

MR Brain Image Segmentation Using an Enhanced Fuzzy C-Means Algorithm

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Abstract—This paper presents a new algorithm for fuzzy segmentation of MR brain images. Starting from the standard FCM [1] and its bias-corrected version BCFCM [2] algorithm, by splitting up the two major steps of the latter, and by introducing a new factor γ , the amount of required calculations is considerably reduced. The algorithm provides good-quality segmented brain images a very quick way, which makes it an excellent tool to support virtual brain endoscopy. This research has been supported by the Hungarian National Research Fund, Grants No. OTKA T042990 and T029830.

Keywords—MR Imaging, Image Segmentation, Fuzzy Logic

I. INTRODUCTION

The standard fuzzy C-means algorithm provides a segmentation of brain images, but it performs no filtering, so the image quality remains poor. The enhanced version BCFCM [2] introduced a filtering inside the cyclical optimization problem, which led to better image quality, but the performance in time is very slow. The main goal of this paper is to reduce the amount of calculations effectuated during the segmentation process, to provide a high-speed good-quality segmentation of MR brain images.

II. METHODOLOGY

The standard FCM algorithm, introduced by Bezdek et al. in [1], groups the values x_k , $k=1..n$ into c clusters, using the objective function

$$J_B = \sum_{i=1}^c \sum_{k=1}^N u_{ik}^p (x_k - v_i)^2, \quad (1)$$

where v_i represents the prototype value of the i th cluster, u_{ik} represents the fuzzy membership of the k th voxel with respect to cluster i , and p is a weighting exponent. By

definition, for any k we have $\sum_{i=1}^c u_{ik} = 1$. To minimize the

objective function, it is necessary to assign high membership values to those voxels, whose intensities are situated close to the prototype values of their particular clusters.

Ahmed et al. proposed a modification to the original objective function by introducing a term that allows the labeling of a voxel to be influenced by the labels in its

immediate neighborhood [2]. This effect acts as a regularizer, and biases the solution toward piecewise-homogeneous labeling. It proved useful in segmenting images corrupted by salt and pepper noise. The modified objective function is given by

$$J_A = \sum_{i=1}^c \sum_{k=1}^N \left[u_{ik}^p (x_k - v_i)^2 + \frac{\alpha}{N_k} \sum_{r=1}^{N_k} u_{ik}^p (x_{k,r} - v_i)^2 \right], \quad (2)$$

where $x_{k,r}$ represents the neighbor voxels of x_k , and N_k stands for the number of voxels in the neighborhood of the k th voxel. The parameter α controls the intensity of the neighboring effect.

In the followings, we will introduce some modifications to this algorithm. MR brain images are stacks of approximately 200 slices, which at their turn represent large matrices of voxels. A set of MR brain image slices contains around ten million (10^7) voxels. The intensity of the voxels is generally encoded with 8 bit resolution, that is, there are only 256 possible levels of intensity for each voxel. To considerably reduce the amount of calculations performed during the segmentation process, we will modify the algorithm the following way.

Step 1. First we apply a local filtering to each voxel. Let us consider the neighborhood of the k th voxel, as described in [2]. Let us denote by ξ_k the filtered intensity level of the k th voxel, and we will compute it as follows:

$$\xi_k = \frac{1}{1+\alpha} \left(x_k + \frac{\alpha}{N_k} \sum_{r=1}^{N_k} x_{k,r} \right). \quad (3)$$

The voxel intensity levels are normalized, they are situated in the $[0,1]$ interval.

Step 2. Let us denote the number of intensity levels by q . As it was previously stated, q is much smaller than N . We denote by γ_l the number of voxels from the whole stack of slices, having the intensity equal to l , where $l=1..q$. By definition, we have $\sum_{l=1}^q \gamma_l = N$.

Step 3. The objective function used for the segmentation of the filtered signal will be:

$$J_S = \sum_{i=1}^c \sum_{l=1}^q \gamma_l u_{il}^p (\xi_l - v_i)^2. \quad (4)$$

We need to find those values of the parameters u_{il} and v_i , for which this objective function has the minimal value. Let us consider the Lagrange multiplier

$$F_S = \sum_{i=1}^c \sum_{l=1}^q [\gamma_l u_{il}^p (\xi_l - v_i)^2] + \sum_{l=1}^q \lambda_l \left(1 - \sum_{i=1}^c u_{il} \right). \quad (5)$$

Step 4. Taking the derivative of F_S with respect to u_{il} , and equaling it to 0, we get:

$$\frac{\delta F_S}{\delta u_{il}} = p \gamma_l u_{il}^{p-1} (\xi_l - v_i)^2 - \lambda_l = 0, \text{ so}$$

$$u_{il} = \left(\frac{\lambda_l}{p \gamma_l} \right)^{\frac{1}{p-1}} (\xi_l - v_i)^{\frac{-2}{p-1}}$$

From $\sum_{j=1}^c u_{jl} = 1$, we obtain

$$\lambda_l = p \gamma_l \left[\sum_{j=1}^c (\xi_l - v_j)^{\frac{-2}{p-1}} \right]^{1-p}, \text{ and so}$$

$$u_{il} = \left[\sum_{j=1}^c \left(\frac{\xi_l - v_i}{\xi_l - v_j} \right)^{\frac{2}{p-1}} \right]^{-1}. \quad (6)$$

Step 5. Taking the derivative of F_S with respect to v_i , and equaling it to 0, we get:

$$\frac{\delta F}{\delta v_i} = -2 \cdot \sum_{l=1}^q (\gamma_l u_{il}^p (\xi_l - v_i)) = 0, \text{ so}$$

$$v_i = \left(\sum_{l=1}^q \gamma_l u_{il}^p \xi_l \right) \left(\sum_{l=1}^q \gamma_l u_{il}^p \right)^{-1}. \quad (7)$$

The enhanced FCM algorithm for MR brain image segmentation can be summarized as follows:

- Determine the values of $\{\gamma_l\}_{l=1}^q$, select initial cluster prototypes $\{v_i = (2i-1)/(2c)\}_{i=1}^c$.
- Update membership function values according to (6).
- Compute the new values for cluster prototypes according to (7).
- Repeat b.-c. until the Euclidean norm of the change of the prototype vector is smaller than a previously set small positive number ε .

III. RESULTS AND DISCUSSION

Medical applications generally use segmentation into three clusters, corresponding to background, gray matter, and white matter. Fig. 1-3. present a brain MRI example, an original image with 256 grey levels, and two segmented versions of the same image, one of them performed using the BCFCM algorithm [2], and the other with the proposed method.

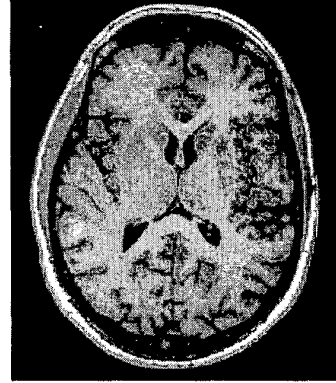


Fig. 1. Original MR brain image



Fig. 2. Segmented brain image (BCFCM)

The parameters introduced in the previous section, namely p , α , N_k , largely influence the efficiency of the algorithm. For example, if the exponent p is smaller than 1, the algorithm will not converge at all. BCFCM algorithm uses $p = 2$, which slightly simplifies the calculations, but this value does not assure the quickest convergence. Fig. 5. shows the relation between the objective functions J_A (with $p = 2$) and J_S (with $p = 1.2$). It can be seen, that after two or more cycles $|J_A - J_{A_{\min}}| / |J_S - J_{S_{\min}}| \approx 5$, which means

the proposed algorithm needs less cycles to get the same accuracy.

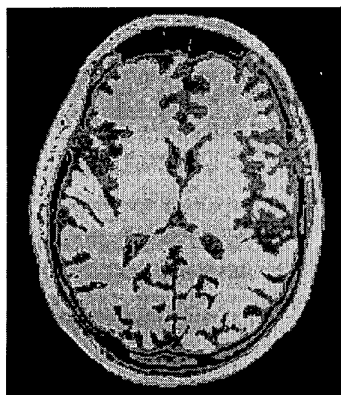


Fig. 3. Segmented brain image (proposed method)

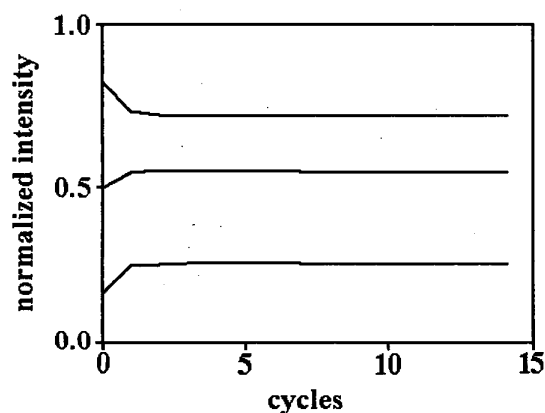


Fig. 4. Convergence of cluster prototypes

The quality of filtering depends on the chosen neighborhooding effect, and its intensity value α . The value of α has to be great enough so that it eliminates most of the salt and pepper noise, but it also has to be small enough, so that the image will not lose much of its sharpness. The optimal value of α varies between 0.5 and 1.2. The cardinality of the neighborhood taken in consideration for each voxel also influences the quality of the obtained image. The best results are obtained, if the neighborhood contains the 8 immediate neighbors of the voxel.

Because of the considerable difference between the number of voxels in an MR slice (or the whole brain volume) (N), and the number of grey intensity levels of the original image (q), the amount of calculation, that is needed to perform during each cycle, is reduced by the new method approximately 40 times.

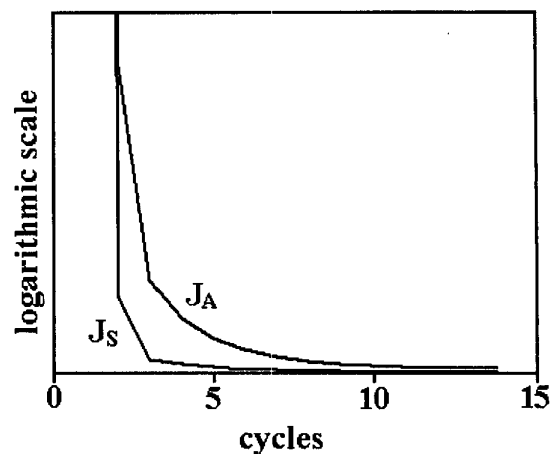


Fig. 5. Convergence of objective function

IV. CONCLUSION

The proposed algorithm provides a slight improvement in the quality of segmented brain images, and it performs significantly quicker than its ancestors. These make it a useful tool to support virtual brain endoscopy.

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