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Surge and Rotating Stall in Axial Flow Compressors

Part I: Theoretical Compression System Model

This paper reports a theoretical study of axial compressor surge. A nonlinear model is developed to predict the transient response of a compression system subsequent to a perturbation from steady operating conditions. It is found that for the system investigated there is an important nondimensional parameter on which this response depends. Whether this parameter is above or below a critical value determines which mode of compressor instability, rotating stall or surge, will be encountered at the stall line. For values above the critical, the system will exhibit the large amplitude oscillatory behavior characteristic of surge; while for values below the critical it will move toward operation in rotating stall, at a substantially reduced flow rate and pressure ratio. Numerical results are presented to show the motion of the compression system operating point during these two basic modes of instability, and a physical explanation is given for the mechanism associated with the generation of surge cycle oscillations.

Introduction

As the flow through an axial compressor is throttled from the design point to the stall limit, the steady, axisymmetric, flow pattern that exists becomes unstable. The phenomenon resulting from this instability can take one of two forms. These are known as surge and rotating stall [1].[1] The two types of behavior are illustrated schematically in Fig. 1. Surge is a large amplitude oscillation of the total annulus averaged flow through the compressor; whereas in rotating stall, one finds from one to several cells of severely stalled flow rotating around the circumference, although the annulus averaged mass flow remains constant in time once the pattern is fully developed. The frequencies of surge oscillations are typically over an order of magnitude less than those associated with the passage of the rotating stall cells, and, in fact, during a surge cycle, the compressor may pass in and out of rotating stall as the mass flow changes with time.

To the compressor or engine designer, it is important to know which of these modes of instability will occur since their consequences can be quite different. For example, once rotating stall is encountered, it may not be possible to return to an unstalled condition merely by opening the throttle, because of system hysteresis effects. In this situation the only way to come out of stall may be to decrease rotational speed considerably, resulting in a sizeable loss in pressure ratio. Also, because of the extremely low efficiencies associated with the presence of rotating stall (efficiencies below twenty percent have been measured) operation for any substantial length of time in this mode can result in excessive internal temperatures which have an adverse effect on blade life, as do the large stresses to which the unsteady flow field subjects the blading. In addition, an even more serious consequence that can occur in an engine is that the low flow rates obtained during rotating stall can lead to substantial overtemperatures in the burner and turbine.

On the other hand, if surge occurs, the transient consequences, such as large inlet overpressures, can also be severe. However, the circumstances may well be more favorable for returning to unstalled operation by opening either the throttle or internal bleeds, since the compressor can be operating in an unstalled condition over part of each surge cycle. For this reason, one of the important problems associated with compressor stability is the determination of which of these two types of behavior will occur with a given axial flow compression system.

Up to now, investigations of this topic have yielded only qualitative guidelines concerning this point. From the experiments described by Pearson [2], Huppert and Benser [3], or Huppert [4] it can be inferred that a compressor may exhibit surge at "high" speeds but not at "low" speeds, where only rotating stall is encountered at the stall limit. In addition, the volume of the exit plenum into which the compressor discharges has also been found to be important, in that at a given speed a compressor can surge with a large plenum but not with a smaller one. However, general quanti-

[1] Numbers in brackets designate References at end of paper.

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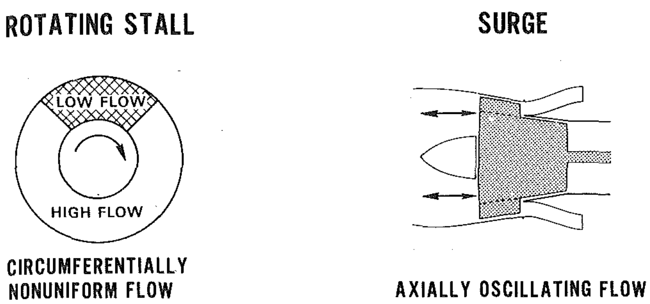

Fig. 1 Possible modes of instability on the stall line

tative rules have not been formulated concerning the relative magnitude of high and low or large and small. Further, the linearized analyses, such as that of Emmons, et al. [1], which have been set up to examine compression system instability, are restricted to infinitesimal departures from an equilibrium situation, and are fundamentally unable to describe the large amplitude pulsations encountered during a surge cycle.

The present study was therefore undertaken to provide a more detailed look at the phenomenon of surge than in previous investigations. In particular, what was desired was a quantitative criterion to decide which of the two types of transient behavior would be exhibited. The basic question to be addressed could be phrased in the following manner: Suppose that one has a compressor operating in a steady fashion with the operating point determined by a given value of exit throttle setting. If the throttle area is now decreased slightly, does the compressor operating point also change only slightly, does the operating point shift rapidly to a new (steady) value at a substantially reduced flow and pressure (a behavior indicative of a compressor operating in rotating stall), or does the system exhibit the large amplitude oscillations of mass flow and pressure rise that are identified by the term surge.

To answer this adequately it is desirable not only to establish which of the physical parameters characterizing the compression system are important for determining the mode of instability, but to understand the fundamental mechanisms that are responsible for the behavior. These dual objectives call for the development of a nonlinear mathematical model of transient compression system behavior. In addition, as a means to experimentally assess the validity of the model, it is necessary to obtain detailed measurements of the compressor operating point, i.e., mass flow and pressure rise, during the large amplitude transients that are associated with the inception of rotating stall or surge, or with a surge cycle.

In this paper, the theoretical model of the time dependent behavior of a compression system will be developed. Numerical results will be shown and a physical explanation for the system behavior will be given. A companion paper, which is presented as Part II [5] will then discuss the experimental program and compare the experimental results to the theoretical predictions.

Fluid Dynamic Model

Many compression systems consist of a compressor working in an annular duct which is connected to an exit plenum of much larger diameter. The discharge from the plenum is then via a throttle in an exit duct whose diameter is again much smaller than that of the plenum. In this situation, it seems natural to model the oscillations occurring in such a system in a manner analogous to those of a Helmholtz resonator, and Emmons, et al., have used this idea in their linearized analysis. This assumption implies that all the kinetic energy of the oscillation is associated with the motion of the fluid in the compressor and throttle ducts, and the potential energy is associated with the compression of the gas in the plenum. The compression systems that are analyzed here are confined to those having low inlet Mach numbers and pressure rises which are small compared to the ambient pressure. However, no restrictions are placed on the amplitude of the oscillations in pressure rise, mass flow, etc., compared to the steady-state values of these quantities, so that the essential strongly nonlinear behavior of the system, which arises from the sharp differences in compressor output as the mass flow changes, is retained.

A schematic of the model compression system used in the analysis is shown in Fig. 2. The compressor and its ducting are replaced by an actuator disk[2] to account for the pressure rise due to the compressor and a length of constant area pipe to account for the dynamics of the fluid in the compressor duct. Similarly the throttle (which in practice may often be just a variable area annular nozzle) is also replaced by this combination of actuator disk, across which the pressure drops, plus a constant area duct.

The oscillations associated with compressor surge can generally be regarded as having quite low frequency. This fact, coupled with the two previously mentioned constraints on the flows that will be examined, implies that to a very good approximation, the flow in the ducts can be considered to be incompressible, with the density taken equal to the ambient value. At any instant, therefore, all the fluid in one of these equivalent ducts will have the same axial velocity. The geometry of the equivalent ducts is determined by requiring that a given rate of change of mass flow produces the same unsteady pressure difference in the actual duct and in the model (including a correction for end effects) and by matching the area of the model duct with a characteristic area of the actual duct. In the compressor, this can be taken as the inlet area, and in the throttle the flow-through area in the discharge plane. As is shown in [1] this requirement leads to the relation

[2] Alternatively, one could phrase the analysis in terms of an axial distribution of body forces analogous to those used (in a different context) in reference [6] and arrive at the same equations to describe the system.

Nomenclature

a = speed of sound
A = flow-through area
B = dimensionless number; defined in equation (15)
C = compressor pressure rise
C_{ss} = steady-state compressor pressure rise
C_x = axial velocity
F = throttle pressure drop
G = dimensionless number; defined in equation (16)
k = polytropic exponent
L = effective length of equivalent duct
$\dot{m}$ = mass flow
$N/\sqrt{\theta}$ = corrected rotor rotational speed (rpm)
P = pressure
ΔP = plenum pressure rise, ($P_{\text{plenum}} - P_{\text{ambient}}$)
R = compressor rotor mean radius
s = streamwise direction
t = time
T = period of surge oscillation
U = mean rotor velocity
V_P = exit plenum volume
γ = specific heat ratio
ρ = density
τ = compressor flow field time constant
ω = Helmholtz resonator frequency; defined in equation (9)

Subscripts

No subscript refers to ambient conditions
p = plenum
c = compressor
T = throttle

Superscripts

$\sim$ = nondimensionalized variable
$'$ = differentiation with respect to $\tilde{\dot{m}}$

$$\left(\frac{L}{A}\right)_{\text{model}} = \int_{\substack{\text{actual}\\\text{ducting}}} \frac{ds}{A(s)} \tag{1}$$

The integration is assumed carried out over all regions of the actual ducting in which the flow has significant kinetic energy.

Equations of Motion

The rate of change of mass flow in the compressor duct can be related to the pressure difference across the duct ($\Delta P = P_p - P$) and the pressure rise across the compressor, C,

$$(P - P_p) + C = \rho L_c \frac{dC_x}{dt} \tag{2a}$$

In terms of the compressor mass flow this is,

$$-\Delta P + C = \frac{L_c}{A_c} \frac{d\dot{m}_c}{dt} \tag{2b}$$

Equation (2a) is essentially the first integral of the one-dimensional momentum equation.

An analogous equation can be written to describe the flow in the throttle duct:

$$\Delta P - F = \frac{L_T}{A_T} \frac{d\dot{m}_T}{dt} \tag{3}$$

where F is the pressure drop across the throttle.

Note that in both of the ducts the equation of continuity has entered implicitly in the definition of the form of the equivalent duct lengths, L_c and L_T.

In the plenum the fluid velocities are negligible. Further, plenum dimensions are typically very much smaller than the wavelength of an acoustic wave having a frequency on the order of that associated with surge. Hence, the static pressure will be uniform throughout the plenum at any instant of time.

The continuity equation for the plenum is:

$$\dot{m}_c - \dot{m}_T = V_p \frac{d\rho_p}{dt} \tag{4}$$

If the process in the plenum is polytropic, then the density change will be related to the changes in plenum pressure by

$$\frac{d\rho_p}{dt} = \frac{\rho_p}{kP_p} \frac{dP_p}{dt} \tag{5}$$

where k is the polytropic exponent. It is shown in Part II that for the present set of experiments it is an extremely good assumption to take k equal to γ, the specific heat ratio. In addition, since the overall pressure and temperature ratios of the compression systems that are studied are near unity, the quantity (ρ_p/P_p) is not appreciably different from (ρ/P) and we can substitute the latter in the pressure-density relationship. The expression for mass conservation in the plenum can thus be written:

$$\dot{m}_c - \dot{m}_T = \frac{\rho V_p}{\gamma P} \frac{dP_p}{dt} \tag{6}$$

Up to now, nothing has been said about the behavior of the pressure rise across the compressor, C, which is a highly nonlinear function of the mass flow through the compressor. Since it is not possible at present to predict the form of this curve over the varied flow regimes (including rotating stall and possibly reverse flow) that are encountered during a surge cycle, the form of the curve must be obtained by experiment. The method by which this was done will not be discussed here, as it is covered in detail in Part II. It will merely be noted that one result of the experiment was to furnish a curve of compressor pressure rise versus mass flow which is obtained during steady-state operation (defined as operation with annulus averaged mass flow and plenum pressure constant in time).

In previous linearized analyses, the assumption has been made that this pressure rise-compressor mass flow relationship is the same transiently as it is in this steady state—in other words, that the compressor responds quasi-steadily to changes in mass flow. However, this is not actually the case. In particular, when the stall limit line is reached and an axisymmetric compressor flow field becomes unstable, there is a definite time lag between the onset of instability and the establishment of the fully developed rotating stall pattern. This time is equal to or longer than the order of several rotor revolutions or equivalently the order of one or more revolutions of a stall cell. As an example, it has been stated by Marsh and Lakhwani [7] that approximately ten rotor revolutions (roughly five stall cell revolutions) were necessary for a stall cell to become fully developed. Some detailed data on stall cell growth from a test of a three stage P&WA compressor which had total pressure probes at a number of circumferential locations is presented in Fig. 3. It can be seen that approximately eight revolutions are needed for full growth from the first noted inception of the cell. The results of the present series of experiments as well as previous experiments with the same compressor showed that approximately two stall cell revolutions, i.e., approximately seven rotor revolutions, were needed for full growth.

The times needed for development of the stall cell can therefore be long enough so that the compressor mass flow undergoes a significant change during this process. Under these conditions, a quasi-steady approximation will not be adequate. We therefore adopt a simple, first order transient response model to simulate this lag in compressor response. The value of the time constant that is used can be taken from examination of data typified by Fig. 3. Explicitly, the approximation for the transient compressor response can be written as:

$$\tau \frac{dC}{dt} = (C_{ss} - C) \tag{7}$$

with C_{ss} denoting the steady-state measured compressor curve.

If the throttle is either a variable area nozzle or a valve, an explicit form for F can be written in terms of the velocity at the throttle discharge plane, where the static pressure is ambient:

$$F = \frac{1}{2} \rho C_{x_T}^2$$

In terms of the throttle mass flow this is:

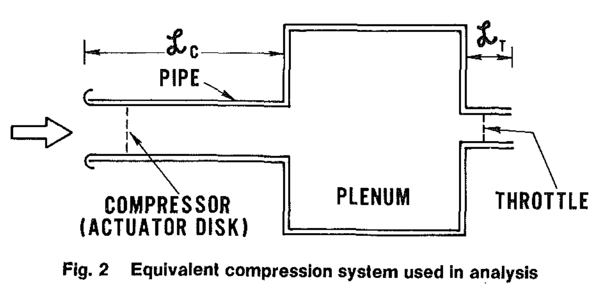

Fig. 2 Equivalent compression system used in analysis

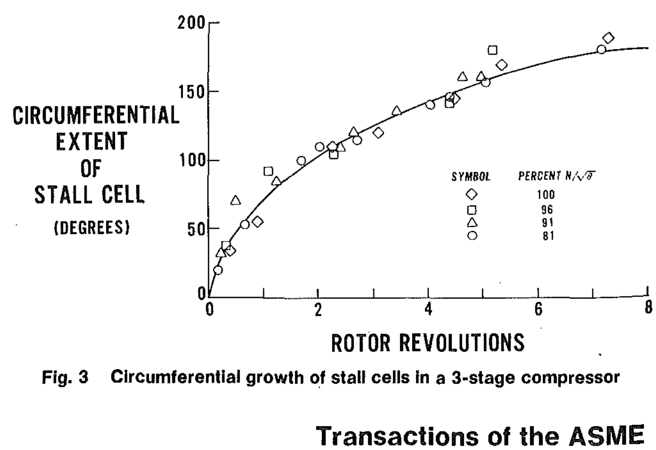

Fig. 3 Circumferential growth of stall cells in a 3-stage compressor

$$F = \frac{\dot{m}_T^2}{2\rho A_T^2} \tag{8}$$

Equations (2), (3), (6), (7), and (8) are the equations describing the dynamics of the compressor system.

It is helpful to nondimensionalize these equations. We nondimensionalize the mass flows using the quantity $\rho U A_c$, the pressure differences using $\frac{1}{2}\rho U^2$ as a representative pressure, and the time variable using the characteristic time $1/\omega$. The Helmholtz frequency, ω, is defined here using the length and area of the equivalent compressor duct:

$$\omega = a\sqrt{\frac{A_c}{V_p L_c}} \tag{9}$$

Note that the nondimensional mass flow is just the axial velocity parameter, C_x/U, i.e.,

$$\tilde{\dot{m}}_c = \frac{\rho C_x A_c}{\rho U A_c} = C_x/U \tag{10}$$

Using the symbol ($\sim$) to designate the nondimensionalized variables, the resulting four equations are:

$$\frac{d\tilde{\dot{m}}_c}{d\tilde{t}} = B(\tilde{C} - \Delta\tilde{P}) \tag{11}$$

$$\frac{d\tilde{\dot{m}}_T}{d\tilde{t}} = \left(\frac{B}{G}\right)(\Delta\tilde{P} - \tilde{F}) \tag{12}$$

$$\frac{d\Delta\tilde{P}}{dt} = \left(\frac{1}{B}\right)(\tilde{\dot{m}}_c - \tilde{\dot{m}}_T) \tag{13}$$

$$\frac{d\tilde{C}}{d\tilde{t}} = \left(\frac{1}{\tilde{\tau}}\right)(\tilde{C}_{ss} - \tilde{C}) \tag{14}$$

In addition to the nondimensionalized compressor time constant, $\tilde{\tau}$, there are two other nondimensional parameters in these equations: B and G. The first is defined as:

$$B = \frac{U}{2\omega L_c} \tag{15a}$$

which can also be written in terms of the physical parameters of the system as

$$B = \frac{U}{2a}\sqrt{\frac{V_p}{A_c L_c}} \tag{15b}$$

The second is:

$$G = \frac{L_T A_c}{L_c A_T} \tag{16}$$

It is possible to relate the first two nondimensional parameters; $\tilde{\tau}$ and B. This is done using the fact that data indicates that, for a given compressor, τ is proportional to the time for some number, N, of rotor revolutions. Thus,

$$\tau = \frac{N \cdot 2\pi R}{U} \tag{17}$$

The nondimensional time lag, $\tilde{\tau}$, is therefore

$$\tilde{\tau} = \frac{2\pi N R \omega}{U} \tag{18}$$

Using the definition of B,

$$\tilde{\tau} = \left(\frac{\pi R}{L_c}\right)\left(\frac{N}{B}\right) \tag{19}$$

For a given compressor, since N, R, and L_c are constant, the nondimensional time lag is proportional to $1/B$.

The coupled nonlinear equations (11) through (14), plus the nondimensionalized counterpart of equation (8) (which gives the explicit form of $\tilde{F}$), are the equations that are to be solved in order to predict the transient behavior of the compressor system.

Calculation Procedure

To determine the dynamic behavior of the compressor system, equations (11)–(14) were solved numerically for different compressor and throttle characteristic curves, and for a range of values of the parameters B and G. The method used was a fourth order predictor-corrector. In the examples shown in the following, the compressor characteristic that was used was obtained from the experiments with a three stage axial compressor that are described in Part II. This characteristic is of the "abrupt stall" type [3], with the pressure rise dropping off sharply when the rotating stall limit is reached. In addition, this compressor performance curve is one in which a hysteresis exists so that the compressor pressure rise is a double valued function of the mass flow over a certain range of mass flows. In order to take this into account, the calculation was set up so that the proper value of steady-state compressor pressure rise would be chosen when the instantaneous mass flow entered the hysteresis range. This branch of the curve would then be used until the mass flow had changed sufficiently so that $\tilde{C}_{ss}(\tilde{\dot{m}}_c)$ became again single valued.

One would expect that the use of a time lag associated with the onset and cessation of rotating stall to model the unsteady performance of the compressor would not be in accord with the basic fluid mechanics of the problem when the compressor is operating far from the stall line. For example, when the compressor is unstalled, the time scale for the lag effects will be based rather on the unsteady response of the compressor blading, and will therefore be much shorter than the times shown in Fig. 3. In this region the compressor performance can be traced out in a basically quasi-steady fashion [5]. Therefore, to more closely model the physical phenomenon, the calculation was carried out with the time constant in equation (14) being set equal to a very small value when the compressor was operating on the unstalled branch of the characteristic and the rate of change of mass flow was less than zero, ($d\tilde{\dot{m}}_c / d\tilde{t} < 0$), i.e., when the flow was decreasing from the maximum value. This was also done when the compressor flow was negative, ($\tilde{\dot{m}}_c < 0$).[3]

To start the solution, the initial conditions must be specified. The method of input was chosen so as to be consistent with compressor test procedure. It was assumed that the compressor was running stably under known operating conditions corresponding to a given throttle setting. The throttle line was then moved a small amount, such as would be accomplished by closing or opening an actual throttle slightly, and the system allowed to seek a new equilibrium point.

Numerical Results

From preliminary analytical investigations, it was initially suspected that the most critical parameter would be B, with G having only minor effect. This was found to be the case and for physically plausible (which means less than unity) values of G, any changes in the latter parameter had negligible effect on the system behavior. Therefore G was kept constant at the experimental value of 0.36 for all of the subsequent calculations that are described in this part of the paper. Since experimental results with different compressors have yielded somewhat differing values of N, the influence of this parameter was also investigated. It was found that for a range of values of N corresponding to the considerable amount of data seen by the author, the effect of changes in N is also rather weak. Therefore, the value of N is kept at 2 in all of the numerical results shown, except for Fig. 8 as is noted below.

By far, the most important parameter in determining the dynamics of the system is B. The changes that result as B is varied over a relatively small range can be seen in Figs. 4–6. These figures

[3] Large pressure differences can occur during reverse flow due to the very high losses in the blading. The response of the compressor in generating these losses appears to be substantially more rapid than that associated with the formation of a fully developed stall cell [5].

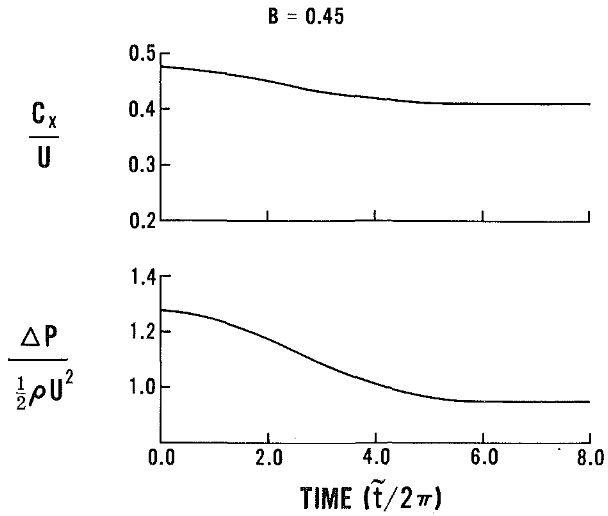

Fig. 4 *(a)* Transient compression system behavior: $B = 0.45$

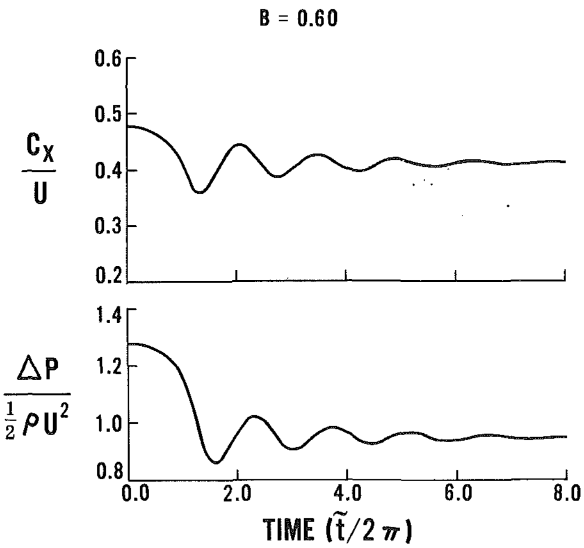

Fig. 5 *(a)* Transient compression system behavior: $B = 0.60$

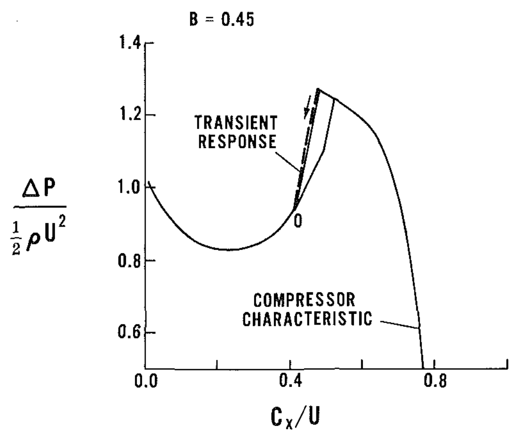

Fig. 4 *(b)* Transient compression system behavior: $B = 0.45$

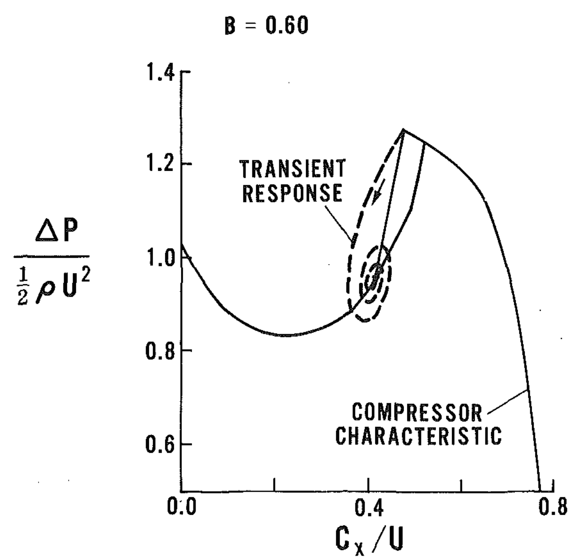

Fig. 5 *(b)* Transient compression system behavior: $B = 0.60$

show the compression system behavior when the throttle is closed so that a uniform flow equilibrium point is no longer possible—i.e., they represent the stall transients of the system. The *(a)* sections of the figure show the compressor axial velocity parameter (or nondimensional mass flow) and nondimensional plenum pressure, respectively, versus time. The units of time are Helmholtz resonator periods ($\tilde{t}/2\pi$). In the *(b)* parts of these figures, the results are displayed in a compressor map type of format in which the instantaneous pressure rise is plotted against the compressor axial velocity parameter. The locus of these transient system operating points is shown as the dashed curves. This representation is somewhat analogous to the phase-plane diagrams which are often used for illustrating nonlinear oscillations [9], but it is more pertinent for the present situation, since the primary variables to the compressor user are mass flow and delivery pressure.

In Fig. 4 we see the system response at a relatively low value of B, corresponding to running the compressor at low speed or with a small volume behind it or some combination of the two. Once the system is perturbed past the stall limit, the plenum pressure and mass flow decrease continuously from their initial value to a new steady value. The value is determined by the intersection of compressor and throttle, and is at a substantially reduced pressure rise and flow from the initial conditions. This behavior characterizes a machine that exhibits rotating stall when the stall limit line is crossed.

Fig. 5 shows the behavior at a larger value of B, i.e., at a higher speed, say. It can be seen that, although the system does not appear to be critically damped as before, the eventual result is the same. Subsequent to the initial drop-off, the pressure and mass flow exhibit oscillations with a decaying amplitude. After some time the system operating point reaches the same value as before, again giving steady operation in rotatiing stall as the end result of the initial instability. However, increasing B to 0.7 brings an entirely different result, as shown in Fig. 6. The initial oscillations grow to a large amplitude and the operating point of the compression system undergoes a limit cycle type of motion. This is surge. During the cycle the compressor axial velocity parameter swings to values that are considerably larger than that necessary for the cessation of rotating stall. One would thus tend to associate these high flow rates with instantaneous operation in an unstalled condition over part of the cycle, and this is precisely what hot wire records show is happening with the actual machine.

The model thus predicts that for this compressor, surge will occur when the value of B is approximately 0.7 and that below this value the system will exhibit a stable, in the sense of being steady in time, rotating stall behavior. It should be noted how quickly the operating point reaches the limit cycle, the approach taking rough-

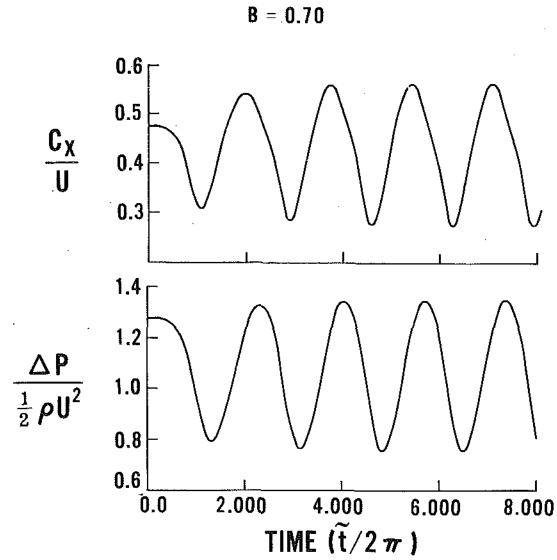

Fig. 6(a) **Transient compression system behavior: $B = 0.70$**

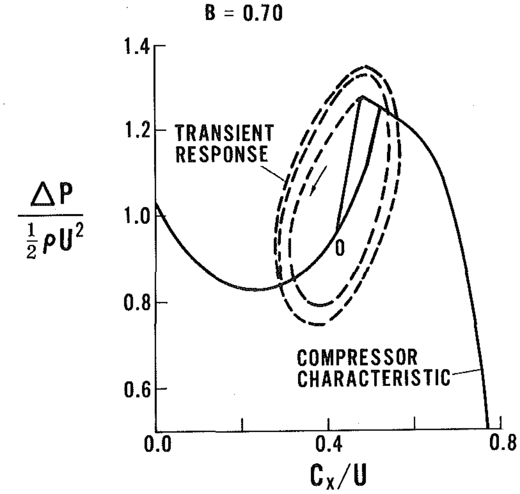

Fig. 6(b) **Transient compression system behavior: $B = 0.70$**

ly one cycle. This is an indication of the strongly nonlinear aspects of the surge oscillations.

As B is increased further, the frequency and shape of the limit cycles will also change. For example, Figs. 7 and 8 show results for $B = 1.58$ and $B = 5.0$, respectively. The former is the highest value obtained in the present series of experiments and the latter is considerably larger than any tested. It can be seen that the character of the oscillations changes markedly from the quasi-sinusoidal form that is observed at values of B near the dividing line between rotating stall and surge. At large values of B the relevant time scales of the motions are no longer characterized by the Helmholtz resonator frequency. The oscillations are of the relaxation type [9, 10], and have two distinct time scales—a long one in which the plenum pressure is built up (or discharged) slowly, the mass flow changes gradually, and the transient operating point moves along the steady-state characteristic, and a much shorter one in which the mass flow changes rapidly at almost constant pressure. These two phases of the oscillations can be distinguished in Figs. 8*(a)* and *(b)*. It should be mentioned that the calculation for Fig. 8 was carried out with no time lag in the compressor response, since the approximate first order model is not adequate to describe the unsteady compressor behavior during the very abrupt mass flow excursions that characterize the relaxation type of oscillation.

In this type of limit cycle, which occurs at large values of B, it is the resistive elements in the system that control the period of the motion. More precisely, the longer time scale is set by a balance between resistive and restoring forces, the latter being associated with the potential energy of the compressed fluid in the plenum. For example, in the oscillation shown in Fig. 8, the "blowdown" time for the plenum pressure to fall, as the system operating point moves along the negative flow portion of the compressor characteristic, is determined by how fast the plenum can empty through the throttle and compressor—in other words, how fast the potential energy stored in the compressed fluid in the plenum can be dissipated in the throttle and compressor resistances. This can be contrasted with the limit cycle behavior at the lower values of B, where the time scale of the motions is set basically by a balance between the inertial and restoring forces.

Overall Surge Cycle Properties and Physical Mechanism

To complement the numerical results that have been presented, it is useful to examine analytically the overall properties of the oscil-

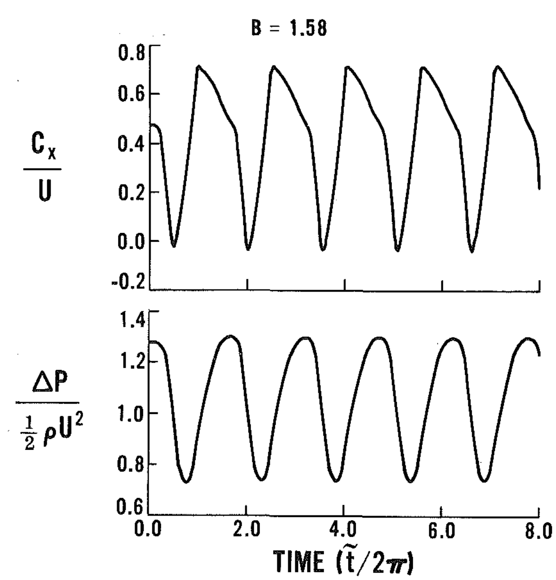

Fig. 7(a) **Transient compression system behavior: $B = 1.58$**

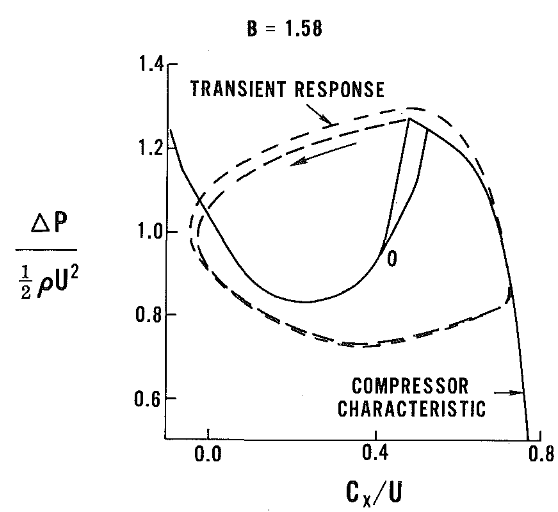

Fig. 7(b) **Transient compression system behavior: $B = 1.58$**

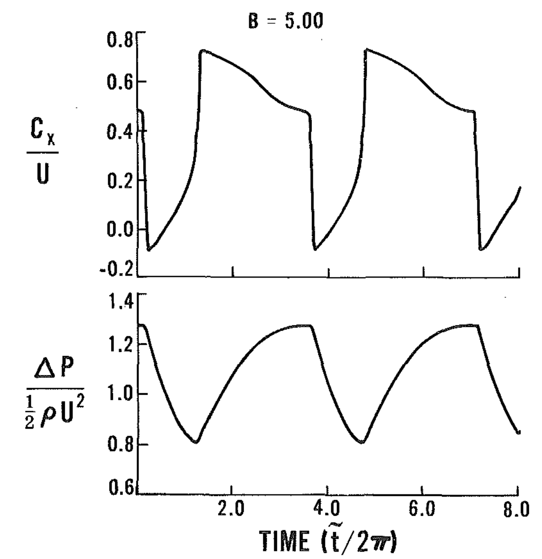

Fig. 8(a) **Transient compression system behavior: $B = 5.00$**

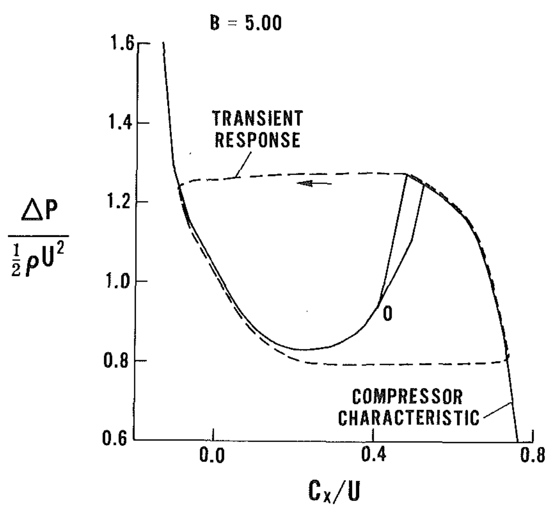

Fig. 8(b) **Transient compression system behavior: $B = 5.00$**

lations that have been encountered, including the physical mechanism by which they are generated and maintained. In order to do this with a minimum of complexity, we can consider in this section a somewhat simpler system, which nevertheless exhibits qualitatively the same type of behavior as the one discussed in the foregoing. Therefore, we now examine a compression system where the inertial forces due to the small mass of fluid in the throttle duct can be neglected, and, in addition, the compressor response is taken to be quasi-steady. This is a basic compression system model that has been considered by several authors (e.g., [8]), using a linearized treatment. Some overall conclusions on the occurrence of surge cycles can be readily derived from this system, and these may then be extended to the more general case where they will still be qualitatively valid.

It is usually found that the throttle curve is rather steep at flows near the stall limit, and as a result the fluctuations in *throttle* mass flow are substantially smaller than those through the compressor. Due to the relatively small changes in the slope of the throttle curve over this limited range of mass flows, it is a reasonable assumption to use a linear representation for the *throttle* curve. The throttle pressure drop can therefore be written in terms of a reference nondimensional mean flow, $\dot{M}$, as:

$$\tilde{F}(\tilde{\dot{m}}_T) = \tilde{F}(\tilde{\dot{M}}) + \tilde{F}'(\tilde{\dot{M}})\,[\tilde{\dot{m}}_T - \tilde{\dot{M}}] \tag{20}$$

The prime denotes differentiation with respect to $\tilde{\dot{m}}_T$.

Since the inertial forces in the throttle are negligible, the difference between plenum and ambient pressures is equal to the quasi-steady throttle pressure drop.

$$\Delta\tilde{P} = \tilde{F}(\tilde{\dot{m}}_T) \tag{21}$$

The rate of change of plenum pressure can therefore be expressed as,

$$\frac{d\Delta\tilde{P}}{d\tilde{t}} = \tilde{F}'\frac{d\tilde{\dot{m}}_T}{d\tilde{t}} \tag{22}$$

The four equations describing the behavior of the more general system (equations (11)–(14)) are now reduced to two:

$$\tilde{C}(\tilde{\dot{m}}_c) - \tilde{F}(\tilde{\dot{m}}_T) = \frac{1}{B}\frac{d\tilde{\dot{m}}_c}{d\tilde{t}} \tag{23}$$

$$\tilde{\dot{m}}_c - \tilde{\dot{m}}_T = B\tilde{F}'\frac{d\tilde{\dot{m}}_T}{d\tilde{t}} \tag{24}$$

Using equation (20) we can combine these into a single equation for $\tilde{\dot{m}}_c (= C_x/U)$, the compressor axial velocity parameter:

$$\frac{d^2\tilde{\dot{m}}_c}{d\tilde{t}^2} + \left[\frac{1}{B\tilde{F}'(\tilde{\dot{M}})} - B\tilde{C}'(\tilde{\dot{m}}_c)\right]\frac{d\tilde{\dot{m}}_c}{d\tilde{t}} + \left[\tilde{\dot{m}}_c - \tilde{\dot{M}} + \frac{\tilde{F}(\tilde{\dot{M}}) - \tilde{C}(\tilde{\dot{m}}_c)}{\tilde{F}'(\tilde{\dot{M}})}\right] = 0 \tag{25}$$

It should be emphasized that equation (25) is still strongly nonlinear because the coefficients depend on the instantaneous value of $\tilde{\dot{m}}_c$ through the form of the compressor curve, $\tilde{C}$.

Equation (25) is analogous to that describing the behavior of a second order mass-spring-damper system with nonlinear restoring force and damping. We could therefore regard $\tilde{\dot{m}}_c$ as the "displacement," $d\tilde{\dot{m}}_c/dt$ as the "velocity," and $d^2\tilde{\dot{m}}_c/d\tilde{t}^2$ as the "acceleration," and make use of concepts that have been developed to analyze this type of nonlinear system. In particular, we can discuss the behavior of the solution in terms of a limit cycle oscillation [9, 10]. This type of periodic motion occurs in nonlinear, nonconservative systems where energy is fed into the oscillation over part of the cycle so that the motion is maintained even though dissipative forces are acting. To derive the conditions necessary for periodic behavior, we can examine the total "mechanical energy" of the analogous system described by equation (25). To do that, we multiply this equation by the "velocity," so as to make the first and third terms exact differentials and integrate over a time interval $\Delta\tilde{t}$. As shown in [9] or [11], the result is to obtain an expression for the change in total mechanical energy of the analogous system during the time interval,

$$\Delta[\text{total mechanical energy}] = \int_{\tilde{t}}^{\tilde{t}+\Delta\tilde{t}} \left\{\left[B\tilde{C}'(\tilde{\dot{m}}_c) - 1/B\tilde{F}'(\tilde{\dot{M}})\right]\left(\frac{d\tilde{\dot{m}}_c}{d\tilde{t}}\right)^2\right\} d\tilde{t} \tag{26}$$

The mechanical energy will increase, decrease, or remain constant, depending on whether the integral is positive, negative, or zero. In terms of the analogous system this integral represents the work done by the damping forces. These forces can be instantaneously negative, corresponding to work put into the oscillation, or positive, corresponding to dissipation.

Using equation (26) we can also point out the basic difference between a linear and a nonlinear analysis of the compression system behavior. In the former, the quantity $(B\tilde{C}' - 1/B\tilde{F}')$ is evaluated at one value of $\tilde{\dot{m}}_c$ only. Whether the amplitudes of perturbations about this point grow or decay depends on the value of $(B\tilde{C}'$

$- 1/B\tilde{F}')$ *at that point;* if it is positive they grow exponentially, if negative they decay. In the nonlinear analysis, it is the integral of the quantity over a finite range of values of $\dot{\tilde{m}}_c$ that determines what will happen. The response of the system does not depend on conditions at a single point, but rather on conditions in a (possibly large) region surrounding this point. In particular there is not, in general, a critical value of the slope of the compressor characteristic at one point which determines whether surge (oscillatory behavior) or rotating stall occurs.

For a periodic surge cycle, the mechanical energy of the analogous system must have the same value after a complete period, $\tilde{T}$, so

$$\int_{\tilde{t}}^{\tilde{t}+\tilde{T}} \{[B\tilde{C}'(\dot{\tilde{m}}_c) - {}^1/B\tilde{F}'(\mathring{\tilde{M}})](\frac{d\dot{\tilde{m}}_c}{d\tilde{t}})^2\}\, d\tilde{t} = 0 \tag{27}$$

The quantities $(d\dot{\tilde{m}}_c/d\tilde{t})^2$, $\tilde{F}'$, and B are always positive. Thus, for surge to occur the compressor characteristic must have a positive slope greater than $1/B^2\tilde{F}'$ over some part of the cycle.

The meaning of the necessity for a positive compressor characteristic can be understood in terms of the following physical arguments, which are based on calculations reported in [12]. Consider a compression system undergoing cyclic motion about a mean operating point. Since dissipation is occurring due to the presence of the throttle, there must be energy put into the system to sustain the oscillation, and the only possible source available to do this is the compressor. Favorable conditions for this energy addition occur when the compressor slope is positive. In this case high mass flow rate and high rate of mechanical energy (in the form of pressure rise) addition per unit mass go together, as do low flow and low rate of energy addition per unit mass. Because of this, over a complete cycle, the net amount of mechanical energy that the compressor puts into the flow will be higher than if the system were in steady operation at the mean mass flow. In a similar fashion, the net dissipation in the throttle will also be higher than if the system were in steady operation. When the energy input over a cycle balances the dissipation, a periodic oscillation can be maintained. To summarize: surge oscillations are possible only when the mechanical energy input from the compressor is greater during the oscillatory flow than during a (mean) steady flow, and this can occur only if the characteristic is somewhere positively sloped so that high mass flow and high mechanical energy input per unit mass go together.

Note that because the occurrence of a surge cycle is dependent upon work being done on the system by negative "damping" forces, the presence of the oscillations is associated with a *dynamic* instability. However the compression system can also be statically unstable. This is the classical type of instability that occurs when the compressor characteristic is locally steeper than the throttle curve [13]:

$$\tilde{C}' > \tilde{F}' \tag{28}$$

In terms of the analogous second order system this would correspond to a negative restoring force. This *static* type of instability is not sufficient for a surge cycle, since the negative restoring force cannot introduce energy into the system to counteract the dissipation that occurs during the cycle, although the system can exhibit local static instability over part of the cycle. For example, an initial stall line transient could be from a statically unstable operating point. However, the subsequent system behavior would then be determined from dynamic considerations, since it is the relative amplitude of the mass flow fluctuations in the compressor and the throttle that determine the relative sizes of the energy input and dissipation, and these quantities must be calculated from the system dynamics. Thus, one cannot predict whether surge or rotating stall will occur just from knowing the shape of the compressor and throttle curves. An illustration of this is seen in Figs. 4–8 where the compressor and throttle curves are the same in all the figures, but the compression system behavior is quite different.

Although the foregoing arguments have been focused on the behavior of the simplified compressions system, the basic qualitative conclusions can be extended to the more general case described by equations (11)–(14). First note that for physically reasonable throttle configurations the throttle mass is usually small and the throttle pressure drop curve is rather steep, so that inclusion of the finite throttle mass has little overall effect. The differences that do exist between the two systems are therefore due almost wholly to the nonquasi-steady compressor behavior. This has the general effect of "flattening out," as far as the system response is concerned, the sharp changes in pressure rise that exist in the steady state compressor characteristic; in other words, of yielding an "effective" compressor curve with more gradual slopes. During a stall transient, however, these sharp changes would be found in the regions of large positive slope that are associated with the onset and cessation of stall. The decrease in effective (positive) slope would therefore be expected to have a stabilizing influence on the calculated transient system response. There would thus be a greater tendency towards rotating stall (rather than surge) than would be predicted from the criterion expressed in equation (27). This is borne out by calculations.

The surge cycle criterion of equation (27) will thus not be sufficient for the general system, as the steady-state compressor curve will now have to be more positively sloped than this criterion would suggest. However, even though the criterion is no longer quantitatively correct, the basic arguments about the physical mechanism are still valid. That is, it is still necessary to have a compressor characteristic with a certain positive slope over some part of the cycle so that the compressor provides a net work input. The difference is that the compressor characteristic that is now referred to is based on the instantaneous pressure rise and is a function not only of the steady state characteristic, but also of the time history of the motion of the compressor operating point. Therefore, the fundamental idea of the surge cycle being maintained by a net energy input from the compressor being balanced against the throttle dissipation is essentially the same whether one considers the simple or the more general system.

Summary and Conclusions

1 A nonlinear model has been developed to predict the transient response of an axial compression system subsequent to a perturbation in operating conditions.

2 This response is strongly dependent on the nondimensional parameter $B[=U/(2\omega L_c)]$, and the shape of the characteristic curve.

3 With a given compressor, there is a critical value of B. For larger values the mode of instability encountered at the stall line will be the large amplitude oscillations that characterize surge; for smaller values the result of the instability will be a transient to a new operating point in rotating stall at a considerably reduced flow and pressure rise.

4 The occurrence of surge cycles in a compression system is associated with a dynamic instability, in which the energy fed into the oscillations by the compressor is analogous to the work done on a mechanical system by a negative damping force. Surge cycles are therefore possible only when the net mechanical energy input from the compressor over a period is greater during the oscillatory flow than during a (mean) steady flow.

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