Optimal flow problems, the Edmonds-Karp algorithm, and birdsong

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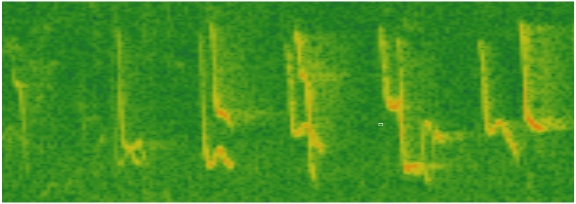
Intro

My research:

- "Machine listening"
  = machine learning & signal processing
  for understanding sound
- Automatic analysis of bird sounds
  (which species, how many, how often...)



Intro

Algorithm: Edmonds-Karp

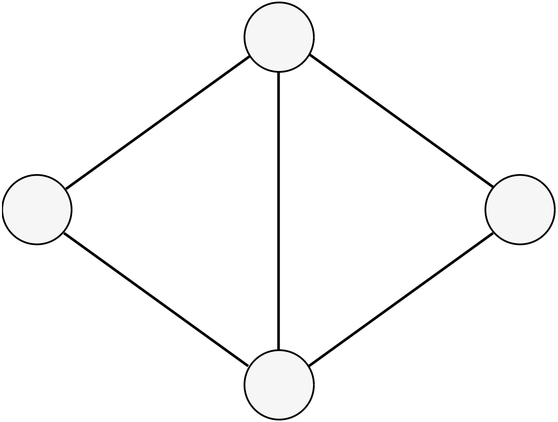
- Graph theory, "flow networks"
- We'll consider the *maximum flow* problem
- Related to shortest-path problems
  (SatNav routing, travelling salesman)
  ... but harder
- Imagine you have to send as many trains-per-hour as possible from London to Edinburgh, and that different segments of the route have different maximum capacities.

Intro

What does that have to do with birdsong?

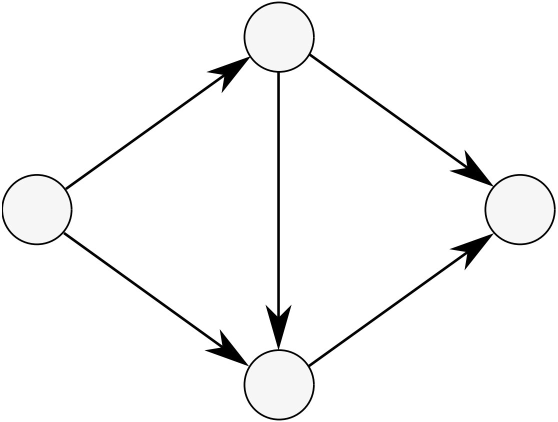
Definitions

Undirected graph



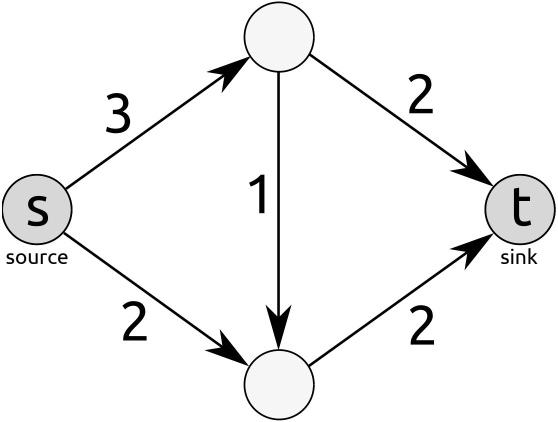
Definitions

Directed graph (or “digraph”)



Definitions

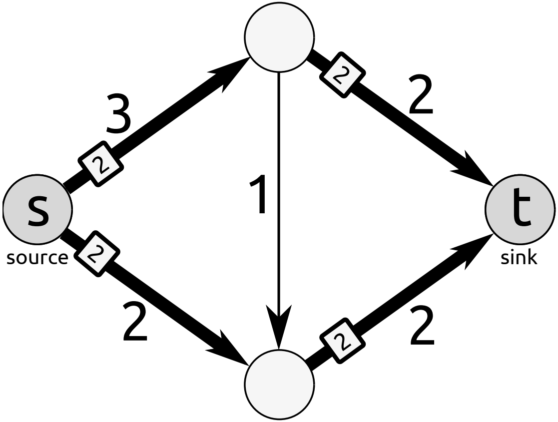
Network



Capacities on each arc

Definitions

Flow



Flow values must balance at each node (except s, t)

Maximum flow

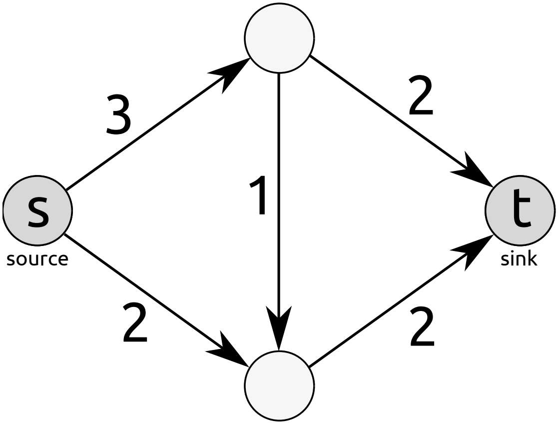
The **maximum flow** problem:

> *For a given network,*
> *find the flow of maximum value*
> *that is compatible with the network capacities*
> *(a flow that is "feasible")*

Flow might be integer or real-valued; I'll focus on integer.

Maximum flow

Greedy approach: use a single-path algorithm, and iterate



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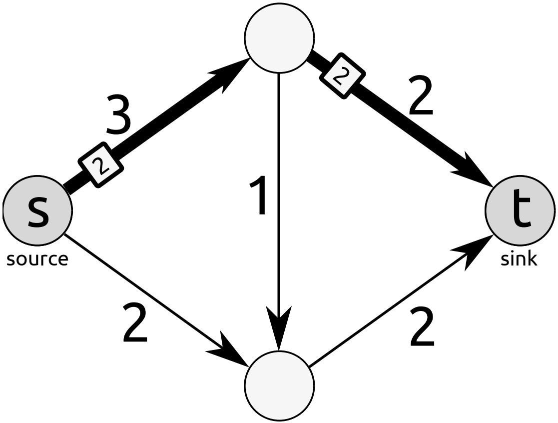

Maximum flow

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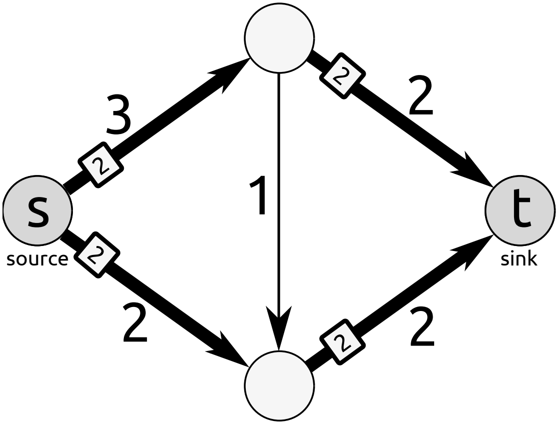

Maximum flow

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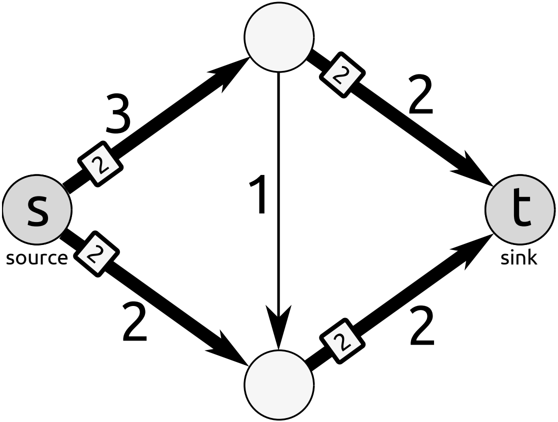
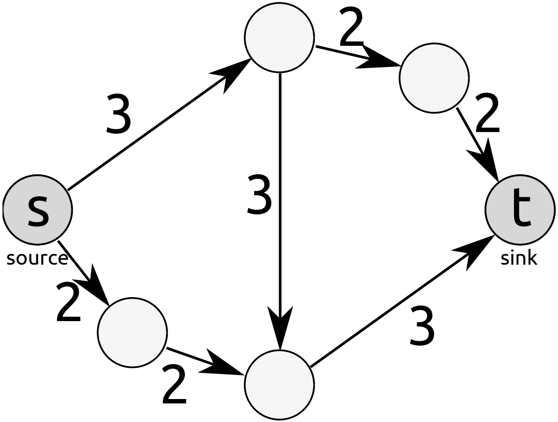


Complexity: $O(|V| + |A|)$ per iteration

Greedy approach may fail

Many networks lead to local optima



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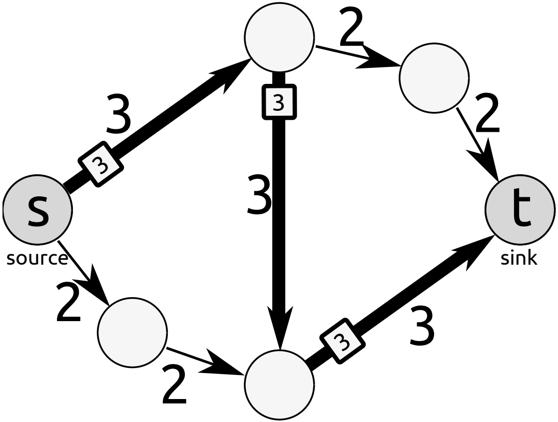


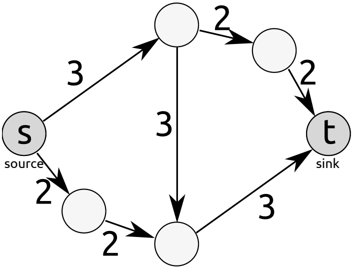
Edmonds-Karp algorithm

The **Edmonds-Karp algorithm** is guaranteed to find the exact solution.
Based on the Ford-Fulkerson algorithm (plus shortest-path).

Key concept: the *residual network*.

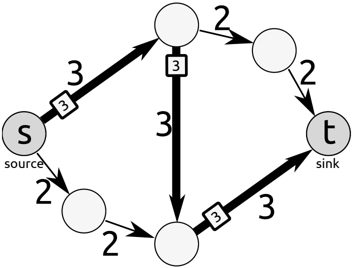
Edmonds-Karp algorithm

Network



Edmonds-Karp algorithm

Network



Edmonds-Karp algorithm

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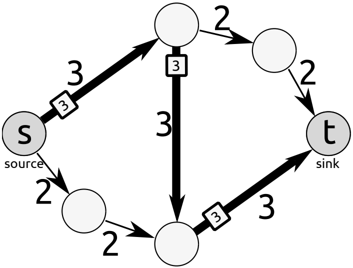
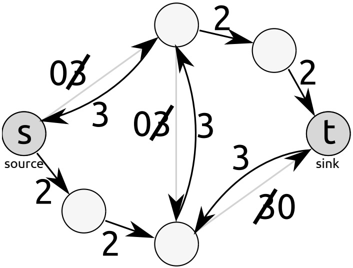


Residual network



Edmonds-Karp algorithm

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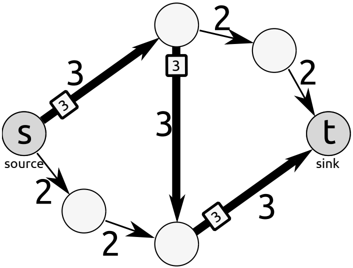

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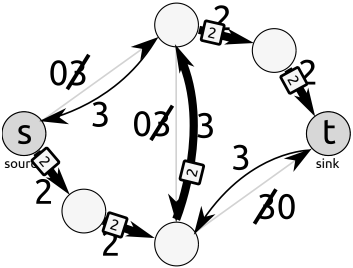

Edmonds-Karp algorithm

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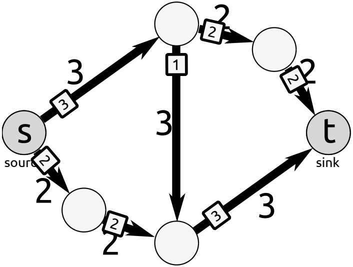

Edmonds-Karp algorithm

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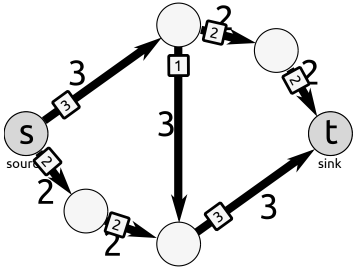
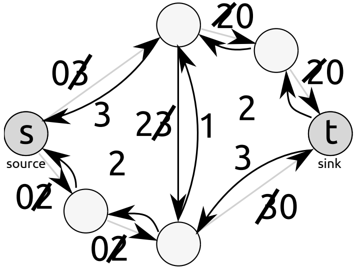


Residual network

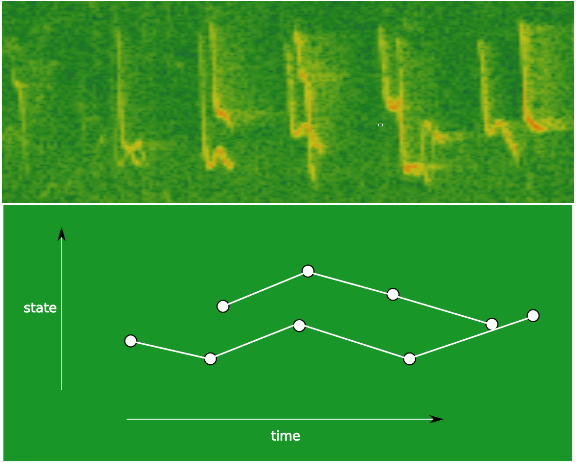


Edmonds-Karp algorithm

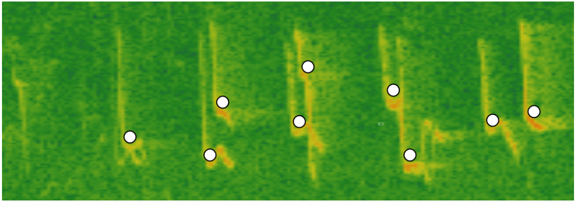
- Guaranteed to have no local optima
- Guaranteed to terminate in bounded number of iterations (as long as all capacities are rational)
- Complexity: $O(|V||A|^2)$

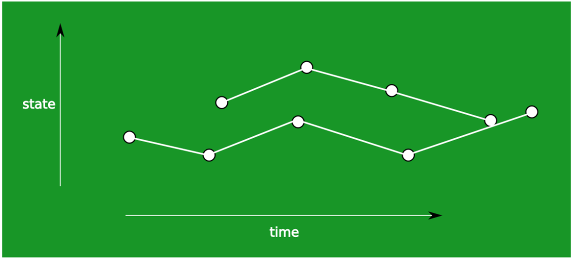
Pause

Disentagling birds



Disentagling birds



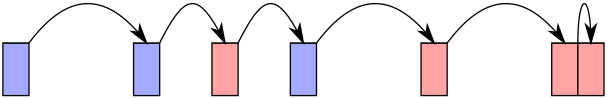


Modelling an intermittent source

Markov renewal process ("MRP"):

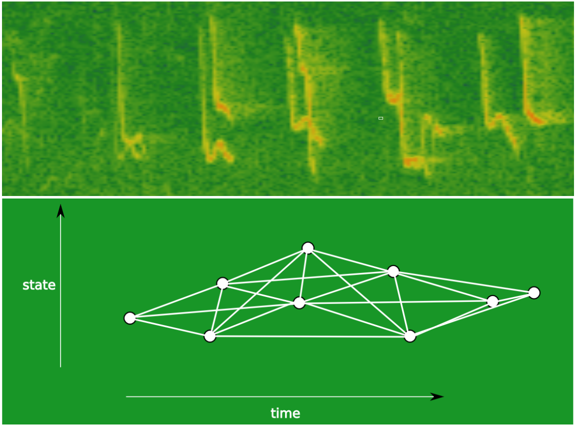
$$
\begin{aligned}
&P(\tau_{n+1} \leq t, X_{n+1} = j \,|\, (X_1, T_1), \ldots, (X_n = i, T_n) \qquad)\\
=&P(\tau_{n+1} \leq t, X_{n+1} = j \,| \qquad\qquad\qquad X_n = i \qquad\qquad)
\end{aligned}
$$

where τ_{n+1} is the time difference $T_{n+1} - T_n$.



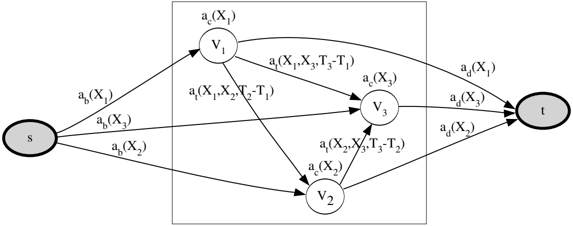
Multiple MRPs

Problem sketch: assume multiple MRPs, plus potential “clutter”.



Given transition probabilities, find the most likely set of paths. (Max 1 path per node)

Convert max-likelihood problem to a flow problem



Convert likelihood expression to flow “costs”:

$$
\begin{aligned}
a_b(X) &= -\log p_b(X) \\
a_d(X) &= -\log p_d(X) \\
a_t(X, X', \tau) &= -\log f_X(X', \tau) \\
a_c(X) &= \log p_c(X)
\end{aligned}
$$

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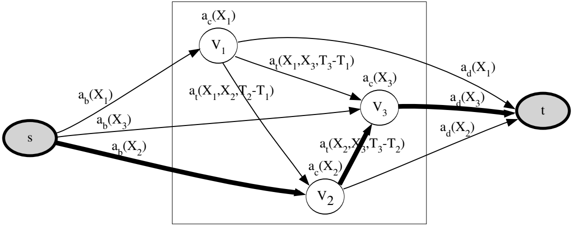

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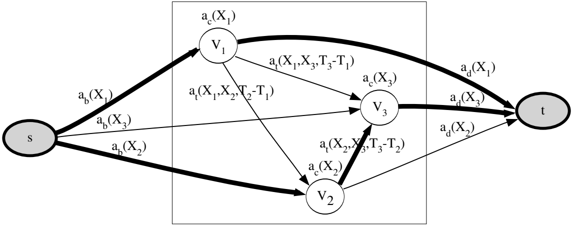

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$$a_c(X) = \log p_c(X)$$

Minimum cost flow

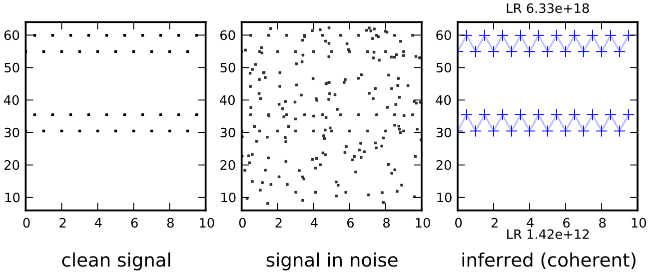
Minimum cost flow problem, with binary capacities

is solved the same way as a

Maximum flow problem

- Optimal minimum-cost flow: Edmonds-Karp algorithm, asymptotic time complexity $O(|V||A|^2)$.
- Or use inexact (greedy) algorithm: $O(|V||A|)$ or lower.

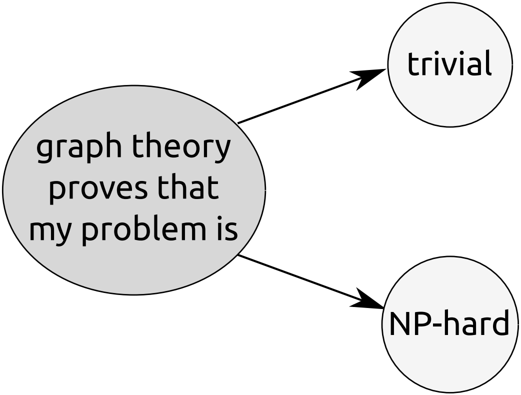
Synthetic example



Lesson

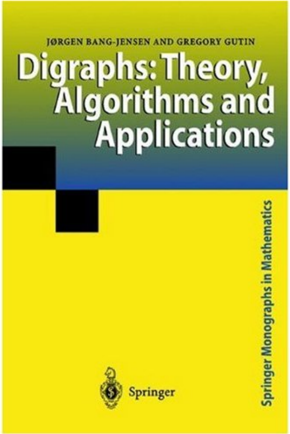
Lesson

If your problem can be represented on a graph/network, you can probably transform it into a well-studied problem.



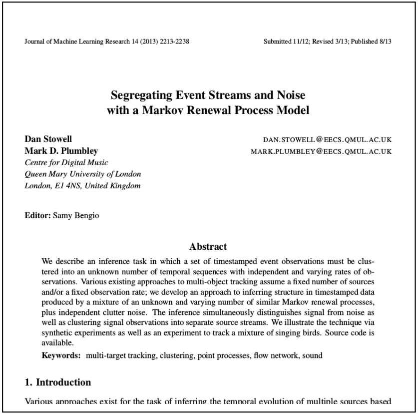
Further reading

A good textbook:
Bang-Jensen and Gutin



(free PDF on website)

Journal paper: JMLR,
Stowell & Plumbley (2013)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Segregating Event Streams and Noise with a Markov Renewal Process Model

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Editor: Samy Bengio

Abstract

We describe an inference task in which a set of timestamped event observations must be clustered into an unknown number of temporal sequences with independent and varying rates of observations. Various existing approaches to multi-object tracking assume a fixed number of sources and/or a fixed observation rate; we develop an approach to inferring structure in timestamped data produced by a mixture of an unknown and varying number of similar Markov renewal processes, plus independent clutter noise. The inference simultaneously distinguishes signal from noise as well as clustering signal observations into separate source streams. We illustrate the technique via synthetic experiments as well as an experiment to track a mixture of singing birds. Source code is available.

Keywords: multi-target tracking, clustering, point processes, flow network, sound

1. Introduction

Various approaches exist for the task of inferring the temporal evolution of multiple sources based

(free, open-access) plus...

Funded **PhD position** available next year!

Working with me at QMUL
on machine learning, signal processing and bird sounds

Drop me a line:
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