ECS230 Applied Numerical Linear Algebra 2016-02-29

Homework4 due 2016-03-14 at 23:55

PageRank algorithm and the power method

The following problems refer to the paper by K. Bryan and T. Leise, SIAM Review 48, pp. 569-581 (2006), hereafter noted [1].

1) Write a proof of the following statement: If λ is an eigenvalue of a real square matrix A, it is also an eigenvalue of the matrix A^T. Give a proof of all statements you make.

2) Write a program that reads a column-stochastic link matrix and computes the eigenvector associated to the eigenvalue $\lambda = 1$. The link matrix is specified by an input file with the format

n
n1 i1 i2 ... i_n1
n2 i1 i2 ... i_n2
...

where n is the total number of nodes in the network, and each subsequent line describes the non-zero entries in a column. The first number on line j is the number n_j of non-zero entries in the column. The numbers $i_1, i_2, \ldots, i_{n_j}$ are the row indices of the non-zero entries. All non-zero elements in column j of the link matrix are set to the value $1/n_j$, consistently with Ref. [1]. It is assumed that there are no dangling nodes. The program should use the power iteration to compute the dominant eigenvector. Iterations should stop when individual elements of the normalized eigenvector change by less than 1.0e-6.

3) Use the program to reproduce the eigenvector associated with Figure 1 of Ref. [1] and verify that the result corresponds to the values given at the top of p. 572.

4) Consider the strategy described in Exercise 1 of [1]. Use the program to determine whether the strategy improves the score of node 3. Comment on your results.

5) Consider a network formed by a linear chain of length n in which each node has a link to its two nearest neighbors (except for nodes 1 and n, which only have one link to their only neighbor). Write the link matrix for n=4. Use the program to compute the eigenvector associated with the eigenvalue $\lambda = 1$. The results depend on the initial vector used. Try the initial vectors (1,1,1,1) and (1,2,3,4). Why are the results different? Discuss your observations.

6) Discuss how the problem observed in 5) may be solved using a shifted power method. How should the shift be chosen? Implement the shifted power method by modifying your program and compute the dominant eigenvector. Discuss your results.

7) For situations such as 5), discuss the relative merits of the shifted power method applied to the link matrix and the power method discussed in Ref[1] using the matrix S. How would these methods compare (in terms of convergence rate) in the limit of large n for the linear chain.

Attach source files for all programs and provide your answers in a pdf file on SmartSite.