Project Gutenberg's The Golden Mean or Ratio [(1+sqrt(5))/2], by Anonymous

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Title: The Golden Mean or Ratio [(1+sqrt(5))/2] to 20,000 places

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\*\*\* START OF THIS PROJECT GUTENBERG EBOOK THE GOLDEN MEAN \*\*\*

The Golden Mean [Golden Ratio] (1+sqrt(5))/2 to 20000 places.

This etext also contains these mathematical items:

Catalan evaluation
170000 digits of gamma or Euler constant
The Artin's Constant
The Backhouse Constant
Zeta(3) or Apery Constant
Zeta(1,2) or the derivative of Zeta function at 2
Feigenbaum reduction parameter
Feigenbaum bifurcation velocity constant
Franson-Robinson constant
The Gauss-Kusmin-Wirsing constant
Khinchin constant
Landau-Ramanujan constant
The twin primes constant
The Lengyel constant
The Levy constant

The golden ratio: (1+sqrt(5))/2 to 20000 places.

As calculated by Greg Fee using Maple Release 3 standard Catalan evaluation. This implementation uses 1 bit/term series of Ramanujan. Calculated on April 25 1996 in approx. 10 hours of CPU on a SGI R4000 machine.

To do the same on your machine just type this.

> catalan := evalf(Catalan,50100):

bytes used=37569782748, alloc=5372968, time=38078.95

here are the 50000 digits (1000 lines of 50 digits each).

It comes from formula 34.1 of page 293 of Ramanujan Notebooks, part I, the series used is by putting x—> -1/2 . In other words the formula used is: the ordinary formula for Catalan sum((-1)\*\* (n+1)/(2\*n+1)\*\*2,n=0...infinity) and then you apply the Euler Transform to it: ref: Abramowitz & Stegun page 16. The article of Greg Fee that took those formulas appear in Computation of Catalan's constant using Ramanujan's Formula, by Greg Fee, ACM 1990, Proceedings of the ISAAC conference, 1990 (MAYBE 1989), held in Tokyo.

170000 digits of gamma, as calculated from a value furnished by Jon Borwein. gamma or Euler constant is  $Lim(n-\sinh(1/k,k=1..n) - \log(n))$ 

The Artin's Constant.

= product(1-1/(p\*\*2-p),p=prime)

0.373955813619202288054728054346516415111629249

The Backhouse constant calculated by Philippe Flajolet INRIA Paris to 1300 places.

1.4560749485826896713995953511165435576531783748471315402707024 89794974645089663963739786981613361814875;

Zeta(3) or Apery constant to 2000 places.

1.2020569031595942853997381615114499907649862923404988817922715553418382057863

Zeta(1,2) of the derivative of Zeta function at 2.

 $-0.9375482543158437537025740945678649778978602886148299258854334803\\6044381131270752279368941514115151749311382116241638535059404171\\5961733247197185174912402688214443700163931015045107160373574873\\1352956057133552593318050514872534799984717397570317550302619073$ 

Feigenbaum reduction parameter

2.502907875095892822283902873218215786381271376727149977336192056

Feigenbaum bifurcation velocity constant

 $4.669201609102990671853203820466201617258185577475768632745651343\ 00413433021131473$ 

Fransen-Robinson constant.

2.80777024202851936522150118655777293230808592093019829122005

ref:

Math of Computation, vol 34 1980 pp 553-566 Math of Computation vol 37 1981 pp 233-235

The Gauss-Kuzmin-Wirsing constant.

0.303663002898732658597448121901

Khinchin constant to 1024 digits.

2.685452001065306445309714835481795693820382293994462953051152345 5572188595371520028011411749318476979951534659052880900828976777 1641096305179253348325966838185231542133211949962603932852204481 9409618068664166428930847788062036073705350103367263357728904990 4270702723451702625237023545810686318501032374655803775026442524 8528694682341899491573066189872079941372355000579357366989339508 7902124464207528974145914769301844905060179349938522547040420337 7985639831015709022233910000220772509651332460444439191691460859 6823482128324622829271012690697418234847767545734898625420339266 2351862086778136650969658314699527183744805401219536666604964826 9890827548115254721177330319675947383719393578106059230401890711 3496246737068412217946810740608918276695667117166837405904739368 8095345048999704717639045134323237715103219651503824698888324870 9353994696082647818120566349467125784366645797409778483662049777 7486827656970871631929385128993141995186116737926546205635059513 8571376169712687229980532767327871051376395637190231452890030581

Landau-Ramanujan constant calculated by Philippe Flajolet INRIA Paris and Paul Zimmermann

 $.76422365358922066299069873125009232811679054139340951472168667374961464165873\\285883840150501313123372193726912079259263418742064678084323063315434629380531\\605171169636177508819961243824994277683469051623513921871962056905329564467041\\917634977065956990571293866028938589982961051662960890991779298360729736972006\\403169851286365173473921065768550978681981674707359066921830288751501689624646\\710918081710618090086517493799082420450570666204898612757713333895484325083035\\682950407721597524121430942470953115765559404064229125772724071563491218723272\\555640889999512705135849728552347645942418505999635800934732669411548076911671\\455813028066898593167493626295259560163215843892463887558347193993864581698751\\045893518777945872755226448709943505595943671299977780669880564555921300690852\\242867691102264527531455816088116296997029876937094388422089495290791626363527\\791432286156863284215944899347183748322904155863814951281527102068249218645827\\978145098870379211809629840943604891233924014852514327407923660178532707078811$ 

The twin primes constant.

0.660161815846869573927812110014555778432623

The Lengyel constant.

1.09868580552518701

The Levy constant.

3.275822918721811159787681882453843863608475525982374149405198924 1907232156449603551812775404791745294926985262434016333281898085 1150341709970823046646564670370807129022418613959423772012981792 4251087697614930028806824926170594041290808697054441234922379888 4427089726916409835535854804837478582876252691844500764338370387 5993822298382263233915155285779861204684944725214872290148093612 0890454342092099519978162400515039051381538771475429139307394617 8045527838277727414847221513498826672911215448270452018633487650 01974618041826296536528801168151105751350871152301630294274883364626099362896606336377631721650883480193611175632669841114675716 4673788536609062436703325968873753578142654195992507552782054563 7159407017761632115357055923392379438429270901774392301637440604 1267849616074783099029968056083369047909232336111034927147379005 2538954916571473419972293167664630764119832764653881696204634629 7332994286181358520050377154266892424005731518557308952871781294

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