Project Gutenberg's Catalan's Constant [Ramanujan's Formula], by Greg Fee

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Produced by Greg Fee

Catalan's Constant [Ramanujan's Formula]

Catalan constant to 300000 digits computed on September 29, 1996 by using a Sun Ultra-Sparc in 1 day 8 hour 15 min 15 sec 55 hsec. The algorithm used is the standard series for Catalan, accelerated by an Euler transform. The algorithm was implemented using the LiDIA library for computational number theory and it is part of the multiprecision floating-point arithmetic of the package.

LiDIA is available from

ftp://crypt1.cs.uni-sb.de/pub/systems/LiDIA/LiDIA-1.2.1.tgz http://www-jb.cs.uni-sb.de/LiDIA/linkhtml/lidia/lidia.html

The implementation of the algorithm is:

```
inline void
const_catalan (bigfloat & y)
{
  bigfloat p;
  bigfloat t;
  int i = 1, j = 3; // j = 2*i+1

// y = t = p = 1/2 divide (y, 1, 2); t.assign (y); p.assign (y);
```

```
// while t is greater than the desired accuracy
while (!t.is_approx_zero ())
{
    // do
    // p = p * (i/j);
    // t = (t * i + p) / j;
    // y = y + t;
    // i++; j+=2;
    multiply (p, p, i);
    divide (p, p, j);
    multiply (t, t, i);
    add (t, t, p);
    divide (t, t, j);
    add (y, y, t);
    i++;
    j += 2;
}
```

Here is the output of the program:

Calculating Catalan's constant to 300000 decimals

Time required: 1 day 8 hour 15 min 15 sec 55 hsec

Additional REFERENCES:

Catalan constant is: $sum((-1)^{**}(n+1)/(2^*n-1)^{**}2,n=1..infinity)$ also known under the name beta(2), see ?catalan in Maple for more details.

The previous record was 200000 digits, also from Thomas Papanikolaou and before that: 100000 digits was due to Greg Fee and Simon Plouffe on August 14, 1996, by using a SGI r10000 Power Challenge with 194 Mhz in 5.63 hours and the standard implementation of Catalan on MapleV, Release 4. (which uses Greg's idea).

Euler Tranform: References, Abramowitz and Stegun, formula 3.6.27 page 16 in Handbook of Mathematical Functions and Tables, Dover 1964.

Ramanujan Notebooks, part I formula 34.1 of page 293.

The series used is by putting $x \rightarrow -1/2$. In other words the formula used is: the ordinary formula for Catalan

```
sum((-1)**(n+1)/(2*n+1)**2,n=0..infinity)
```

and then you apply the Euler Transform to it.

Computation of Catalan's constant using Ramanujan's Formula, by Greg Fee, ACM 1990, Proceedings of the ISAAC conference, 1990, p. 157.

Catalan constant to 300000 digits

 $580594253162330504135345441243730679356290951594486527411441001762026150749539\\ 779523450850067140195326700687372683342304951807483713651358549671866675099013\\ 829351728309200638642485984342014228695482705430932134583565849346307176696114\\ 645527408685396309531779809839339467562597483727150015110318725104115044737409\\ 656404094693565178949164702736819227759379694799077728370813738643592423723643\\ 432568999414277671149482737656695972837510170120731021202853085289585580262839\\ 009038589909633411329463088278787461042577068922850898191537791785946970991510\\ 951691571196251071871059641004166660572567197085658059074859369510151513239134\\ 7010467539889561309527815738732772683974390361306712262666274315817155149983369\\ 255232611835020702025334380328028447809317987826912290482102307728881699091027\\ 238217624823814141166942609035417856997584043316823744116630923803965016670561\\ 130179842030708619496906015870141452658258605005599198260333820012987567795243\\ 3428285033676$

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