

# Flow instability inception prediction using eigenvalue analysis

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The compression system in turbomachines, e.g., aircraft engines and gas turbines, when operating under off-design conditions, exhibits flow instability such as surge, rotating stall leading to performance deterioration, noises and structural damages. Inability to accurately predict the condition under which such flow instability will happen severe hinders the development of high performance compression system. In this work, an eigenvalue analysis method is developed to predict flow stability commonly seen for turbomachines operating at near stall conditions. The eigenvalue analysis is fully based on the steady state three-dimensional Reynolds-averaged Navier-Stokes equations and thus the stability boundary is fully consistent with the one that is predicted by the time-accurate flow simulation, i.e., URANS, but two to three times faster. The method is applied to the computation of stability boundary of (i) the laminar flow around a two-dimensional circular cylinder, (ii) the flow around a quasi-three-dimensional compressor annular cascade, and (iii) flow around a three-dimensional compressor rotor. The method developed here has the potential to revive the once-popular eigenvalue method for prediction rotating stall and surge, which was based on lower-fidelity flow models and provide industry with tools to accurately predict the stall line in the early design stage.

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## **I. The nonlinear flow solver**

### **A. Governing equations**

### **B. Spatial discretization**

## **II. Stability analysis**

### **A. Time-domain unsteady approach**

### **B. Eigenvalue approach**

## **III. Eigenvalue analysis for large sparse matrices**

## **IV. Results**

### **A. Laminar flow around a two-dimensional circular cylinder**

#### *1. Steady state calculation*

#### *2. Unsteady calculation*

#### *3. Eigenvalue analysis*

### **B. Rotating stall for an annular compressor cascade**

#### *1. Steady state calculation*

#### *2. Unsteady calculation*

#### *3. Eigenvalue analysis*

### **C. Rotating instability for an axial compressor rotor**

#### *1. Steady state calculation*

#### *2. Unsteady calculation*

#### *3. Eigenvalue analysis*

## **V. Conclusion**

## **Acknowledgements**

## **References**

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