}{2 \text{mash}(\mathcal{E}_{\sigma}) + 2 \text{mash}(\mathcal{E}_{\sigma}^{\dagger})}$$

introducing the scattering length  $a_s$ , leading to the following regularized 3D gap equation:

$$\frac{1}{g} = \frac{1}{2} \sum_{\sigma} \left( \frac{1}{\mathcal{E}_{\sigma}} \frac{\text{mash}(\mathcal{E}_{\sigma})}{2 \text{mash}(\mathcal{E}_{\sigma}) + 2 \text{mash}(\mathcal{E}_{\sigma}^{\dagger})} \right)$$

### FURTHER WORK

Further work will focus on these topics:

- Relevance of the order parameter fluctuations (beyond mean-field approach) in deriving the phase diagrams and in calculating the condensate fraction, especially on the BCS side of the transition.
- For the experimental data in [5] was taken to a range of temperatures  $0 \leq T \leq T_c$ , while computing it with zero-temperature condensate fraction calculation provides a good approximation, we should nonetheless account for finite temperature effects.
- Realistic modeling of the potential trap confining fermions: local density approximation.

### REFERENCES

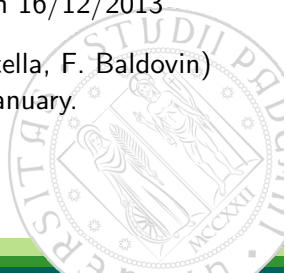
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- [3] L. Bighini, G. Mazzarella, L. Salasnich, Phys. Rev. A 82, 032403 (2010)
- [4] S. Sachdev, Quantum phase transitions - Cambridge University Press, 2011
- [5] N. Jakubowski, Y. Chabbi, L. Taylor, A. Griffin, Phys. Rev. A 75, 033604 (2007)
- [6] L. Salasnich, N. Mastai, Phys. Rev. A 72, 022402 (2005)

- Granada (Spain), September 15th-19th, 2013
- I presented a poster there: "Condensate fraction for an unbalanced Fermi gas".

# Exams

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- Fisica statistica dei fenomeni emergenti: dalla geometria delle reti fluviali alla biogeografia. (A. Maritan) ✓ 28/10/2013
- Risposta dei Sistemi Complessi: Teoria ed Esperimenti (F. Baldovin, M. Pierno) ✓ 12/11/2013
- Standard Model (M. Passera, E. Torassa) due on 16/12/2013
- Processi stocastici e dinamica dei mercati (A. Stella, F. Baldovin) attending now, exam due in late December or January.



## Future work and plans

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- Ultracold Fermi gases: role of fluctuations at finite temperature in 3D.
- Ultracold Fermi gases: two-dimensional case.
  - Phase (as opposed to modulus) fluctuations should play a key role in the finite-temperature behavior.
  - Vortices in 2D.
  - Finite temperature effects.
- The theoretical methods used for cuprates can also be applied in describing the dynamics of ultracold gases. The pseudogap region.
- Collaboration with Prof. Jacques Tempère, Universiteit Antwerpen, Belgium

Thanks for your attention.

