Finding Optimal Triadic Transformational Spaces with Dijkstra's Shortest Path Algorithm

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Abstract. This paper presents a computational approach to a particular theory in the work of Julian Hook—Uniform Triadic Transformations (UTTs). A UTT defines a function for transforming one chord into another, and is useful for explaining triadic transitions that circumvent traditional harmonic theory. By combining two UTTs and extrapolating, it is possible to create a two-dimensional chord graph. Meanwhile, graph theory has long been studied in the field of Computer Science. Dijkstra's shortest-path algorithm, for example, provides a method for finding the shortest path between two points on a weighted graph. This work describes a software tool which can compute the shortest path between two points in a two-dimensional transformational chord space. Utilizing computational techniques, it is then possible to find the optimal chord space for a given musical piece. The musical work of Michael Nyman is analyzed computationally, and the implications of a weighted chord graph are explored.

Keywords: Neo-Riemannian Theory, Shortest-path algorithm, Transformational Spaces, Computational Musicology

1 Introduction

Innovation has always been the impetus for more innovation; the creation of new concepts and new ideas immediately demands new methods of analysis, and new tools for that analysis. Contemporary Art Music is no exception. Indeed, innovative compositions of the late 20^{th} century have thwarted the methods of analysis on which many theorists rely. Some composers have created compositions that utilize advanced tools for the creation process, as well as for the performance. Others have aimed to create new concepts of harmony and melody, as well as explored the manipulation of other compositional parameters. In the late 1980s, a new sub-discipline of music theory arose out of the need to describe triadic and intervallic motions irrespective of a common key tonic. This sub-discipline has been labelled as transformational theory [8], and has inspired a handful of different theories that are concerned with triadic and intervallic relationships

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outside of the more common diatonic framework–those that relate pitch-based units directly by transforming one into the other.

The somewhat narrower field of neo-Riemannian theory is similarly focused on the transformation of musical objects, specifically triads. Neo-Riemannian theory encapsulates the work of Richard Cohn [2], and more recently Julian Hook [7]. All of these theorists are concerned with the relationships between musical objects unique of a centralized tonic reference. Neo-Riemannian theory presents a different perspective on the concept of harmonic distance, and create transformations based on shared tones between triads, and voice-leading concerns. Distance in a neo-Riemannian triadic space is defined by the number of unit operations (i.e. the shortest path) required to get from one musical object to another in the transformational space [6]. Together, the theories have since been extended to analyze composers in different eras, for example Milton Babbitt, Bela Bartók and György Ligeti. [9,5,1].

Transformational spaces are a flexible, yet formal tool for defining the harmonic spaces of certain innovative pieces. Lewin's definition of transformational theory was founded on a mathematical technique called graph theory. Likewise, Hook's theory of Uniform Triadic Transformations (UTTs) utilizes 2-dimensional graphs of functional transformations (chord spaces, or chord graphs). For the purposes of this research, the definition of an optimal chord space is the space which most efficiently represents the transformational transitions of the triadic sequence of an entire piece. Since there are many harmonic spaces that can be created with these 2-dimensional chord graphs, this paper seeks to find the optimal space for a given piece. It is a logical extension, then, to employ one of the most effective tools in computational analysis to analyse the possible harmonic spaces of new music: the computer. Specifically, this essay will outline the application of UTTs to the first three pieces of Michael Nyman's *Six Celan Songs*, and a computational approach to discovering the most appropriate space.

2 Transformational Theory

In Lewin's Generalized Musical Intervals and Transformations [8], he defines sets of pitches (labelled "spaces"), as well as corresponding groups of transformations. A simply transitive, or STRANS system is a space-and-group couple in which each relationship between two pitches in the pitch space can be described only by a single, unique transformation from the group (satisfying the simply transitive property). The theory immediately distinguishes itself from atonal theory, because although there is a similar focus on transformations (e.g., transpositions, inversions and retrogrades), these transformations are required to be unique within a group. The fundamental difference in perspective for Lewin is that transformations can also be considered measurements. If that is the case, then the need to provide a unique measurement is immediately apparent. As Satyendra summarizes in his introduction to Lewin's theory [10]:

> . . . [It] is intuitive to say that both T_3 and I_3 transform C to E♭, but it is counterintuitive to think of the interval between C and E♭ as both T_3 and I_3

It is for this reasoning that the argument is made for group and space sets which have a unique set of actions which define the relationships between every pair in the pitch space, and no more.

3 Uniform Triadic Transformations

Julian Hook similarly uses transformational spaces to define movement between triads [7]. He developed the concept of Uniform Triadic Transformations (UTT), which is based on Riemann's fundamental idea of major and minor triads being mirror images of one another. The format for defining a UTT is <+/−, m, n>. The plus / minus sign defines whether or not the resulting triad will be the same (+) or the opposite (-) type of triad, and the m and n variables define the movement of the triads root for the major and minor groups, respectively [3].

Hook refined his UTT spaces, and employed Lewin's idea of simply transitive spaces to the triadic graphs he had created. Specifically, a simply transitive UTT group is a group in which any pair of triads can be represented only by a single transformation in the group. Simply transitive groups can be created by the repeated application of certain members. Hook describes the groups that embody the simply transitive property as K(a,b) groups. Take the K(1,1) subgroup, for example. Its UTT members are listed in Figure 1 [3]. Through repeated application of the transformation represented by members with a certain property, the entire group can be generated. The property necessary is that for any UTT in the form of <-, m, m+1>, $2m$ + 1 must equal a number in the set [1, 5, 7, 11]. If the member does not satisfy this property, it will not generate the entire K(1,1) subgroup. The UTT <-, 2, 3> ($2m + 1 = 5$) will generate all members, for instance, while the UTT <-, 4, 5> ($2m + 1 = 9$) will not [3].

Fig. 1: The UTTs which comprise the K(1,1) simply transitive subgroup.

Mode Preserving:			Mode Reversing:		
<+, 0, 0>	<+, 4, 4>	<+, 8, 8>	<–, 0, 1>	<–, 4, 5>	<–, 8, 9>
<+, 1, 1>	<+, 5, 5>	<+, 9, 9>	<–, 1, 2>	<–, 5, 6>	<–, 9, T>
<+, 2, 2>	<+, 6, 6>	<+, T, T>	<–, 2, 3>	<–, 6, 7>	<–, T, E>
<+, 3, 3>	<+, 7, 7>	<+, E, E>	<–, 3, 4>	<–, 7, 8>	<–, E, 0>

4 Dijkstras Shortest Path Algorithm

A natural extension of graph theory is to employ methods to traverse the triadic space. Graphical models for computation have been studied since the infancy of computer science. The algorithms that were designed in the earliest days of computing science are still widely employed in computing systems today. Arguably the best such example of this is Dijkstras Shortest Path algorithm. Conceived in 1956 by Edsger Dijkstra, the shortest path algorithm will find the shortest path between two vertices in a graph, as long as a path exists between the two vertices [4]. The graph is required to have edges linking nodes, such that every vertex must have at least one edge, otherwise that vertex would not be considered part of the graph. Each edge is assigned a certain distance, so that vertices that are closer to one another are assigned an edge with a smaller distance. The distance metric can be arbitrarily assigned to represent whatever metric is most useful for the graph representation.

4.1 The Algorithm

Given a properly formed weighted graph, a start vertex and an end vertex, Dijkstra's algorithm will traverse the graph in a way that allows the shortest path between the start and end vertex to be found in the least amount of movements. The method for finding this shortest path is relatively intuitive. First, assign the distance to the starting vertex to zero. Also, mark every other node in the graph as unvisited, and assign it the distance of infinity. Then, traverse through every unvisited vertex in the graph until the newly visited vertex is the desired end vertex. It is best to demonstrate with a concrete example. Suppose we are at the starting vertex, and it has edges of distances 3, 4, and 5. In our initialization step, we assigned the starting vertex a distance of zero. We would then loop through all of the starting vertexs edges, and assign the value of (0 + <distance of edge to neighbor>) to each neighboring vertex. Then, we would mark our starting vertex as visited. Now, we search for the unvisited vertex of the minimum distance. In our case, the vertex with distance 3 would be visited next. Suppose, then, that the vertex of distance 3 had neighbors with edges of .5 and 2. We would assign the distances of (3 + .5) = 3.5 and (3 + 2) = 5 to those new neighbors, and mark the vertex with distance 3 as visited. Our set of unvisited vertices with non-infinite distances would then be the set (3.5, 5, 4, 5). The next selection for an unvisited vertex to visit would be 3.5, even though it is 2 vertices from our original vertex. That is, the distance assigned to the edges of the vertices (3 and .5) is more important, since that vertex is closer than our starting vertexs neighbors of distances 4 and 5. With this method, we can assign varying distances between vertices, and still traverse the graph to find the shortest path between two vertices in a minimally efficient way.

4.2 Song Harmonies

Nymans Six Celan songs are very triadic in their harmonic structure, but also don't follow standard tonal syntax. Thus, they are a good starting point for the utilization of digital UTT spaces. The first step is to analyze the underlying triads of each piece and to create a digital representation that can be ingested by the algorithm. Most of the harmonic content in Nyman's songs have relatively regular durations, which is ideal since the UTT space does not consider durations of the triads. One drawback of the UTT space is that only major or minor triads can be considered. Therefore, if certain parts of the piece contain more complicated harmonies than simple triads, they must be simplified to the closest approximation of a major or minor triad. This is an unfortunate drawback, however most of the content in the pieces are simple triads, so the majority of the harmonies will be represented correctly.

The representation chosen for chord sequences is the same representation that was chosen for the annotation of a large data set of Rock n Roll music [11]. Simply put, bar boundaries are notated by the pipe symbol: '|', and the '.' symbol denotes the continuation of the previous triad. The following is the annotated harmonies of *Chanson einer Dame im Schatten*:

```
| Dm  | Fm | Am | Am . F Am |
| Dm  | Fm | Am | Am . . C |
| C7   | Ebm  | Gm |  Eb Gm |
| C7   | Ebm  | Gm |  Gm . Eb Gm7 |
| Gm | Bm | Eb | G . Eb Eb |
| Gm | Bm | Eb | Eb . Eb F |
| Eb | F#m | Bb | Bb . Dm F |
| Eb | F#m | Bb | Bb . Dm F  |
| C#m | Em |  Fm | D . D# C |
| C#m | Em | C | C . C C |
| Em | F#m | Dm | Dm | Em | Em A7 Em7 |
|F#m | F#m | Ebm | F#m . Dm F |
| Bb | B . . . Em . | Bb | Eb | Em | F#m | Dm | Fm | Db | G . G G |
| Ebm | Fm | C |
| A# | Em  | G#m | G#m E G |
| C#m | Em | G#m | G . . . . . |
| C7 | Am | Gm | Eb |
| C7 | Am | Gm | Eb |
| Em | C | Gm | Eb |
| C | Am | Gm | Eb |
| C | Am | Gm | Eb |
| Em | C | Gm | Eb |
| Eb | A | Bb | Bb Dm |
| Fb | A | Bb | Bb|
| Em | F#m | Dm | Gm |
| Eb | F#m | Dm | Gm |
| Dm | Fm | Am | Am . F Am |
| Dm | Fm | Am | Am . A Am |
|G|
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The reason this piece is so relevant is that there are chord movements which cannot be ascribed to traditional forms of functional harmony. For example, the opening progression of Dm→ Fm→Am, suggest that the tonic is Am, since it rests on Am for more than 1 measure. However, this movement could only be described as a vi→iv→i movement in harmonic minor, which is not a very common or functional progression. The song tends to group progressions in phrase couples, so that every other phrase tends to repeat the last, before moving on to new harmonic locations.

For *Es war Erde in ihnen*, a little more estimation is necessary. Nyman begins to include augmented and diminished chords in his triadic harmonies. Unfortunately, these triads also must be simplified to their corresponding major or minor triads. For the purpose of including them in the triadic space, the augmented triad has been simplified to its major triad of the same root, and the diminished triad has been simplified to the minor triad of the same root. This leaves a little to be desired from the UTT space, however it is a reasonable approximation, and only occurs for roughly 20% (20/99) of the triads in the piece.

4.3 Finding the Optimal Harmonic Space of a Song

Now that the triadic representations of each song are available, how does one then discover a set of UTTs that best describe a song's harmonic space? Perhaps one should find the most common intervallic movements for each type of transformation, and combine the two UTTs that represent those movements. In order to do so, it is necessary to separate the transitions of triads into their corresponding categories, then consider the most frequent pitch-class interval among common categories. For a detailed description, the process is presented for the first of the three songs.

The first step is to categorize the triadic transitions in the chord sequence into two possible categories: mode-reversing (major to minor, or minor to major) and mode-preserving (minor to minor, major to major). Then, for each triad in the piece, sum each occurrence of the transition from the given triad to every other triad in the song. This process is shown for *Chanson einer Dame im Schatten* in Figure 2a.

In order to translate this into the most likely UTT, one must also consider which root movements these transitions represent. For each of our transitions in the previous graph, the difference in semitones must be computed. The results are presented in Figure 2b. Notice that for the two sets of chord transitions which have a changing triadic type, minor-to-major and major-to-minor, the pitch-class root movements are aligned vertically. UTTs can be applied as their regular form of transformations, but they can also be applied inversely. We consider the inverse movements to be equivalent, and sum the number of times each movement happens. The results are shown in Figure 3. It is evident that for mode-reversing transformations, the best solution is the UTT <-, 4, 3>. For UTTs with mode-preserving transformations, there is a tie for the most common movements between UTT <+, 2, 2> and UTT <+, 3, 3>.

(a) Chord transitions.

Minor chords	Major chords transitioned to (occurrences)					Minor chords transitioned to (occurrences)				
Dm	F(3)	Fb(1)				Fm(5)	Em(1)	Gm(2)		
Fm						Am(4)				
Am	F(2)	C(1)	G(1)			Dm(2)	Gm(4)			
Ebm	X					Gm(2)	F#m(1)	Fm(1)		
Gm	C7(1)	Eb(9)				Bm(1)	Dm(1)			
Bm	Eb(2)					X				
F#m	Bb(1)					Dm(5)	Ebm(1)			
C#m	X					Em(3)				
Em	C(3)	A7(1)	Bb(1)			Fm(1)	F#m(4)	G#m(2)		
G#m	E(1)					X				
Major Chords										
F	Eb(2)	Bb(1)				Am(2)	C#m(1)			
C/7	A#[1]					Ebm(2)	C#m[1]	Em[1]	Gm(2)	Am(4)
Eb	G(1)	F(1)	C[3]	A(1)		Gm(3]	F#m(3)	Em(3)		
G	X					Ebm[1]	C#m[1]			
D	C					X				
Bb	B(1)	Eb(1)				Dm(3)	Em[1]			
B	X					Em(1)				
Db	G(1)					X				
A#	Em(1)					X				
Fb	A(1)					X				
A	Bb(2)					Em(1)				

(b) Root movement (in pitch class) for every chord transition that occurs.

Minor Chords	Minor to major root movements (occurrences)					Minor to minor root movements (occurrences)			
Dm	3(3)	2(1)				3(5)	2(1)	5(2)	
Fm						4(4)			
Am	8(2)	3(1)	10(1)			5(2)	10(4)		
Ebm	X					4(2)	3(1)	2(1)	
Gm	5(1)	8(9)				4(1)	7(1)		
Bm	4(2)					X			
F#m	4(1)					8(5)	9(1)		
C#m	X					3(3)			
Em	8(3)	5(1)	6(1)			1(1)	2(4)	3(2)	
G#m	8(1)					X			
Major Chords	Major to minor root movements (occurrences)					Major to major root movements (occurrences)			
F	4(2)	8(1)				10(2)	5(1)		
C/7	3(2)	1[1]	4[1]	7(2)	9(4)	10[1]			
Eb	4(3]	3(3)	1(3)			4(1)	2(1)	9[3]	6(1)
G	8[1]	6[1]				X			
D	X					2(1)			
Bb	4(3)	6[1]				1(1)	5(1)		
B	5(1)					X			
Db	X					6(1)			
A#	X					6(1)			
Fb	X					5(1)			
A	7(1)					1(2)			

Fig. 2: Chord transition analysis in *Chanson einer Dame im Schatten*

There is a distinct drawback with this approach, however. The interaction of the combinations of the two UTTs, as our triadic space allows, is not being considered. Thus, there may a pair of UTTs that better represent the harmonic space than the ones computed above, because the combination of the two UTTs is more common, and better describes a wider range of transitions.

Pitch class	Minor to major root movement	Inverse pitch class	Major to minor root movement	Sum
1	0	11	0	0
2	1	10	0	1
3	4	9	4	8
4	3	8	2	5
5	1	7	3	4
6	1	6	2	3
7	0	5	1	1
8	15	4	9	24
9	0	3	5	5
10	1	2	0	1
11	0	1	4	4

(a) Mode-reversing (highest values high-lighted).

Pitch class	Minor to minor root movement	Major to major root movement	Sum	Sum (movement, movement inverse)
1	1	3	4	4
2	6	2	8	15
3	11	0	11	15
4	7	1	8	13
5	4	3	7	8
6	0	3	3	
7	1	0	1	
8	5	0	5	
9	1	3	4	
10	4	3	7	
11	0	0	0	

(b) Mode-preserving

Fig. 3: Root movements of all chord transitions found in Song 1, summed by pitch class equivalence.

Advantage: Computer Instead of adding up the mode-preserving and mode-reversing UTTs separately and considering a space that combines the two arbitrarily, it may be more useful to consider pairs of UTTs from a subgroup of UTTs which has certain beneficial properties. Considering inverse pitch-class transformations to be equivalent may not be necessary, if we utilize a *simply transitive* subgroup of UTTs, and consider UTTs only in that group. The advantage of this, is that for any transition between a pair of triads, only one UTT there is only one possible transformation. We consider the group K(1,1) as defined by Hook, shown previously in Figure 1. To select a pair of UTTs from the K(1,1) we look to the shortest-path algorithm. Intuitively, the pair of UTTs that best represent a sequence of triads should have the shortest cumulative distance between each pair of triads in the sequence. This is a purely computational challenge. We have a list of UTTs provided. So, for every UTT, we must do the following procedure:

1. Loop through every UTT that is not identical to the current UTT.

2. Create the 2-dimensional UTT space that represents all the triads that can possibly be visited given the two current transformations.
   a. Ensure that the created space contains every triad in the set of all major and minor triads.
3. Step through the triads in the given piece, and compute the shortest distance between each pair of triads, sequentially.
4. Add up the total transformations that were required for every triadic transition in the piece.

Performing this on a piece would find the UTT pairs that represent the minimum required transformations for every triad transition in the piece. Surely, this must be the best harmonic space to represent the piece. A potential problem arises when computing every possible UTT space in the K(1,1) subgroup. As mentioned previously, not every UTT pair will create the entire set of possible K(1,1) UTT members, and thus the entire set of major and minor triads. When combining two UTTs to create the space, there need be a step in the process to evaluate the chord coverage of a generated space. For this reason, step $2a$ is critical in evaluating eligible spaces.

Returning to Nyman, every possible space created from pairs in the K(1,1) subgroup was computed for each piece, and the distances between each triadic transition computed. For each song, there is a set of UTT pairs that have identical total distances for a given song—those that are equivalent in terms of their possible inverse transformations. The best UTT pair can simply be represented by choosing the pairs with the smaller interval values for each song. For example, in *Chanson einer Dame im Schatten*, the UTT pair of [(<-, 4, 5>, 1, A), (<+, 2, 2>, 1, B)] was chosen instead of [(<-, 4, 5>, 1, A), (<+, 10, 10>, 1, B)] or [(<-, 7, 8>, 1, A), (<+, 2, 2>, 1, B)], since they all had the same cumulative distance. The results for the three analyzed Nyman songs are:

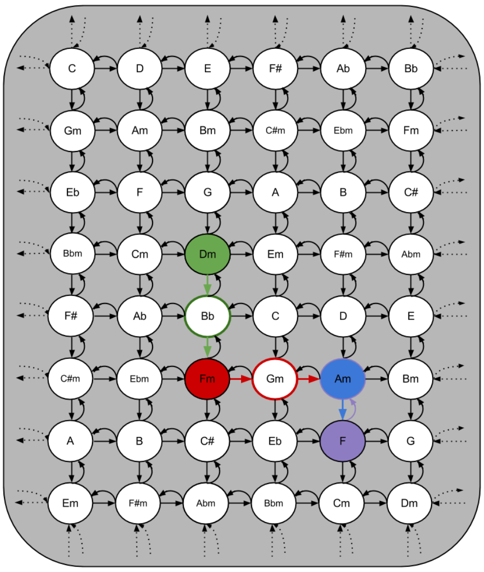
Song Title	Optimal UTT Pair	Dist.
Chanson einer Dame im Schatten	(<-, 4, 5>, 1, A), (<+, 2, 2>, 1, B)	246
Es war Erde in ihnen	(<-, 1, 2>, 1, A), (<-, 2, 3>, 1, B)	172
Psalm	(<-, 1, 2>, 1, A), (<+, 2, 2>, 1, B)	107

5 Application and Investigation

The efficient spaces that have been discovered for each song can then be used to explore the transformational motion going on in each song. Take the first song, *Chanson einer Dame im Schatten*, for example. First, it is useful to also visualize the space. In Figure 4, the opening phrase of *Chanson einer Dame im Schatten* is shown with its transformations in the UTT space of [(<-, 4, 5>, 1, A), (<+, 2, 2>, 1, B)].

It is difficult to tell how effective this new space is at first glance. However, with deeper inspection, one can start to understand the effectiveness of representing the harmony in such a way. Take, for example, the ending of most phrases. Often, each chord phrase ends in two successive chords, often on the last

Fig. 4: UTT space of [(<-, 4, 5>, 1, A), (<+, 2, 2>, 1, B)], with the transformations of the opening chord sequence [Dm, Fm, Am, F, Am] shown in alternating colors.



two beats of the last bar of the phrase. The opening phrase defines this clearly with its F→Am movement. Given the quickness of the transition between these two chords, it seems that an accurate harmonic space would likely find these chords to be close in harmonic distance. Grouping each of these quick transitions with their corresponding transformation(s) show that this is, indeed, the case (Figure 5).

Two things are interesting about the transformations that the phrase-ending chord transitions create. For one, they are mostly a single transformation. The transitions with a single transformation also represent single transformations in *both* axes of the UTT space (i.e., there exists both a single 'A' transformation, as well as a single 'B' transformation), which implies that both UTTs are effective. Secondly, these transformations occur from different starting points, showing that it is not just the same chord sequence occurring repeatedly in the song.

The space created from *Psalm*, namely [(<-, 1, 2>, 1, A), (<+, 2, 2>, 1, B)] has a common UTT with the space from *Chanson einer Dame im Schatten*. Also, because the UTT space for *Es war Erde in ihnen* is necessarily an approximation, it is more useful to consider the space of *Psalm*, and to investigate the commonalities between the space of *Psalm* and *Chanson einer Dame im Schatten*. One of the most striking characteristics of the transformations of the chord transitions in *Psalm* is the combination of three identical transformations. For example, for a single chord transition in the song, the combination of three 'B'

Fig. 5: Table of quick, phrase-ending chord transitions with their corresponding transformations

Start Chord	End Chord	Transformation
F	Am	A
Am	C	$B^{-1}\vert A$
Eb	Gm	A
Gm	Eb	A^{-1}
Eb	F	B
Dm	F	$B^{-1}\vert A$
Am	Em	A^{-1}
Bb	Dm	A

(<+, 2, 2>) transformations in sequence, or the inversely equivalent transformation of three ‘B^{-1}’ transformations in sequence occurs a total of 8 times in the song. Quite similarly, the total number of times the combination of three ‘A’ transformations (‘A|A|A’) or three ‘A^{-1}’ transformations (‘$\text{A}^{-1}|\text{A}^{-1}|\text{A}^{-1}$’) occurs is also 8. This is very convincing evidence that the UTT space is appropriate, since similar movements in both axes are equally likely. Furthermore, simple movements of ‘A’ and ‘A^{-1}’ occur a total sum of 13 times. However, the one exception is the simple movement of a single ‘B’ or a single ‘B^{-1}’ transformation, both of which are completely missing from the song.

It might make sense to consider changing the UTT space to instead use a UTT that represents the application of three consecutive ‘B’ transformations. That is, instead of using the unrepresented single transformation of <+, 2, 2>, <+, 6, 6> could theoretically be used instead. The problem here is that the space of [(<-, 1, 2>, 1, A), (<+, 6, 6>, 1, B)] is an incomplete triadic space, where not all triads are represented.

Still, there is a very distinct motion of three consecutive, identical transformations in both the ‘A’ and ‘B’ axes for the UTT space [(<-, 1, 2>, 1, A), (<+, 2, 2>, 1, B)] in *Psalm*. To what could this be ascribed? Comparing the UTT spaces of *Psalm* and *Chanson einer Dame im Schatten* illuminates the mystery. Let us consider the UTT created when applying the UTT of <-, 1, 2> three times consecutively. When starting on a major chord, the root movement will be 1, then 2 (since the mode was reversed), then 1 again, for a total of 4. Starting on a minor chord, the root movement will be 2, then 1, then 2, for a total of 5, with the mode changing three times (which is equivalent to one reversal). The resulting UTT is exactly that from *Chanson einer Dame im Schatten*, <-, 4, 5>!

6 Conclusion

These new methods for transforming triads and intervals provide a new perspective on the possible harmonic spaces in which songs may have been composed.

Furthermore, computational models of harmonic spaces provide immense power when considering multiple spaces. Commonly, the approach for utilizing these new transformational spaces has been a top-down approach, where the musicologist would identify what she thought were the most important harmonic movements, and shape the harmonic space around those. The computational method, on the other hand, provides a bottom-up approach, where every transition is considered when deciding on an optimal space. However, one must not rely entirely on this new tool set; still, there is a need for the application of musical intuition. As was evident in the analysis of the first three of Nyman's *Six Celan Songs*, the shortest distance algorithm found the most efficient harmonic space for each song. In the end, it was still up to the analyst to investigate the possible consequences of the identified spaces.

There is no doubt that the computer can serve music theorists for representing complex and abstract concepts. The current state of computer technologies affords a wide range of tools for the application of a computational approach to even the most cutting-edge of musical theories. It is with these tools that one can gain a new perspective on the music, and lead music analysts to new theories and innovative insights.

The code for the shortest path algorithm and the generation of UTT spaces can be found at Ryan Groves' github landing page at http://www.github.com.

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