A Theory of Post-Stall Transients in Axial Compression Systems: Part I—Development of Equations

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An approximate theory is presented for post-stall transients in multistage axial compression systems. The theory leads to a set of three simultaneous nonlinear third-order partial differential equations for pressure rise, and average and disturbed values of flow coefficient, as functions of time and angle around the compressor. By a Galerkin procedure, angular dependence is averaged, and the equations become first order in time. These final equations are capable of describing the growth and possible decay of a rotating-stall cell during a compressor mass-flow transient. It is shown how rotating-stall-like and surgelike motions are coupled through these equations, and also how the instantaneous compressor pumping characteristic changes during the transient stall process.

Introduction

Background. Problems of compressor instability have been of concern to aircraft engine designers for a number of years, and the provision of sufficient stall margin is an important consideration in any compressor design. In addition to problems associated with the inception of stall, however, another aspect of this general topic has been of increasing import in recent years. This is the behavior of the compression system subsequent to the onset of stall, i.e., subsequent to the onset of compressor or compression system instability.

The reason for this increased interest in post-stall behavior has to do with the phenomenon of nonrecoverable stall (or stall "stagnation"). If the engine enters this condition, described in [1], the only remedy may be to shut the engine down and restart it.

Clearly, therefore, it is important to gain the ability to predict post-stall behavior, as a basis for rational design of stagnation-resistant compressors. This is true not only for quasi-steady operation but also for the unsteady features of the compression system response. In the first instance, one may be able to consider the compressor in isolation, without coupling it to the system. This is essentially the pure rotating-stall problem treated by Day, Greitzer, and Cumpsty [2] semi-empirically and, more recently, by Moore [3] in a more fundamental manner. To understand engine behavior more generally, however, one must deal with the unsteady behavior of the compression system during the mass flow oscillations that characterize post-stall transients. Thus, one must describe the possible formation and growth in the compressor of a rotating-stall cell during a transient process when the compressor is interacting dynamically with other components of the system, as, for example, in a surge cycle.

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It is to be stressed from the outset that this task cannot today be carried out by exact or numerical solution of the basic partial differential equations of fluid motion. The prediction of even steady-state rotating-stall performance has not yet been carried out on this level. However, many of the salient features of rotating stall have recently been predicted [3,4] using certain simplifying assumptions, and we have found that an effective model for general transients of compressor operation can be formulated on the basis of these same assumptions. We believe that the resulting general theory, to be described in this paper (Part I) and its companion (Part II) will prove helpful for general physical insight, give guidance to experimental studies, and also will help to unravel and assess the various interactive effects of many important design parameters.

Before plunging into the theoretical development, it seems appropriate to outline certain expectations, based on past studies [5,6] and a consideration of relevant time scales. First, we expect that a general theory of transients should show rotating stall [2,3,4] developing and ultimately predominating in some circumstances, and surge [5,6] ultimately predominating in others. Those two types of oscillation differ fundamentally in that pure rotating stall is steady (in the proper rotating frame of reference) but not axisymmetric. Pure surge, on the other hand, we will define to be axisymmetric but unsteady. We will assume that a transient is initiated by some disturbance which is generally both unsteady and nonaxisymmetric; thus, we expect surgelike and rotating-stall-like features to be coupled in the transient as it develops.

As discussed in [7], in many practical compression systems the time needed to form a rotating-stall cell and the time for mass flow to change (as in surge) are comparable. Therefore, rotating stall will not have time to fully form during a typical transient. This has significant implications for the modeling of post-stall transients, because during the transient, the compressor is not performing in a quasi-steady manner.

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Therefore, information obtained from a steady-state compressor test with the machine operating in rotating stall will generally not represent performance during a rapid post-stall transient. This being so, it is imperative that the theory developed herein should predict how the growth or decay of rotating stall affects the instantaneous compressor pumping characteristic.

We may also note that the expected asymmetric flow patterns will have length scales on the order of compressor diameter. They will therefore change on a time scale which is typically very much longer than that associated with the unsteady response of a single-blade passage. Thus, the precise modeling of the unsteady aerodynamic performance of the blades would not be expected to be important for a successful theory of stall-cell growth [7].

In this paper, we will first derive a set of coupled nonlinear equations which describe the time-dependent state of the assumed compression system. These equations will be simplified by use of an approximate (Galerkin) procedure, and then specialized by introduction of a particular compressor performance curve. The general nature of the resulting equations will be briefly discussed, concluding the present paper (Part I). In the companion paper (Part II—Application), some general features of the solutions will be analyzed, and numerical solutions will be presented which indicate the effects of certain parameters of practical interest. Part II will conclude with comments about future avenues of research, both theoretical and experimental, that are thought to be needed to extend and exploit the theory.

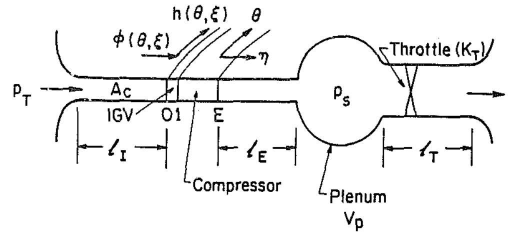

Fig. 1 Schematic of compressor and compression system geometry showing nondimensional lengths

Fluid Dynamic Model

The basic compression system to be analyzed is shown in Fig. 1. It consists of a compressor operating in a duct and discharging to a downstream plenum. The plenum dimensions are large compared to those of the compressor and its ducts, so that the velocities and fluid accelerations in the plenum can be considered negligible. Hence the pressure in the plenum can be considered to be uniform spatially, although possibly varying in time. The flow through the system is controlled by a throttle at the plenum exit.

The systems that we consider will have overall pressure rises, atmosphere to plenum, that are small compared to the ambient level. If, also, we assume Mach numbers to be small, and oscillations to have frequencies well below those of acoustic resonance, then we can assume incompressible flow in the compressor itself. The gas in the plenum must be

Nomenclature

A = amplitude function of first-harmonic angular disturbance, equation (53)
A_c = compressor duct area
a = reciprocal time-lag parameter of blade passage, equation (6)
a_s = sound speed
B = $(U/2a_s)\sqrt{V_p/A_cL_c}$
F = pressure-rise coefficient in blade passage, equation (2)
F_T = throttle characteristic function, equation (39); inverse is $F_T^{-1}(\Psi)$
f = nondimensional speed coefficient, relative to propagation of laboratory, of angular disturbance
f_0 = value of f for single harmonic disturbance, equation (57)
g = disturbance of axial flow coefficient
H = semi-height of cubic axisymmetric characteristic, Fig. 3
h = circumferential velocity coefficient
J = square of amplitude of angular disturbance of axial-flow coefficient, equation (58); J_e for pure rotating stall
K_G = loss coefficient at IGV entrance, equation (10)
K_T = throttle coefficient, equation (39)
l_c = total aerodynamic length (L_c/R) of compressor and ducts, in wheel radii, equation (24)
l_E, l_I, l_T = length of exit, entrance, and throttle ducts, in wheel radii, Fig. 1
m = compressor-duct flow parameter, equation (20)
N = number of stages of core compressor
P = pressure coefficient, equation (17)
p_0 = static pressure at entrance to IGV, Fig. 1
p_1, p_E = static pressures at entrance and exit of core compressor, Fig. 1
p_S = static pressure at end of exit (diffuser) duct, and pressure in the plenum, Fig. 1
p_T = total pressure ahead of entrance and following the throttle duct, Fig. 1
R = mean wheel radius
r = time-dependent phase angle, equation (53)
t = time
U = wheel speed at mean diameter
V_p = volume of plenum, Fig. 1
K_T = throttle coefficient, equation (39)
W = semi-width of cubic characteristic, Fig. 3
Y = disturbance potential at compressor entrance, equation (31)
η = axial distance measured in wheel radii, Fig. 1
θ = angular coordinate around wheel, Fig. 1
θ^* = angular coordinate around wheel, measured relative to a rotating-stall cell, equation (46)
λ = rotating-stall parameter, equation (47)
ξ = time, referred to time for wheel to rotate one radian
ρ = density
τ = coefficient of pressure-rise lag, equation (2)
Φ = axial flow coefficient in compressor, annulus averaged; axial velocity divided by wheel speed
Φ_T = flow coefficient of throttle duct, referred to entrance-duct area, Fig. 1
ϕ = local axial flow coefficient, a function of θ and ξ
$\tilde{\phi}$ = velocity potential in entrance duct, equation (11)
$\tilde{\phi}'$ = disturbance velocity potential, equation (14)
Ψ = total-to-static pressure-rise coefficient, equation (22)
ψ_c = axisymmetric pressure-rise coefficient, equation (23)
ψ_{c0} = shut-off value of axisymmetric characteristic, Fig. 3

Subscripts

0 = at the entrance to the compressor
E = at the exit of the compressor

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considered compressible, however, because it acts dynamically as a gas spring.

This basic compression system model has been used in many investigations of surge. Where we will depart from previous practice in analysis of compression system transients, however, is in the coupling of an analysis of the two-dimensional unsteady flow in the compressor with the lumped-parameter system model. We therefore begin by describing our representation of the compressor.

Flow Disturbance in the Compressor. The basic compressor model is that of [3]. The compressor is considered to have a high hub-to-tip radius ratio, so that a two-dimensional description can be used. The compressor duct is modeled as an inlet contraction followed by a constant-area section upstream of the compressor. The annulus immediately downstream of the compressor also has constant area. These restrictions simplify the present analysis, but could be removed in a more general treatment.

The following nomenclature will be used (see Fig. 1 and the list provided at the beginning of this paper). All distances will be nondimensionalized by the mean compressor radius (R). The nondimensional circumferential coordinate is then simply the wheel angle θ, while the axial coordinate, also referred to R, will be denoted by η. Time will be denoted by radians of wheel travel:

$$\xi = Ut/R \tag{1}$$

As discussed in [3], the basic model we adopt for the unsteady performance of a compressor blade row is that the pressure rise across a single row is given by

$$\frac{\Delta P}{\frac{1}{2}\rho U^2} = F(\phi) - \tau \frac{d\phi}{dt} \tag{2}$$

where $\phi (= C_x/U)$ is the local, unsteady axial velocity coefficient at the compressor. The quantity $F(\phi)$ represents the axisymmetric steady performance of the blade row, and τ can be viewed as a "time constant" associated with the internal lags in the compressor. In [3], it was shown that a reasonable value for τ can be based on the inertia of fluid in the passage.

The unsteadiness of the flow through a stator passage reflects the accelerations associated with transient effects. For a rotor, however, there is also unsteadiness due to the rotor blades moving (with velocity U) through a circumferentially nonuniform flow. Therefore, using equation (1),

$$\left(\frac{d\phi}{dt}\right)_{\text{stator}} = \frac{U}{R}\frac{\partial\phi}{\partial\xi} \tag{3}$$

$$\left(\frac{d\phi}{dt}\right)_{\text{rotor}} = \frac{U}{R}\left(\frac{\partial\phi}{\partial\xi} + \frac{\partial\phi}{\partial\theta}\right) \tag{4}$$

We note that in the present situation, because of the possibility of growth or decay of velocity nonuniformities, there is not necessarily a Galilean equivalence between the changes in time and the spatial derivatives, as there was for the pure traveling wave analyzed in [3].

Applying these ideas to a compressor of N stages, we record the pressure rise across a compressor (not including the inlet and exit guide vanes which we will discuss subsequently) comprising N rotor–stator pairs:

$$\frac{p_E - p_1}{\rho U^2} = NF(\phi) - \frac{1}{2a}\left(2\frac{\partial\phi}{\partial\xi} + \frac{\partial\phi}{\partial\theta}\right) \tag{5}$$

where we refer to Fig. 1 to identify the pressures p_E and p_1, and, for convenience, we define

$$a \equiv R/(N\tau U). \tag{6}$$

Returning to the overall sketch provided in Fig. 1, an irrotational, inviscid flow is imagined to proceed from an upstream (atmospheric) reservoir at stagnation pressure p_T through an entrance duct to the IGV entrance at 0. We note that ϕ, the axial flow coefficient at 0, can depend on both angle θ around the wheel and time ξ, even though the reservoir pressure is constant. As a result of the flow process in the approach duct, if ϕ varies with θ, then we will find that a circumferential coefficient $h(\xi,\theta)$ must also appear at the IGV entrance. It should be noted that the present coordinate system is fixed in the laboratory, whereas in [3, 4], the system traveled with a supposed permanent rotating-stall pattern.

We define an average of ϕ around the wheel

$$\frac{1}{2\pi}\int_0^{2\pi} \phi(\xi,\theta)\,d\theta \equiv \Phi(\xi) \tag{7}$$

Proceeding just as in [3, 4], we suppose that any circumferential nonuniformity of axial velocity within the compressor must, by a continuity argument and by evidence cited in [2], go straight through the machine. That is, we write

$$\phi = \Phi(\xi) + g(\xi,\theta); \quad h = h(\xi,\theta) \tag{8}$$

and note that, by definition, the angle average of g must vanish. Also, because under the stated assumptions no circulation can arise in the entrance duct, h must have vanishing average as well. Thus,

$$\int_0^{2\pi} g(\xi,\theta)\,d\theta = 0; \quad \int_0^{2\pi} h(\xi,\theta)\,d\theta = 0 \tag{9}$$

We could now introduce equation (8) into equation (5) to describe pressure rise in terms of Φ and g. We defer this step until after entrance and exit duct pressure changes have also been found.

Flow Disturbance in the Entrance Duct and Guide Vanes. The overall pressure rise from compressor inlet to exit will include the pressure difference associated with the circumferential velocity component just ahead of the inlet guide vanes. The pressure difference from station 0 to station 1 (where the flow is axial) can be written as:

$$\frac{p_1 - p_0}{\rho U^2} = \frac{1}{2}K_G h^2 \tag{10}$$

If the IGV entrance is lossless, the entrance recovery coefficient K_G is unity, but if a loss does occur there, then $K_G < 1$.

In the entrance duct upstream of the IGV, we assume irrotational flow, so that a velocity potential exists which, though unsteady, will satisfy Laplace's equation. Specifically, we define a velocity potential $\tilde{\phi}$ the gradient of which will give axial and circumferential velocity coefficients everywhere in the entrance duct; that is,

$$\tilde{\phi}_\eta = v/U; \quad \tilde{\phi}_\theta = u/U \tag{11}$$

Henceforward, we will use subscripts to denote partial differentiation. Accordingly, at the IGV entrance (point 0 on Fig. 1, where v and u are described in terms of g and h),

$$(\tilde{\phi}_\eta)_0 = \Phi(\xi) + g(\xi,\theta); \quad (\tilde{\phi}_\theta)_0 = h(\xi,\theta) \tag{12}$$

Far upstream, in the reservoir, we take $\tilde{\phi}$ itself to be zero because the flow is at rest.

As a general matter, we must solve Laplace's equation for $\tilde{\phi}$ and then apply Bernouilli's equation to evaluate pressure at the point 0. For unsteady flow, with our particular definitions of variables, the result is

$$\frac{P_T - P_0}{\rho U^2} = \frac{1}{2}(\phi^2 + h^2) + (\tilde{\phi}_\xi)_0 \tag{13}$$

The unsteady contribution, $(\tilde{\phi}_\xi)_0$, is due to unsteadiness in both Φ and g. The complexity of this term is reduced if we consider a straight inlet duct, preceded by a much shorter contracting passage, as sketched in Fig. 1. In such a duct, of

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dimensionless length l_I, the angle-averaged coefficient $\Phi(\xi)$ will be constant along the duct, and, in view of equation (11), the velocity potential may be immediately written as

$$\tilde{\phi}=(\eta+l_I)\Phi(\xi)+\tilde{\phi}'(\xi,\eta) \tag{14}$$

where $\tilde{\phi}'$ is a disturbance velocity potential vanishing at $\eta = -l_I$, and giving g and h at point 0:

$$(\tilde{\phi}'_\eta)_0=g(\xi,\theta);\quad (\tilde{\phi}'_\theta)_0=h(\xi,\theta) \tag{15}$$

The term needed for equation (13) is simply

$$(\tilde{\phi}_\xi)_0=l_I\frac{d\Phi}{d\xi}+(\tilde{\phi}'_\xi)_0 \tag{16}$$

Flow Disturbance in the Exit Duct and Guide Vanes. In the exit duct, a complicated rotational flow appears even with axial OGV's, when axial flow varies with θ. We will use an approximation, somewhat generalized, developed in [3]. We consider the function P, defined below, to be sufficiently small that it satisfies Laplace's equation.

$$P\equiv\frac{p_s(\xi)-p}{\rho U^2};\quad \nabla^2 P=0 \tag{17}$$

That is, pressure in the exit duct is assumed to differ (owing to circumferential flow nonuniformities) only slightly from static pressure at discharge (p_s); and in such a case, one may show from the Euler equations that it then satisfies Laplace's equation. Now, the axial Euler equation, evaluated at the compressor discharge (point E of Fig. 1), gives exactly

$$(P_\eta)_E=(\tilde{\phi}_{\eta\xi})_0=\frac{d\Phi}{d\xi}+(\tilde{\phi}'_{\eta\xi})_0 \tag{18}$$

Thus, the potential problem for P is the same as the one already mentioned for $-\tilde{\phi}'_\xi$; the minus sign is necessary because, in the exit duct, η increases away from the compressor face, whereas in the entrance duct, η increases toward the compressor face. Choosing a constant to make $P = 0$ at the duct exit, $\eta = l_E$, we find

$$P=(\eta-l_E)\frac{d\Phi}{d\xi}-\tilde{\phi}'_\xi \tag{19}$$

The foregoing simplified analysis assumes that l_E is not smaller than the distance (conservatively, l_I) at which the entrance flow disturbance potential vanishes. If it were much shorter, presumably the second term of equation (19) should be omitted. Therefore, just as in [3], we introduce a parameter m in the final result for exit pressure change

$$\frac{p_S-p_E}{\rho U^2}=(P)_E=-l_E\frac{d\Phi}{d\xi}-(m-1)(\tilde{\phi}'_\xi)_0 \tag{20}$$

where $m = 2$ would refer to a "long enough" exit duct, and $m = 1$ would refer to a very short one.

Net Pressure Rise to End of Compressor Exit Duct. Between the upstream reservoir (p_T) and the discharge from the exit duct (p_S) is the zone where circumferential pressure variations may arise. Further downstream are the plenum and throttle, which we assume to contain only axisymmetric disturbances. We can now form the net pressure rise in the zone of possible angular variations by combining equations (5, 10, 13, and 19). After introducing equations (8, 16), the result may be written

$$\frac{p_S-p_T}{\rho U^2}=\left(NF-\frac{1}{2}\phi^2\right)-\left(l_I+\frac{1}{a}+l_E\right)\frac{d\Phi}{d\xi}-m(\tilde{\phi}'_\xi)_0$$
$$-\frac{1}{2a}(2\tilde{\phi}'_{\xi\eta}+\tilde{\phi}'_{\theta\eta})_0-\frac{1}{2}(1-K_G)h^2 \tag{21}$$

In effect, this is the integral of the axial momentum equation from the upstream reservoir to the plenum.

Following [3], we identify the left side, which is the total-to-static pressure rise coefficient, as $\Psi(\xi)$, and the first parenthesis on the right as the quasisteady, axisymmetric compressor characteristic, also familiar from [3]; we notice that for steady flow with no disturbance, all other terms on the right vanish, leaving the obvious identity $\Psi = \psi_c$. The foregoing definitions are

$$\Psi(\xi)\equiv\frac{p_S-p_T}{\rho U^2} \tag{22}$$

$$\psi_c(\phi)\equiv NF(\phi)-\frac{1}{2}\phi^2 \tag{23}$$

It should be emphasized that the "axisymmetric characteristic," ψ_c, is the compressor performance that would be realized if no angle or time dependence were permitted, even in a stalled condition.

The second parenthesis on the right of equation (21) is the effective flow-passage length through the compressor and its ducts, measured in radii of the compressor wheel. This we denote as l_c

$$l_c\equiv l_I+\frac{1}{a}+l_E \tag{24}$$

As explained in [3], if the compressor lag τ is considered to be purely inertial, one could write

$$\frac{1}{a}=(2NL_R)\left(\frac{1}{R}\right)\frac{k}{\cos^2\gamma} \tag{25}$$

The first factor in parentheses is simply the axial length of the compressor, L_R being the axial length of a row; the factor $1/R$ puts the distance into wheel radians; k is a factor which could account for interrow spacing and possible unsteady losses; and $\cos^2\gamma$ accounts for the tortuous flow path through the compressor, γ being the effective stagger angle of the blades. Thus, the effective (inertial) path length for the system is longer than the axial length.

The last term of equation (21) will be neglected in the present treatment, in effect assuming that the recovery coefficient K_G at the IGV entrance is 1. Finally, therefore, we may write equation (21) in the form

$$\Psi(\xi)=\psi_c(\Phi+(\tilde{\phi}'_\eta)_0)-l_c\frac{d\Phi}{d\xi}-m(\tilde{\phi}'_\xi)_0-\frac{1}{2a}(2\tilde{\phi}'_{\xi\eta}+\tilde{\phi}'_{\theta\eta})_0 \tag{26}$$

In this we have used equations (8, 15) to express the argument of ψ_c in terms of $\tilde{\phi}'$:

$$\phi=\Phi+(\tilde{\phi}'_\eta)_0 \tag{27}$$

The next step will be to place equation (26) in the context of the complete compression system, with plenum and throttle, as sketched in Fig. 1. Before doing so, we simplify equation (26) by introducing a generalization of an approximation used in the previous study of pure rotating stall [3].

A Simplifying Approximation of the $dh/d\theta = -g$ Type. The upstream disturbance potential $\tilde{\phi}'$ introduced by equation (14) satisfies Laplace's equation

$$\tilde{\phi}'_{\theta\theta}+\tilde{\phi}'_{\eta\eta}=0 \tag{28}$$

Its value and normal derivative ϕ'_η at the compressor entrance are needed, which implies that equation (28) must be solved as part of our analysis. We would like to avoid the difficulties of such a procedure, if possible.

We know that g is a periodic function of θ, and, by definition, must have a vanishing average over 2π. Therefore, it has a Fourier series and the solution of equation (28), vanishing at $\eta = -\infty$, is

$$\tilde{\phi}'=\sum_{n=1}^{\infty}\frac{1}{n}e^{n\eta}(a_n\sin n\theta+b_n\cos n\theta);\ \eta\le 0 \tag{29}$$

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If, as approximation, we were to keep only the first term of equation (29), we would infer that

$$(\tilde{\phi}_\eta')_0 = -(\tilde{\phi}_{\theta\theta}')_0 \tag{30}$$

In view of equation (15), this is precisely the relation $dh/d\theta = -g$ introduced and discussed in [3].

Thus, it will be expedient to use the value of $\tilde{\phi}'$ at $\eta = 0$ as dependent variable, using the notation Y for simplicity. Equation (30) becomes

$$(\tilde{\phi}')_0 \equiv Y(\xi,\theta);\ (\tilde{\phi}_\eta')_0 = -Y_{\theta\theta} \tag{31}$$

Equation (31) gives the pressure-rise equation (26) the form

$$\Psi(\xi) = \psi_c(\Phi - Y_{\theta\theta}) - l_c \frac{d\Phi}{d\xi} - mY_\xi + \frac{1}{2a}(2Y_{\xi\theta\theta} + Y_{\theta\theta\theta}) \tag{32}$$

This equation is far easier to apply than equation (26), because only derivatives in time and angle (ξ,θ) are involved, whereas equation (26) requires solution in the axial coordinate (η) as well.

Equations (12) and (31) imply that

$$h = Y_\theta;\ g = -Y_{\theta\theta} \tag{33}$$

Therefore, if Y is simply required to be periodic in θ, as it obviously must be to describe a physical quantity in the compressor,

$$Y(\xi,\theta+2\pi) = Y(\xi,\theta) \tag{34}$$

then the cyclic integrals of g and h vanish, as equation (9) requires. It is also obvious from the general solution of equation (29) that the cyclic integral of ϕ' itself must vanish, so we know that Y should have the property

$$\int_0^{2\pi} Y(\xi,\theta)\,d\theta = 0 \tag{35}$$

In summary, we have formulated the approximate governing equation (32) to describe the dynamic pressure rise of our system, subject to equations (34) and (35) as requirements on the unknown $Y(\xi,\theta)$.

An idea described in Appendix A of [4], to improve the accuracy of the $dh/d\theta = -g$ approximation for rotating stall, could be adapted for the present more general case. We will omit this possible refinement, because recent numerical studies have shown that the present approximation (equation (33)) is quite accurate over a wide range of circumstances.

Overall Pressure Balance of the Compression System. We now return to Fig. 1 and account for dynamic pressure changes downstream of the compressor exit, in the plenum and across the throttle, following closely ideas used in previous studies of pure surge [5].

As we have already mentioned, the plenum will eliminate any spatial variations. It will receive mass at a rate $\rho UA_c\Phi(\xi)$, and discharge mass at a generally different rate $\rho UA_c\Phi_T(\xi)$. Duct areas at entrance and discharge of the plenum would both presumably be different from the compressor area A_c; the difference is accounted for in the definition of Φ_T as in [5]. If Φ and Φ_T are different, then mass must accumulate in the plenum, changing the density there. As discussed in [5], we can take this change to be isentropic for the situations under consideration, and the plenum will act as a gas spring.

We assume that the compressor exit discharges as a free jet into the plenum, so that its pressure is $p_S(\xi)$. The isentropic relation between density and pressure then provides that the rate of density change in the plenum is the ratio of dp_S/dt to the square of sound speed, a_S. The rate of increase of stored mass in the plenum volume (V_p) is thus proportional to the dimensionless time derivative of the pressure-rise coefficient $\Psi(\xi)$, defined in equation (22). After some manipulation, the balance of entering, leaving, and stored mass of the plenum can then be written in the following convenient form:

$$l_c \frac{d\Psi}{d\xi} = \frac{1}{4B^2}[\Phi(\xi) - \Phi_T(\xi)] \tag{36}$$

in which the familiar "B-parameter" [5] appears

$$B \equiv \frac{U}{2a_S}\sqrt{\frac{V_p}{A_cL_c}} \tag{37}$$

The throttle then discharges to an infinite reservoir which, in the present study, we take to be at the same pressure p_T as the system inlet (Fig. 1).

Therefore, the pressure difference $p_S - p_T$ balances the throttle loss, and also provides any acceleration of the mass in the throttle duct. In coefficient form,

$$\Psi(\xi) = F_T(\Phi_T) + l_T \frac{d\Phi_T}{d\xi} \tag{38}$$

In the present paper, we will take the throttle duct to be short enough to justify neglecting l_T. This is generally quite a good assumption for realistic throttle ducts. A realistic throttle characteristic might be parabolic in form:

$$F_T = \frac{1}{2}K_T\Phi_T^2 \tag{39}$$

with K_T being a constant throttle coefficient. In any case, we write that Φ_T is simply the inversion of F_T, according to equation (39):

$$\Phi_T = F_T^{-1}(\Psi) \tag{40}$$

In principle, equations (36, 40) complete our system of equations for disturbances of the compression system. We imagine that Φ_T may be eliminated between these two equations, and the resulting equation connecting Ψ and Φ can be combined in some way with equation (32). However, there are only two equations, so far, for the three unknowns $\Psi(\xi)$, $\Phi(\xi)$, and $Y(\xi,\theta)$. The third equation comes from realizing that equation (32) involves terms which are functions only of time. If that equation is integrated over a cycle of θ, and equations (34) and (35) are applied, one finds

$$\Psi(\xi) + l_c \frac{d\Phi}{d\xi} = \frac{1}{2\pi}\int_0^{2\pi} \Psi_c(\Phi - Y_{\theta\theta})\,d\theta \tag{41}$$

which will serve as the needed third equation. In fact, equation (41) is the annulus average of the axial momentum equation.

Equations of a General Disturbance. For convenience, we assemble below the three pressure-balance equations we have just derived and will use in subsequent analysis, giving them new numbers.

$$\Psi(\xi) + l_c \frac{d\Phi}{d\xi} = \psi_c(\Phi - Y_{\theta\theta}) - mY_\xi + \frac{1}{2a}(2Y_{\xi\theta\theta} + Y_{\theta\theta\theta}) \tag{42}$$

$$\Psi(\xi) + l_c \frac{d\Phi}{d\xi} = \frac{1}{2\pi}\int_0^{2\pi} \psi_c(\Phi - Y_{\theta\theta})\,d\theta \tag{43}$$

$$l_c \frac{d\Psi}{d\xi} = \frac{1}{4B^2}[\Phi(\xi) - F_T^{-1}(\Psi)] \tag{44}$$

The first of these is the local (in θ) momentum balance of the system; the second is annulus-averaged momentum balance; and the third is the mass balance of the plenum. Our task is to find suitable solutions for Φ, Ψ, and Y under the influences of the various system parameters appearing in the equations.

Pure Rotating Stall and Pure Surge as Special Cases

Before attempting any general application of equations (42–44), we will find it instructive to see how those equations describe the special cases of pure rotating stall and pure surge, which have already been analyzed [3, 5]. Surge is the simplest;

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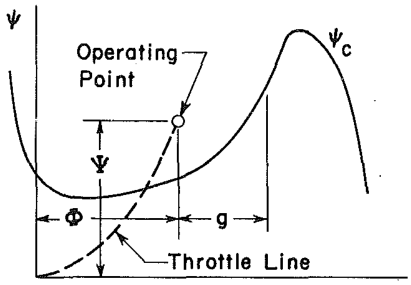

Fig. 2 Notation used for compressor performance characteristics; total-to-static pressure rise (ψ_c) versus flow coefficient (ϕ) in absence of rotating stall or surge; operating point lies on throttle line if $\Phi = \Phi_T$

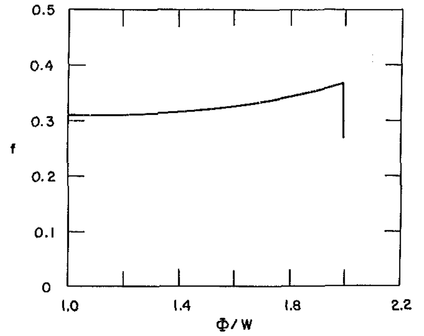

Fig. 4 Nondimensional stall cell speed (f) versus normalized throttle setting (Φ/W), calculated numerically for cubic characteristic with H/W = 0.72; pure rotating stall

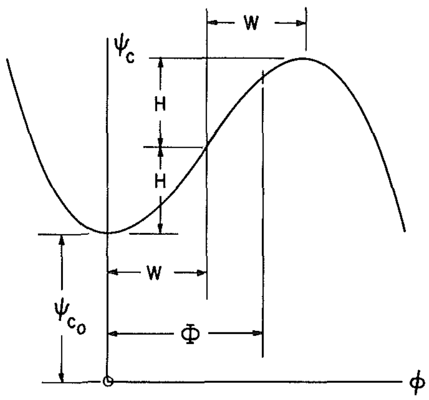

Fig. 3 Notation used in definition of cubic axisymmetric compressor characteristic

in that case there is no θ variation, and $Y(\xi, \theta)$ may therefore be set equal to zero. Equations (42) and (43) then become the same equation, namely

$$\Psi(\xi) + l_c \frac{d\Phi}{d\xi} = \psi_c(\Phi) \tag{45}$$

This equation, together with equation (45), defines the surge problem according to the analysis of [5].

In contrast, for pure rotating stall, the plenum pressure and the angle-averaged through flow are constant with time; therefore, $d\Psi/d\xi$ and $d\Phi/d\xi$ both vanish. Equation (44) then reduces to a statement that $\Phi = \Phi_T$, or that the operating point must lie on the throttle characteristic. It now becomes possible to represent Y as a wave traveling at some constant speed f; that is, as a function of a single variable:

$$Y(\xi,\theta) \equiv Y(\theta^*), \text{ where } \theta^* \equiv \theta - f\xi \tag{46}$$

The variable θ^* is then an angle measured relative to the disturbance. After making a convenient definition,

$$\lambda \equiv \frac{1}{a}\left(\frac{1}{2} - f\right) \tag{47}$$

equations (42) and (43) become

$$\mu \frac{d^3 Y}{d\theta^{*3}} + mf\frac{dY}{d\theta^*} - \left[\Psi - \psi_c\left(\Phi - \frac{d^2 Y}{d\theta^{*2}}\right)\right] = 0 \tag{48}$$

$$\Psi = \frac{1}{2\pi}\int_0^{2\pi} \Psi_c\left(\Phi - \frac{d^2 Y}{d\theta^{*2}}\right) d\theta^* \tag{49}$$

Next, we recall equation (33) and see that, now,

$$\frac{dY}{d\theta^*} = h; \quad \frac{d^2 Y}{d\theta^{*2}} = -g \tag{50}$$

These last equations convert equation (48) precisely into the governing equation of pure rotating stall to be found in [3]. After g is found, equation (49) would give the performance effect of rotating stall [3], a matter discussed more fully below.

A Cubic Characteristic

In order to pursue our line of analysis further, we need first to specify the axisymmetric characteristic function $\psi_c(\phi)$ for use in equations (42–44), and then we need to devise a method of solution. First, we will introduce a particular characteristic function, and show its effect for pure rotating stall.

In our general theory, we are making a necessary distinction between Ψ, the actual pressure rise, and ψ_c, the axisymmetric, steady characteristic that would hypothetically be measured for a compressor with rotating stall and surge absent. Figure 2 schematically shows the difference. At a given flow coefficient ϕ, the value of Ψ may differ from ψ_c even if, as is true in pure rotating stall, Ψ is constant at an operating point fixed by a given throttle setting. This is a nonlinear consequence of rotating stall [3], should it occur.

It is to be emphasized that it is $\psi_c(\phi)$ which we must choose, because it is an inherent feature of the compressor, independent of any disturbance motions. In [8] and [9] it is argued that the axisymmetric characteristic is typically a smooth S-shaped curve. (In those references, the axisymmetric curve is estimated for a three-stage compressor using a procedure based on corrections to transient data.) If this idea is correct, a physically realistic choice could be a simple cubic, as shown in Fig. 3. We expect that the rotating-stall or transient characteristic calculated using this cubic will be a good representation of that calculated using an exact, but as yet unknown, axisymmetric diagram.

The formula for the curve shown in Fig. 3 is

$$\psi_c(\phi) = \psi_{c_0} + H\left[1 + \frac{3}{2}\left(\frac{\phi}{W} - 1\right) - \frac{1}{2}\left(\frac{\phi}{W} - 1\right)^3\right] \tag{51}$$

where ψ_{c_0}, H, and W are parameters. We locate a throttle point at the angle-averaged flow coefficient, Φ. The local flow coefficient to be used in equation (51) includes departures (g or $-Y_{\theta\theta}$) from that value. Thus, for use in equation (51), we set

$$\phi = \Phi - Y_{\theta\theta} \tag{52}$$

The resulting expressions for $\psi_c(\phi)$ will be used in equations (42, 43) or (48), as appropriate.

To show the effect of the cubic characteristic in pure rotating stall, Chue has carried out calculations for a particular cubic using a numerical procedure [8, 9] for solving equation (48). The value used for m was 1.75. H and W were chosen so that the cubic would have the the same peak and valley points as a measured curve; their values were 0.18 and 0.25 respectively. Rather than equation (47), a somewhat more elaborate relation between λ and f was used, in which

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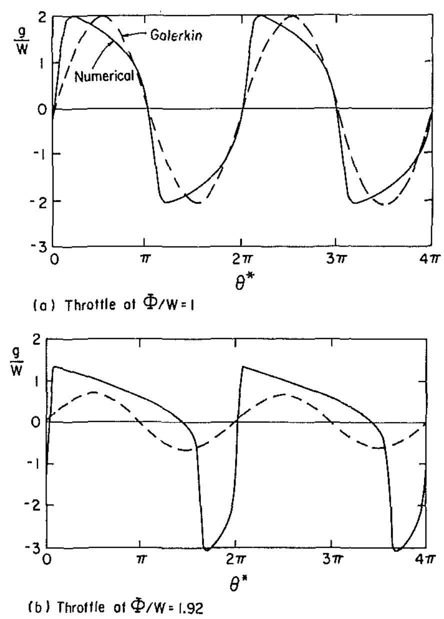

Fig. 5 Comparison between numerical calculation (solid line) and single harmonic Galerkin representation (dashed line) of circumferential variation of axial velocity coefficient for cubic characteristic, $H/W = 0.72$, and pure rotating stall

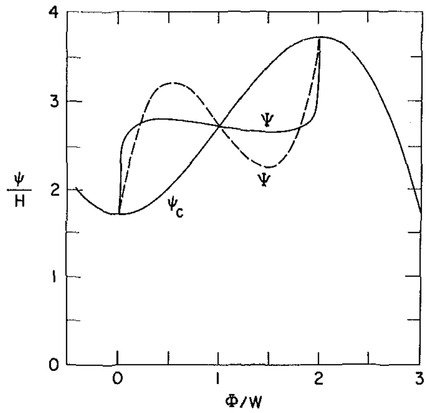

Fig. 6 Numerical (solid) and single-harmonic Galerkin (dashed) solutions for compressor performance (Ψ) in pure rotating stall, for a given axisymmetric characteristic (ψ_c) with $H/W = 0.72$ and $\psi_{c_0}/H = 1.67$

guide-vane effects were included, following [9]. Numerically, the relation was, in effect,

$$\lambda = 4.34(0.38 - f)$$

Figure 4 shows how the rotating-stall propagation speed f increases when the throttle is opened (i.e., as Φ is increased), as was predicted in [3]. Figures 5(a, b) show how the axial velocity disturbance $g(\theta^*)$ changes as the throttle is opened. When $\Phi = W$, the symmetry features shown in Fig. 5(a) must appear because of the symmetry assumed for ψ_c, and we also note that $g(\theta^*)$ swings between extremes which lie on the unstalled and reverse-flow legs of the characteristic. As Φ increases, these extrema remain nearly the same, but more time is spent unstalled, so to speak. Figure 5(b) shows the result for $\Phi = 0.48$. There is no solution beyond $\Phi = 0.50$, which corresponds to the peak point of the curve. The dashed lines on Figs. 5(a, b) will be discussed later.

The most interesting result is shown in Fig. 6. There, the cubic axisymmetric characteristic is shown with the calculated Ψ (equation (49)) superposed as a solid line to form a prediction of how the actual in-stall pressure rise varies with throttle setting (Φ). Again, the dashed line will be discussed later.

This graph dramatically illustrates the idea of [8], that the usually observed sudden break in performance at the stall point, and subsequent drop as the throttle is closed, can be the result of an appearance of rotating stall, rather than a direct effect of the inherent axisymmetric characteristic of the compressor. The latter might, in fact, be quite smooth and gradual, with no break at all. The reasonableness of these results gives us confidence to adopt the cubic characteristic for use in the theoretical development to follow.

A Galerkin Procedure

To treat a general transient disturbance which may have both angular variation like rotating stall and time-dependent mean flow like surge will require solution of the complete system of equations (42–44), which include derivatives which are third order in angle, but only first order in time. Therefore, at this initial stage of investigation, we would like to devise a simple method of solution which effectively reduces the order of the equations in angle θ.

If the function $\psi_c(\phi)$ is smooth, the Galerkin method of nonlinear mechanics [10] will perhaps serve this purpose. In fact, this method has been used successfully for the van der Pol problem, which is just the symmetric case ($\Phi = W$) in the present theory of pure rotating stall with cubic ψ_c.

In a Galerkin procedure, the solution to the differential equation is represented by a suitable sequence of basic functions. If the functions are well chosen, and enough terms in the sequence are taken, the "true" solution can be represented very accurately. Fourier series or spectral methods are examples of this procedure. For present purposes, we are seeking the simplest possible wave representation, capable of describing the transient phenomena. Accordingly we have chosen a single harmonic wave to serve as our representational function. The sequence is truncated at one term, in effect.

For the present exploratory purposes, this approximation will prove to be reasonable. However, it is very important to recognize the limitations of this approximation. The single-term Galerkin procedure is most reliably accurate for weakly nonlinear systems. The present set of equations is strongly nonlinear. Thus, although we shall see that the amplitudes of axial velocity disturbance (g) calculated by the single-term Galerkin method for pure rotating stall are in reasonable agreement with exact calculations, as are the corresponding pressure-rise curves, which are averaged functions of g, the method does not give a good description of the derivatives of g. This is to be expected, because as the Figs. 5(a, b) show, the exact solutions are not of harmonic shape.

In this study, we wish to emphasize the amplitudes of disturbances and the integrated effects on performance. Thus, the information that is of most interest at present is just that information which should be least sensitive to the higher harmonics of the stall-cell description. We proceed, then, to develop a single-term Galerkin method for our general transient equations (42–44), and we will note the simple results obtained in the special case of pure rotating stall.

We begin by representing Y by a single harmonic function, of unknown amplitude A

$$Y = WA(\xi)\sin(\theta - r(\xi)) \tag{53}$$

We include an unknown phase angle $r(\xi)$ in this equation, because we must not expect nodes of the disturbance always to reamin in the same angular location. We note that the question of validity of the "$h' = -g$" assumption (equation (33)) does not arise when our Galerkin method is used; for a single harmonic wave, equation (33) may be shown to be exact.

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We substitute equation (53) into equation (42), to formulate a residual (which would vanish if the solution were exact). We then set various moments of that residual equal to 0. First its integral must vanish; this accounts directly for one of the basic equations, equation (43):

$$\frac{1}{2\pi}\int_0^{2\pi}\psi_c(\Phi+WA\sin\zeta)d\zeta=\Psi+l_c\frac{d\Phi}{d\xi} \tag{54}$$

The $\sin(\theta - r)$ moment of the residual gives

$$\frac{1}{\pi W}\int_0^{2\pi}\sin\zeta\,\psi_c(\Phi+WA\sin\zeta)d\zeta=\left(m+\frac{1}{a}\right)\frac{dA}{d\xi} \tag{55}$$

and the $\cos(\theta - r)$ moment gives

$$\frac{1}{\pi W}\int_0^{2\pi}\cos\psi_c(\Phi+WA\sin\zeta)d\zeta= -\left[\left(m+\frac{1}{a}\right)\frac{dr}{d\xi}-\frac{1}{2a}\right]A \tag{56}$$

Even before specifying the characteristic function ψ_c, we see that if it is a regular function of its argument, the integral on the left of equation (56) must vanish when carried out over a complete cycle. Thus, very generally, it is clear that $dr/d\xi$ must be a particular constant if A is not to be zero. Calling that constant f_0,

$$r=\xi f_0;\quad f_0\equiv\frac{1/2}{1+ma} \tag{57}$$

In fact, f_0 is identical to the propagation speed found in [3] for small-disturbance rotating stall (which is a simple harmonic wave). We conclude that, even though the amplitude A varies with time, the one-term approximation provides that nodes of the circumferential disturbance will travel at a constant speed comparable to that of pure rotating stall. This is a result of the Galerkin method we are using; Fig. 4 tells us that propagation speed is actually a rising function of Φ, in rotating stall.

We are left with equations (44, 54, 55) which contain the unknown functions of time, Ψ, Φ, and A. These equations apply for any characteristic function ψ_c.

Final Simplified Equations. In the foregoing equations, it remains only to specify the axisymmetric characteristic; we adopt the cubic function defined in equation (51). It is obvious that if ψ_c can be expressed as a sum of powers of ϕ, only even powers of $A\ \sin\zeta$ will contribute to the integral in equation (54), and only odd powers in equation (55). Inspection of those equations then shows that A must enter as the square, no matter what the particular form of ψ_c. For convenience, therefore, we define a new variable J to replace A:

$$J(\xi)\equiv A^2(\xi) \tag{58}$$

and we note that J must always be positive.

Carrying out the indicated integrals, we find the final governing equations to be

$$\frac{d\Psi}{d\xi}=\frac{W/H}{4B^2}\left[\frac{\Phi}{W}-\frac{1}{W}F_T^{-1}(\Psi)\right]\frac{H}{l_c} \tag{59}$$

$$\frac{d\Phi}{d\xi}=\left[-\frac{\Psi-\psi_{c_0}}{H}+1+\frac{3}{2}\left(\frac{\Phi}{W}-1\right)\left(1-\frac{1}{2}J\right) -\frac{1}{2}\left(\frac{\Phi}{W}-1\right)^3\right]\frac{H}{l_c} \tag{60}$$

$$\frac{dJ}{d\xi}=J\left[1-\left(\frac{\Phi}{W}-1\right)^2-\frac{1}{4}J\right]\frac{3aH}{(1+ma)W} \tag{61}$$

These are our final equations for instantaneous values of circumferentially averaged flow coefficient (Φ), total-to-static pressure rise (Ψ), and squared amplitude of angular variation (J), all as functions of time (ξ). Compressor and system parameters which will govern the solutions are diagram steepness (H/W), shut-off head (ψ_{c_0}/H), compressor duct length (l_c) and slope (m), internal compressor lag (a), plenum volume and compressor annulus area (B), and the throttle pressure characteristic function $F_T(\Phi)$.

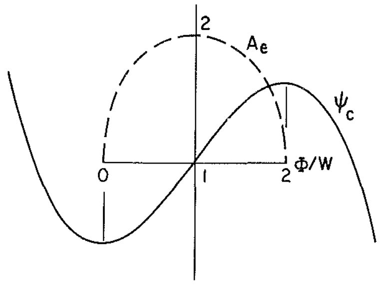

Fig. 7 Amplitude (A_e) of single harmonic Galerkin solution for pure rotating stall as a function of Φ/W; the cubic axisymmetric compressor characteristic (ψ_c) is also shown for reference

Galerkin Solution for Pure Rotating Stall. For rotating stall, time derivatives in equations (59–61) must vanish. Equation (61) then requires that J either vanish or have the constant "equilibrium" value

$$J_e=4\left[1-\left(\frac{\Phi}{W}-1\right)^2\right] \tag{62}$$

Figure 7 displays this result (in terms of $A = \sqrt{J}$), which quite reasonably shows the rotating-stall amplitude to vanish at the neutral-stability points $\Phi = 0$, $2W$, and have its maximum at the midvalue $\Phi = W$. These features are essentially correct, according to the exact calculations displayed in Fig. 5(a, b).

Equations (33, 46, 53, and 58) combine to give a representation of $g(\theta)$, for rotating stall; it appears as the dashed harmonic waves on Figs. 5(a, b), for two values of Φ. As we expect, these Galerkin results correctly suggest the trend of amplitude with Φ, but are incapable of representing the exact wave shape. Obviously, the simple harmonic wave cannot represent the sudden drop into, and recovery from, reversed flow which occurs when Φ is near $2W$.

When J_e from equation (62) is substituted into the right side of equation (60), which must vanish for rotating stall, one finds the steady performance Ψ:

$$\Psi=\psi_{c_0}+H\left[1-\frac{3}{2}\left(\frac{\Phi}{W}-1\right)+\frac{5}{2}\left(\frac{\Phi}{W}-1\right)^3\right] \tag{63}$$

which may be compared with equation (51) to infer the performance effect of rotating stall. Indeed, equation (63) is shown in Fig. 6 as the dashed line, which may be compared with the exact (solid line) result. Although the two curves do not agree numerically, it is remarkable that the qualitative features of the performance effect are well predicted by the simple, first-harmonic Galerkin theory, especially the steep rise near $\Phi = 2W$, and the equally steep drop near $\Phi = 0$.

This examination of the special case of rotating stall gives us confidence that the single harmonic Galerkin method will prove to be a useful approximation for the general transients contemplated in the final equations (59–61). It correctly forbids any solution beyond the stability limits of the compressor characteristic, and it gives a qualitatively correct picture of the nonlinear effect of rotating stall on performance. However, it cannot accurately represent wave shape of the relaxation type of transition which arises in fully developed rotating stall, nor can it show effects of extreme diagram steepness, as in [3].

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Discussion of Simplified Equations

Pure Modes and Their Growth. Our intention is to use the foregoing equations (59–61) to compute the consequences of an initial disturbance of a compression system. Before doing that, we should examine these equations to discover what we can about their general features.

First, we have already observed how these equations describe pure surge or rotating stall. In the case of pure surge, $J = 0$ and equation (60) becomes simply equation (49) with cubic ψ_c (equation (51)). In pure rotating stall, amplitude J is constant at an "equilibrium" level defined by J_e (equation (62)).

Our equations thus appear to permit the existence of pure modes; that is, either surge or rotating stall without the other. However, we should question whether such modes could evolve from initially infinitesimal disturbances. Clearly, such a process is possible for pure surge; if J is zero it will remain zero forever, according to equation (61), and the other two equations will deal with the evolution of pure surge disturbances, as described in [5, 6].

It appears to be impossible, however, for rotating stall to evolve without producing at least some disturbance of Φ or ψ; if J changes, then equation (60) says the Ψ or Φ must also change. A disturbance of Φ induced in this way might of course be a noncyclic transient, not to be identified as "surge." Further comments on these points are given in Part II.

Nature of the Coupling Process. We have seen that equation (61) describes how angular-disturbance amplitude J grows at a rate which initially depends only on the disturbance itself, but then tries to approach an equilibrium condition defined by equation (62). In general, the flow coefficient and therefore J_e will change with time. Therefore, during a coupled oscillation, $J_e(\xi)$ will be a moving target, so to speak, for J. Whether J actually achieves and stays on that target, which is quasi-steady equilibrium with $\Phi(\xi)$, will be known only from simultaneous solution of the equations, that is, a trajectory in the three-dimensional space Ψ, Φ, J. This behavior may remind one of the relaxation of an internal degree of freedom in physical gas dynamics.

Inspection of equation (61) also shows that excursions of Φ away from the central point $\Phi = W$ will diminish the rate of change of J. Thus, for a throttle setting of $\Phi = W$, the presence of surgelike variations of Φ will tend to suppress circumferential variations. Furthermore, if B is small, equation (59) implies that swings of Φ will tend to be small, which, relatively, would encourage circumferential variation, according to equation (61). This is in accordance with our expectation [5, 6] that small B will favor rotating stall.

Turning to equation (60), we see that the presence of a circumferential angular variation in velocity will diminish the term in which J appears (recalling that J must always be positive). That term describes the central slope of the characteristic, at the midpoint of Fig. 3, thus reducing the basic amplifying effect of the characteristic in the stall region. We can conclude that this variation (i.e., the presence of a rotating-stall-like disturbance) would tend to inhibit surgelike disturbances of Φ.

The foregoing arguments, which are certainly in qualitative agreement with the many experimental observations that have been made of these phenomena, only suggest how surgelike and rotating-stall-like disturbances are coupled through equations (60) and (61). The coupling is mediated by equation (59), and one must carry out trajectory calculations to discover the actual consequences of a given initial disturbance.

Conclusions

We are now ready to trace the evolution of disturbances in an axial compression system, in order to predict when pure surge or pure rotating stall, or perhaps some composite motion, prevails. Calculations can proceed using equations which have a known basis in the differential equations of fluid mechanics; while assumptions have been made, all assumptions are identified, and, ultimately, can be evaluated. In this sense, the present theory is a "first-principle" one, as distinct from a "model" which proceeds from commonly observed features of the phenomena themselves.

The equations show that pure modes of surge (time variation) or rotating stall (angle variation) each can exist without the other. However, in general, time and angle fluctuations are coupled. Angular variations must grow toward an equilibrium, or target value; this target depends on the strength of any time variation in progress. Conversely, the damping rate of any time variation of flow coefficient depends on the strength of any angular variation in progress.

The equations presented here account, approximately, for many important compression-system features, including system volumes, inlet and diffuser shapes, axisymmetric compressor characteristic, throttle characteristic, system hysteresis (because the actual system pressure rise depends on the existence of rotating stall), compressor geometry and speed, and internal lag processes in the compressor. Some important effects have not yet been included; in this category are inlet distortion, compressibility, and combustor phenomena.

In the companion paper to this one, Part II, we will further study the nature of the general transient equations, show the results of illustrative calculations, and discuss needs for future research.

Acknowledgments

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