Some Stall and Surge Phenomena in Axial-Flow Compressors

MERLE C. HUPPERT* AND WILLIAM A. BENSER†

Lewis Flight Propulsion Laboratory, NACA

SUMMARY

Observations of rotating stall have shown that a wide variety of stall patterns is possible.

Hot-wire anemometer data on a multistage compressor have shown a progressive-type stall at low speeds. The amplitude of the flow fluctuations increases in magnitude through the first few stages and then diminishes rapidly to a small value in the latter stages. A stage-stacking analysis has shown that rotating stall will exist over a large portion of the compressor map at low speeds but will be instigated almost simultaneously with compressor surge at high speeds.

Blades failures attributable to resonant vibrations excited by rotating stall have been experienced in single- and multistage compressors.

In the stage-stacking analysis no deterioration of stage performance due to unsteady flow resulting from stall of adjacent stages was considered. In general, the pressure drop at the stall point is believed to be much larger than indicated by an analytical formulation of compressor performance. Compressor surge is attributed to a limit cycle operation about the compressor stall point, and, as indicated in a few compressor tests and in jet-engine tests, a small compressor discharge receiver volume may result simply in stall of the compressor without the cyclic characteristics of compressor surge. In this event, engine operation will be limited because of the large drop in performance which accompanies compressor stall.

INTRODUCTION

THE GOALS OF INCREASED FUEL ECONOMY and increased altitude operational ceiling for aircraft jet engines have led to engines having compressor pressure ratios of the order of 7 to 1 and higher. At the same time, minimum engine weight and frontal area are essential for operational range of the aircraft. Because of its advantages of high flow capacity, high efficiency, and simplicity of staging, the axial-flow-type compressor is particularly adapted to the high-performance jet engine. The stall or surge limit of the high-pressure-ratio axial-flow compressor, however, presents a major problem with regard to engine acceleration and off-design performance of the engine. This limit, which represents the minimum weight flow obtainable consistent with useful operation at any compressor rotational speed, is alternately defined as stall limit or surge limit. In this paper the stall limit of the multistage compressor will be referred to as compressor stall as differentiated from individual stage stall.

The limitations resulting from compressor stall can be illustrated by superimposing an engine equilibrium operating line on a conventional compressor map (Fig. 1). In this figure compressor pressure ratio is plotted against equivalent weight flow for constant values of equivalent or aerodynamic speed. The equilibrium operating line shown passes through the region of high compressor efficiency at design speed and represents steady-state operating conditions for the compressor in a jet engine with a fixed exhaust-nozzle area. The compressor stall limit is also shown.

The various off-design performance problems resulting from compressor stall can now be discussed. The first problem concerns that of accelerating the engine from idle to full speed. When the fuel flow is increased from the value required for idling, the turbine-inlet temperature increases and results in a transient decrease in weight flow. The compressor operating point thus moves closer to the compressor stall limit. The distance between the equilibrium operating line and the compressor stall limit determines the permissible acceleration and is defined as the acceleration margin. Large values of this margin permit large increases in fuel flow and, therefore, rapid engine acceleration. On the other hand, if the acceleration margin is small, large increases in fuel flow result in compressor stall or surge, and the engine acceleration is diminished; in fact, in some cases the compressor performance may deteriorate so much that the engine will decelerate. Stall-limit lines for high-pressure-ratio axial-flow compressors frequently have kinks that intersect the equilibrium operating line and, thus, give zero or nega-

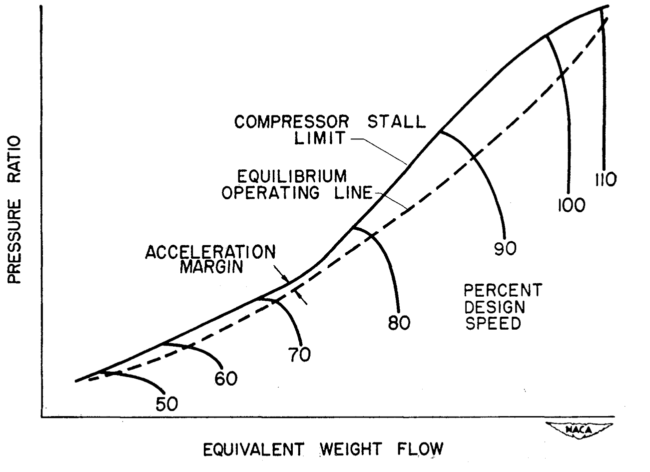

FIG. 1. Overall performance of a typical axial-flow compressor.

Presented at the Flight Propulsion Session, Twenty-First Annual Meeting, IAS, New York, January 26–29, 1953.

* Aeronautical Research Scientist.

† Chief, Compressor Research Branch.

tive acceleration margins at intermediate speeds. Twin-spool engines or auxiliary means of acceleration, such as adjustable exhaust nozzles, compressor bleed, or adjustable compressor-inlet guide vanes that increase engine weight and complexity, are then required in order to obtain satisfactory acceleration. The acceleration margin can be increased by overdesigning the compressor and turbine, but this technique is uneconomical with respect to weight and design-speed fuel consumption. The frequently encountered adverse intermediate-speed stall characteristics of high-pressure-ratio multistage axial-flow compressors thus impose a serious acceleration limit for the high-performance, high-pressure-ratio jet engine.

The second off-design problem associated with compressor stall concerns the development of high engine thrust with engines having adjustable jet nozzles or other thrust-augmentation devices. Closing the jet nozzle and increasing the fuel flow again imposes an increased throttling on the compressor, and the operating point moves toward the stall limit. The thrust increase obtainable is thus determined by the margin between the compressor stall-limit line and the cruise or open jet nozzle condition. Inasmuch as the peak efficiency range for the compressor is usually relatively close to the stall limit, it is desirable to match the engine components so that the compressor operates at its peak efficiency for a cruise or open jet nozzle condition. The thrust increase by jet nozzle adjustment may, thus, be seriously limited by the compressor stall limit. A similar problem exists for high-thrust, high-altitude flight, for which the compressor-inlet temperature may be lower than standard sea-level temperature. For an engine operating at a constant mechanical speed, the aerodynamic or equivalent speed is then higher than the design value; and the compressor stall limit may also be encountered for operation of the engine at high thrust, greater than design equivalent speed. Thus, the compressor stall limit is important over most of the engine speed range from idle to maximum aerodynamic speed at altitude.

Because of the importance of the compressor stall limit, the source of this limit is a pertinent matter. It can readily be shown that stall of the multistage compressor results from stall of one or more of the individual stages; and the stages that instigate overall compressor stall will vary with the operating condition, as can be seen from the following considerations. At peak efficiency at design speed, each of the stages of the compressor will operate at or in the region of its peak performance. As the pressure ratio is increased at design speed, the operating condition of the front stages changes only slightly; whereas, the rear stages approach their stall points very rapidly because of the progressive increase above the design value of density of the air passing through the compressor. The small change in inlet-stage operating conditions is indicated by the nearly vertical characteristic of pressure ratio against weight flow as shown on the compressor map. Therefore, at speeds near the design value, stall of the multistage compressor is associated with deterioration of performance of the latter stages.

Next the problem of low-speed compressor stall is considered. Because of the small stage pressure and density ratios associated with low compressor speeds, the axial velocity increases excessively through the compressor, and the rear stages operate at or near their choke or turbining condition. The flow restriction of the rear stages causes the front stages to operate in a high angle of attack or stalled condition. Thus, low-speed stall of the multistage compressor is associated with deterioration of the performance of the inlet or front stages. This is borne out by the fact that the flow at 50 per cent of design speed is on the order of 30 per cent of design speed flow instead of the 50 per cent that would be required for design angles of attack for the inlet stages. At intermediate speeds, stall will be approached more or less simultaneously in several stages. It should be noted that increasing the overall pressure ratio of the compressor increases the deviations from the design vector diagrams for a given off-design speed or weight flow and renders the off-design performance and overall compressor stall problems more acute.

Inasmuch as multistage compressor stall is a result of individual stage stall, a study of compressor stall requires an understanding of individual stage characteristics and the stage operating conditions for off-design operation. For this paper the discussion of this phenomenon has been divided into two phases. The first is a study of stall characteristics of multistage compressors and the effects of the individual stage characteristics on multistage compressor stall. Inasmuch as the experimental technique of studying multistage compressor stall may be hampered by surge and stage interactions, it is difficult to evaluate stage characteristics in the proximity of overall compressor stall. Therefore, much of the investigation reported herein has been done by analytical techniques. In addition, an analysis of multistage compressor surge characteristics is considered, and the effects of compressor stall and external system characteristics on the compressor surge phenomenon are discussed.

Stall of Single-Stage Axial-Flow Compressors

Single-stage stall characteristics must be evaluated in order to understand multistage compressor stall characteristics. Performance of a single-stage axial-flow compressor is generally measured with steady-state instrumentation that has sufficient damping to attenuate fluctuations with frequencies over 5 or 10 cycles per sec. As the flow is decreased beyond the stall point, the flow in all blade passages consequently appears to be uniform with time. With some stages, regions of flow rate exist where surge or low-frequency pulsation is encountered, but steady-state conditions are again indicated if further reductions in flow rate are made. Recent studies with high-frequency-response instrumentation, such as hot-wire anemometers and pressure transducers, have shown stage stall to be a nonsteady type of phenomenon in which some sectors of the annulus are operating at an extremely low mass-flow rate as compared with the remainder of the annulus; these low-flow sectors rotate about the compressor axis. A typical stall pattern is illustrated in Fig. 2 for a single-stage compressor consisting of guide vanes, rotor, and stator. In this example, four stall or low-flow zones at the blade tips are depicted by the hazy areas. These low-flow zones extend completely through the stage and rotate in the direction of the rotor but at a lower speed. The mass-flow rate measured on a fixed hot-wire anemometer at the discharge of the stage is also presented on this figure. This instrument was so oriented as to read principally the component of flow in the design direction. The alternate high- and low-flow zones passing the fixed instrument produce the wave form indicated. Similar patterns are observed some distance ahead of the guide vanes or behind the stators. Temperature and pressure variations across the stall zone were also observed but were not evaluated.

At some point far upstream of the stall zone there exists a uniform flow. The stream lines immediately upstream of the blades must, therefore, diverge to maintain continuity around the low-flow or stall zone. This spillage of flow to either side of the stall zones, which tends to increase the angle of attack on one side of the stall and decrease it on the other, results in a tangential propagation of the stall zone. The propagation rate of the stall region relative to the rotor is generally opposite in direction but lower in magnitude than the rotor-blade speed. On the absolute basis, therefore, the stall rotates in the direction of the rotor but at a lower speed. It appears that in principle the rotating or propagating stall may be instigated by a stator as well as a rotor.

To date, no definite relation between the number of stall regions in the stall pattern and compressor geometry has been determined. Two fairly distinct categories of stall patterns, however, have been observed: one, a progressive stall as shown in Fig. 2, originates at the blade ends and consists of multiple stall zones that extend over only a portion of the blade span, and the other, a root-to-tip stall, generally consists of only one zone that extends over the entire blade span.

Progressive Stage Stall.—The progressive-type stall has been observed on several low hub-tip ratio stages of the basic solid-body rotation type. For this type stage operating with uniform inlet flow, the rotor-tip angle of attack varies much more rapidly with flow rate than does the root angle of attack, and the rotor-blade tip naturally stalls first, while the remainder of the blade is inherently unstalled. The circumferential and radial extent of the low-flow zones at the blade tip

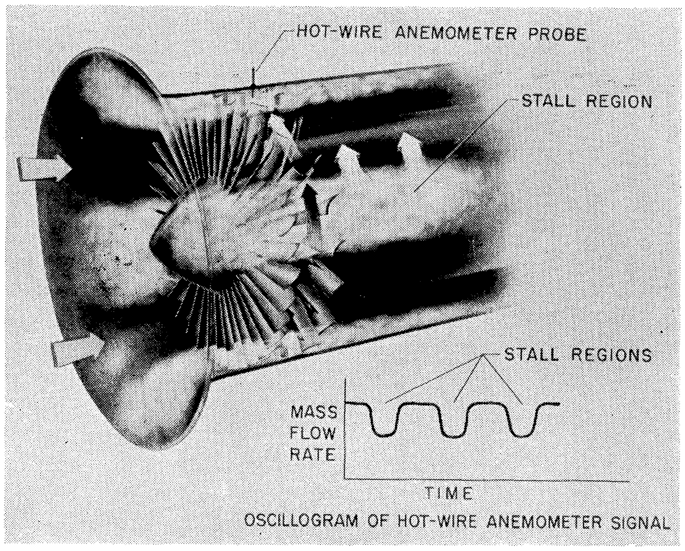

Fig. 2.

increases with further decreases in flow rate, while the root-section operating condition remains relatively unchanged. Other types of stage diagram, such as a free-vortex flow, might have a progressive-type stall originating at the root section of the blade. In general, however, the jet-engine requirements of high flow capacity and high stage-pressure ratio lead to inlet-stage designs in which the rate of change of loading with flow at the blade tip is much greater than that at the blade root.

A typical performance curve for a stage that is critical at the blade tip is shown in Fig. 3 as a plot of pressure coefficient ψ against flow coefficient φ. As stall first occurred (point A), three stall zones were found. On the left side of the figure, the oscillograms of mass-flow rate obtained by hot-wire anemometers for the tip, mean radius, and root section are shown. The curves of flow rate against time show the greatest flow-rate variations at the tip, a smaller variation at the mean radius section, and only minor flow variations at the root. As the flow is further reduced from the stall point, the area of each stall zone increases. Reductions in flow to point B (Fig. 3) resulted in a sudden transition from three to four stall zones, and still further reductions of flow resulted in a transition to five stall zones at point C. Thus, the changes in area of individual stall

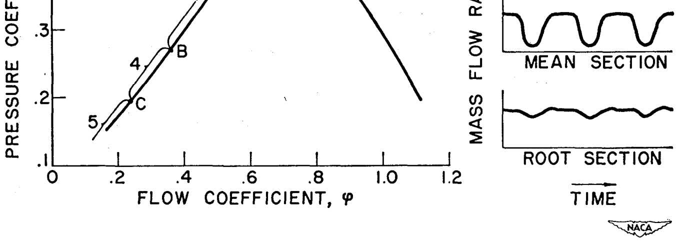

Fig. 3. Performance of low hub-tip ratio stage with progressive stall.

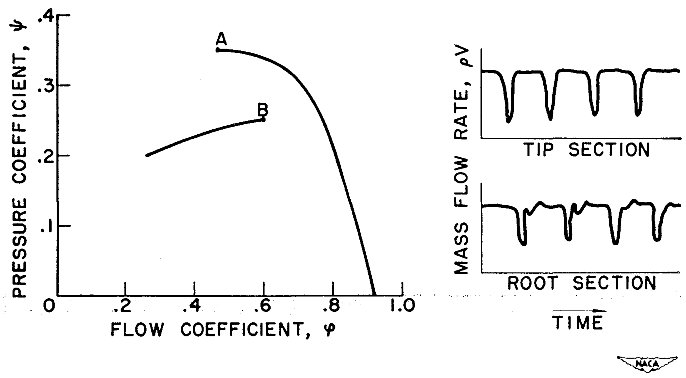

FIG. 4. Performance of high hub-tip ratio stage with root-to-tip stall.

zones and changes in number of stall zones result in a relatively uniform rate of change of stalled flow area with flow rate. Progressive-type stall patterns having from three to eight stall zones with rotational speeds of 40 to 85 per cent of rotor speed have been observed. For each stall pattern the rotational speed of the stall zones varied directly with rotor speed. It should be noted that progressive-type rotating stall results in a gradual drop in pressure coefficient as the severity of stall increases.

Root-to-Tip Stage Stall.—When the stall angle of attack is approached nearly simultaneously over the entire span of the blade, as in high hub-tip ratio stages, the resultant flow-rate variations are relatively uniform in intensity from root to tip. Observations of this type stall, defined as a root-to-tip stall, show only one stall zone. A typical curve of performance in terms of pressure coefficient against flow coefficient is shown in Fig. 4, together with oscillograms of mass-flow rate against time for the tip and root sections. There is little radial variation of stall pattern, and from the pressure-coefficient curve it is seen that the instigation of stall is accompanied by a sharp drop in pressure ratio as shown at point A, as compared with the gradual decrease in pressure ratio for the progressive-type stall. The drop in pressure rise is on the order of one-third of the stage-pressure rise prior to stall. Further reductions in flow result in an increase in the circumferential extent of the stall zone and a gradual decrease in pressure coefficient. The stall zone may cover one-third or more of the annulus (in circumferential extent) and has an absolute rotational speed of 30 to 40 per cent of the rotor speed. A root-to-tip stall zone having been established, the flow may be considered to be divided in two parts, a low-flow or stalled part and a high-flow or unstalled part. Changes in compressor weight flow may then result in changes in area or weight-flow rate of the two zones. In order to unstall this type stage, the stalled zone must be completely eliminated. Because of the freedom of change of area or of flow rate in the stalled zones, these zones will persist when the compressor weight flow is increased above the value for instigation of stall. Unstalling of the stage shown on Fig. 4 is obtained at the point indicated as B, and a hysteresis effect results. Thus, the stage performance curve has discontinuities between the branches.

The value of the pressure coefficient on the stalled branch of the curve in Fig. 4 was determined by conventional steady-state instruments and represents some average value of the pressure coefficient for the stalled and unstalled portions of the annulus. Because of the response characteristics of the instrumentation used, the pressure coefficient in the stall region does not necessarily represent a true average of the stage performance. The existence of this double-branch type of stage characteristic is frequently unsuspected or ignored in the presentation of single-stage performance. Discontinuties and double-valued pressure coefficients, however, must be considered in computations of multistage compressor performance from single-stage data and are extremely important with regard to studies of compressor stall and surge, inasmuch as a double-valued single-stage characteristic must effect a double-valued multistage characteristic.

General Discussion of Rotating Stall.—In general, low hub-tip ratio stages or stages that approach critical conditions much more rapidly at one end of the blade span than at the other exhibit progressive-type stall. High hub-tip ratio or exit stages in which angle of attack variations along the blade span are small exhibit root-to-tip stall. Available data on intermediate hub-tip ratio or typical middle stages of a multistage compressor exhibit first a progressive stall that, with further reductions in flow, changes to a root-to-tip stall with the associated sharp drop in pressure and the hysteresis characteristic. An important difference between progressive and root-to-tip rotating stall is the gradual decrease in pressure coefficient associated with progressive stall as compared with the discontinuity of pressure coefficient associated with root-to-tip stall. Of secondary importance is the fact that the frequencies of flow variation are generally much higher for progressive than for root-to-tip stall.

The rotational velocities of the stall zones are approximately proportional to the compressor speed, and in the useful speed range the multiple-zone progressive stall may result in flow-rate variations with frequencies relative to the rotor or stator blading which are of the same order as the resonant bending frequencies of typical inlet-stage blading. Inasmuch as frequent blade failures have resulted from brief periods of operation in stall at intermediate compressor speeds, indications are that rotating stall is a possible source of resonant excitation that may lead to blade failure.

It should be noted that the rotating-stall pattern is not always as sharply defined as indicated in the stages discussed, and at least one stage has been tested in which no rotating stall existed. This particular stage has an extremely high camber at the blade tip. Thus, although rotating stall appears to be prevalent, it is not necessarily the only type of stall which can exist.

Stall of Multistage Compressors

The second phase of this study of axial-flow-compressor stall is consideration of the effects of the single-stage stall characteristics on the multistage stall problem. As was discussed earlier in this paper, for operation of high-pressure-ratio multistage compressors at 50 per cent of design speed, flow restrictions in the latter stages limit the flow and result in stalling of the front stages. In spite of front-stage stalling, however, the compressors do have an appreciable flow range of usable operation at low speeds. It is evident, therefore, that stage stall is a necessary but not a sufficient condition for multistage compressor stall.

Experimental Multistage Study.—Some understanding of the flow processes prevalent at low and intermediate speeds is given by some limited hot-wire anemometer data recently taken on a multistage research compressor. These data indicate that a rotating stall of the progressive type existed during the surge-free and useful range of compressor operation at low speeds. A summary of the radial and axial extent of these rotating-stall patterns is given in Fig. 5. The ordinate of the curves is the ratio of the amplitude of the flow variations $\Delta\rho V$ of rotating stall to the average ρV for the tip, mean, and root sections. The abscissa is the axial distance through the compressor. A rotating stall of the progressive type existed in the earlier stages. The amplitude of the flow variations in the stall zones increased through the first few stages and then decreased rapidly to relatively small values in the rear stages. The amplitude of the flow variations decreases rapidly with radius and is relatively small at the root section even in the inlet stages. Stall patterns having three to seven stalled zones were found with the number of stalls increasing with a decrease in flow and speed. A marked similarity exists between the stall patterns measured in the front stages of this compressor at low speed and those obtained with the inlet-type single-stage compressor discussed previously. The stall zones extended nearly axially through the compressor, and the harmonics of the flow variations were damped in the latter stages, leaving a sinusoidal pattern at the compressor discharge. As was mentioned in the single-stage discussion, the stall frequencies measured for this progressive-type stall were of the same order of magnitude as the natural bending frequency of the blading of the first few stages. Therefore, resonant excitation of rotor and stator blades in the front stages as a result of rotating stall is a potential source of blade failure. Although these studies were of a preliminary nature and the results are only qualitative, they clearly indicate that, even though rotating stalls instigated by the inlet stages persist throughout the compressor, the pressure-ratio producing capacity of the compressor is not excessively inhibited so long as the flow coefficients of middle and latter stages are sufficiently above their stall values.

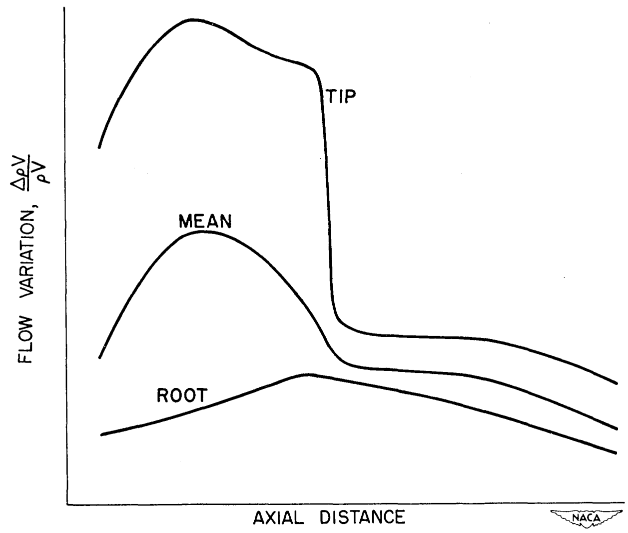

Fig. 5. Flow variation in a multistage compressor with progressive stall in inlet stages.

Computed Multistage Performance.—The results of these preliminary experimental studies suggest an analytical study of overall multistage compressor stall and off-design operation based on a knowledge of individual stage characteristics. An approximate knowledge of variations in operating conditions of each stage can be obtained over the complete range of multistage operation by a stage-stacking technique. A representative analysis was made, therefore, for a hypothetical 12-stage compressor having an overall pressure ratio of 7.75 at the reference or match point. For this analysis, stage performance was generalized in the form of pressure coefficient and adiabatic efficiency against a flow coefficient based on mean radius velocities, and the overall compressor performance was computed with a stage-by-stage technique. Single-stage data suggest that high-flow, high-pressure-ratio inlet stages will probably have progressive stall; whereas middle stages will have progressive stall followed by root-to-tip stall, and exit stages will have only root-to-tip stall. The first four stages, therefore, were assumed to have a progressive stall, and the stage characteristics shown in Fig. 6 were chosen. For stages 5 through 8, an initial progressive stall followed by a root-to-tip stall at a lower flow coefficient was assumed; this assumption gave the stage characteristics indicated. The last four stages were assumed to have only a root-to-tip stall, and thus the selection of the rear-stage-performance characteristics was as shown.

The curves for all stages were identical except for the stall characteristics. A reference speed was chosen and the area ratio through the compressor determined so that all stages operated at a flow coefficient of 0.69 at the reference condition. Overall compressor performance was then computed for speeds from 50 to 100 per cent

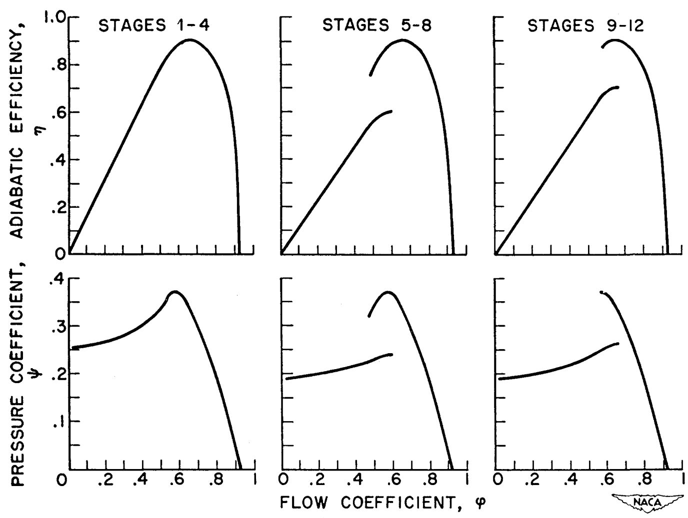

FIG. 6. Stage performance parameters for stacking analysis.

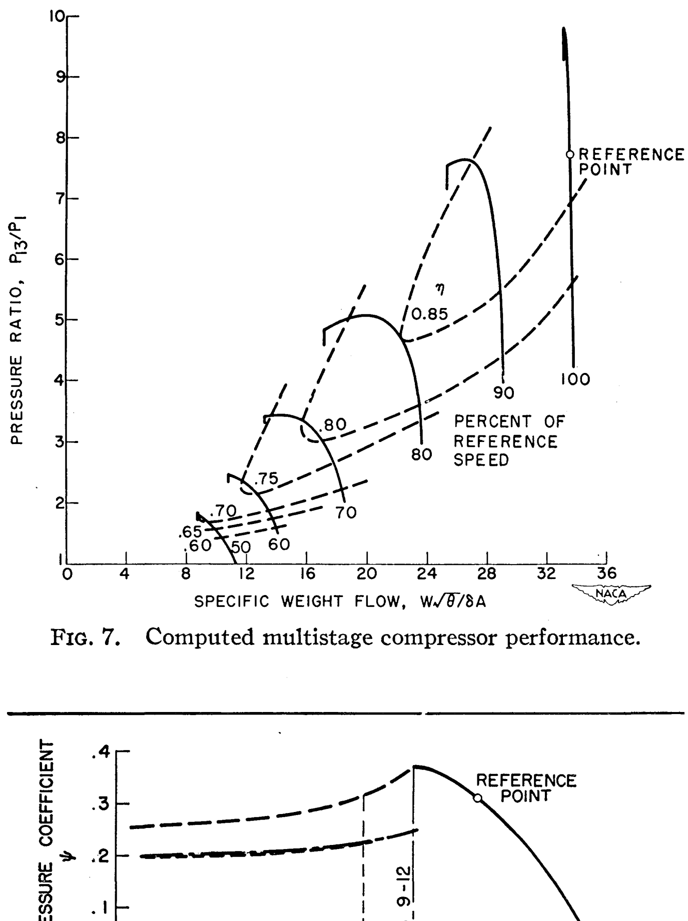

FIG. 7. Computed multistage compressor performance.

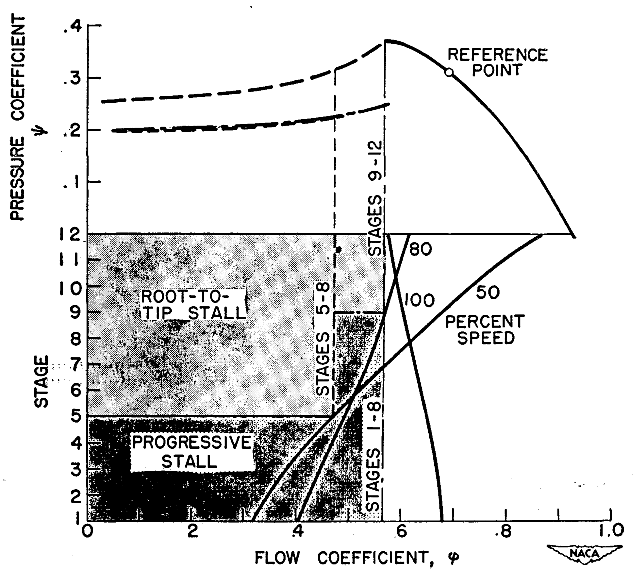

FIG. 8. Stagewise variation of flow at discontinuity points.

of the reference speed in 10 per cent intervals. It should be noted that the stacking technique based on generalized stage curves does not consider decreases in either the stage range or efficiency at extremely high Mach Numbers, nor does it account for changes in performance of a given stage as a result of nonsteady or nonuniform flow imposed by adjacent stages. This technique is used primarily to determine the relative matching of the stages for off-design performance points for the multistage compressor and, thus, to determine how peculiarities in stage characteristics resulting from instigation of rotating stall may affect overall performance.

The computed compressor map in terms of pressure ratio against specific weight flow (flow rate per unit of annulus area) is presented in Fig. 7. An appreciable difference in peak efficiency at high and low speeds is indicated by the contours of constant efficiency. Efficiencies of slightly over 0.70 were computed for 50 per cent speed, whereas a peak efficiency of over 0.85 was obtained for the reference speed. The islands of high efficiency at the reference speed are broader than those generally measured experimentally; this difference can be attributed to the fact that neither the range of high individual stage efficiency nor the maximum of stage efficiency was reduced as Mach Number was increased.

The lowest flow rate considered at each speed was the one at which some stage had a discontinuity in pressure coefficient resulting from the instigation of root-to-tip stall. For these minimum flow points, the overall pressure ratio of the multistage compressor was computed for both the upper and lower branches of the individual-stage curve. For each speed, therefore, a discontinuity was computed for the overall multistage pressure-ratio curve, as shown in Fig. 8.

Stagewise Variation of Flow Coefficient.—One of the principal results of the stage-stacking analysis is the determination of the particular stage responsible for the discontinuity in overall pressure ratio at each speed. The study of the sources of these discontinuities is provided by plotting the stagewise variation of flow coefficient for the points where the discontinuities occur for three representative speeds, 50, 80, and 100 per cent of the reference value (Fig. 8). The originally assumed pressure-coefficient curves are superimposed at the top of the figure, and the reference flow coefficient of 0.69 for all stages is indicated. The progressive stall limit for stages 1 through 8, at a flow coefficient of 0.565, is indicated by the dashed line on Fig. 8. The root-to-tip stall for stages 5 through 8, at a value of flow coefficient of 0.470, and the associated discontinuity in pressure coefficient are indicated in the curve at the top of the figure. Similarly, the discontinuity of pressure coefficient resulting from root-to-tip stall for stages 9 through 12 occurs at a value of flow coefficient of 0.565. The flow coefficient at the inlet stage for the reference-speed discontinuity point is only slightly lower than the value used in the reference point computation. Succeeding stages, however, have a progressive decrease in flow coefficient from the value for the inlet stage, and the stall of the twelfth stage is responsible for the discontinuity in the overall pressure-ratio curve that was shown on the previous figure. Effects of compressibility and radial variations on stage range and efficiency, which were neglected in this analysis, may reduce the maximum pressure ratio from the value computed here, but the general shape of the overall pressure-ratio curve and the determination of the instigating stage are valid.

As compressor speed is reduced, there is a general decrease in flow coefficient for the inlet stages and an increase for the rear stages. This is borne out by the stagewise variations of flow coefficient for the discontinuity point at 80 per cent of reference speed. At this speed all stages up to stage 9 are operating in the range where progressive rotating stall without discontinuities exists, and the discontinuity in the overall performance curve results from instigation of root-to-tip stall in the ninth stage. Again, consideration of effects of root-to-tip stall on the performance of other stages would result in a large drop in the attainable overall pressure ratio at 80 per cent of reference speed.

For the 50 per cent speed point, the trend is similar to that for 80 per cent speed, except that the discontinuity of pressure rise results from the instigation of root-to-tip stall in the fifth stage. As a result of the progressive stagewise variations of flow coefficient and the grouping of stage types, root-to-tip stall was encountered only in stages 5, 9, and 12. With other designs, this situation will, of course, be altered. In general, however, early stages will instigate stall at low speeds, and latter stages at high speeds.

In order to study the speed ranges over which various stages are stalled, the overall performance map has been replotted (Fig. 9), and lines of first-, fifth-, ninth-, and twelfth-stage stall have been added. The discontinuities in overall pressure ratio for speeds of 50, 60, and 70 per cent of the reference value are the result of instigation of root-to-tip stall in the fifth stage. For speeds of 80 and 90 per cent, the discontinuity is caused by the ninth stage. At higher speeds, root-to-tip stall of the twelfth stage causes the discontinuity in the overall pressure-ratio curve. The magnitude of pressure drop shown is the result of stall of only one stage. As a result of flow fluctuations resulting from root-to-tip stall, however, adjacent stages that are close to stall would probably be adversely affected, and the magnitude of pressure discontinuity would be much greater than indicated.

All points to the left of the inlet-stage stall-limit line indicate the existence of a progressive stall in one or more of the inlet stages. All the flow range for 50, 60, and 70 per cent speeds and most of the flow range for 80 per cent speed are in the range of inlet-stage stall. The fifth-stage progressive-stall limit line shows that

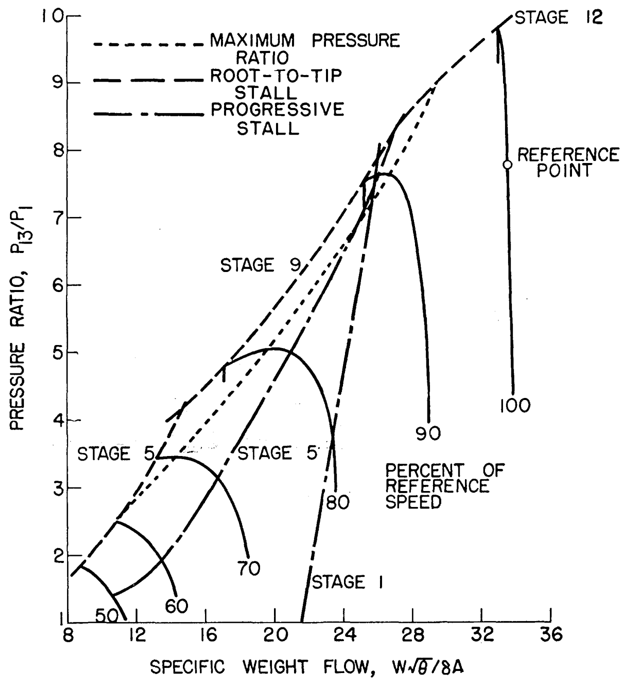

FIG. 9. Stage stall limits for multistage compressor.

progressive stall exists at least back to the fifth stage over the low-flow range for speed up to 90 per cent of reference. As shown on Fig. 9, therefore, progressive stall exists in many of the inlet stages at low speeds for all flows, and it also exists for low flows at intermediate speeds. Discontinuities in overall pressure are instigated by root-to-tip stall of the middle stages at low speed and the latter stages at high speed.

General Considerations.—Effects of progressive-type stall on altering the effective performance curves of succeeding stages have not been considered in this analysis. Such interactions would probably result in some deterioration of stage performance but would not appreciably alter the general trends of the pressure ratio against weight flow at a constant speed. At intermediate speeds, where a large number of stages are approaching stall simultaneously as flow is decreased, root-to-tip stall of middle stages may occur at a higher value of flow coefficient than normally as a result of the progressive stall of early stages. This situation would cause the discontinuity of overall pressure ratio to occur at a higher value of specific weight flow and at a lower pressure ratio than indicated by this analysis.

The relations of the sharp discontinuities of pressure ratio shown to compressor surge on Fig. 9 will be discussed later; but whether or not the compressor surges, these points represent a lower limit to the useful flow range at any speed. It will be noted that, at speeds of 80 and 90 per cent, the discontinuities in pressure ratio occur at values of pressure ratio less than the peak. On the basis of stability criteria, surge may be encountered at approximately the flow for peak pressure ratio; therefore, a curve through the peak pressure-ratio points at various speeds has been included in Fig. 9. The curves of neither overall stall point nor peak pressure ratio indicate the kinks in the compressor stall or surge line which are prevalent for high-pressure-ratio axial-flow compressors. Observation of regions where progressive-type rotating stall prevails, however, suggests that stage interactions may induce root-to-tip stall in inlet stages, which might readily cause kinks in the compressor stall-limit line. Variations in stage matching and stage characteristics may also be responsible for the undesirable kinks. As indicated in the analysis presented herein, a large group of stages in the middle of the compressor are operating near their individual stall points at intermediate speeds; and, if any one stage stalls, a serious deterioration of many stages may be precipitated. Further studies of stages interactions are necessary to predict completely compressor stall characteristics at low and intermediate speeds. A stall may cause discontinuities in individual stage characteristics and result in sharp discontinuities in the overall compressor performance curve. A definite limit of useful operating range of the compressor at any speed, regardless of whether surge occurs, is thus directly attributable to the discontinuity in the performance of one or more of the individual stages.

Compressor Surge

Tests of multistage axial-flow compressors are conventionally made in test rigs having large inlet and discharge tanks. Violent surge is commonly encountered in these rigs as the compressor weight flow is decreased to a limiting value. This surge is a severe audible pulsation of compressor pressure rise and weight-flow rate and normally has a frequency on the order of 0.5 to 2 cycles per sec., although much higher values have been observed on small rigs. In some jet-engine tests and in multistage tests with low receiver volumes, stall with a severe drop in pressure ratio and efficiency has been observed without the audible flow pulsation normally defined as surge. Many investigators have studied the criteria for surge as the criteria for stability, considering the response of the system to small variations from an equilibrium operating point. A continuous pressure-ratio, weight-flow characteristic was implied in these analyses.[1, 2]

Classical Surge.—In the previously reported investigations, in which the compressor characteristic of pressure rise was assumed to be a continuous function of weight flow for a constant compressor speed, linearization of the compressor and discharge throttle characteristics and simplification of the external system result in a solution of the differential equation of flow of the following form:

$$\Delta W = K_1 e^{-\alpha t} \sin(\omega t + K_2) \tag{1}$$

where

ΔW = the variation of weight flow from the equilibrium value
K_1 = a constant
α = damping coefficient
t = time
ω = a function of the frequency of oscillation
K_2 = a constant expressing the phase relation

From this equation it can readily be seen that the damping coefficient α defines the stability of the system—that is, its response to a small deviation from the equilibrium operating condition. If the damping coefficient has a positive value, the system will be stable. If the damping coefficient has a negative value, however, the system will be unstable and the magnitude of the resulting oscillations will continue to increase until the original assumptions of linearity are no longer valid. The effects of the neglected nonlinearity, therefore, will limit the amplitude of oscillation to some finite value, and the cyclic pulsation of surge results. For convenience, this type surge will be referred to as classical in this paper. It should be noted that the analyses of references 1 and 2 merely define the conditions of stability. Determination of amplitudes of flow pulsation are dependent on nonlinearities of the compressor characteristics in the stall region, and the procurement of accurate data of this type is difficult because of the existence of compressor surge and rotating stall. When compressor characteristics are discontinuous, however, a different approach to the analysis of surge may be utilized.

Limit-Cycle Surge.—As discussed earlier in this paper, single-stage tests have shown pressure-ratio, weight-flow characteristics that are discontinuous at stall (Fig. 4). This characteristic indicates that, as weight flow is even slightly decreased below the value for stall, the pressure ratio drops discontinuously to the lower branch of the curve. Further decreases in weight flow, however, cause a continuous change in pressure ratio. Once stall has been encountered, the flow must be increased beyond the initial value for stall in order to relieve the stalled condition. This results in the hysteresis effect shown. Discontinuities in stage curves cause discontinuities in overall pressure-ratio curves for multistage compressors, as indicated in the calculated performance map presented. Double-branch performance curves have been obtained in one multistage compressor test in which a small discharge receiver volume was used and also in a research jet engine.[3] Operation on the stalled branch of the compressor characteristic apparently did not result in the classical instabilities of surge. These compressors, however, surged violently when operated in standard component test rigs that had large receiver volumes. Inasmuch as the compressor characteristics on the stalled branch of the curve apparently satisfied the stability requirements of Eq. (1), the question arises as to the reason for violent surge when large receiver volumes were utilized. The discontinuity in pressure associated with stall will undoubtedly result in a damped, transient flow-rate fluctuation if normal stability requirements are satisfied. If, however, the compressor flow rate reaches or exceeds the value for stall recovery during the transient, the stall condition will be relieved, and the compressor will again seek equilibrium at the original point of stall on the upper or unstalled branch of its characteristic. Cyclic stalling and unstalling of the compressor, resulting from transient flow variations following an initial stall, thus provide a mechanism for surge. The discontinuous-type compressor characteristic, therefore, suggests solving the differential equation of motion for the compressor and external system for the transient flow-rate response to a step change in compressor discharge pressure at stall.

For the analysis of surge in a compressor with a discontinuous pressure-ratio, weight-flow characteristic, the simplified system indicated in Fig. 10 is considered in which there are a compressor, discharge receiver, and discharge throttle. The compressor draws air from the atmosphere at station 1 and discharges to the receiver, station 2. The flow is then discharged through the outlet throttle to station 3. Similar to the technique of references 1 and 2, the compressor characteristics and throttle characteristics are linearized as shown in Fig. 11. The upper or unstalled branch of the pressure characteristic has a negative slope and stalls at point 0. The pressure drop at stall is represented by ΔP_s, and the lower or stalled branch of the characteristic curve is assumed to have a zero slope. For this analysis, the throttle characteristic passes through the compressor stall point indicated by 0 in Fig. 11. The increment of weight flow between the stall point and the intersection of the throttle line and the stalled branch of the compressor curve is defined as ΔW_s. The flow must be increased above the original stall value by an increment ΔW_{sr} to permit stall recovery. On the basis of the surge criteria previously discussed, this system is stable on the upper or unstalled branch of the compressor characteristic except at the stall point 0. The system is also stable on the lower or stalled branch of the compressor characteristic. In the solution of the system differential equation, the compressor is assumed to be operating at the stall point, point 0 on Fig. 11, and a step change in pressure ratio is imposed. This results in a receiver pressure that is greater than the compressor is capable of producing in the stalled condition, and the solution of the linearized flow equations for the resulting flow variation with time is of the following form:

$$\Delta W = -|\Delta W_s| \, [1 + e^{-\alpha t} \, (\Lambda \sin \omega t - \cos \omega t)] \quad (2)$$

where

ΔW = the variation of weight flow from the initial or stall value

$|\Delta W_s|$ = absolute value of the weight-flow increment between equilibrium points on the upper and lower branches of the compressor characteristic

α = the damping coefficient determined by compressor and system characteristics

t = time

Λ = a constant determined by compressor and system characteristics

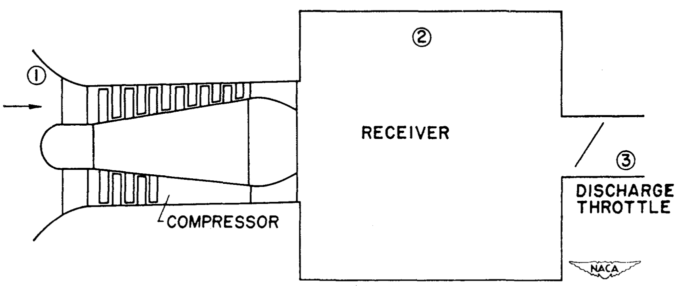

FIG. 10. Compressor and system considered in surge analysis.

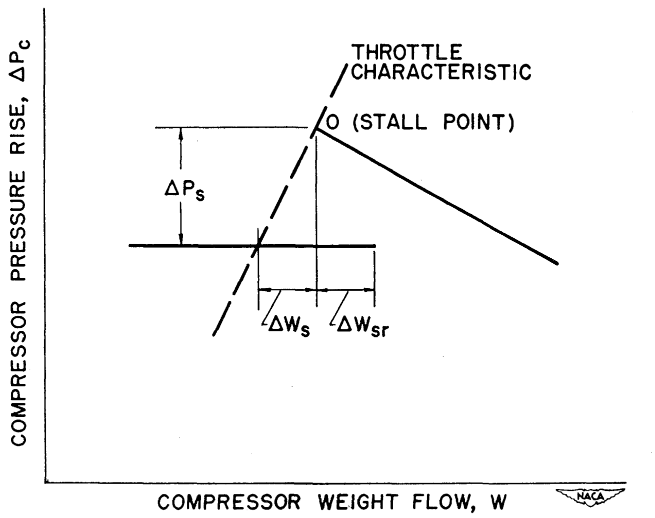

FIG. 11. Linearized compressor and throttle characteristics.

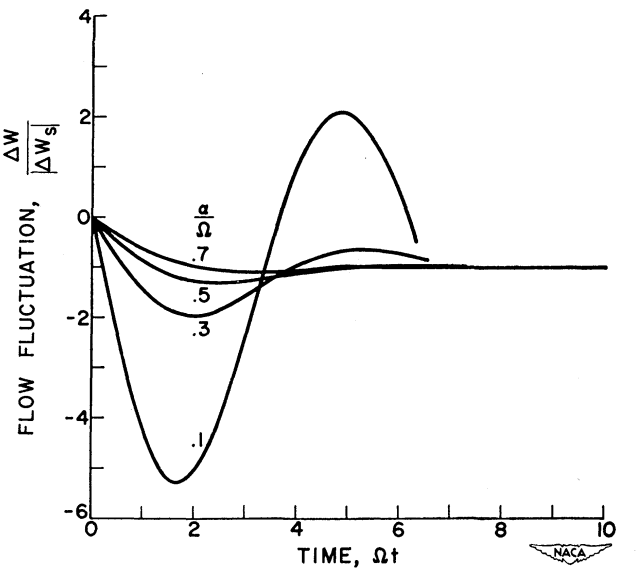

FIG. 12. Transient flow-rate fluctuation following stall.

ω = a system constant that determines the frequency of oscillations

It may be noted that if the weight flow at the intersection of the throttle curve and the stalled branch of the compressor curve is taken as the base for determination of ΔW, the above equation may be expressed in the same form as Eq. 1. As in references 1 and 2, the magnitude and stability of the resulting flow oscillation are dependent on the magnitude and sign of the damping coefficient α.

Solutions to the above equation have been computed for various compressor and system characteristics, and the results are presented with dimensionless variables in Fig. 12. The ordinate for this plot is $\Delta W \div |\Delta W_s|$, or the ratio of the weight-flow increment at any instant measured from the equilibrium point on the unstalled branch of the compressor curve to the absolute value of the difference in weight flow between the equilibrium point on the unstalled and the stalled branch. The variable of time is made dimensionless by multiplying it by a characteristic frequency (or dividing it by a characteristic time). The undamped natural frequency Ω of the compressor and receiver acting as a resonator was selected as the characteristic frequency for this presentation, and, accordingly, Ωt is used as the abscissa for Fig. 12. Similarly, the damping term α, which has the dimensions of frequency and is inversely proportional to the time constant of the receiver and throttle, is divided by Ω to provide the dimensionless parameter α/Ω. The variation of $\Delta W \div |\Delta W_s|$ with Ωt is presented for values of α/Ω of 0.1, 0.3, 0.5, and 0.7 in Fig. 12. Negative values of the ratio $\Delta W/\Delta W_s$ represent decreases in flow from the initial or stall point, and positive values represent increases. For small damping, α/Ω equal to 0.1, the flow decreases, then increases to a value greater than the stall value, and then decreases again. If the flow increases above the value for stall recovery $\Delta W_{sr}/|\Delta W_s|$, the compressor's ability to develop pressure will increase discontinuously to the upper or unstalled branch of the curve. A similar computation for a step increase in pressure would indicate a sharp increase in flow rate, followed by a decrease to the value for stall, and the cycle would repeat. Inasmuch as the main purpose of this computation is to study the potential of surge due to cyclic stalling and unstalling, only the overshoot on the stalled branch of the compressor curve was considered, and the transient following the step increase in pressure resulting from stall recovery was not computed.

This cyclic-type operating condition, caused by a step change in pressure ratio due to complete compressor stall, has been defined as limit-cycle surge. This type surge will result in violent oscillations of flow and pressure ratio. Because of inertia and capacitance effects of the system, the receiver pressure change will lag the flow change during the transient. By again referring to Fig. 12, it is seen that, for larger values of α/Ω such as 0.3 to 0.7 as shown on this chart, the transient flow fluctuations damp out rapidly and the compressor operating point converges on the intersection of the throttle line and the stalled branch of the compressor characteristic ($\Delta W/|\Delta W_s| = -1$), and limit-cycle surge will not result. From a consideration of discontinuous pressure characteristics, therefore, limit-cycle surge due to complete compressor stall may or may not occur, depending on the magnitude of the system damping.

It can be seen that two types of surge are possible: classical surge resulting from system instabilities for a compressor with a continuous characteristic curve, and limit-cycle surge resulting from a discontinuity of the compressor characteristic. If the system damping is sufficiently high at the equilibrium point on the stalled branch, compressors with discontinuous characteristics may simply stall without surging. Classical surge may, however, exist as a result of negative damping on either the unstalled or stalled branch of a double-valued compressor characteristic, and the resulting instabilities may incite a limit-cycle-type surge.

Effects of System Characteristics on Damping Coefficient.—From the preceding analysis, the importance of system damping on compressor surge or stall characteristics is evident. Consideration must, therefore, be made of the effects of system characteristics on the magnitude of the damping coefficient. If the receiver and throttle have a large time constant—that is, if the discharge volume is large or the throttle curve slope is large, or both, as is the case for most compressor component test rigs—the damping coefficient will be small, and the compressor will probably surge. It should also be noted that limit-cycle surge may be encountered more readily at high compressor speeds than at low speeds, because of the increase in slope of the throttle curve and also because of the very narrow range of weight flows for which the two branches overlap. Classical-type surge is also more likely to occur at high than at low speeds.

In jet engines the discharge receiver volume is the combustion-chamber volume and is normally small when compared to the receivers used in compressor test equipment. Moreover, the throttling is done by the turbine nozzle, and the throttle curve is essentially a constant fuel-flow line which has somewhat less slope than the throttles used in compressor test facilities. As a consequence, the compressor in the jet engine will have a relatively large damping coefficient and will tend to stall rather than to surge.

Experimental Considerations.—Further studies of multistage compressor characteristics at the stall point and in the stall region are vital for obtaining a better understanding of surge and stall phenomena. Caution, however, must be exercised in obtaining and analyzing these data. During operation with overall compressor stall, fast-response instruments at either the compressor inlet or discharge may indicate large flow variations due to rotating stall, and these fluctuations may easily be mistaken for compressor surge. Phase relations from two fast-response instruments in a plane normal to the axis of rotation of the compressor will readily identify rotating stall. Furthermore, fluctuations in torque required to drive the compressor will exist during surge, whereas the torque will be relatively constant with stall.

Inasmuch as the flow fluctuations of surge prevent the experimental determination of the slope of the compressor characteristic curve at the stall point, a discontinuity in the compressor characteristic can be found only if surge is prevented—that is, the stall characteristics of a compressor cannot be properly determined if stall results in surge.

Conclusions

From the preceding discussion of the various aspects of stall and surge in axial-flow compressors, the following results are obtained: Stall of single-stage compressors frequently results in zones of high and low flow which rotate about the compressor axis. These rotating stalls may be classed as either progressive stall with multiple-stall zones, or root-to-tip stall, which generally consists of a single-stall zone. The progressive type results in a gradual decrease in pressure ratio with flow decreases below the stall value and is most likely to occur in low hub-tip ratio stages where stalling weight flow is significantly greater for one part of the blade span than the other. Root-to-tip stall results in sharp drops or discontinuities in the pressure-ratio flow characteristic and generally exists in stages of high hub-tip ratio where critical conditions are approached simultaneously over the entire span of the blade. Rotating stall of either type may excite compressor blading to resonant vibration.

Rotating stall has been observed in multistage, as well as single-stage, axial-flow compressors. A stacking analysis for a hypothetical compressor with front stages having a progressive-type stall and with middle and latter stages having the pressure-ratio discontinuities associated with root-to-tip stall indicates multistage compressor performance with discontinuities in the pressure-ratio, weight-flow characteristics at all speeds. At low speeds these discontinuities result from stall of the earliest stages having discontinuous characteristics, and at high speeds from stall of the latter stages. Progressive-type stall of the inlet stages may exist over the entire flow range at low speeds. At intermediate speeds all stages are close to their stall points, and stage interactions resulting from the flow variations of stall of any element may seriously limit the pressure-ratio-producing capacity of the compressor.

In addition to surge that results from classical system instabilities, a limit-cycle-type surge may result from discontinuities in the pressure-ratio, flow characteristic of the compressor. A compressor may stall rather than surge, provided the damping of the compressor and external system is sufficiently high. As was the case for classical surge, the potential of limit-cycle surge is greatest at high compressor rotational speeds and pressure ratios. The potential of limit-cycle surge is also greatest where the slope of the throttle characteristic is large, the incremental flow increase for stall recovery is small, and the discharge receiver volume is large, as in compressor component test rigs. In jet engines the receiver volume is small, the slope of the throttle characteristic is small, and the pressure ratio at which stall will normally be encountered is low; consequently, compressors in jet engines are less likely to surge than are those operated in test rigs.

This paper deals primarily with the definition of the problem of stall and surge. Much research must still be done on such problems as correlations of types of stall with stage geometry, evaluations of stage interactions when rotating stall exists in multistage compressors, and evaluation of the system characteristics governing surge.

References

[1] Bullock, Robert O., Wilcox, Ward W., and Moses, Jason J., *Experimental and Theoretical Studies of Surging in Continuous-Flow Compressors*, NACA Report No. 861, 1946. (Supersedes NACA T.N. No. 1213.)

[2] Pearson, H., and Bower, T., *Surging in Axial Compressors*, The Aeronautical Quarterly, Vol. 1, Part III, November, 1949. (Published by the Roy. Aero. Soc., London.)

[3] Goldstein, Arthur, Alpert, Sumner, Beede, William, and Kovach, Karl, *Analysis of Performance of Jet Engine from Characteristics of Components. II—Interaction of Components as Determined from Engine Operation.* NACA Report No. 928, 1949. (Supersedes NACA T.N. No. 1701.)

Plastic Collapse of Thin Rings

(*Concluded from page 826*)

[10] Drucker, D. C., Prager, W., and Greenberg, H. J., *Extended Limit Design Theorems for Continuous Media*, Quarterly App. Math., Vol. IX, No. 4, January, 1952.

[11] Onat, E. T., and Prager, W., *Limit Analysis of Arches*, J. Mechanics & Physics of Solids, Vol. 1, No. 2, p. 77, January, 1953

[12] Timoshenko, S., *Strength of Materials*, Vol. II, p. 79; D. Van Nostrand, New York, 1940.

[13] Kooharian, A., *Limit Analysis of Voussoir and Concrete Arches*, J. Am. Concrete Inst., Vol. 24, No. 4, p. 317, December, 1952.

[14] Burke, W. F., *Working Charts for the Stress Analysis of Elliptic Rings*, NACA TN No. 444, January, 1933.