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Author: J. Malcolm Bird

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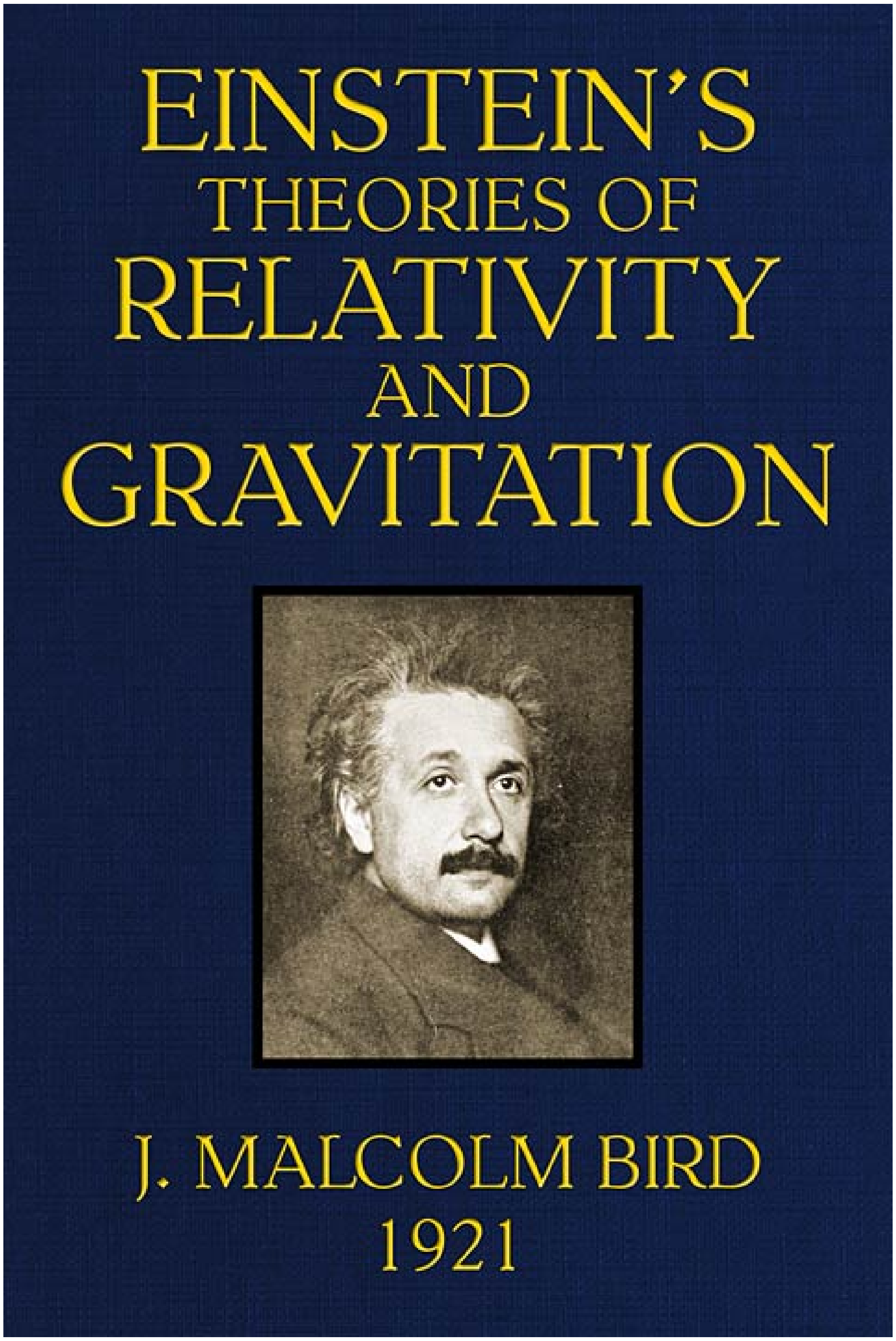
EINSTEIN’S
THEORIES OF
RELATIVITY
AND
GRAVITATION
J. MALCOLM BIRD
1921

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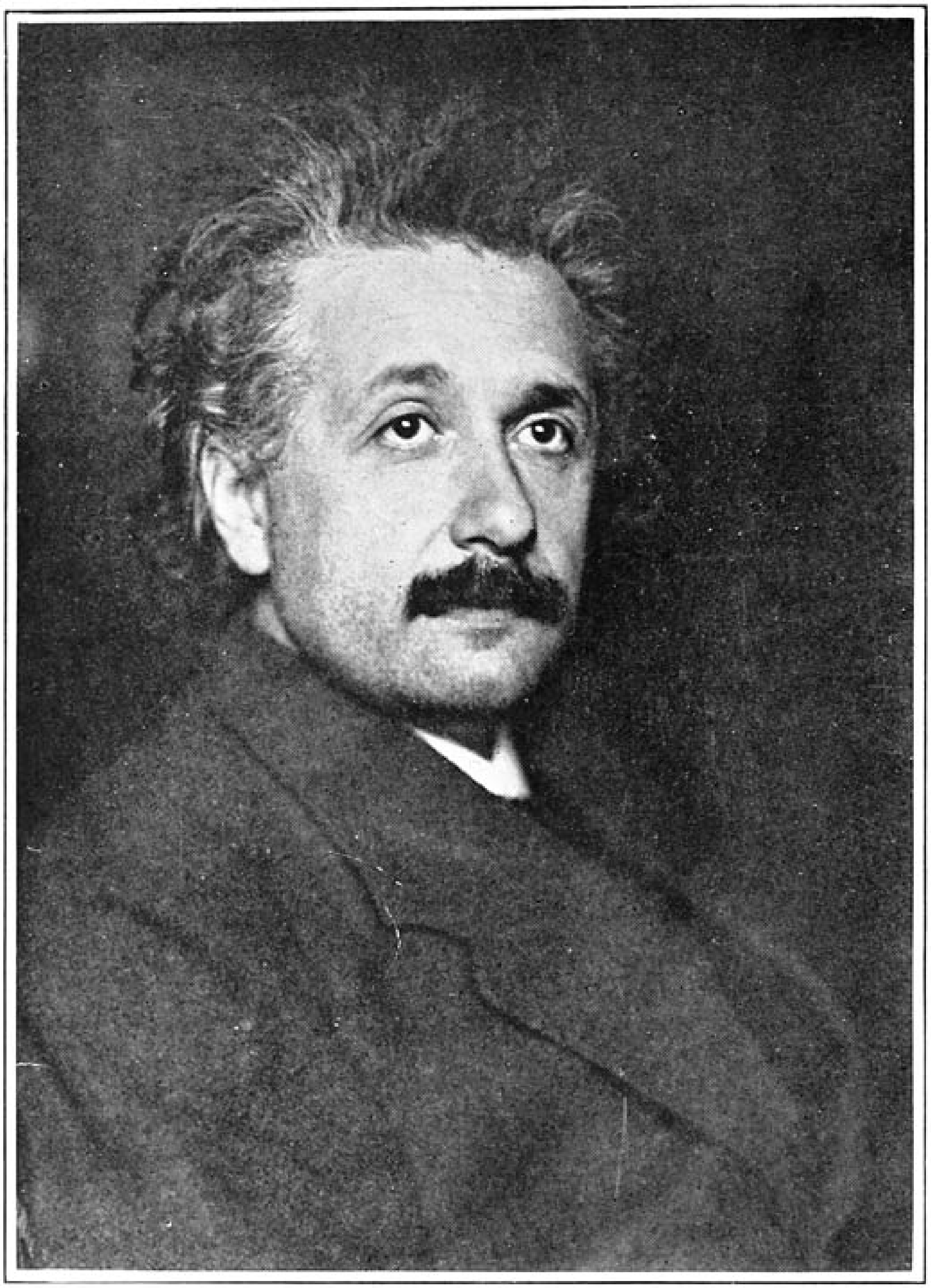

Dr. Albert Einstein,
Originator of the Special and General Theories of Relativity

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Einstein’s Theories of Relativity and Gravitation

A SELECTION OF MATERIAL FROM THE ESSAYS SUBMITTED IN THE COMPETITION FOR THE EUGENE HIGGINS PRIZE OF $5,000

COMPILED AND EDITED,
AND INTRODUCTORY MATTER SUPPLIED
BY
J. MALCOLM BIRD,
Associate Editor, Scientific American

NEW YORK
SCIENTIFIC AMERICAN PUBLISHING CO.,
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PREFACE

The obstacles which the layman finds to understanding Einstein's relativity theories lie not so much in the inherent difficulty of these theories themselves as in the difficulty of preparing the mind for their reception. The theory is no more difficult than any scientific development of comparable depth; it is not so difficult as some of these. But it is a fact that for a decent understanding of it, a large background of scientific knowledge and scientific habit of thought is essential. The bulk of the writers who have attempted to explain Einstein to the general reader have not realized the great gulf which lies between the mental processes of the trained mathematician and those of the man in the street. They have not perceived that the lay reader must be personally conducted for a long distance from the vestibule of the temple of science before he comes to Einstein, and that he cannot by any possibility make this journey unaided. The result has been to pitchfork the reader into the intricacies of the subject without adequate preparation.

The present volume avoids this mistake with the utmost care. It avoids it, in fact, with such deliberation as to make it in order to say a word in explanation of what will at first glance seem an extraordinary arrangement of material. It was to be expected, doubtless, that this book would open with a brief statement of the genesis and the outcome of the Einstein Prize Essay Contest for the [iv] $5,000 prize offered by Mr. Eugene Higgins. It was doubtless to be expected that, after this had been dismissed, the winning essay would be given the post of honor in advance of all other material bearing actually on the Einstein theories. When the reader observes that this has not been done, he will by all means expect a word of explanation; and it is mainly for the purpose of giving this that we make these introductory remarks.

The essays submitted in the contest, and in particular the comments of a few disappointed readers upon Mr. Bolton's prize essay, make quite plain what might have been anticipated—that in the small compass of 3,000 words it is not possible both to prepare the reader's mind for a discussion of Relativity and to give a discussion that shall be adequate. Mr. Bolton himself, in replying to a protest that he had not done all this, has used the word "miracle"—we think it a well-advised one. No miracle was expected as a result of the contest, and none has been achieved. But in awarding the prize, the Judges had to decide whether it was the best preliminary exposition or the best discussion that was wanted. They decided, and rightly we believe, that the award should go to an actual statement of what the Einstein theories are and what they do, rather than to a mere introduction, however well conceived and well executed the latter might be. Nevertheless, we should be closing our eyes to a very obvious fact if we did not recognize that, without something in the way of preparation, the general reader is not going to pursue Mr. Bolton's essay, or [v] any other essay on this subject, with profit. It is in order the more forcefully to hold out inducements to him to subject himself to this preparation that we place at the head of the book the chapters designed to give it to him.

Chapter II. is intended so to bring the mind of the reader into contact with certain philosophical problems presented to us by our experiences with the external world and our efforts to learn the facts about it, that he may approach the subject of relativity with an appreciation of the place it occupies as a phase of human thought and a pillar of the scientific structure. Until the reader is aware of the existence of these problems and the directions taken by the efforts, successful and unsuccessful, to unravel them, he is not equipped to comprehend the doctrine of relativity at all; he is in much the same case as a child whose education had reached only the primer stage, if asked to read the masterpieces of literature. He lacks not alone the vocabulary, but equally the mental background on which the vocabulary is based.

It will be noted that in this and the chapters immediately following it, the Editor has supplied material freely. The obvious interpretation is that satisfactory material covering the desired ground was not found in any of the essays; for we are sure the scope and number of the credited excerpts will make it clear that all contributions were adequately scrutinized in search of available passages. This "inadequacy" of the competing essays has been severely commented upon by several correspondents, and the inference drawn that as a whole the offerings were not up to the mark. Such a viewpoint is [vi] wholly unjust to the contestants. The essays which paid serious attention to the business of paving the way to relativity necessarily did so at the expense of completeness in the later paragraphs where specific explanation of the Einstein theories was in order. Mr. Law, whose essay was by all means the best of those that gave much space to introductory remarks, found himself left with only 600 words in which to tell what it was that he had been introducing. The majority of the contestants appear to have faced the same question as to subject matter which the Judges faced, and to have reached the same decision. They accordingly devoted their attention toward the prize, rather than toward the production of an essay that would best supplement that of the winner. It is for this very reason that, in these preliminary chapters, so large a proportion of the material has had to be supplied by the Editor; and this very circumstance is a tribute to the good judgment of the competitors, rather than ground for criticism of their work.

The general introduction of Chapter II. out of the way, Chapters III. and IV. take up the business of leading the reader into the actual subject of relativity. The subject is here developed in what may be called the historical order—the order in which it took form in Einstein's own mind. Both in and outside the contest of which this book is the outcome, a majority of those who have written on relativity have followed this order, which is indeed a very natural one and one well calculated to give to the rather surprising assumptions of relativity a reasonableness which they might well appear to the lay mind to [vii] lack if laid down more arbitrarily. In these two chapters no effort is made to carry the argument beyond the formulation of the Special Principle of the relativity of uniform motion, but this principle is developed in considerably more detail than would be the case if it were left entirely to the competing essayists. The reason for this is again that we are dealing with a phase of the subject which is of subordinate importance so far as a complete statement of the General Theory of Relativity is concerned, but which is of the greatest significance in connection with the effort of the layman to acquire the proper preliminary orientation toward the larger subject.

Chapter V. goes back again to general ground. Among the ideas which the competing essayists were forced to introduce into their text on a liberal scale is that of non-Euclidean geometry. The entire formulation of the General Theory of Relativity is in fact an exercise in this. The essayists—good, bad and indifferent alike—were quite unanimous in their decision that this was one thing which the reader would have to assume the responsibility of acquiring for himself. Certainly they were justified in this; for the Editor has been able to explain what non-Euclidean geometry is only by using up considerably more space than the contestants had for an entire essay. No effort has been made to set forth any of the details of any of the various non-Euclidean geometries; it has simply been the aim to draw the dividing line between Euclidean and non-Euclidean, and to make the existence of the latter appear reasonable, so that [viii] when the essayists come to talk about it the reader will not feel hopelessly at sea. In other words, this is another case of providing the mental background, but on such a scale that it has seemed necessary to give a separate chapter to it.

Chapter VI. completes the preliminary course in the fundamentals of relativity by tying up together the findings of Chapter V. and those of Chapters III. and IV. It represents more or less of a last-minute change of plan; for while it had been the Editor's intent from the beginning to place the material of Chapters II.–V. in its present position, his preliminary impression would have been that the work of the present Chapter VI. would be adequately done by the essayists themselves. His reading of the essays, however, convinced him that it had not so been done—that with the possible exception of Mr. Francis, the essayists did not make either a serious or a successful effort to show the organic connection between the Special Theory of Relativity and the Minkowski space-time structure, or the utter futility of trying to reconcile ourselves to the results of the former without employing the ideas of the latter. So Chapter VI. was supplied to make good this deficiency, and to complete the mental equipment which the reader requires for his battle with the General Theory.

In laying down a set of general principles to govern the award of the prize, one of the first things considered by the Judges was the relative importance of the Special and the General Theories. It was their opinion that no essay could possibly qualify for the prize which did not very distinctly give to the General [ix] Theory the center of the stage; and that in fact discussion of the Special Theory was pertinent only so long as it contributed, in proportion to the space assigned it, to the attack upon the main subject. The same principle has been employed in selecting essays for complete or substantially complete reproduction in this volume. Writers who dealt with the Special Theory in any other sense than as a preliminary step toward the General Theory have been relegated to the introductory chapters, where such excerpts from their work have been used as were found usable. The distinction of publication under name and title is reserved for those who wrote consistently and specifically upon the larger subject—with the one exception of Dr. Russell, whose exposition of the Special Theory is so far the best of those submitted and at the same time so distinctive that we have concluded it will appear to better advantage by itself than as a part of Chapters III. and IV.

Following after Mr. Bolton's essay we have tried to arrange the various contributions, not at all in any order of merit, but in the order that will make connected reading of the book most nearly possible and profitable. Each essay should be made easier of reading by the examination of those preceding it; at the same time each, by the choice of ground covered and by the emphasis on points not brought out sharply by its predecessors, should throw new light upon these predecessors.

The reader will find that no two of the essays given thus in full duplicate or even come close to duplicating one another. They have of course been selected [x]
with this in view; each represents the best of several essays of substantially the same character. Not all of them require comment here, but concerning some of them a word may well be said.

Mr. Francis, we believe, has succeeded in packing more substance into his 3,000 words than any other competitor. Mr. Elliot has come closer than anybody else to really explaining relativity in terms familiar to everybody, without asking the reader to enlarge his vocabulary and with a minimum demand so far as enlarging his mental outlook is concerned. Were it not for certain conspicuous defects, his essay would probably have taken the prize. In justice to the Judges, we should state that we have taken the liberty of eliminating Mr. Elliot's concluding paragraph, which was the most objectionable feature of his essay.

Dr. Dushman chose for his title the one which we adopted for this book. It became necessary, therefore, for us to find a new title for his essay; aside from this instance, the main titles appearing at the heads of the various complete essays are those of the authors. The subtitles have in practically every instance been supplied editorially.

Dr. Pickering submitted two essays, one written from the viewpoint of the physicist, the other from that of the astronomer. To make each complete, he naturally found it necessary to duplicate between them certain introductory and general material. We have run the two essays together into a single narrative, with the elimination of this duplicated material; aside from this blue-penciling no alteration has been made in Dr. Pickering's text. This text however served [xi]
as the basis of blue-penciling that of several other contestants, as indicated in the foot notes.

For the reader who is qualified or who can qualify to understand it, Dr. Murnaghan's essay is perhaps the most illuminating of all. Even the reader who does not understand it all will realize that its author brings to the subject a freshness of viewpoint and an originality of treatment which are rather lacking in some of the published essays, and which it will readily be understood were conspicuously lacking in a good many of the unpublished ones. Dr. Murnaghan of all the competitors has come closest to making a contribution to science as well as to the semi-popular literature of science.

In the composite chapters, the brackets followed by reference numbers have been used as the most practicable means of identifying the various individual contributions. We believe that this part of the text can be read without allowing the frequent occurrence of these symbols to distract the eye. As to the references themselves, the asterisk marks the contributions of the Editor. The numbers are those attached to the essays in order of and at the time of their receipt; it has been more convenient to use these than to assign consecutive numbers to the quoted essays. The several numbers identify passages from the essays of the following contestants:

10: Frederick W. Shurlock, Derby, England.
18: L. L. Whyte, Cambridge, England.
24: Prof. Moritz Schlick, University of Rostock, Germany.
30: C. E. Rose, M.E., Little Rock, Ark.
33: H. Gartelmann, Bremen, Germany.
35: Prof. Joseph S. Ames, Johns Hopkins University, Baltimore.
47: James O. G. Gibbons, East Orange. N. J.
82: Charles H. Burr, Philadelphia.
101: L. F. H. de Miffonis. B.A., C.E., Ottawa, Canada.
102: Charles A. Brunn, Kansas City. [xii]
106: J. Elias Fries, Fellow A.I.E.E., Birmingham, Ala.
114: Dean W. P. Graham, Syracuse University, Syracuse, N. Y.
115: Rev. George Thomas Manley, London.
116: Prof. J. A. Schouten, Delft, Netherlands.
121: Elwyn F. Burrill, Berkeley, Cal.
125: Dorothy Burr, Bryn Mawr, Pa.
130: C. W. Kanolt, Bureau of Standards, Washington.
135: Robert Stevenson, New York.
139: Leopold Schorsch, New York.
141: Dr. M. C. Mott-Smith, Los Angeles, Calif.
147: Edward A. Clarke, Columbus, O.
149: Edward A. Partridge, Philadelphia.
150: Col. John Millis, U. S. A., Chicago.
152: George F. Marsteller, Detroit.
156: D. B. Hall, Cincinnati.
165: Francis Farquhar, York, Pa.
178: Dr. George de Bothezat, Dayton, O.
179: Professor A. E. Caswell, University of Oregon, Eugene, Ore.
182: C. E. Dimick, New London, Conn.
186: Earl R. Evans, Washington, D. C.
188: Norman E. Gilbert, Dartmouth College, Hanover, N. H.
192: A. d'Abro. New York.
194: L. M. Alexander, Cincinnati.
197: Kenneth W. Reed, East Cleveland, O.
198: Prof. E. N. da C. Andrade, Ordnance College, Woolwich, England.
216: Professor Andrew H. Patterson, University of North Carolina, Chapel Hill, N. C.
220: Prof. Arthur Gordon Webster, Clark College, Worcester, Mass.
221: Walter van B. Roberts, Princeton University, N. J.

223: Paul M. Batchelder, Austin, Tex.

227: Prof. R. W. Wood, Johns Hopkins University, Baltimore.

229: E. P. Fairbairn, M.C., B.Sc., Glasgow.

231: R. F. Deimel, Hoboken, N. J.

232: Lieut. W. Mark Angus, U. S. N., Philadelphia.

235: Edward Adams Richardson, Kansas City.

263: Prof. William Benjamin Smith, Tulane University, New Orleans.

264: James Rice, University of London, London.

267: William Hemmenway Pratt, Lynn, Mass.

272: R. Bruce Lindsay, New Bedford, Mass.

283: Frank E. Law, Montclair, N. J.

In addition to the specific credit given by these references for specifically quoted passages, the Editor feels that he ought to acknowledge his general indebtedness to the competing essayists, collectively, for the many ideas which he has taken away from their text to clothe in his own words. This does not mean that the Editor has undertaken generally to improve upon the language of the competitors, but merely that the reading of all their essays has given him many ideas of such complex origin that he could not assign credit if he would. [xiii]

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THE EINSTEIN $5,000 PRIZE

How the Contest Came to be Held, and Some of the Details of its Conduct

BY THE EDITOR

In January, 1909, an anonymous donor who was interested in the spread of correct scientific ideas offered through the Scientific American a prize of $500 for the best essay explaining, in simple non-technical language, that paradise of mathematicians and bugaboo of plain ordinary folk—the fourth dimension. Many essays were submitted in this competition, and in addition to that of the winner some twenty were adjudged worthy of ultimate publication. It was felt that the competition had added distinctly to the popular understanding of this significant subject; that it had done much to clear up popular misconception of just what the mathematician means when he talks of four or even more dimensions; and that it had therefore been as successful as it was unusual in character.

In November, 1919, the world was startled by the announcement from London that examination of the photographs taken during the total solar eclipse of May 29th had been concluded, and that predictions based upon the Einstein theories of relativity had been verified. In the reaction from the long surfeit of war news an item of this sort was a thoroughly journalistic one. Long cable dispatches were carried in the news columns all over the world; Einstein and [2] his theories were given a prominent place on the front pages day after day; leading scientists in great number were called upon to tell the public through the reportorial medium just what the excitement was all about, just in what way the classical scientific structure had been overthrown.

Instead of being a mere nine days’ wonder, the Einstein theories held their place in the public mind. The more serious periodicals devoted space to them. First and last, a very notable group of scientific men attempted to explain to the general reader the scope and content of Einstein’s system. These efforts, well considered as they were, could be no more than partially successful on account of the very radical character of the revisions which the relativity doctrine demands in our fundamental concepts. Such revisions cannot be made in a day; the average person has not the viewpoint of the mathematician which permits a sudden and complete exchange of one set of fundamentals for another. But the whole subject had caught the popular attention so strongly, that even complete initial failure to discover what it was all about did not discourage the general reader from pursuing the matter with determination to come to some understanding of what had happened to Newton and Newtonian mechanics.

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The Donor and the Prize

In May, 1920, Mr. Eugene Higgins, an American citizen long resident in Paris, a liberal patron of the arts and sciences, and a lifelong friend of the Scientific American and its proprietors, suggested that the success of the Fourth [3] Dimension Prize Contest of 1910 had been so great that it might be desirable to offer another prize in similar fashion for the best popular essay on the Einstein theories. He stated that if in the opinion of the Scientific American these theories were of sufficient importance, and the probability of getting a good number of meritorious essays were sufficiently great, and the public need and desire for enlightenment were sufficiently present, he would feel inclined to offer such a prize, leaving the conduct of the contest to the Scientific American as in the former event. It was the judgment of the editors of the Scientific American that all these provisos should be met with an affirmative, and that Mr. Higgins accordingly could with propriety be encouraged to offer the prize.

In his preliminary letter Mr. Higgins had suggested that in view of the apparent greater importance of the subject to be proposed for discussion by the contestants of 1920, the prize offered should probably be more liberal than in the former instance. This view met with the approval of the editors as well; but they were totally unprepared for the receipt, late in June, of a cablegram from Mr. Higgins stating that he had decided to go ahead with the matter, and that he was forwarding a draft for $5,000 to represent the amount of the prize. Such a sum, exceeding any award open to a professional man with the single exception of the Nobel Prize, for which he cannot specifically compete, fairly took the breath of the Editors, and made it immediately clear that the contest [4] would attract the widest attention, and that it should score the most conspicuous success. It also made it clear that the handling of the contest would be a more serious matter than had been anticipated.

In spite of the fact that it would not for some time be possible to announce the identity of the Judges, it was felt that the prospective contestants should have every opportunity for extensive preparation; so the contest was announced, and the rules governing it printed as far as they could be determined on such short shrift, in the Scientific American for July 10, 1920. Several points of ambiguity had to be cleared up after this initial publication. In particular, it had been Mr. Higgins' suggestion that in the very probable event of the Judges' inability to agree upon the winning essay, the prize might, at their discretion, be divided between the contributors of the best two essays. This condition was actually printed in the first announcement, but the Post Office Department insisted upon its withdrawal, on the ground that with it in force the contestant would not know whether he were competing for $5,000 or for $2,500, and that this would introduce the "element of chance" which alone was necessary, under the Federal statutes, to make the contest a lottery. So this provision was replaced by one to the effect that in the event the Judges were not able to agree, the Einstein Editor should cast the deciding vote between the essays respectively favored by them.

The announcement attracted the widest attention, and was copied in newspapers and magazines all over the world. Inquiries poured in from all quarters, and the Einstein Editor found it almost impossible to keep himself [5] supplied with proofs of the conditions and rules to mail in response to these inquiries. It was immediately clear that there was going to be a large number of essays submitted, and that many distinguished names would be listed among the competitors.

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The Judges

In the Scientific American for September 18, announcement was carried in the following words:

"We are assured with complete certainty that the competition for the five-thousand-dollar prize will be very keen, and that many essays will be submitted which, if they bore the names of their authors, would pass anywhere as authoritative statements. The judges will confront a task of extraordinary difficulty in the effort to determine which of these efforts is the best; and we believe the difficulties are such that multiplication of judges would merely multiply the obstacles to an agreement. It is altogether likely that the initial impressions of two or three or five judges would incline toward two or three or five essays, and that any final decision would be attainable only after much consultation and discussion. It seems to us that by making the committee as small as possible while still preserving the necessary feature that its decision represent a consensus, we shall simplify both the mental and the physical problem of coming to an agreement. We believe that the award should if possible represent a unanimous decision, without any minority report, and that such a requirement is far more likely to be met among two men than among [6] three or five. At the same time, the bringing together of two men and the details of general administration of their work together are far simpler than if there were three or five. So we have finally decided to have but two judges, and in this we have the endorsement of all the competent opinion that we have consulted.

"The gentlemen who have consented to act as Judges are Professors Leigh Page and Edwin Plimpton Adams, of the departments of physics of Yale and Princeton Universities, respectively. Both are of the younger generation of physicists that has paid special attention to those phases of mathematics and physics involved in the Einstein theories, and both have paid special attention to these theories themselves. We are gratified to be able to put forward as Judges two men so eminently qualified to act. We feel that we may here appropriately quote Professor Page, who says in his acceptance: 'As the large prize offers a great inducement, I had thought of entering the contest. However I realize that not many people in this country have made a considerable study of Einstein's theory, and if all who have should enter the contest, it would be difficult to secure suitable Judges.' Without any desire to put the gentleman in the position of pleading for himself, we think this suggests very well the extent to which the Scientific American, the contestants, and the public at large, are indebted to Professors Page and Adams for their willingness to serve in the difficult capacity of Judges."

It might appropriately have been added to this announcement that it was altogether to the credit of science and the scientific spirit that the first two gentlemen approached with the invitation to act as Judges were willing to forego their prospects as contestants in order thus to contribute to the success of the contest. [7]

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Three Thousand Words

Of the conditions, the one which evoked most comment was that stating the word limit. This limit was decided upon after the most careful discussion of the possibilities of the situation. It was not imagined for a moment that any contestant would succeed in getting within 3,000 words a complete discussion of all aspects of the Special and the General Theories of Relativity. It was however felt that for popular reading a single essay should not be much if any longer than this. Moreover, I will say quite frankly that we should never have encouraged Mr. Higgins to offer such a prize if we had supposed that the winning essay was the only thing of value that would come from the contest, or if we had not expected to find in many of the other essays material which would be altogether deserving of the light. From the beginning we had in view the present volume, and the severe restriction in length was deliberately imposed for the purpose of forcing every contestant to stick to what he considered the most significant viewpoints, and to give his best skill to displaying the theories of Einstein to the utmost advantage from these viewpoints. We felt that divergent viewpoints would be more advantageously [8] treated in this manner than if we gave each contestant enough space to discuss the subject from all sides; and that the award of the prize to the essay which, among other requirements, seemed to the Judges to embody the best choice of material, would greatly simplify the working of the contest without effecting any injustice against those contestants who displayed with equal skill less happily chosen material. Perhaps on this point I may again quote with profit the editorial page of the Scientific American:

"An essay of three thousand words is not long enough to lose a reader more than once; if it does lose him it is a failure, and if it doesn't it is a competitor that will go into the final elimination trials for the prize. If we can present, as a result of the contest, six or a dozen essays of this length that will not lose the lay reader at all, we shall have produced something amply worth the expenditure of Mr. Higgins' money and our time. For such a number of essays of such character will of necessity present many different aspects of the Einstein theories, and in many different ways, and in doing so will contribute greatly to the popular enlightenment.

"Really the significant part of what has already appeared is not the part that is intelligible, but rather the part that, being unintelligible, casts the shadow of doubt and suspicion on the whole. The successful competitor for the prize and his close contestants will have written essays that, without any claim to completeness, will emphasize what seems to each author the big outstanding feature; and every one of them will be intelligible. Together they will in all probability be reasonably complete, and will retain the individual characteristic of intelligibility. They will approach the various parts of the field from various directions—we could fill this page with suggestions as to how the one item of the four-dimensional character of Einstein's time-space might be set forth for the general reader. And when a man must say in three thousand words as much as he can of what eminent scientists have said in whole volumes—well, the result in some cases will be sheer failure, and in others a product of the first water. The best of the essays will shine through intelligent selection of what is to be said, and brilliant success in saying it. It is to get a group of essays of this character, not to get the single essay which will earn the palm, that the prize is offered." [9]

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The Competing Essays

At all times after the first announcement the Einstein Editor had a heavy correspondence; but the first real evidence that the contest was under way came with the arrival of the first essay, which wandered into our office in the middle of September. About a week later they began to filter in at the rate of one or two per day—mostly from foreign contestants who were taking no chances on the mails. Heavy returns did not commence until about ten days before the closing date. The great avalanche, however, was reserved for the morning of Monday, November 1st. Here we had the benefit of three days' mail; there were about 120 essays. Among those which were thrown out on the ground of lateness the honors should no doubt go to the man who mailed his [10] offering in The Hague on October 31st.

Essays were received in greater quantity from Germany than from any other foreign country, doubtless because of the staggering value of $5,000 when converted into marks at late 1920 rates. England stood next on the list; and one or more essays were received from Austria, Czechoslovakia, Jugoslavia, France, Switzerland, the Netherlands, Denmark, Italy, Chile, Cuba, Mexico, India, Jamaica, South Africa and the Fiji Islands. Canada, of course, contributed her fair share; and few of our own states were missing on the roll-call.

The general level of English composition among the essays from non-English-speaking sources was about what might have been expected. A man may have a thorough utilitarian knowledge of a foreign tongue, but when he attempts intensive literary competition with a man who was brought up in that tongue he is at a disadvantage. We read French and German with ease and Spanish and Italian without too much difficulty, ourselves; we should never undertake serious writing in any of these languages. Not many of the foreign contributions, of course, were as ludicrous as the one we quote to some extent in our concluding chapter, but most of them were distinctly below par as literary compositions. Drs. De Sitter and Schlick were the notable exceptions to this; both showed the ability to compete on a footing of absolute equality with the best of the native product.

We dare say it was a foregone conclusion that many essays should have been over the limit, and that a few should have been over it to the point of absurdity. The winning essay contains 2,919 words, plus or minus a reasonable allowance for error in counting; that it should come so far from being on the ragged edge should be sufficient answer to those who protested against the severity of the limitation. One inquirer, by the way, wanted to know if 3,000 words was not a misprint for 30,000. Another contestant suggested that instead of disqualifying any essay that was over the line, we amputate the superfluous words at the end. This was a plausible enough suggestion, since any essay able to compete after such amputation must necessarily have been one of extreme worth; but fortunately we did not have to decide whether we should follow the scheme. Perhaps twenty of the essays submitted were so seriously in excess of the limit that it was not even necessary to count their words in detail; most of these offenders ran to 3,500 words or thereabouts, and one—a good one, too, from which we use a good deal of material in this volume—actually had 4,700. On the other extreme were a few competitors who seemed to think that the shortest essay was necessarily the best, and who tried to dismiss the subject with 500 or 1,000 words. [11]

By a curious trick of chance there were submitted in competition for the prize exactly 300 essays. Of course a few of these did not require serious consideration—this is inevitable in a contest of such magnitude. But after excluding all the essays that were admittedly not about the Einstein theories at all, and all those whose English was so execrable as to make them quite out of the question, and all those which took the subject so lightly as not to write reasonably close to the limit of 3,000 words, and all those which were given over to explanation of the manner in which Einstein's theories verify those of the writer, and all those in which the writer attempted to substitute his own cosmic scheme for Einstein's—after all this, there remained some 275 essays which were serious efforts to explain in simple terms the nature and content and consequences of Special and General Relativity. [12]

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Looking for the Winner

The Einstein Editor was in sufficiently close touch with the details of the adjudication of the essays to have every realization of the difficulty of this work. The caliber of the essays submitted was on the whole high. There were many which would have been well worthy of the prize in the absence of others that were distinctly better—many which it was not possible to eliminate on the ground of specific faults, and which could only be adjudged "not the best" by detailed comparison with specific other essays. It was this detailed comparison which took time, and which so delayed the award that we were not able to publish the winning essay any sooner than February 5th. Especially difficult was this process of elimination after the number of surviving essays had been reduced to twenty or less. The advantages of plan possessed by one essay had to be weighed against those of execution exhibited in another. A certain essay had to be critically compared with another so like it in plan that the two might have been written from a common outline, and at the same time with a third as unlike it in scope and content as day and night. And all the time there was [13] present in the background the consciousness that a prize of $5,000 hung upon the decision to be reached. For anyone who regards this as an easy task we have no worse wish than that he may some day have to attack a similar one.

We had anticipated that the bulk of the superior essays would be among those received during the last day or two of the contest; for we felt that the men best equipped to attack the subject would be the most impressed with its seriousness. Here we were quite off the track. The seventeen essays which withstood most stubbornly the Judges' efforts at elimination were, in order of receipt, numbers 8, 18, 28, 40, 41, 43, 92, 95, 97, 130, 181, 194, 198, 223, 267, 270, 275: a fairly even distribution. The winner was the 92nd essay received.

The Judges held their final meeting in the editorial office on January 18, 1921. The four essays which were before the committee at the start of the session were speedily cut to three, and then to two; and after an all-day session the Judges found themselves conscientiously able to agree on one of these as the best. This unanimity was especially gratifying, the more so since it by no means was to be confidently expected, on *a priori* grounds, that it would be possible of attainment. Even the Einstein Editor, who might have been called upon for a final decision but wasn't, can hardly be classed as a dissenter; for with some slight mental reservations in favor of the essay by Mr. Francis [14] which did not enter the Judges' final discussion at all, and which he rather suspects appeals more to his personal taste than to his soundest judgment, he is entirely in accord with the verdict rendered.

The fact that the prize went to England was no surprise to those acquainted with the history of Einstein's theories. The Special Theory, promulgated fifteen years ago, received its fair share of attention from mathematicians all over the world, and is doubtless as well known and as fully appreciated here as elsewhere. But it has never been elevated to a position of any great importance in mathematical theory, simply because of itself, in the absence of its extension to the general case, it deserves little importance. It is merely an interesting bit of abstract speculation.

The General Theory was put out by Einstein in finished form during the war. Owing to the scientific moratorium, his paper, and hence a clear understanding of the new methods and results and of the sweeping consequences if the General Theory should prevail, did not attain general circulation outside Germany until some time in 1918 or even later. Had it not been for Eddington it is doubtful that the British astronomers would have realized that the eclipse expeditions were of particular consequence. Therefore at the time of these expeditions, and even as late as the November announcement of the findings, the general body of scientific men in America had not adequately realized the immense distinction between the Special and the General Theories, had not adequately appreciated that the latter led to distinctive consequences of any [15] import, and we fear in many cases had not even realized explicitly that the deflection of light and the behavior of Mercury were matters strictly of the General and in no sense of the Special Theory. Certainly when the American newspapers were searching frantically for somebody to interpret to their public the great stir made by the British announcement that Einstein's predictions had been verified, they found no one to do this decently; nor were our magazines much more successful in spite of the greater time they had to devote to the search. In a word, there is not the slightest room for doubt that American science was in large measure caught asleep at the switch—perhaps for no reason within its control; and that American writers were in no such favorable case to write convincingly on the subject as were their British and continental contemporaries.

So it was quite in accord with what might have been expected to find, on opening the identifying envelopes, that not alone the winning essay, but its two most immediate rivals, come from members of that school of British thought which had been in contact with the Einstein theories in their entirety for two years longer than the average American of equal competence. This riper familiarity with the subject was bound to yield riper fruit. Indeed, had it not been for the handicap of writing in a strange language, it is reasonable to assume that the scientists of Germany would have made a showing superior to that of either Americans or British—and for the same reason that Britain showed to better advantage than America. [16]

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The Winner of the Prize

Mr. Bolton, the winner of the big prize, we suppose may fairly be referred to as unknown in a strict scientific sense. Indeed, at the time of the publication of his essay in the Scientific American nothing could be learned about him on the American side of the water beyond the bare facts that he was not a young man, and that he had for a good many years occupied a position of rank in the British Patent Office. (It will be recalled that Einstein himself was in the Swiss Patent Office for some time.) In response to the request of the Scientific American for a brief biographical sketch that would serve to introduce him better to our readers, Mr. Bolton supplied such a concise and apparently such a characteristic statement that we can do no better than quote it verbatim.

"I was born in Dublin in 1860, but I have lived in England since 1869. My family belonged to the landed gentry class, but I owe nothing to wealth or position. I was in fact put through school and college on an income which a workman would despise nowadays. After attending sundry small schools, I entered Clifton College in 1873. My career there was checkered, but it ended well. I was always fairly good at natural science and very fond of all sorts of mechanical things. I was an honest worker but no use at classics, and as I did practically nothing else for the first four years at Clifton, I came to consider myself something of a dunce. But a big public school is a little world. Everyone gets an opportunity, often seemingly by accident, and it is up to him [17] to take it. Mine did not come till I was nearly 17. As I was intended for the engineering profession, I was sent to the military side of the school in order to learn some mathematics, at which subject I was then considered very weak. This was certainly true, as at that time I barely knew how to solve a quadratic, I was only about halfway through the third book of Euclid, and I knew no trigonometry. But the teaching was inspiring, and I took readily to mathematics. One day it came out that I had been making quite a good start with the differential calculus on my own without telling anybody. After that all was well. I left Clifton in 1880 with a School Exhibition and a mathematical scholarship at Clare College, Cambridge.

“After taking my degree in 1883 as a Wrangler, I taught science and mathematics at Wellington College, but I was attracted by what I had heard of the Patent Office and I entered it through open competition in 1885. During my official career I have been one of the Comptroller’s private secretaries and I am now a Senior Examiner. During the war I was attached to the Inventions Department of the Ministry of Munitions, where my work related mainly to anti-aircraft gunnery. I have contributed, and am still contributing to official publications on the subject.

“I have written a fair number of essays on various subjects, even on literature, but my only extra-official publications relate to stereoscopic photography. I read a paper on this subject before the Royal Photographic Society in 1903 which was favorably noticed by Dr. von Rohr of Messrs. Zeiss of Jena. I have [18] also written in the *Amateur Photographer*.

“I have been fairly successful at athletics, and I am a member of the Leander Club.”

That Mr. Bolton did not take the prize through default of serious competition should be plain to any reader who examines the text from competing essays which is to be found in this volume. The reference list of these competitors, too, supplemented by the names that appear at the heads of complete essays, shows a notable array of distinguished personalities, and I could mention perhaps a dozen more very well known men of science whose excellent essays have seemed a trifle too advanced for our immediate use, but to whom I am under a good deal of obligation for some of the ideas which I have attempted to clothe in my own language.

Before leaving the subject, we wish to say here a word of appreciation for the manner in which the Judges have discharged their duties. The reader will have difficulty in realizing what it means to read such a number of essays on such a subject. We were fortunate beyond all expectation in finding Judges who combined a thorough scientific grasp of the mathematical and physical and philosophical aspects of the matter with an extremely human viewpoint which precluded any possibility of an award to an essay that was not properly a popular discussion, and with a willingness to go to meet each other’s opinions that is rare, even among those with less ground for confidence in their own views than is possessed by Drs. Page and Adams. [19]

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II.

THE WORLD—AND US

An Introductory Discussion of the Philosophy of Relativity, and of the Mechanism of Our Contact with Time and Space

BY VARIOUS CONTRIBUTORS AND THE EDITOR

From a time beyond the dawn of history, mankind has been seeking to explain the universe. At first the effort did not concern itself further probably than to make a supposition as to what were the causes of the various phenomena presented to the senses. As knowledge increased, first by observation and later by experiment also, the ideas as to these causes passed progressively through three stages—the theological (the causes were thought to be spirits or gods); the metaphysical (the causes were thought in this secondary or intermediate stage to be some inherent, animating, energizing principles); and the scientific (the causes were finally thought of as simply mechanical, chemical, and magneto-electrical attractions and repulsions, qualities or characteristics of matter itself, or of the thing of which matter is itself composed.)

With increase of knowledge, and along with the inquiry as to the nature of causes, there arose an inquiry also as to what reality was. What was the essential nature of the stuff of which the universe was made, what was matter, what were things in themselves, what were the noumena (the realities), lying [20]
back of the phenomena (the appearances)? Gradually ideas explaining motion, force, and energy were developed. At the same time inquiry was made as to the nature of man, the working of his mind, the nature of thought, the relation of his concepts (ideas) to his perceptions (knowledge gained through the sense) and the relations of both to the noumena (realities).][283]

[The general direction taken by this inquiry has been that of a conflict between two schools of thought which we may characterize as those of absolutism and of relativism.]* [The ancient Greek philosophers believed that they could tap a source of knowledge pure and absolute by sitting down in a chair and reasoning about the nature of time and space, and the mechanism of the physical world.][221] [They maintained that the mind holds in its own right certain concepts than which nothing is more fundamental. They considered it proper to conceive of time and space and matter and the other things presented to their senses by the world as having a real existence in the mind, regardless of whether any external reality could be identified with the concept as ultimately put forth. They could even dispute with significance the qualities which were to be ascribed to this abstract conceptual time and space and matter. All this was done without reference to the external reality, often in defiance of that reality. The mind could picture the world as it ought to be; if the recalcitrant facts refused to fit into the picture, so much the worse for them. We all have heard the tale of how generation after generation of Greek philosophers disputed learnedly why and how it was that a live fish could be [21]
added to a brimming pail of water without raising the level of the fluid or increasing the weight; until one day some common person conceived the troublesome idea of trying it out experimentally to learn whether it were so—and found that it was not. True or false, the anecdote admirably illustrates the subordinate place which the externals held in the absolutist system of Greek thought.]*

[Under this system a single observer is competent to examine a single phenomenon, and to write down the absolute law of nature by referring the results to his innate ideas of absolute qualities and states. The root of the word absolute signifies "taking away," and in its philosophical sense the word implies the ability of the mind to subtract away the properties or qualities from things, and to consider these abstract qualities detached from the things; for example, to take away the coldness from ice, and to consider pure or abstract coldness apart from anything that is cold; or to take away motion from a moving body, and to consider pure motion apart from anything that moves. This assumed power is based upon the Socratic theory of innate ideas. According to this theory the mind is endowed by nature with the absolute ideas of hardness, coldness, roundness, equality, motion, and all other absolute qualities and states, and so does not have to learn them. Thus a Socratic philosopher could discuss pure or absolute being, absolute space and absolute time.][121]

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Getting Away from the Greek Ideas

[22] [This Greek mode of thought persisted into the late Middle Ages, at which time it was still altogether in order to dispose of a troublesome fact of the external world by quoting Aristotle against it. During the Renaissance, which intellectually at least marks the transition from ancient to modern, there came into being another type of absolutism, equally extreme, equally arbitrary, equally unjustified. The revolt against the mental slavery to Greek ideas carried the pendulum too far to the other side, and early modern science as a consequence is disfigured by what we must now recognize as gross materialism. The human mind was relegated to the position of a mere innocent bystander. The external reality was everything, and aside from his function as a recorder the observer did not in the least matter. The whole aim of science was to isolate and classify the elusive external fact. The rôle of the observer was in every possible way minimized. It was of course his duty to get the facts right, but so far as any contribution to these was concerned he did not count—he was definitely disqualified. He really played the part of an intruder; from his position outside the phenomena he was searching for the absolute truth about these phenomena. The only difference between his viewpoint and that of Aristotle was that the latter looked entirely inside himself for the elusive "truth," while the "classical" scientist, as we call him now, looked for it entirely outside himself.

Let me illustrate the difference between the two viewpoints which I have discussed, and the third one which I am about to outline, by another concrete instance. [23] The Greeks, and the medievals as well, were fond of discussing a question which embodies the whole of what I have been saying. This question involved, on the part of one who attempted to answer it, a choice between the observer and the external world as the seat of reality. It was put in many forms; a familiar one is the following: "If the wind blew down a great tree at a time and place where there was no conscious being to hear, would there be any noise?" The Greek had to answer this question in the negative because to him the noise was entirely a phenomenon of the listener. The classical scientist had to answer it in the affirmative because to him the noise was entirely a phenomenon of the tree and the air and the ground. Today we answer it in the negative, but for a very different reason from that which swayed the Greek. We believe that the noise is a *joint phenomenon* of the observer and the externals, so that in the absence of *either* it must fail to take existence. We believe there are sound waves produced, and all that; but what of it? There is no noise in the presence of the falling tree and the absence of the observer, any more than there would be in the presence of the observer and the absence of the tree and the wind; the noise, a joint phenomenon of the observer and the externals, exists only in their joint presence.

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Relativism and Reality

This is the viewpoint of relativism. The statue is golden for one observer and silver to the other. The sun is rising here and setting in another part of the world. It is raining here and clear in Chicago. The observer in Delft hears the [24] bombardment of Antwerp and the observer in London does not. If they were to be consistent, both the Greek and the medieval-modern absolutist would have to dispute whether the statue were "really" golden or silver, whether the sun were "really" rising or setting, whether the weather were "really" fair or foul, whether the bombardment were "really" accompanied by loud noises or not; and on each of these questions they would have to come to an agreement or confess their methods inadequate. But to the relativist the answer is simple—whether this or that be true *depends upon the observer*. In simple cases we understand this full well, as we have always realized it. In less simple cases we recognize it less easily or not at all, so that some of our thought is absolutist in its tendencies while the rest is relativistic. Einstein is the first ever to realize this fully—or if not this, then the first ever to realize it so fully as to be moved toward a studied effort to free human thought from the mixture of relativism and absolutism and make it consistently the one or the other.

This brings it about that the observed fact occupies a position of unexpected significance. For when we discuss matters of physical science under a strictly relativistic philosophy, we must put away as metaphysical everything that smacks of a "reality" partly concealed behind our observations. We must focus attention upon the reports of our senses and of the instruments that supplement them. These observations, which join our perceptions to their external objects, afford us our only objective manifestations; them we must accept as final—subject always to such correction as more refined observations may suggest. [25] The question whether a "true" length or area or mass or velocity or duration or temperature exists back of the numerical determination, or in the presence of a determination that is subject to correction, or in the absence of any determination at all, is a metaphysical one and one that the physicist must not ask. Length, area, mass, velocity, duration, temperature—none of these has any meaning other than the number obtained by measurement.]* [If several different determinations are checked over and no error can be found in any of them, the fault must lie not with the observers but with the object, which we must conclude presents different values to different observers.][33]

[We are after all accustomed to this viewpoint; we do not demand that Pittsburgh shall present the same distance from New York and from Philadelphia, or that the New Yorker and the Philadelphian come to any agreement as to the "real" distance of Pittsburgh. The distance of Pittsburgh depends upon the position of the observer. Nor do we demand that the man who locates the magnetic pole in one spot in 1900 and in another in 1921 come to a decision as to where it "really" is; we accept his statement that its position depends upon the time of the observation.

What this really means is that the distance to Pittsburgh and the position of the magnetic pole are joint properties of the observer and the observed—relations between them, as we might put it. This is obvious enough in the case of the distance of Pittsburgh; it is hardly so obvious in the case of the position of the magnetic pole, varying with the lapse of time. But if we reflect that the observation of 1900 and that of 1921 were both valid, and both represented the true position of the pole for the observer of the date in question, we must see that this is the only explanation that shows us the way out. [26]

I do not wish to speak too definitely of the Einstein theories in these introductory remarks, and so shall refrain from mentioning explicitly in this place the situation which they bring up and upon which what I have just said has direct bearing. It will be recognized when it arises. What must be pointed out here, however, is that we are putting the thing which the scientist calls the "observed value" on a footing of vastly greater consequence than we should have been willing offhand to concede to it. So far as any single observer is concerned, his own best observed values are themselves the external world; he cannot properly go behind the conditions surrounding his observations and speak of a real external world beyond these observations. Any world which he may think of as so existing is purely a conceptual world, one which for some reason he infers to exist behind the deceptive observations. Provided he makes this reservation he is quite privileged to speculate about this concealed world, to bestow upon it any characteristics that he pleases; but it can have no real existence for him until he becomes able to observe it. The only reality he knows is the one he can directly observe.

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Laws of Nature

The observations which we have been discussing, and which we have been trying to endow with characteristics of "reality" which they are frequently not realized to possess, are the raw material of physical science. The finished product is the result of bringing together a large number of these observations.]* [The whole underlying thought behind the making of observations, in fact, is to correlate as many as possible of them, to obtain some generalization, and finally to express this in some simple mathematical form. This formulation is then called a "law of nature."][35] [27]

[Much confusion exists because of a misunderstanding in the lay mind of what is meant by a "law of nature." It is perhaps not a well chosen term. One is accustomed to associate the word law with the idea of necessity or compulsion. In the realm of nature the term carries no such meaning. The laws of nature are man's imperfect attempts to explain natural phenomena; they are not inherent in matter and the universe, not an iron bar of necessity running through worlds, systems and suns. Laws of nature are little more than working hypotheses, subject to change or alteration or enlargement or even abandonment, as man's vision widens and deepens. No sanctity attaches to them, and if any one, or all, of them fail to account for any part, or all, of the phenomena of the universe, then it or they must be supplemented or abandoned.][102]

[The test of one of these laws is that it can be shown to include all the related phenomena hitherto known and that it enables us to predict new phenomena which can then be verified. If new facts are discovered that are not in agreement with one of these generalized statements, the assumptions on which [28] the latter is based are examined, those which are not in accordance with the new facts are given up, and the statement is modified so as to include the new facts.][10] [And if one remembers that the laws of physics were formerly based on a range of observations much narrower than at present available, it seems natural that in the light of this widening knowledge one law or another may be seen to be narrow and insufficient. New theories and laws do not necessarily disprove old ones, but explain certain discrepancies in them and penetrate more deeply into their underlying principles, thereby broadening our ideas of the universe. To follow the new reasoning we must rid ourselves of the prejudice behind the old, not because it is wrong but because it is insufficient. The universe will not be distorted to fit our rules, but will teach us the rules of existence.][125]

[Always, however, we must guard against the too easy error of attributing to these rules anything like absolute truth.]* [The modern scientist has attained a very business-like point of view toward his "laws of nature." To him a law is fundamentally nothing but a short-hand way of expressing the results of a large number of experiments in a single statement. And it is important to remember that this mere shortening of the description of a lot of diverse occurrences is by no means any real *explanation* of how and why they happened. In other words, the aim of science is not ultimately to *explain* but only to discover the relations that hold good among physical quantities and to embody all these relations in as few and as simple physical laws as possible.][221] [This is inherently the [29] method of relativism.]* [Under it a set of phenomena is observed. There are two or many observers, and they write down their several findings. These are reviewed by a final observer or judge, who strains out the bias due to the different viewpoints of the original observers. He then writes down, not any absolute law of nature governing the observed phenomena, but a law as general as possible expressing their interrelations.][121] [And through this procedure modern science and philosophy reveal with increasing emphasis that we superimpose our human qualities on external nature to such an extent that][106] [we have at once the strongest practical justification, in addition to the arguments of reason, for our insistence that the contact between objective and subjective represented by the observation is the only thing which we shall ever be able to recognize as real. We may indulge in abstract metaphysical speculation to our heart's content, if we be metaphysically inclined; we may not attempt to impose the dicta of metaphysics upon the physical scientist.]*

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Concepts and Realities

[From the inquiry and criticism which have gone on for centuries has emerged the following present-day attitude of mind toward the sum total of our knowledge. The conceptual universe in our minds in some mysterious way parallels the real universe, but is totally unlike it. Our conceptions (ideas) of matter, molecules, atoms, corpuscles, electrons, the ether, motion, force, energy, space, and time stand in the same or similar relation to reality as the x's and y's of the mathematician do to the entities of his problem. Matter, [30] molecules, atoms, corpuscles, electrons, the ether, motion, force, energy, space, and time do not exist actually and really as we conceive them, nor do they have actually and really the qualities and characteristics with which we endow them. The concepts are simply representations of things outside ourselves; things which, while real, have an essential nature not known to us. Matter, molecules, atoms, corpuscles, electrons, the ether, motion, force, energy, space and time are merely devices, symbols, which enable us to reason about reality. They are parts of a conceptual mechanism in our minds which operates, or enables our minds to operate, in the same sequence of events as the sequence of phenomena in the external universe, so that when we perceive by our senses a group of phenomena in the external universe, we can reason out what result will flow from the interaction of the realities involved, and thus predict what the situation will be at a given stage in the sequence.

But while our conceptual universe has thus a mechanical aspect, we do not regard the real universe as mechanical in its nature.][283] [This may be illustrated by a little story. Entering his friend's house, a gentleman is seized unawares from behind. He turns his head but sees nothing. His hat and coat are removed and deposited in their proper places by some invisible agent, seats and tables and refreshments appear in due time where they are required, all without any apparent cause. The visitor shivers with horror and asks his host for an explanation. He is then told that the ideas "order" and "regularity" are at [31] work, and that it is they who acquit themselves so well of their tasks. These ideas cannot be seen nor felt nor seized nor weighed; they reveal their existence only by their thoughtful care for the welfare of mankind. I think the guest, coming home, will relate that his friend's house is haunted. The ghosts may be kind, benevolent, even useful; yet ghosts they are. Now in Newtonian mechanics, absolute space and absolute time and force and inertia and all the other apparatus, altogether imperceptible, appearing only at the proper time to make possible a proper building up of the theory, play the same mysterious part as the ideas "order" and "regularity" in my story. Classical mechanics is haunted.][116]

[As a matter of fact, we realize this and do not allow ourselves to be imposed upon with regard to the true nature of these agencies.]* [We use a mechanistic terminology and a mechanistic mode of reasoning only because we have found by experience that they facilitate our reasoning. They are the tools which we find produce results. They are adapted to our minds, but perhaps it would be better to say that our minds are so constructed as to render our conceptual universe necessarily mechanical in its aspect in order that our minds may reason at all. Two things antithetic are involved—subject (our perceiving mind which builds up concepts) and object (the external reality); and having neither complete nor absolute knowledge of either, we cannot affirm which is more truly to be said to be mechanistic in its nature, though we may suspect that really neither is. We no longer think of cause and effect as dictated by inherent [32] necessity, we simply regard them as sequences in the routine of our sense-impressions of phenomena. In a word, we have at length grasped the idea that our notions of reality, at present at least, whatever they may become ultimately, are not absolute, but simply relative. We see, too, that we do not *explain* the universe, but only *describe* our perceptions of its contents.

The so-called laws of nature are simply statements of formulæ which resume or sum up the relationships and sequences of phenomena. Our effort is constantly to find formulæ which will describe the widest possible range of phenomena. As our knowledge increases, that is, as we perceive new phenomena, our laws or formulæ break down, that is, they fail to afford a description in brief terms of all of our perceptions. It is not that the old laws are untrue, but simply that they are not comprehensive enough to include all of our perceptions. The old laws are often particular or limiting instances of the new laws.][283]

[From what we have said of the reality of observations it follows that we must support that school of psychology, and the parallel school of philosophy, which hold that concepts originate in perceptions. But this does not impose so strong a restriction upon conceptions as might appear. The elements of all our concepts do come to us from outside; we manufacture nothing out of whole cloth. But when perception has supplied a sufficient volume of raw material, we may group its elements in ways foreign to actual occurrence in the perceptual world, and in so doing get conceptual results so entirely different [33] from what we have consciously perceived that we are strongly tempted to look upon them as having certainly been manufactured in our minds without reference to the externals. Of even more significance is our ability to abstract from concrete objects and concrete incidents the essential features which make them alike and different. But unlike the Greeks, we see that our concept of coldness is not something with which we were endowed from the beginning, but merely an abstraction from concrete experiences with concrete objects that have been cold.

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The Concepts of Space and Time

When we have formed the abstract ideas of coldness and warmth, and have had experience indicating that the occurrence of these properties varies in degree, we are in a position to form the secondary abstract notion covered by the word "temperature." When we have formed the abstract ideas of size and position and separation, we are similarly in a position to form a secondary abstraction to which we give the name "space." Not quite so easy to trace to its definite source but none the less clearly an abstraction based on experience, is our idea of what we call "time." None of us are deceived as to the reality of these abstractions.]* [We do not regard space as real in the sense that we regard a chair as real; it is merely an abstract idea convenient for the location of material objects like the chair.][198] [Nor do we regard time as real in this sense. Things occupy space, events occupy time; space and time themselves we realize are immaterial and unreal; space does not exist and time does not [34] happen in the same sense that material objects exist and events occur. But we find it absolutely necessary to have, among the mental machinery mentioned above as the apparatus by aid of which we keep track of the external world, these vessels for that world to exist in and move in.

Space and time, then, are concepts.]* [It is not strange, however, that when confronted with the vast and bewildering complexity of the universe and the difficulty of keeping separate and distinct in our minds our perceptions and conceptions, we should at times and as respects certain things project our conceptions illegitimately into the perpetual universe and mistake them for perceptions. The most notable example perhaps of this projection has occurred in the very case of space and time, most fundamental of all of our concepts. We got to think of these as absolute, as independent of each other and of all other things, and as always existing and continuing to exist whether or not we or anything else existed—space as a three-dimensional, uniform continuum, having the same properties in all directions; time as a one-dimensional, irreversible continuum, flowing in one direction. It is difficult to get back to the idea that space and time so described and defined are concepts merely, for the idea of their absolute existence is ingrained in us as the result probably of long ancestral experience.][283]

[Newton's definitions of course represent the classical idea of time and space. He tells us that "absolute, true and mathematical time flows in virtue of its own nature, uniformly and without reference to any external object;" and that [35] "absolute space, by virtue of its own nature and without reference to any external object, always remains the same and is immovable." Of course from modern standpoints it is absurd to call either of these pronouncements a definition; but they represent about as well as any words can the ideas which Newton had about time and space, and they make it clear enough that he regarded both as having real existence in the external world.

If space and time are to be the vessels of our universe, and if the only thing that really matters is measured results, it is plain enough that we must have, from the very beginning, means of measuring space and time. Whether we believe space and time to have real existence or not, it is obvious that we can measure neither directly. We shall have to measure space by measuring from one material object to another; we shall have to measure time by some similar convention based on events. We shall later have something further to say about the measurement of time; for the present we need only point out that]* [Newtonian time is measured independently of space; and the existence is presupposed of a suitable timekeeper.][10]

[The space of Galileo and Newton was conceived of as empty, except in so far as certain parts of it were occupied by matter. Positions of bodies in this space were in general determined by reference to][283] [a "coordinate system" of some kind. This is again something that demands a certain amount of discussion. [36]

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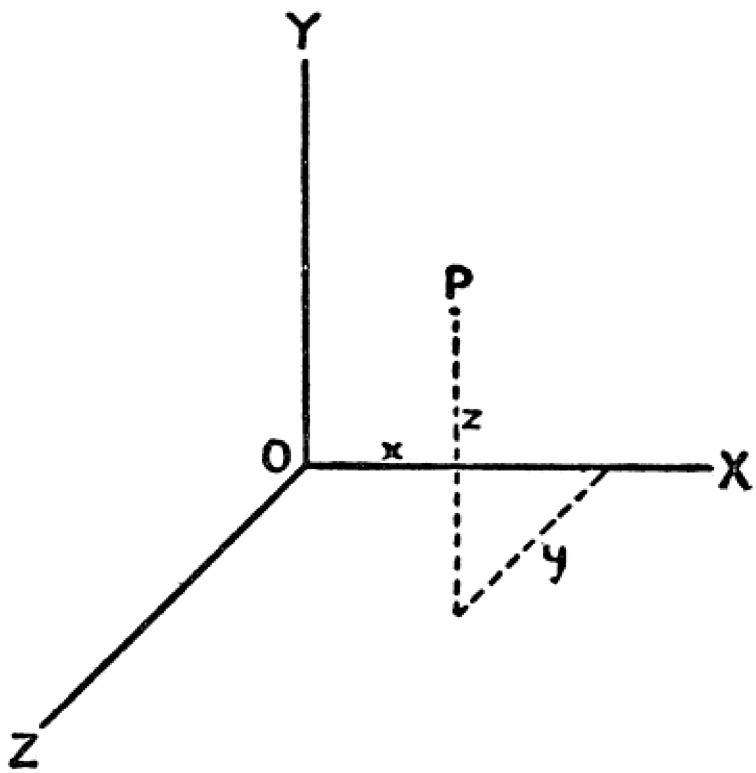
The Reference Frame for Space

The mathematician, following the lead of the great French all-around genius, Descartes, shows us very clearly how to set up, for the measurement of space, the framework known as the Cartesian coordinate system. The person of most ordinary mathematical attainments will realize that to locate a point in a plane we must have two measurements; and we could probably show this person, without too serious difficulty, that we can locate a point in *any* surface by two measurements. An example of this is the location of points on the earth's surface by means of their latitude and longitude. It is equally clear that if we add a third dimension and attempt to locate points in space, we must add a third measurement. In the case of points on the earth's surface, this might be the elevation above sea level, which would define the point not as part of the spherical surface of the earth but as part of the solid sphere. Or we may fall back on Dr. Slosson's suggestion that in order to define completely the position of his laboratory, we must make a statement about Broadway, and one about 116th Street, and one telling how many flights of stairs there are to climb. In any event, it should be clear enough that the complete definition of a point in space calls for three measurements.

The mathematician formulates all this with the utmost precision. He asks us to]* [pick out any point whatever in space and call it *O*. We then draw or conceive to be drawn through this point three mutually perpendicular lines called coordinate axes, which we may designate *OX*, *OY* and *OZ*, respectively. [37]
Finally, we consider the three planes also mutually perpendicular like the two walls and the floor of a room that meet in one common corner, which are formed by the lines *OX* and *OY*, *OY* and *OZ*, and *OZ* and *OX*, respectively. These three planes are called coordinate planes. And then any other point *P* in space can be represented with respect to *O* by its perpendicular distances from each of the three coordinate planes—the distances *x*, *y*, *z* in the figure. These quantities are called the coordinates of the point.][272]



[To the layman there seems something altogether naive in this notion of the scientist's setting up the three sides of a box in space and using them as the basis of all his work. The layman somehow feels that while it is perfectly all right for him to tell us that he lives at 1065 (one coordinate) 156th Street (two coordinates) on the third floor (three coordinates), it is rather trivial business for the serious-minded scientist to consider the up-and-down, the forward-and- [38]
back, the right-and-left of every point with which he has occasion to deal. There seems to the layman something particularly inane and foolish and altogether puerile about a set of coordinate axes, and you simply can't make

him believe that the serious-minded scientist has to monkey with any such funny business. He can't be induced to take this coordinate-axis business seriously. Nevertheless, the fact is that the scientist takes it with the utmost seriousness. It *is* necessary for him to define the positions of points; and he *does* do it by means of a set of coordinate axes.

The scientist, however, is not interested in points of empty space. The point is to him merely part again of the conceptual machinery which he uses in his effort to run along with the external world. He knows there are no real points, but it suits his convenience to keep track of certain things that *are* real by representing them as points. But these things are in practically every instance material bodies; and in practically every instance, instead of staying put in one spot, they insist upon moving about through space. The scientist has to use his coordinate system, not merely to define a single position of such a "point," but to keep track of the path over which it moves and to define its position in that path at given moments.

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Time and the Coordinate System

This introduces the concept of time into intimate relationship with the spatial coordinate system. And at once we feel the lack of a concrete, visualized [39] fourth dimension.]* [If we want to fix objects in the floor alone, the edge of the room running toward the ceiling would become unnecessary and could be dropped from our coordinate system. That is, we need only two coordinates to fix the position of a point in a plane. Suppose instead of discarding the third coordinate, we use it to represent units of time. It then enables us to record the *time* it took a moving point in the floor to pass from position to position. Certain points in the room would be vertically above the corresponding points occupied by the moving point in its path across the floor; and the vertical height above the floor of such points corresponds to a value of the time-coordinate which indicates the time it took the point to move from position to position.][152] [Just as the path of the point across the floor is a continuous curve (for the mathematician, it should be understood, this term "curve" includes the straight line, as a special case in which the curvature happens to be zero); so the series of points above these in the room forms a continuous curve which records for us, not merely the path of the point across the floor, but in addition the time of its arrival at each of its successive positions. In the algebraic work connected with such a problem, the third coordinate behaves exactly the same, regardless of whether we consider it to represent time or a third spatial dimension; we cannot even tell from the algebra what it does represent.

When we come to the more general case of a point moving freely through space, we have but three coordinates at our disposal; there is not a fourth one by aid of which we can actually diagram its time-space record. Nevertheless, [40] we can write down the numerical and algebraic relations between its three space-coordinates and the time which it takes to pass from one position to another; and by this means we can make all necessary calculations. Its motion is completely defined with regard both to space and to time. We are very apt to call attention to the fact that if we *did* have at our disposal a fourth, space-coordinate, we could use it to represent the time graphically, as before, and actually construct a geometric picture of the path of our moving point with regard to space and time. And on this account we are very apt to speak as though the time measurements constituted a fourth coordinate, regardless of any question of our ability to construct a picture of this coordinate. The arrival of a point in a given position constitutes an event; and this event is completely defined by means of four coordinates—three in space, which we can picture on our coordinate axes, and one in time which we cannot.

The set of coordinate axes in space, together with the zero point from which we measure time, constitute what we call a frame of reference. If we are not going to pay any attention to time, we can think of the space coordinate system alone as constituting our reference frame. This expression appears freely throughout the subsequent text, and always with one or the other of these interpretations.

We see, then, how we can keep track of a moving point by keeping track of the successive positions which it occupies in our reference frame.]* [Now we have implied that these coordinate axes are fixed in space; but there is nothing to [41]
prevent us from supposing that they move.][272] [If they do, they carry with them all their points; and any motion of these points which we may speak about will be merely motion with reference to the coordinate system. If we find something outside our coordinate system that is not moving, the motion of points in our system with regard to those outside it will be a combination of their motion with regard to our coordinate axes and that of these axes with regard to the external points. This will be a great nuisance; and it represents a state of affairs which we shall try to avoid. We shall avoid it, if at all, by selecting a coordinate system with reference to which we, ourselves, are not moving; one which partakes of any motion which we may have. Or perhaps we shall sometimes wish to reverse the process, in studying the behavior of some group of bodies, and seek a set of axes which is at rest with respect to these bodies; one which partakes of any motion *they* may have.

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The Choice of a Coordinate Frame

All this emphasizes the fact that our coordinate axes are not picked out for us in advance by nature, and set down in some one particular spot. We select them for ourselves, and we select them in the most convenient way. But different observers, or perhaps the same observer studying different problems, will find it advantageous to utilize different coordinate systems.]* [The astronomer has found it possible, and highly convenient, to select a coordinate frame such that the great majority of the stars have, on the whole, no motion [42]
with respect to it.][283] [Such a system would be most unsuited for investigations confined to the earth; for these we naturally select a framework attached to the earth, with its origin O at the earth's center if our investigation covers the entire globe and at some more convenient point if it does not, and in either event accompanying the earth in its rotation and revolution. But such a framework, as well as the one attached to the fixed stars, would be highly

inconvenient for an investigator of the motions of the planets; he would doubtless attach his reference frame to the sun.][101]

[In this connection a vital question suggests itself. Is the expression of natural law independent of or dependent upon the choice of a system of coordinates? And to what extent shall we be able to reconcile the results of one observer using one reference frame, and a second observer using a different one? The answer to the second question is obvious.]* [True, if any series of events is described using two different sets of axes, the descriptions will be different, depending upon the time system adopted and the relative motion of the axes. But if the connection between the reference systems is known, it is possible by mathematical processes to deduce the quantities observed in one system if those observed in the other are known.][35] [This process of translating the results of one observer into those of another is known as a transformation; and the mathematical statement of the rule governing the transformation is called the equation or the equations (there are usually several of them) of the transformation.]* [Transformations of this character constitute a well-developed branch of mathematics.][35] [43]

[When we inquire about the invariance of natural law it is necessary to be rather sure of just what we mean by this expression. The statement that a given body is moving with a velocity of 75 miles per hour is of course not a natural law; it is a mere numerical observation. But aside from such numerical results, we have a large number of mathematical relations which give us a more or less general statement of the relations that exist between velocities, accelerations, masses, forces, times, lengths, temperatures, pressures, etc., etc. There are some of these which we would be prepared to state at once as universally valid —distance travelled equals velocity multiplied by time, for instance. We do not believe that any conceivable change of reference systems could bring about a condition in which the product of velocity and time, as measured from a certain framework, would fail to equal distance as measured from this same framework. There are other relations more or less of the same sort which we probably believe to be in the same invariant category; there are others, perhaps, of which we might be doubtful; and presumably there are still others which we should suspect of restricted validity, holding in certain reference systems only and not in others.

The question of invariance of natural law, then, may turn out to be one which may be answered in the large by a single statement; it may equally turn out to be one that has to be answered in the small, by considering particular laws in connection with particular transformations between particular reference systems. Or, perhaps, we may find ourselves justified in taking the stand that [44] an alleged "law of nature" is truly such a law only in the event that it *is* independent of the change from one reference system to another. In any event, the question may be formulated as follows:

Observer A, using the reference system R, measures certain quantities t, w, x, y, z. Observer B, using the reference system S, measures the same items and gets the values t', w', x', y', z'. The appropriate transformation equations for calculating the one set of values from the other is found. If a mathematical relation of any sort is found to exist between the values t, w, x, y, z, will the same relation exist between the values t', w', x', y', z'? If it does not, are we justified in still calling it a law of nature? And if it does not, and we refrain from calling it such a law, may we expect in every case to find some relation that *will be* invariant under the transformation, and that may therefore be recognized as the natural law connecting *t*, *w*, *x*, *y* and *z*?

I have found it advisable to discuss this point in such detail because here more than in any other single place the competing essayists betray uncertainty of thought and sloppiness of expression. It doesn't amount to much to talk about the invariance of natural laws and their persistence as we pass from one coordinate system to another, unless we are fairly well fortified with respect to just what we mean by invariance and by natural law. We don't expect the velocity of a train to be 60 miles per hour alike when we measure it with respect to a signal tower along the line and with respect to a moving train on the other track. We don't expect the angular displacement of Mars to change as [45]
rapidly when he is on the other side of the sun as when he is on our side. But we do, I think, rather expect that in any phenomenon which we may observe, we shall find a natural law of some sort which is dependent for its validity neither upon the units we employ, nor the place from which we make our measurements, nor anything else external to the phenomenon itself. We shall see, later, whether this expectation is justified, or whether it will have to be discarded in the final unravelling of the absolutist from the relativistic philosophy which, with Einstein, we are to undertake.]* [46]

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III

THE RELATIVITY OF UNIFORM MOTION

Classical Ideas on the Subject; the Ether and the Apparent Possibility of Absolute Motion; the Michelson-Morley Experiment and the Final Negation of This Possibility

BY VARIOUS CONTRIBUTORS AND THE EDITOR

When we speak of a body as being "in motion," we mean that this body is changing its position "in space." Now it is clear that the position of an object can only be determined with reference to other objects: in order to describe the place of a material thing we must, for example, state its distances from other things. If there were no such bodies of reference, the words "position in space" would have no definite meaning for us.][24] [The number of such external bodies of reference which it is necessary to cite in order to define completely the position of a given body in space depends upon the character of the space dealt with. We have seen that when we visualize the space of our experience as a surface of any character, two citations are sufficient; and that when we conceive of it as surrounding us in three dimensions we require three. It will be realized that the mathematician is merely meeting this requirement when he [47]
sets up his system of coordinate axes to serve as a reference frame.]*

[What is true of "place" must be true also of "motion," since the latter is nothing but change of place. In fact, it would be impossible to ascribe a state of motion or of rest to a body poised all alone in empty space. Whether a body is to be regarded as resting or as moving, and if the latter at what speed, depends entirely upon the objects to which we refer its positions in space.][24] [As Einstein sits at his desk he appears to us to be at rest; but we know that he is moving with the rotation of the earth on its axis, with the earth in its orbit about the sun, and with the solar system in its path through space—a complex motion of which the parts or the whole can be detected only by reference to appropriately chosen ones of the heavenly bodies. No mechanical test has ever been devised which will detect this motion,][182] [if we reserve for discussion in its proper place the Foucault pendulum experiment which will reveal the axial rotation of our globe.]* [No savage, if he were to "stand still," could be convinced that he was moving with a very high velocity or in fact that he was moving at all.][30] [You drop a coin straight down a ship's side: from the land its path appears parabolic; to a polar onlooker it whirls circle-wise; to dwellers on Mars it darts spirally about the sun; to a stellar observer it gyrates through the sky][263] [in a path of many complications. To you it drops in a straight line from the deck to the sea.]* [Yet its various tracks in ship-space, sea-space, earth-space, sun-space, star-space, are all equally real,][263] [and the one which [48] will be singled out for attention depends entirely upon the observer, and the objects to which he refers the motion.]* [The earth moves in the solar system, which is itself approaching a distant star-cluster. But we cannot say whether we are moving toward the cluster, or the cluster toward us,][18] [or both, or whether we are conducting a successful stern chase of it, or it of us,]* [unless we have in mind some third body with reference to which the motions of earth and star-cluster are measured.][18] [And if we have this, the measurements made with reference to it are of significance with regard to it, rather than with regard to the earth and the star-cluster alone.]*

[We can express all this by saying "All motions are *relative*; there is no such thing as *absolute* motion." This line of argument has in fact been followed by many natural philosophers. But is its result in agreement with actual experience? Is it really impossible to distinguish between rest and motion of a body if we do not take into consideration its relations to other objects? In fact it can easily be seen that, at least in many cases, no such distinction is possible.

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Who Is Moving?

Imagine yourself sitting in a railroad car with veiled windows and running on a perfectly straight track with unchanging velocity: you would find it absolutely impossible to ascertain by any mechanical means whether the car were moving or not. All mechanical instruments behave exactly the same, whether the car be [49] standing still or in motion.][24] [If you drop a ball you will see it fall to the floor in a straight line, just as though you had dropped it while standing on the station platform. Furthermore, if you drop the ball from the same height in the two cases, and measure the velocities with which it strikes the car floor and the station platform, or the times which it requires for the descent, you will find these identical in the two cases.][182]

[Any *changes* of speed or of direction (as when the car speeds up or slows down or rounds a curve) can be detected by observing the behavior of bodies in the car, without apparent reference to any outside objects. This becomes particularly obvious with sudden irregularities of motion, which manifest themselves by shaking everything in the car. But a uniform motion in a straight line does not reveal itself by any phenomenon within the vehicle.][24]

[Moreover, if we remove the veil from our window to the extent that we may observe the train on the adjoining track, we shall be able to make no decision as to whether we or it be moving. This is indeed an experience which we have all had.]* [Often when seated in a train about to leave the station, we have thought ourselves under way, only to perceive as the motion becomes no longer uniform that another train has been backing into the station on the adjoining track. Again, as we were hurried on our journey, we have, raising suddenly our eyes, been puzzled to say whether the passing train were moving with us or against us or indeed standing still; or more rarely we have had the impression that both it and we seemed to be at rest, when in truth both were [50]
moving rapidly with the same speed.][82] [Even this phrase "in truth" is a relative one, for it arises through using the earth as an absolute reference body. We are indeed naive if we cannot appreciate that there is no reason for doing this beyond convenience, and that to an observer detached from the earth it were just as reasonable to say that the rails are sliding under the train as that the train is advancing along the rails. One of my own most vivid childhood recollections is of the terror with which, riding on a train that passed through a narrow cut, I hid my head in the maternal lap to shut out the horrid sight of the earth rushing past my window. The absence of a background in relatively slow retrograde motion was sufficient to prevent my consciousness from drawing the accustomed conclusion that after all it was really the train that was moving.]*

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Mechanical Relativity

[So we can enunciate the following principle: When a body is in uniform rectilinear motion relatively to a second body, then all phenomena take place on the first in exactly the same manner as on the second; the physical laws for the happenings on both bodies are identical.][24] [And between a system of bodies, nothing but relative motion may be detected by any mechanical means whatever; any attempt to discuss absolute motion presupposes a super-observer on some body external to the system. Even then, the "absolute" motion is nothing but motion relative to this super-observer. By no mechanical means is [51]
uniform straight-line motion of any other than relative character to be detected. This is the Principle of Mechanical Relativity.

There is nothing new in this. It was known to Galileo, it was known to Newton, it has been known ever since. But the curious persistence of the human mind in habits of thought which confuse relativity with absolutism brought about a state of affairs where we attempted to know this and to ignore it at the same time. We shall have to return to the mathematical mode of reasoning to see how this happened. The mathematician has a way all his own of putting the statement of relativity which we have made. He recalls, what we have already seen, that the observer on the earth who is measuring his "absolute" motion with respect to the earth has merely attached his reference framework to the earth; that the passenger in the train who measures all motion naively with respect to his train is merely carrying his coordinate axes along with his baggage, instead of leaving them on the solid ground; that the astronomer who deals with the motion of the earth about the sun, or with that of the "fixed" stars against one another, does so simply by the artifice of hitching his frame of reference to the sun or to one of the fixed stars. So the mathematician points out that dispute as to which of two bodies is in motion comes right down to dispute as to which of two sets of coordinate axes is the better one, the more nearly "natural" or "absolute." He therefore phrases the mechanical principle of relativity as follows:

Among all coordinate systems that are merely in uniform straight-line motion to one another, no one occupies any position of unique natural advantage; all such systems are equivalent for the investigation of natural laws; all systems lead to the same laws and the same results. [52]

The mathematician has thus removed the statement of relativity from its intimate association with the external observed phenomena, and transferred it to the observer and his reference frame. We must either accept the principle of relativity, or seek a set of coordinate axes that have been singled out by nature as an absolute reference frame. These axes must be in some way unique, so that when we refer phenomena to them, the laws of nature take a form of exceptional simplicity not attained through reference to ordinary axes. Where shall we look for such a preferred coordinate system?]*

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The Search for the Absolute

[Older theory clung to the belief that there was such a thing as absolute motion in space.][197] [As the body of scientific law developed from the sixteenth century onward, the not unnatural hypothesis crept in, that these laws (that is to say, their mathematical formulations rather than their verbal statements) would reveal themselves in especially simple forms, were it possible for experimenters to make their observations from some absolute standpoint; from an absolutely fixed position in space rather than from the moving earth.][264] [Somewhere a set of coordinate axes incapable of motion was to be found,][197] [a fixed set of axes for measuring absolute motion; and for two hundred years [53] the world of science strove to find it,][147] [in spite of what should have been assurance that it did not exist. But the search failed, and gradually the universal applicability of the principle of relativity, so far as it concerned mechanical phenomena, grew into general acceptance.]* [And after the development, by the great mathematicians of the eighteenth century, of Newton's laws of motion into their most complete mathematical form, it was seen that so far as these laws are concerned the absolutist hypothesis mentioned is quite unsupported. No complication is introduced into Newton's laws if the observer has to make his measurements in a frame of reference moving uniformly through space; and for measurements in a frame like the earth, which moves with changing speed and direction about the sun and rotates on its axis at the same time, the complication is not of so decisive a nature as to give us any clue to the earth's absolute motion in space.

But mechanics, albeit the oldest, is yet only one of the physical sciences. The great advance made in the mathematical formulation of optical and electromagnetic theory during the nineteenth century revived the hope of discovering absolute motion in space by means of the laws derived from this theory.][264] [Newton had supposed light to be a material emanation, and if it were so, its passage across "empty space" from sun and stars to the earth raised no problem. But against Newton's theory Huyghens, the Dutch astronomer, advanced the idea that light was a wave motion of some sort. During the Newtonian period and for many years after, the corpuscular theory [54] prevailed; but eventually the tables were turned.]* [Men made rays of light interfere, producing darkness (see page 61). From this, and from other phenomena like polarization, they had deduced that light was a form of wave motion similar to water ripples; for these interfere, producing level surfaces, or reinforce each other, producing waves of abnormal height. But if light were to be regarded as a form of wave motion—and the phenomena could apparently be explained on no other basis—then there must be some medium capable of undergoing this form of motion.][135] [Transmission of waves across empty space without the aid of an intermediary material medium would be "action at a distance," an idea repugnant to us. Trammeled by our tactual, wire-pulling conceptions of a material universe, we could not accustom ourselves to the idea of something—even so immaterial a something as a wave—being transmitted by nothing. We needed a word—ether—to carry light if not to shed it; just as we need a word—inertia—to carry a projectile in its flight.][231] [It was necessary to invest this medium with properties to account for the observed facts. On the whole it was regarded as the perfect fluid.][235] [The ether was imagined as an all-pervading, imponderable substance filling the vast emptiness through which light reaches us, and as well the intermolecular spaces of all matter. Nothing more was known definitely, yet this much served as a good working hypothesis on the basis of which Maxwell was enabled to predict the possibility of radio communication. By its fruits the ether hypothesis justified itself; but does the ether exist?][231] [55]

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The Ether and Absolute Motion

[If it does exist, it seems quite necessary, on mere philosophical grounds, that it shall be eligible to serve as the long-sought reference frame for absolute motion. Surely it does not make sense to speak of a homogeneous medium filling all space, sufficiently material to serve as a means of communication between remote worlds, and in the next breath to deny that motion with respect to this medium is a concept of significance.]* [Such a system of reference as was offered by the ether, coextensive with the entire known region of the universe, must necessarily serve for all motions within our perceptions.][186] [The conclusion seems inescapable that motion with respect to the ether ought to be of a sufficiently unique character to stand out above all other motion. In particular, we ought to be able to use the ether to define, somewhere, a system of axes *fixed with respect to the ether*, the use of which would lead to natural laws of a uniquely simply description.

Maxwell's work added fuel to this hope.]* [During the last century, after the units of electricity had been defined, one set for static electrical calculations and one for electromagnetic calculations, it was found that the ratio of the metric units of capacity for the two systems was numerically equal to what had already been found as the velocity with which light is transmitted through the hypothetical ether. One definition refers to electricity at rest, the other to [56] electricity in motion. Maxwell, with little more working basis than this, undertook to prove that electrical and optical phenomena were merely two aspects of a common cause,][235] [to which the general designation of "electromagnetic waves" was applied. Maxwell treated this topic in great fullness and with complete success. In particular, he derived certain equations giving the relations between the various electrical quantities involved in a given phenomenon. But it was found, extraordinarily enough, that these relations were of such character that, when we subject the quantities involved to a change of coordinate axes, the transformed quantities did not preserve these relations if the new axes happened to be in motion with respect to the original ones. This, of course, was taken to indicate that motion really *is* absolute when we come to deal with electromagnetic phenomena, and that the ether which carries the electromagnetic waves really *may be* looked to to display the properties of an absolute reference frame.

Reference to the phenomenon of aberration, which Dr. Pickering has discussed adequately in his essay and which I need therefore mention here only by name, indicated that the ether was not dragged along by material bodies over and through which it might pass. It seemed that it must filter through such bodies, presumably via the molecular interstices, without appreciable opposition. Were this not the case, we should be in some doubt as to the possibility of observing the velocity through the ether of material bodies; if the ether adjacent to such bodies is not dragged along or thrown into eddies, but "stands still" while the [57] bodies pass, there seems no imaginable reason for anything other than the complete success of such observations. And of course these are of the utmost importance, the moment we assign to the ether the rôle of absolute reference frame.

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The Earth and the Ether

One body in motion with respect to the ether is our earth itself. We do not know in advance in what direction to expect this motion or what magnitude to anticipate that it will have. But one thing is clear.]* [In its motion around the sun, the earth has, at opposite points on its orbit, a difference in velocity with respect to the surrounding medium which is double its orbital velocity with respect to the sun. This difference comes to 37 miles per second. The earth should therefore, at some time in the year, show a velocity equal to or greater than 18½ miles per second, with reference to the universal medium. The famous Michelson-Morley experiment of 1887 was carried out with the expectation of observing this velocity.][267]

[The ether, of course, and hence velocities through it, cannot be observed directly. But it acts as the medium for the transmission of light.]* [If the velocity of light through the ether is *C* and that of the earth through the ether is *v*, then the velocity of light past the earth, so the argument runs, must vary from $C - v$ to $C + v$, according as the light is moving exactly in the same direction as the earth, or in the opposite direction,][182] [or diagonally across the earth's path so as to get the influence only of a part of the earth's motion. This of course assumes that *C* has always the same value; an assumption that impresses one as inherently probable, and one that is at the same time in accord with ordinary astronomical observation. [58]

It is not possible to measure directly the velocity of light (186,330 miles per second, more or less) with sufficient accuracy to give any meaning to the variation in this velocity which might be effected by adding or subtracting that of the earth in its orbit (a mere 18½ miles per second). It is, however, possible to play a trick on the light by sending it back and forth over several paths, and comparing (*not* measuring absolutely, but merely *comparing*) with great minuteness the times consumed in these several round trips.

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A Journey Upstream and Back

The number of letters the Scientific American has received questioning the Michelson-Morley experiment indicates that many people are not acquainted with the fundamental principle on which it is based. So let us look at a simple analogous case. Suppose a swimmer or a rower make a return trip upstream and down, contending with the current as he goes up and getting its benefit when he comes down. Obviously, says snap judgment, since the two legs of the journey are equal, he derives exactly as much benefit from the current when he goes with it as he suffers handicap from it when he goes against it; so the round trip must take exactly the same time as a journey of the same length in still water, the argument applying equally in the case where the "swimmer" is a wave of light in the ether stream. [59]

But let us look now at a numerical case. A man can row in still water at four miles per hour. He rows twelve miles upstream and back, in a current of two miles per hour. At a net speed of two miles per hour he arrives at his turning point in six hours. At a net speed of six miles per hour he makes the downstream leg in two hours. The elapsed time for the journey is eight hours; in still water he would row the twenty-four miles in six hours.

If we were to attempt an explanation of this result in words we should say that by virtue of the very fact that it *does* delay him, the adverse current prolongs the time during which it operates; while by virtue of the very fact that it accelerates his progress, the favoring current shortens its venue. The careless observer realizes that distances are equal between the two legs of the journey, and unconsciously assumes that times are equal.

If the journey be made directly with and directly against the stream of water or ether or what not, retardation is effected to its fullest extent. If the course be a diagonal one, retardation is felt to an extent measurable as a component, and depending for its exact value upon the exact angle of the path. Felt, however, it must always be.

Here is where we begin to get a grip on the problem of the earth and the ether. In any problem involving the return-trip principle, there will enter two velocities—that of the swimmer and that of the medium; and the time of retardation. If we know any two of these items we can calculate the third. When the swimmer is a ray of light and the velocity of the medium is that of the ether as it flows past the earth, we know the first of these two; we hope to observe the retardation so that we may calculate the second velocity. The apparatus for the experiment is ingenious and demands description. [60]

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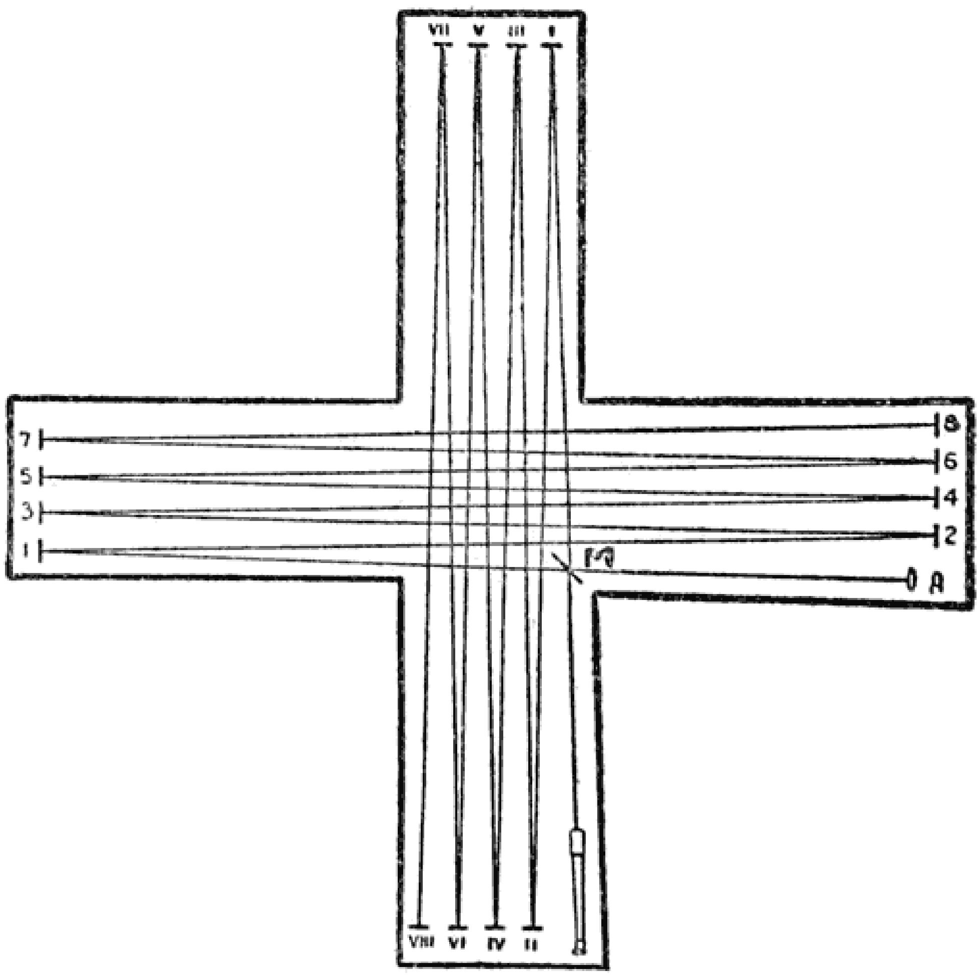
The Michelson-Morley Experiment

The machine is of structural steel, weighing 1,900 pounds. It has two arms which form a Greek cross. Each arm is 14 feet in length. The whole apparatus is floated in a trough containing 800 pounds of mercury.



Four mirrors are arranged on the end of each arm, sixteen in all, with a seventeenth mirror, M, set at one of the inside corners of the cross, as diagrammed. A source of light (in this case a calcium flame) is provided, and its rays directed by a lens toward the mirror M. Part of the light is allowed to [61] pass straight through M to the opposite arm of the cross, where it strikes mirror 1. It is reflected back across the arm to mirror 2, thence to 3, and so on until it reaches mirror 8. Thence it is reflected back to mirror 7, to 6, and so on, retracing its former path, and finally is caught by the reverse side of the mirror M and is sent to an observer at O. In retracing its path the light sets up an interference phenomenon (see below) and the interference bands are visible to the observer, who is provided with a telescope to magnify the results.

A second part of the original light-beam is reflected off at right angles by the mirror M, and is passed to and fro on the adjacent arms of the machine, in exactly the same manner and over a similar path, by means of the mirrors I, II, III, … VIII. This light finally reaches the observer at the telescope, setting up a second set of interference bands, parallel to the first.

A word now about this business of light interference. Light is a wave motion. The length of a wave is but a few millionths of an inch, and the amplitude is correspondingly minute; but none the less, these waves behave in a thoroughly wave-like manner. In particular, if the crests of two waves are superposed, there is a double effect; while if a crest of one wave falls with a trough of another, there is a killing-off or "interference".

Under ordinary circumstances interference of light waves does not occur. This is simply because under ordinary circumstances light waves are not piled up on one another. But sometimes this piling up occurs; and then, just so sure as the piled-up waves are in the same phase they reinforce one another, while if they are in opposite phase they interfere. And the conditions which we have outlined above, with the telescope and the mirrors and the ray of light retracing the path over which it went out, are conditions under which interference *does* occur. If the returning wave is in exact phase with the outgoing one, the effect is that of uniform double illumination; if it is in exactly opposite phase the effect is that of complete extinguishing of the light, the reversed wave exactly cancelling out the original one. If the two rays are partly in phase, there is partial reinforcement or partial cancelling out, according to whether they are nearly in phase or nearly out of phase. Finally, if the mirrors are not set absolutely parallel—as must in practice be the case when we attempt to measure their parallelism in terms of the wave-length of light—adjacent parts of the light ray will vary in the extent to which they are out of phase, since they will have travelled a fraction of a wave-length further to get to and from this, that or the other mirror. There will then appear in the telescope alternate bands of illumination and darkness, whose width and spacing depend upon all the factors entering into the problem. [62]

If it were possible for us to make the apparatus with such a degree of refinement that the path from mirror M via mirrors 1, 2, 3, etc., back through M and into the telescope, were exactly the same length as that from flame to telescope by way of the mirrors I, II, III, etc.—exactly the same to a margin of error materially less than a single wave-length of light—why, then, the two sets of interference fringes would come out exactly superposed provided the motion of the earth through the "ether" turn out to have no influence upon the velocity of light; or, if such influence exist, these fringes would be displaced from one another to an extent measuring the influence in question. But our ability to set up this complicated pattern of mirrors at predetermined distances falls far short of the wave-length as a measure of error. So in practice all that we can say is that having once set the instrument up, and passed a beam of [63] light through it, there will be produced two sets of parallel interference fringes. These sets will fail of superposition—each fringe of one set will be removed from the corresponding fringe of the other set—by some definite distance. Then, any subsequent variation in the speed of light along the two arms will at once be detected by a shifting of the interference bands through a distance which we shall be able to measure.

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The Verdict

Under the theories and assumptions governing at the time of the original performance of this experiment, it will be readily seen that if this machine be set up in an "ether stream" with one arm parallel to the direction of the stream and the other at right angles thereto, there will be a difference in the speed of the light along the two arms. Then if the apparatus be shifted to a position [64] oblique to the ether stream, the excess velocity of the light in the one arm would be diminished, and gradually come to zero at the 45-degree angle, after which the light traveling along the other arm would assume the greater speed. In making observations, therefore, the entire apparatus was slowly rotated, the observers walking with it, so that changes of the sort anticipated would be observed.

The investigators were, however, ignorant of the position in which the apparatus ought to be set to insure that one of the arms lie across the ether drift; and they were ignorant of the time of year at which the earth's maximum velocity through the ether was to be looked for. In particular, it is plain that if the solar system as a whole is moving through the ether at a rate *less* than the earth's orbital velocity, there is a point in our orbit where our velocity through the ether and that around the sun just cancel out and leave us temporarily in a state of "absolute rest." So it was anticipated that the experiment might have to be repeated in many orientations of the machine and at many seasons of the year in order to give a series of readings from which the true motion of the earth through the ether might be deduced.

For those who have a little algebra the demonstration which Dr. Russell gives on a subsequent page will be interesting as showing the situation in perfectly general terms. It will be realized that the more complicated arrangement of mirrors in the experiment as just described is simply an eightfold repetition of the simple experiment as outlined by Dr. Russell, and that it was done so for [65] the mere sake of multiplying by eight the distances travelled and hence the difference in time and in phase.

And now for the grand climax. The experiment was repeated many times, with the original and with other apparatus, indoors and outdoors, at all seasons of the year, with variation of every condition that could imaginably affect the result. The apparatus was ordinarily such that a shift in the fringes of anywhere from one-tenth to one one-hundredth of that which would have followed from any reasonable value for the earth's motion through the ether would have been systematically apparent. The result was uniformly negative. At all times and in all directions the velocity of light past the earth-bound observer was the same. The earth has no motion with reference to the ether!

[The amazing character of this result is not by any possibility to be exaggerated.]* [According to one experiment the ether was carried along by a rapidly moving body and according to another equally well-planned and well-executed experiment a rapidly moving body did not disturb the ether at all. This was the blind alley into which science had been led.][232]

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The "Contraction" Hypothesis

[Numerous efforts were made to explain the contradiction.]* [It is indeed a very puzzling one, and it gave physicists no end of trouble. However Lorentz and Fitzgerald finally put forward an ingenious explanation, to the effect that the actual motion of the earth through the ether is balanced, as far as the ability [66] of our measuring instruments is concerned, by a contraction of these same instruments in the direction of their motion. This contraction obviously cannot be observed directly because all bodies, including the measuring instruments themselves (which after all are only arbitrary guides), will suffer the contraction equally. According to this theory, called the Lorentz-Fitzgerald contraction theory,][272] [all bodies in motion suffer such contraction of their length in the direction of their motion;][283] [the contraction being made evident by our inability to observe the absolute motion of the earth, which it is assumed must exist.][272] [This would suffice to show why the Michelson-Morley experiment gave a negative result, and would preserve the concept of absolute motion with reference to the ether.][283]

[This proposal of Lorentz and Fitzgerald loses its startling aspect when we consider that all matter appears to be an electrical structure, and that the dimensions of the electric and magnetic fields which accompany the electrons of which it is constituted change with the velocity of motion.][267] [The forces of cohesion which determine the form of a rigid body are held to be electromagnetic in nature; the contraction may be regarded as due to a change in the electromagnetic forces between the molecules.][10] [As one writer has put it, the orientation, in the electromagnetic medium, of a body depending for its very existence upon electromagnetic forces is not necessarily a matter of indifference.]*

[Granting the plausibility of all this, on the basis of an electromagnetic theory of matter, it leaves us in an unsatisfactory position. We are left with a fixed [67] ether with reference to which absolute motion has a meaning, but that motion remains undetected and apparently undetectable. Further, if we on shore measure the length of a moving ship, using a yard-stick which is stationary on shore, we shall obtain one result. If we take our stick aboard it contracts, and so we obtain a greater length for the ship. Not knowing our "real" motion through the ether, we cannot say which is the "true" length. Is it not, then, more satisfactory to discard all notion of true length as an inherent quality of bodies, and, by regarding length as the measure of a relation between a particular object and a particular observer, to make one length as true as the other?][182] [The opponents of such a viewpoint contend that Michelson's result was due to a fluke; some mysterious counterbalancing influence was for some reason at work, concealing the result which should normally have been expected. Einstein refuses to accept this explanation;][192] [he refuses to believe that all nature is in a contemptible conspiracy to delude us.]*

[The Fitzgerald suggestion is further unsatisfactory because it assumes all substances, of whatever density, to undergo the same contraction; and above all for the reason that it sheds no light upon other phenomena.][194] [It is indeed a very *special* explanation; that is, it applies only to the particular experiment in question. And indeed it is only one of many *possible* explanations. Einstein conceived the notion that it might be infinitely more valuable to take the most general explanation possible, and then try to find from this its logical consequences. This "most general explanation" is, of course, simply that it is [68] impossible in any way whatever to measure the absolute motion of a body in space.][272] [Accordingly Einstein enunciated, first the Special Theory of Relativity, and later the General Theory of Relativity. The special theory was so called because it was, limited to uniform rectilinear and non-rotary motions. The general theory, on the other hand, dealt not only with uniform rectilinear motions, but with any arbitrary motion whatever.

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Taking the Bull by the Horns

The hypothesis of relativity asserts that there can be no such concept as absolute position, absolute motion, absolute time; that space and time are inter-dependent, not independent; that everything is relative to something else. It thus accords with the philosophical notion of the relativity of all knowledge.][283] [Knowledge is based, ultimately, upon measurement; and clearly all measurement is relative, consisting merely in the application of a standard to the magnitude measured. All metric numbers are relative; dividing the unit multiplies the metric number. Moreover, if measure and measured change proportionately, the measuring number is unchanged. Should space with all its contents swell in fixed ratio throughout, no measurement could detect this; nor even should it *pulse* uniformly throughout. Furthermore, were space and space-contents in any way systematically *transformed* (as by reflection in curved mirrors) point for point, continuously, without rending, no measurement could reveal this distortion; experience would proceed [69] undisturbed.][263]

[Mark Twain said that the street in Damascus "which is called straight," is so called because while it is not as straight as a rainbow it is straighter than a corkscrew. This expresses the basic idea of relativity—the idea of *comparison*. All our knowledge is *relative*, not *absolute*. Things are big or little, long or short, light or heavy, fast or slow, only by comparison. An atom may be as large, compared to an electron, as is a cathedral compared to a fly. The relativity theory of Einstein emphasizes two cases of relative knowledge; our knowledge of *time and space*, and our knowledge of *motion*.][216] [And in each case, instead of allowing the notions of relativity to guide us only so far as it pleases us to follow them, there abandoning them for ideas more in accord with what we find it easy to take for granted, Einstein builds his structure on the thesis that relativity must be admitted, must be followed out to the bitter end, in spite of anything that it may do to our preconceived notions. If relativity is to be admitted at all, it must be admitted *in toto*; no matter what else it contradicts, we have no appeal from its conclusions so long as it refrains from contradicting itself.]*

[The hypothesis of relativity was developed by Einstein through *a priori* methods, not the more usual *a posteriori* ones. That is, certain principles were enunciated as probably true, the consequences of these were developed, and these deductions tested by comparison of the predicted and the observed phenomena. It was in no sense attained by the more usual procedure of [70] observing groups of phenomena and formulating a law or formula which would embrace them and correctly describe the routine or sequence of phenomena.

The first principle thus enunciated is that it is impossible to measure or detect absolute translatory motion through space, under any circumstances or by any means. The second is that the velocity of light in free space appears the same to all observers regardless of the relative motion of the source of light and the observer. This velocity is not affected by motion of the source toward or away from the observer,][283] [if we may for the moment use this expression with its implication of absolute motion.]* [But universal relativity insists that motion of the source toward the observer is identical with motion of the observer toward the source.][283]

[It will be seen that we are at once on the horns of a dilemma. Either we must give up relativity before we get fairly started on it, or we must overturn the foundations of common sense by admitting that time and space are so constituted that when we go to meet an advancing light-impulse, or when we retreat from it, it still reaches us with the same velocity as though we stood still waiting for it. We shall find when we are through with our investigation that common sense is at fault; that our fixed impression of the absurdity of the state of affairs just outlined springs from a confusion between relativism and absolutism which has heretofore dominated our thought and gone unquestioned. The impression of absurdity will vanish when we have resolved this confusion.]* [71]

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Questions of Common Sense

[But it is obvious from what has just been said that if we are to adopt Einstein's theory, we must make very radical changes in some of our fundamental notions, changes that seem in violent conflict with common sense. It is unfortunate that many popularizers of relativity have been more concerned to astonish their readers with incredible paradoxes than to give an account such as would appeal to sound judgment. Many of these paradoxes do not belong essentially to the theory at all. There is nothing in the latter that an enlarged and enlightened common sense would not readily endorse. But common sense must be educated up to the necessary level.][141]

[There was a time when it was believed, as a result of centuries of experience, that the world was flat. This belief checked up with the known facts, and it could be used as the basis for a system of science which would account for things that had happened and that were to happen. It was entirely sufficient for the time in which it prevailed.

Then one day a man arose to point out that all the known facts were equally accounted for on the theory that the earth was a sphere. It was in order for his contemporaries to admit this, to say that so far as the facts in hand were concerned they could not tell whether the earth was flat or round—that new facts would have to be sought that would contradict one or the other hypothesis. Instead of this the world laughed and insisted that the earth could not be round because it was flat; that it could not be round because then the people would fall off the other side. [72]

But the field of experimentation widened, and men were able to observe facts that had been hidden from them. Presently a man sailed west and arrived east; and it became clear that in spite of previously accepted "facts" to the contrary, the earth was really round. The previously accepted "facts" were then revised to fit the newly discovered truth; and finally a new system of science came into being, which accounted for all the old facts and all the new ones.

At intervals this sort of thing has been repeated. A Galileo shows that preconceived ideas with regard to the heavens are wrong, and must be revised to accord with his newly promulgated principles. A Newton does the same for physics—and people unlearn the "fact" that motion has to be supported by continued application of force, substituting the new idea that it actually requires force to stop a moving body. A Harvey shows that the things which have been "known" for generations about the human body are not so. A Lyell and a Darwin force men to throw overboard the things they have always believed about the way in which the earth and its creatures came into being. Every science we possess has passed through one or more of these periods of readjustment to new facts.

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Shifting the Mental Gears

Now we are apt to lose sight of the true significance of this. It is not alone our opinions that are altered; it is our fundamental concepts. *We get concepts wholly from our perceptions*, making them to fit those perceptions. Whenever a new vista is opened to our perceptions, we find facts that we never could have suspected from the restricted viewpoint. We must then actually alter our concepts to make the new facts fit in with the greatest degree of harmony. And we must not hesitate to undertake this alteration, through any feeling that fundamental concepts are more sacred and less freely to be tampered with than derived facts.]* [We do, to be sure, want fundamental concepts that are easy for a human mind to conceive; but we also want our laws of nature to be simple. If the laws begin to become, intricate, why not reshape, somewhat, the fundamental concepts, in order to simplify the scientific laws? Ultimately it is the simplicity of the scientific system as a whole that is our principal aim.][178] [73]

[As a fair example, see what the acceptance of the earth's sphericity did to the idea represented by the word "down." With a flat earth, "down" is a single direction, the same throughout the universe; with a round earth, "down" becomes merely the direction leading toward the center of the particular heavenly body on which we happen to be located. It is so with every concept we have. No matter how intrinsic a part of nature and of our being a certain notion may seem, we can never know that new facts will not develop which will show it to be a mistaken one. Today we are merely confronted by a gigantic example of this sort of thing. Einstein tells us that when velocities are attained which have just now come within the range of our close investigation, extraordinary things happen—things quite irreconcilable with our present [74] concepts of time and space and mass and dimension. We are tempted to laugh at him, to tell him that the phenomena he suggests are absurd because they contradict these concepts. Nothing could be more rash than this.

When we consider the results which follow from physical velocities comparable with that of light, we must confess that here are conditions which have never before been carefully investigated. We must be quite as well prepared to have these conditions reveal some epoch-making fact as was Galileo when he turned the first telescope upon the skies. And if this fact requires that we discard present ideas of time and space and mass and dimension, we must be prepared to do so quite as thoroughly as our medieval fathers had to discard their notions of celestial "perfection" which demanded that there be but seven major heavenly bodies and that everything center about the earth as a common universal hub. We must be prepared to revise our concepts of these or any other fundamentals quite as severely as did the first philosopher who realized that "down" in London was not parallel to "down" in Bagdad or on Mars.]*

[In all ordinary terrestrial matters we take the earth as a fixed body, light as instantaneous. This is perfectly proper, for such matters. But we carry our earth-acquired habits with us into the celestial regions. Though we have no longer the earth to stand on, yet we assume, as on the earth, that all measurements and movements must be referred to some fixed body, and are only then valid. We cling to our earth-bound notion that there *is* an absolute [75] up-and-down, back-and-forth, right-and-left, in space. We may admit that we can never find it, but we still *think it is there*, and seek to approach it as nearly as possible. And similarly from our earth experiences, which are sufficiently in a single place to make possible this simplifying assumption, we get the idea that there is *one* universal time, applicable at once to the entire universe.][141] [The difficulty in accepting Einstein is entirely the difficulty in getting away from these earth-bound habits of thought.]* [76]

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IV

THE SPECIAL THEORY OF RELATIVITY

What Einstein's Study of Uniform Motion Tells Us About Time and Space and the Nature of the External Reality

BY VARIOUS CONTRIBUTORS AND THE EDITOR

Whatever the explanation adopted for the negative result of the Michelson-Morley experiment, one thing stands out clearly: the attempt to isolate absolute motion has again failed.]* [Einstein generalizes this with all the other and older negative results of similar sort into a negative deduction to the effect that no experiment is possible upon two systems which will determine that one of them is in motion and the other at rest.][121] [He elevates the repeated failure to detect absolute motion through space into the principle that experiment will never reveal anything in the nature of absolute velocities. He postulates that all laws of nature can and should be enunciated in such forms that they are as true in these forms for one observer as for another, even though these observers with their frames of reference be in motion relative to one another.][264]

[There are various ways of stating the principle of the relativity of uniform motion which has been thus arrived at, and which forms the basis of the [77] Special Theory of Einstein. If we care to emphasize the rôle of mathematics and the reference frame we may say that]* [any coordinate system having a uniform rectilinear motion with respect to the bodies under observation may be interchangeably used with any other such system in describing their motions;][232] [or that the unaccelerated motion of a system of reference cannot be detected by observations made on this system alone.][194] [Or we can let this aspect of the matter go, and state the relativity postulate in a form more intelligible to the non-mathematician by simply insisting that it is impossible by any means whatever to distinguish any other than the relative motion between two systems that are moving uniformly. As Dr. Russell puts it on a later page, we can assume boldly that the universe is so constituted that uniform straight-ahead motion of an observer and all his apparatus will not produce any difference whatever in the result of any physical process or experiment of any kind.

As we have seen, this is entirely reasonable, on philosophical grounds, until we come to consider the assumptions of the past century with regard to light and its propagation. On the basis of these assumptions we had expected the Michelson-Morley experiment to produce a result negativing the notion of universal relativity. It refused to do this, and we agree with Einstein that the best explanation is to return to the notion of relativity, rather than to invent a forced and special hypothesis to account for the experiment's failure. But we must now investigate the assumptions underlying the theory of light, and [78] remove the one that requires the ether to serve as a universal standard of absolute motion.

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Light and the Ether

It is among the possibilities that the wave theory of light itself will in the end be more or less seriously modified. It is even more definitely among the possibilities that the ether will be discarded.]* [Certainly when Lord Kelvin estimates that its mass per cubic centimeter is .000,000,000,000,000,001 gram, while Sir Oliver Lodge insists that the correct figure is 1,000,000,000,000,000 grams, it is quite evident that we know so little about it that it is better to get along without it if we can.][216] [But to avoid confusion we must emphasize that

Einstein makes no mention whatsoever of the ether; his theory is absolutely independent of any theory of the ether.][139] [Save as he forbids us to employ the ether as a standard of absolute motion, Einstein does not in the least care what qualities we assign to it, or whether we retain it at all. His demands are going to be made upon light itself, not upon the alleged medium of light transmission.

When two observers in relative motion to one another measure their velocities with respect to a third material object, they expect to get different results. Their velocities with regard to this object properly differ, for it is no more to be taken as a universal super-observer than either of them. But if they get different results when they come to measure the velocity with which light passes their respective systems, relativity is challenged. Light *is* with some propriety to be [79] regarded as a universal observer; and if it will measure our velocities against each other we cannot deny it rank as an absolute standard. If we are not prepared to abandon universal relativity, and adopt one of the "fluke" explanations for the Michelson-Morley result, we must boldly postulate that in free space light presents the same velocity *C* to all observers—whatever the source of the light, whatever the relative motion between source and observer, whatever the relative motion between the several observers. The departure here from the old assumption lies in the circumstance that the old physics with its ether assigned to light a velocity universally constant *in this ether*; we have stopped talking about the medium and have made the constant *C* refer to the observer's measured value of the velocity of light with regard to himself.

We are fortified in this assumption by the Michelson-Morley result and by all other observations bearing directly upon the matter. Nevertheless, as Mr. Francis says in his essay, we feel instinctively that space and time are not so constituted as to make it possible, if I pass you at 100 miles per hour, for the same light-impulse to pass us both at the same speed *C*.]* [The implicit assumptions underlying this feeling, be they true or false, are now so interwoven with the commonly received notions of space and time that any theory which questions them has all the appearance of a fantastic and unthinkable thing.][115] [We cannot, however, go back on our relativity; so when]* [Einstein shows us that an entirely new set of time and space concepts [80] is necessary to reconcile universe relativity with this fundamental fact of the absolute constancy of the observed velocity of light *in vacuo*,][18] [all that is left for us to do is to inquire what revisions are necessary, and submit to them.]*

[The conceptual difficulties of the theory arise principally from attributing to space and time the properties of things. No portion of space can be compared with another, save by convention; it is things which we compare. No interval of time can be compared with another, save by convention. The first has gone when the second becomes "now".][149] [It is events that we compare, through the intervention of things. Our measurements are never of space or of time, but only of the things and the events that occupy space and time. And since the measurements which we deal with as though they were of space and of time lie at the foundation of all physical science, while at the same time themselves constituting, as we have seen, the only reality of which we are entitled to speak, it is in order to examine with the utmost care the assumptions underlying them. That there are such assumptions is clear—the very possibility of making measurements is itself an assumption, and every technique for carrying them out rests on an assumption. Let us inquire which of these it is that relativity asks us to revise.]*

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The Measurement of Time and Space

[Time is generally conceived as perfectly uniform. How do we judge about it? What tells us that the second just elapsed is equal to the one following? By the very nature of time the superposition of its successive intervals is impossible. How then can we talk about the relative duration of these intervals? It is clear that any relationship between them can only be conventional.][178] [As a matter of fact, we habitually measure time in terms of moving bodies. The simplest method is to agree that some entity moves with uniform velocity. It will be considered as travelling equal distances in equal intervals of time, the distances to be measured as may be specified by our assumptions governing this department of investigation.][179] [The motions of the earth through which we ultimately define the length of day and year, the division of the former into 86,400 "equal" intervals as defined by the motions of pendulum or balance wheel through equal distances, are examples of this convention of time measurement. Even when we correct the motions of the earth, on the basis of what our clocks tell us of these motions, we are following this lead; the earth and the clocks fall out, it is plain that one of them does not satisfy our assumption of equal lengths in equal times, and we decide to believe the clock.]* [81]

[The foregoing concerning time may be accepted as inherent in time itself. But concerning lengths it may be thought that we are able to verify absolutely their equality and especially their invariability. Let us have the audacity to verify this statement. We have two lengths, in the shape of two rods, which coincide perfectly when brought together. What may we conclude from this coincidence? Only that the two rods so considered have equal lengths at the [82] same place in space and at the same moment. It may very well be that each rod has a different length at different locations in space and at different times; that their equality is purely a local matter. Such changes could never be detected if they affected all objects in the universe. We cannot even ascertain that both rods remain straight when we transport them to another location, for both can very well take the same curvature and we shall have no means of detecting it.

Euclidean geometry assumes that geometrical objects have sizes and shapes independent of position and of orientation in space, and equally invariable in time. But the properties thus presupposed are only conventional and in no way subject to direct verification. We cannot even ascertain space to be independent of time, because when comparing geometrical objects we have to conceive them as brought to the same place in space and in time.][178] [Even the statement that when they are made to coincide their lengths are equal is, after all, itself an assumption inherent in our ideas of what constitutes length. And certainly the notion that we can shift them from place to place and from moment to moment, for purposes of comparison, is an assumption; even Euclid, loose as he was from modern standards in this business of "axioms," knew this and included a superposition axiom among his assumptions.

As a matter of fact, this procedure for determining equality of lengths is not always available. It assumes, it will be noted, that we have free access to the object which is to be measured—which is to say, it assumes that this object is at rest with respect to us. If it is not so at rest, we must employ at least a modification of this method; a modification that will in some manner involve the sending of signals. Even when we employ the Euclidean method of superposition directly, we must be assured that the respective ends of the lengths under comparison coincide at the same time. The observer cannot be present at both ends simultaneously; at best he can only be present at one end and receive a signal from the other end. [83]

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The Problem of Communication

Accordingly, in making the necessary assumptions to cover the matter of measuring lengths, we must make one with regard to the character of the signals which are to be employed for this purpose. If we could assume a system of signalling that would consume no time in transmission all would be simple enough. But we have no experience with such a system. Even if we believe that it ought to be possible thus to transmit signals at infinite velocity, we may not, in the absence of our present ability to do this, assume that it is possible. So we may only assume, with Einstein, that for our signals we shall employ the speediest messenger with which we are at present acquainted. This of course is light, the term including any of the electromagnetic impulses that travel at the speed *C*.

Of course in the vast majority of cases the distance that any light signal in which we are interested must go to reach us is so small that the time taken by [84] its transmission can by no means be measured. We are then, to all intents and purposes, at both places—the point of origin of the signal and the point of receipt—simultaneously. But this is not the question at all. Waiving the fact that in astronomical investigations this approximation no longer holds, the fact remains that it is, in every case, merely an approximation. Approximations are all right in observations, where we know that they are approximations and act accordingly. But in the conceptual universe that parallels the external reality, computation is as good an agent of observation as visual or auditory or tactile sensation; if we can compute the error involved in a wrong procedure the error is there, regardless of whether we can see it or not. We must have methods which are conceptually free from error; and if we attempt to ignore the velocity of our light signals we do not meet this condition.

The measurement of lengths demands that we have a criterion of simultaneity between two remote points—remote in inches or remote in light-years, it does not matter which. There is no difficulty in defining simultaneity of two events that fall in the same point—or rather, in agreeing that we know what we mean by such simultaneity. But with regard to two events that occur in remote places there may be a question. A scientific definition differs from a mere description in that it must afford us a means of testing whether a given item comes under the definition or not. There is some difficulty in setting up a definition of simultaneity between distant events that satisfies this requirement. If we try [85] simply to fall back upon our inherent ideas of what we mean by "the same instant" we see that this is not adequate. We must lay down a procedure for determining whether two events at remote points occur at "the same instant," and check up alleged simultaneity by means of this procedure.

Einstein says, and we must agree with him, that he can find but one reasonable definition to cover this ground. An observer can tell whether he is located half way between two points of his observation; he can have mirrors set up at these points, send out light-signals, and note the time at which he gets back the reflection. He knows that the velocity of both signals, going and coming, is the same; if he observes that they return to him together so that their time of transit for the round trip is the same, he must accept the distances as equal. He is then at the mid-point of the line joining the two points under observation; and he may define simultaneity as follows, without introducing anything new or indeterminate: Two events are simultaneous if an observer midway between them sees them at the same instant, by means, of course, of light originating at the points of occurrence.]*

[It is this definition of simultaneity, coupled with the assumption that all observers, on whatever uniformly moving systems, would obtain the same experimental value for the velocity of light, that leads to the apparent paradoxes of the Special Theory of Relativity. If it be asked why we adopt it, we must in turn ask the inquirer to propose a better system for defining simultaneous events on different moving bodies.][198] [86]

[There is nothing in this definition to indicate, directly, whether simultaneity persists for all observers, or whether it is relative, so that events simultaneous to one observer are not so to another. The question must then be investigated; and the answer, of course, will hinge upon the possibility of making proper allowances for the time of transit of the light signals that may be involved. It seems as though this ought to be possible; but a simple experiment will indicate that it is not, unless the observers involved are at rest with respect to one another.

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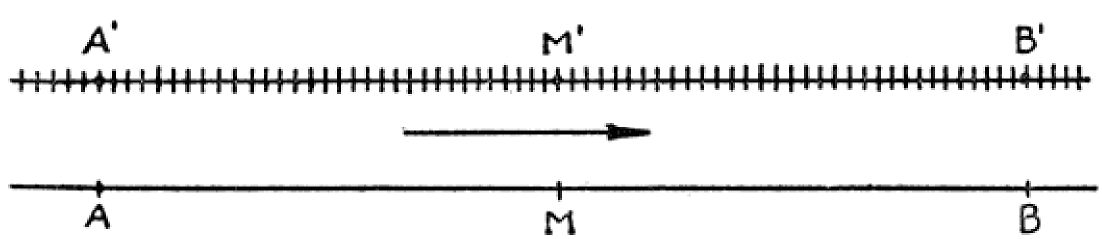
An Einsteinian Experiment

Let us imagine an indefinitely long, straight railroad track, with an observer located somewhere along it at the point *M*. According to the convention suggested above, he has determined points *A* and *B* in opposite directions from him along the



track, and equally distant from him. We shall imagine, further, than a beneficent Providence supplies two lightning flashes, one striking at *A* and one at *B*, in such a way that observer *M* finds them to be simultaneous.

While all this is going on, a train is passing—a very long train, amply long enough to overlap the section *AMB* of the track. Among the passengers there is [87] one, whom we may call *M′*, who is directly opposite *M* at the instant when, according to *M*, the lightning strikes. Observe he is not opposite *M* when *M* *sees* the flashes, but a brief time earlier—at the instant when, according to *M*'s computation, the simultaneous flashes *occurred*. At this instant there are definitely determined the points *A′* and *B′*, on the train; and since we may quite well think of the two systems—train-system and track-system—as in coincidence at this instant, *M′* is midway between *A′* and *B′*, and likewise is midway between *A* and *B*.

Now if we think of the train as moving over the track in the direction of the arrow, we see very easily that *M′* is running away from the light from *A* and toward that from *B*, and that, despite—or if you prefer because of—the uniform velocity of these light signals, the one from *B* reaches him, over a slightly shorter course, sooner than the one from *A*, over the slightly longer course. When the light signals reach *M*, *M′* is no longer abreast of him but has moved along a wee bit, so that at this instant when *M* has the two signals, one of these has passed *M′* and the other has yet to reach him. The upshot is that the events which were simultaneous to *M* are not so to *M′*.

It will probably be felt that this result is due to our having, somewhat unjustifiably and inconsistently, localized *on the train* the relative motion between train and track. But if we think of the track as sliding back under the train in the direction opposite to the arrow, and carrying with it the points *A* and *B*; and if we remember that this in no way affects *M*'s observed velocity of [88] light or the distances *AM* and *BM* as he observes them: we can still accept his claim that the flashes were simultaneous. Then we have again the same situation: when the flashes from *A* and from *B* reach *M* at the same moment, in his new position a trifle to the left of his initial position of the diagram, the flash from *A* has not yet reached *M′* in his original position while that from *B* has passed him. Regardless of what assumption we make concerning the motion between train-system and track-system, or more elegantly regardless of what coordinate system we use to define that motion, the event at *B* precedes that at *A* in the observation of *M′*. If we introduce a second train moving on the other track in the opposite direction, the observer on it will of course find that the flash at *A* precedes that at *B*—a disagreement not merely as to simultaneity but actually as to the order of two events! If we conceive the lightning as striking at the points *A′* and *B′ on the train*, these points travel with *M′* instead of with *M*; they are fixed to his coordinate system instead of to the other. If you carry out the argument now, you will find that when the flashes are simultaneous to *M′*, the one at *A* precedes that at *B* in *M*'s observation.

A large number of experiments more or less similar in outline to this one can be set up to demonstrate the consequences, with regard to measured values of time and space, of relative motion between two observers. I do not believe that a multiplicity of such demonstrations contributes to the intelligibility of the subject, and it is for this reason that I have cut loose from immediate dependence upon the essayists in this part of the discussion, concentrating [89] upon the single experiment to which Einstein himself gives the place of importance.

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Who Is Right?

We may permit Mr. Francis to remind us here that neither *M* nor *M′* may correct his observation to make it accord with the other fellow's. The one who does this is admitting that the other is at absolute rest and that he is himself in absolute motion; and this cannot be. They are simply in disagreement as to the simultaneity of two events, just as two observers might be in disagreement about the distance or the direction of a single event. This can mean nothing else than that, under the assumptions we have made, simultaneity is not an absolute characteristic as we had supposed it to be, but, like distance and direction, is in fact merely a relation between observer and objective, and therefore depends upon the particular observer who happens to be operating and upon the reference frame he is using.

But this is serious. My time measurements depend ultimately upon my space measurements; the latter, and hence both, depend closely upon my ideas of simultaneity. Yours depend upon your reading of simultaneity in precisely the same way.]* [Suppose the observer on the track, in the above experiment, wants to measure the length of something on the car, or the observer on the car something on the track. The observer, or his assistant, must be at both ends of the length to be measured at the same time, or get simultaneous reports in some way from these ends; else they will obtain false results. It is plain, then, [90] that with different criteria of what the "same time" is, the observers in the two systems may get different values for the measured lengths in question.][220]

[Who is right? According to the principle of relativity a decision on this question is absolutely impossible. Both parties are right from their own points of view; and we must admit that two events in two different places may be simultaneous for certain observers, and yet not simultaneous for other observers who move with respect to the first ones. There is no contradiction in this statement, although it is not in accordance with common opinion, which believes simultaneousness to be something absolute. But this common opinion lacks foundation. It cannot be proved by direct perception, for simultaneity of events can be perceived directly,][24] [and in a manner involving none of our arbitrary assumptions,]* [only if they happen at the same place; if the events are distant from each other, their simultaneity or succession can be stated only through some method of communicating by signals. There is no logical reason why such a method should not lead to different results for observers who move with regard to one another.

From what we have said, it follows immediately that in the new theory not only the concept of simultaneousness but also that of duration is revealed as dependent on the motion of the observer.][24] [Demonstration of this should be superfluous; it ought to be plain without argument that if two observers cannot agree whether two instants are the same instant or not, they cannot agree on [91] the interval of time between instants. In the very example which we have already examined, one observer says that a certain time-interval is zero, and another gives it a value different from zero. The same thing happens whenever the observers are in relative motion.]* [Two physicists who measure the duration of a physical process will not obtain the same result if they are in relative motion with regard to one another.

They will also find different results for the length of a body. An observer who wants to measure the length of a body which is moving past him must in one way or another hold a measuring rod parallel to its motion and mark those points on his rod with which the ends of the body come into *simultaneous coincidence*. The distance between the two marks will then indicate the length of the body. But if the two markings are simultaneous for one observer, they will not be so for another one who moves with a different velocity, or who is at rest, with regard to the body under observation. He will have to ascribe a different length to it. And there will be no sense in asking which of them is right: length is a purely relative concept, just as well as duration.][24]

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The Relativity of Time and Space

[The degree to which distance and time become relative instead of absolute quantities under the Special Theory of Relativity can be stated very definitely. In the first place, we must point out that the relativity of lengths applies with [92] full force only to lengths that lie parallel to the direction of relative motion. Those that lie exactly perpendicular to that direction come out the same for both observers; those that lie obliquely to it show an effect, depending upon the angle, which of course becomes greater and greater as the direction of parallelism is approached.

The magnitude of the effect is easily demonstrated, but with this demonstration we do not need to be concerned here. It turns out that if an observer moving with a system finds that a certain time interval in the system is T seconds and that a certain length in the system is L inches, then an observer moving parallel with L and with a velocity v relative to the system will find for these the respective values $T \div K$ and $L \times K$, where

$$K = \sqrt{1 - v^2/C^2}.$$

C in this expression of course represents the velocity of light. It will be noted that the fraction v^2/C^2 is ordinarily very small; that the expression under the radical is therefore less than 1 but by a very slight margin; and that the entire expression K is itself therefore less than 1 but by an even slighter margin. This means, then, that the observer outside the system finds the lengths in the system to be a wee bit shorter and the time intervals a wee bit longer than does the observer in the system. Another way of putting the matter is based, ultimately, upon the fact that in order for the observer *in the system* to get the larger value for distance and the smaller value for time, his measuring rod [93] must go into the distance under measurement more times than that of the moving observer, while his clock must beat a longer second in order that less of them shall be recorded in a given interval between two events. So it is often said that the measuring rod as observed from without is contracted and the clock runs slow. This does not impress me as a happy statement, either in form or in content.]*

[The argument that these formulae are contradicted by human experience can be refuted by examining a concrete instance. If a train is 1,000 feet long at rest, how long will it be when running a mile a minute?][232] [I have quoted this question exactly as it appears in the essay from which it is taken, because it is such a capital example of the objectionable way in which this business is customarily put. For the statement that lengths decrease and time-intervals increase "with velocity" is not true in just this form. The velocity, to have meaning, must be relative to some external system; and it is the observations *from that external system* that are affected. So long as we confine ourselves to the system in which the alleged modifications of size are stated as having taken place, there is nothing to observe that is any different from what is usual; there is no way to establish that we are enjoying a velocity, and in fact within the intent of the relativity theory we are *not* enjoying a velocity, for we are moving with the objects which we are observing. It is inter-systemic observations, and these alone, that show the effect. When we travel with the system under observation, we get the same results as any other observer on this [94] system; when we do not so travel, we must conduct our observations from our own system, in relative motion to the other, and refer our results to our system.

Now when no particular observer is specified, we must of course assume an observer connected with the train, or with whatever the body mentioned. To that observer it doesn't make the slightest difference what the train does; it may stand at rest with respect to some external system or it may move at any velocity whatsoever; its length remains always 1,000 feet. In order for this question to have the significance which its propounder means it to have, I must restate it as follows: A train is 1,000 feet long as measured by an observer travelling with it. If it passes a second observer at 60 miles per hour, what is its length as observed by him? The answer is now easy.]* [According to the formula the length of the moving train as seen from the ground will be

$$1,000 \times \sqrt{1-(88)^2/(186,000 \times 5,280)^2} = 999.999,999,999,996$$

feet, a change entirely too small for detection by the most delicate instruments. Examination of the expression K shows that in so far as terrestrial movements of material objects are concerned it is equal to 1][232] [within a far smaller margin than we can ever hope to make our observations. Even the diameter of the earth, as many of the essayists point out, will be shortened only 2½ inches for an outside observer past whom it rushes with its orbital speed of 18.5 miles [95] per second. But slight as the difference may be in these familiar cases, its scientific importance remains the same.]*

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Relativity and Reality

[A simple computation shows that this effect is exactly the amount suggested by Lorentz and Fitzgerald to explain the Michelson-Morley experiment.][188] [This ought not to surprise us, since both that explanation and the present one are got up with the same purpose. If they both achieve that purpose they must, numerically, come to the same thing in any numerical case. It is, however, most emphatically to be insisted that the present "shortening" of lengths]* [no longer appears as a "physical" shortening caused by absolute motion through the ether but is simply a result of our methods of measuring space and time.][188] [Where Fitzgerald and Lorentz had assumed that a body in motion has its dimensions shortened in the direction of its motion,][220] [this very form of statement ceases to possess significance under the relativity assumption.]* [For if we cannot tell which of two bodies is moving, which one is shortened? The answer is, both—*for the other fellow*. For each frame of reference there is a scale of length and a scale of time, and these scales for different frames are related in a manner involving *both* the length and the time.][220] [But we must not yield to the temptation to say that all this is not real; the confinement of a certain scale of length and of time to a single observation system does not in the least make it unreal.]* [The situation *is* real—as real as any other physical event.][165] [96]

[The word physical is used in two senses in the above paragraph. It is denied that the observed variability in lengths indicates any "physical" contraction or shrinkage; and on the heels of this it is asserted that this observed variability is of itself an actual "physical" event. It is difficult to express in words the distinction between the two senses in which the term physical is employed in these two statements, but I think this distinction ought to be clear once its existence is emphasized. There is no material contraction; it is not right to say that objects in motion contract or are shorter; they are not shorter to an observer in motion with them. The whole thing is a phenomenon of observation. The definitions which we are obliged to lay down and the assumptions which we are obliged to make in order, first, that we shall be able to measure at all, and second, that we shall be able to escape the inadmissible concept of absolute motion, are such that certain realities which we had supposed ought to be the same for all observers turn out not to be the same for observers who are in relative motion with respect to *one another*. We have found this out, and we have found out the numerical relation which holds between the reality of the one observer and that of the other. We have found that this relation depends upon nothing save the relative velocity of the two observers. As good a way of emphasizing this as any is to point out that two observers who have the same velocity with respect to the system under examination (and whose mutual relative velocity is therefore zero) will always get the same results when measuring lengths and times on that system. The [97] object does not go through any process of contraction; it is simply shorter because it is observed from a station with respect to which it is moving. Similar remarks might be made about the time effect; but the time-interval is not so easily visualized as a concrete thing and hence does not offer such temptation for loose statement.

The purely relative aspect of the matter is further brought out if we consider a single example both backwards and forwards. Systems S and S' are in relative motion. An object in S which to an observer in S is L units long, is shorter for an observer in S'—shorter by an amount indicated through the "correction factor" K. Now if we have, in the first instance, made the objectionable statement that objects are shorter in system S' than they are in S, it will be quite natural for us to infer from this that objects in S must be longer than those in S'; and from this to assert that when the observer in S measures objects lying in S', he gets for them greater lengths than does the home observer in S'. But if we have, in the first instance, avoided the objectionable statement referred to, we shall be much better able to realize that the whole business is quite reciprocal; that the phenomena are symmetric with respect to the two systems, to the extent that we can interchange the systems in any of our statements without modifying the statements in any other way.

Objects in *S* appear shorter and times in *S* appear longer to the external "moving" observer in *S′* than they do to the domestic observer in *S*. Exactly in the same way, objects in *S′* appear shorter to observers in the foreign system *S* [98]
than to the home observer in *S′*, who remains at rest with respect to them. I think that when we get the right angle upon this situation, it loses the alleged startling character which has been imposed upon it by many writers. The "apparent size" of the astronomer is an analogy in point. Objects on the moon, by virtue of their great distance, look smaller to observers on the earth than to observers on the moon. Do objects on the earth, on this account, look larger to a moon observer than they do to us? They do not; any suggestion that they do we should receive with appropriate scorn. The variation in size introduced by distance is reciprocal, and this reciprocity does not in the least puzzle us. Why, then, should that introduced by relative motion puzzle us?

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Time and Space in a Single Package

Our old, accustomed concepts of time and space, which have grown up through countless generations of our ancestors, and been handed down to us in the form in which we are familiar with them, leave no room for a condition where time intervals and space intervals are not universally fixed and invariant. They leave no room for us to say that]* [one cannot know the time until he knows where he is, nor where he is until he knows the time,][220] [nor either time or place until he knows something about velocity. But in this concise formulation of the difference between what we have always believed and what we have seen to be among the consequences of Einstein's postulates of the universal relativity of uniform motion, we may at once locate the [99]
assumption which, underlying all the old ideas, is the root of all the trouble. The fact is we have always supposed time and space to be absolutely distinct and independent entities.]*

[The concept of time has ever been one of the most absolute of all the categories. It is true that there is much of the mysterious about time; and philosophers have spent much effort trying to clear up the mystery—with unsatisfactory results. However, to most persons it has seemed possible to adopt an arbitrary measure or unit of duration and to say that this is absolute, independent of the state of the body or bodies on which it is used for practical purposes.][272] [Time has thus been regarded as something which of itself flows on regularly and continuously, regardless of physical events concerning matter.][150] [In other words, according to this view, time is not affected by conditions or motions in space.][272] [We have deliberately chosen to ignore the obvious fact that time can never appear to us, be measured by us, or have the least significance for us, save as a measure of something that is closely tied up with space and with material space-dimensions. Not merely have we supposed that time and space are separated in nature as in our easiest perceptions, but we have supposed that they are of such fundamentally distinct character that they can never be tied up together. In no way whatever, assumes the Euclidean and

Newtonian intellect, may space ever depend upon time or time upon space. This is the assumption which we must remove in order to attain universal relativity; and while it may come hard, it will not come so hard as the [100] alternative. For this alternative is nothing other than to abandon universal relativity. This course would leave us with logical contradictions and discrepancies that could not be resolved by any revision of fundamental concepts or by any cleaning out of the Augean stables of old assumptions; whereas the relativity doctrine as built up by Einstein requires only such a cleaning out in order to leave us with a strictly logical and consistent whole. The rôle of Hercules is a very difficult one for us to play. Einstein has played it for the race at large, but each of us must follow him in playing it for himself.

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Some Further Consequences

I need not trespass upon the subject matter of those essays which appear in full by going here into any details with regard to the manner in which time and space are finally found to depend upon one another and to form the parts of a single universal whole. But I may appropriately point out that if time and space are found to be relative, we may surely expect some of the less fundamental concepts that depend upon them to be relative also. In this expectation we are not disappointed. For one thing,]* [mass has always been assumed to be a constant, independent of any motion or energy which it might possess. Just as lengths and times depend upon relative motion, however, it is found that mass, which is the remaining factor in the expression for energy due to motion, also depends upon relative velocities. The dependence is such that if a body takes [101] up an amount of energy *E with respect to a certain system*, the body behaves, to measurements made *from that system*, as though its mass had been increased by an amount E/C^2, where *C* is as usual the velocity of light.][194]

[This should not startle us. The key to the situation lies in the italicized words above, which indicate that the answer to the query whether a body has taken up energy or not depends upon the seat of observation. If I take up my location on the system *S*, and you on the system *S′*, and if we find that we are in relative motion, we must make some assumption about the energy which was necessary, initially, to get us into this condition. Suppose we are on two passing trains.]* [The chances are that either of us will assume that he is at rest and that it is the other train which moves, although if sufficiently sophisticated one of us may assume that he is moving and that the other train is at rest.][272] [Whatever our assumption, whatever the system, the localization of the energy that is carried in latent form by our systems depends upon this assumption. Indeed, if our systems are of differing mass, our assumptions will even govern our ideas of the *amount* of energy which is represented by our relative motion; if your system be the more massive, more energy would have to be localized in it than in mine to produce our relative motion. If we did not have the universal principle of relativity to forbid, we might make an arbitrary assumption about our motions and hence about our respective latent energies; in the presence of this veto, the only chance of adjustment lies in our masses, which must differ according to whether you or I observe them.]* [102]

[For most of the velocities with which we are familiar E/C^2 is, like the difference between *K* and unity, such an extremely small quantity that the most delicate measurements fail to detect it. But the electrons in a highly evacuated tube and the particles shot out from radioactive materials attain in some cases velocities as high as eight-tenths that of light. When we measure the mass of such particles at different velocities we find that it actually increases with the velocity, and in accordance with the foregoing law.][194] [This observation, in fact, antedates Einstein's explanation, which is far more satisfactory than the earlier differentiation between "normal mass" and "electrical mass" which was called upon to account for the increase.]*

[But if the quantity E/C^2 is to be considered as an actual increase in mass, may it not be possible that all mass is energy? This would lead to the conclusion that the energy stored up in *any* mass is mC^2. The value is very great, since *C* is so large; but it is in good agreement with the internal energy of the atom as calculated from other considerations. It is obvious that conservation of mass and of momentum cannot both hold good under a theory that translates the one into the other. Mass is then not considered by Einstein as conservative in the ordinary sense, but it is the total quantity of mass plus energy in any closed system that remains constant. Small amounts of energy may be transformed into mass, and *vice versa*.][194]

[Other features of the theory which are often displayed as consequences are really more in the nature of assumptions. It will be recalled that when we had [103] agreed upon the necessity of employing signals of some sort, we selected as the means of signalling the speediest messenger with which we happened to be acquainted. Our subsequent difficulties were largely due to the impossibility of making a proper allowance for this messenger's speed, even though we knew its numerical value; and as a consequence, this speed enters into our formulae. Now we have not said in so many words that *C* is the greatest speed attainable, but we have tacitly assumed that it is. We need not, therefore, be surprised if our formulae give us absurd results for speeds higher than *C*, and indicate the impossibility of ever attaining these. Whatever we put into a problem the algebra is bound to give us back. If we look at our formula for *K*, we see that in the event of *v* equalling *C*, lengths become zero and times infinite. The light messenger itself, then, has no dimension; and for it time stands still.

If we suppose *v* to be greater than *C*, we get even more bizarre results, for then the factor *K* is the square root of a negative number, or as the mathematician calls it an "imaginary" quantity; and with it, lengths and times become imaginary too.

The fact that time stops for it, and the fact that it is the limiting velocity, give to *C* certain of the attributes of the mathematician's infinity. Certainly if it can never be exceeded, we must have a new formula for the composition of velocities. Otherwise when my system passes yours at a speed of 100,000 miles per second, while yours passes a third in the same direction at the same velocity, I shall be passing this third framework at the forbidden velocity of [104] 200,000 miles per second—greater than *C*. In fact Einstein is able to show that an old formula, which had already been found to connect the speed of light in a *material* medium with the speed of that medium, will now serve universally for the composition of velocities. When we combine the velocities *v* and *u*, instead of getting the resultant $v + u$ as we would have supposed, we get the resultant

$$(v + u)/(1 + (uv/C^2)),$$

or

$$C^2(v + u)/(C^2 + uv)$$

This need not surprise us either, if we will but reflect that the second velocity effects a second revision of length and time measurements between the systems involved. And now, if we let either *v*, or *u*, or even both of them, take the value *C*, the resultant still is *C*. In another way we have found *C* to behave like the mathematician's infinity, to which, in the words of the blind poet, if we add untold thousands, we effect no real increment.

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Assumption and Consequence

A good many correspondents who have given the subject sufficient thought to realize that the limiting character of the velocity *C* is really read into Einstein's system by assumption have written, in more or less perturbed inquiry, to know whether this does not invalidate the whole structure. The answer, of course, is yes—provided you can show this assumption to be invalid. The same answer [105] may be made of any scientific doctrine whatever, and in reference to any one of the multitudinous assumptions underlying it. If we were to discover, tomorrow, a way of sending signals absolutely instantaneously, Einstein's whole structure would collapse as soon as we had agreed to use this new method. If we were to discover a signalling agent with finite velocity greater than that of light, relativity would persist with this velocity written in its formulae in the place of *C*.

It is a mistake to quote Einstein's theory in support of the statement that such a velocity can never be. An assumption proves its consequences, but never can prove itself; it must remain always an assumption. But in the presence of long human experience supporting Einstein's assumption that no velocity in excess of *C* can be found, it is fair to demand that it be disputed not with argument but with demonstration. The one line of argument that would hold out *a priori* hope of reducing the assumption to an absurdity would be one based on the familiar idea of adding velocities; but Einstein has spiked this argument before it is started by replacing the direct addition of velocities with another method of combining them that fits in with his assumption and as well with the observed facts. The burden of proof is then on the prosecution; anyone who would contradict our assertion that *C* is the greatest velocity attainable may do so only by showing us a greater one. Until this has been done, the admission that it may properly be attempted can in no way be construed as a confession of weakness on the part of Einstein. [106]

It may be well to point out that in no event may analogy be drawn with sound, as many have tried to do. In the first place sound requires a material medium and its velocity with regard to *this* rather than relative to the observer we know to be fixed; in the second place, requiring a material medium, sound is not a universal signalling agent; in the third place, we *know* definitely that its velocity *can* be exceeded, and are therefore barred from making the assumption necessary to establish the analogy. The very extraordinary behavior of light in presenting a velocity that *is* the same for all observers, and in refusing to betray the least material evidence of any medium for its transmission, rather fortifies us in believing that Einstein's assumption regarding the ultimate character of this velocity is in accord with the nature of things.

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Relativity and the Layman

A great deal can be said in the direction of general comment making the Special Theory and its surprising accompaniments easier of acceptance, and we shall conclude the present discussion by saying some of these things.]* [It has been objected that the various effects catalogued above are only apparent, due to the finite velocity of light—that the *real* shape and size of a body or the *real* time of an event cannot be affected by the point of view or the motion of an observer. This argument would be perfectly valid, if there *were* real times and distances; but there are not. These are earth-bound notions, due to our experience on an apparently motionless platform, with slow-moving bodies. [107] Under these circumstances different observations of the same thing or of the same event agree. But when we no longer have the solid earth to stand on, and are dealing with velocities so high that the relativity effects become appreciable, there is no standard by which to resolve the disagreements. No one of the observations can claim to be nearer reality than any other. To demand the real size of a thing is to demand a stationary observer or an instantaneous means of information. Both are impossible.

When relativity asks us to give up our earth-bound notions of absolute space and absolute time the sensation, at first, is that we have nothing left to stand on. So must the contemporaries of Columbus have felt when told that the earth rested on—nothing. The remedy too is similar. Just as they had to be taught that falling is a local affair, that the earth is self-contained, and needs no external support—so we must be taught that space and time standards are local affairs. Each moving body carries its own space and time standards with it; it is self-contained. It does not need to reach out for eternal support, for an absolute space and time that can never quite be attained. All we ever need to know is the relation of the other fellow's space and time standards to our own. This is the first thing relativity teaches us.][141]

[The consequences of Einstein's assumptions have led many to reject the theory of relativity, on the ground that its conclusions are contrary to common sense—as they undoubtedly are. But to the contemporaries of Copernicus and Galileo the theory that the earth rotates on its axis and revolves around the sun [108] was contrary to common sense; yet this theory prevailed. There is nothing sacred about common sense; in the last analysis its judgments are based on the accumulated experience of the human race. From the beginning of the world up to the present generation, no bodies were known whose velocities were not extremely small compared with that of light. The development of modern physics has led to discovery of very much larger velocities, some as high as 165,000 miles per second. It is not to be wondered at that such an enlargement of our experience requires a corresponding enlargement or generalization of the concepts of space and time. Just as the presupposition of primitive man that the earth was flat had to be given up in the light of advancing knowledge, so we are now called upon to give up our presupposition that space and time are absolute and independent in their nature.

The reader must not expect to understand the theory of relativity in the sense of making it fit in with his previous ideas. If the theory be right these ideas are wrong and must be modified, a process apt to be painful.][223] [All the reader can do is to become familiar with the new concepts, just as a child gets used to the simple relations and quantities he meets until he "understands" them.][221] [Mr. Francis has said something of the utmost significance when he points out that "understanding" really means nothing in the world except familiarity and accustomedness.]* [The one thing about the relativity doctrine that we can hope thus to understand at once and without pain is the logical process used in [109] arriving at our results.][221] [Particularly is it hard to give a satisfactory explanation of the theory in popular language, because the language itself is based on the old concepts; the only language which is really adequate is that of mathematics.][223] [Unless we have, in addition to the terms of our ordinary knowledge, a set of definitions that comes with a wide knowledge of mathematics and a lively sense of the reality of mathematical constructions, we are likely to view the theory of relativity through a fog of familiar terms suddenly become self-contradictory and deceptive. Not that we are unfamiliar with the idea that some of our habitual notions may be wrong; but knowledge of their illusory nature arises and becomes convincing only with time. We may now be ready to grant that the earth, seemingly so solid, is really a whirling globe rushing through space; but we are no more ready immediately to accept the bald assertion that this space is not what it seems than our ancestors were to accept the idea that the earth was round or that it moved.][156] [What we must have, if we are to comprehend relativity with any degree of thoroughness, is the mathematician's attitude toward his assumptions, and his complete readiness to swap one set of assumptions for another as a mere part of the day's work, the spirit of which I have endeavored to convey in the chapter on non-Euclidean geometry.]*

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Physics vs. Metaphysics

[The ideas of relativity may seem, at first sight, to be giving us a new and metaphysical theory of time and space. New, doubtless; but certainly the [110] theory was meant by its author to be quite the opposite of metaphysical. Our actual perception of space is by measurement, real and imagined, of distances between objects, just as our actual perception of time is by measurement. Is it not less metaphysical to accept space and time as our measurements present them to us, than to invent hypotheses to force our perceptual space into an absolute space that is forever hidden from us?][182] [In order not to be metaphysical, we must eliminate our preconceived notions of space and time and motion, and focus our attention upon the indications of our instruments of observation, as affording the only objective manifestations of these qualities and therefore the only attributes which we can consider as functions of observed phenomena.][47] [Einstein has consistently followed the teachings of experience, and completely freed himself from metaphysics.][114] [That this is not always easy to do is clear, I think, if we will recall the highly metaphysical character often taken by the objections to action-at-a-distance theories and concepts; and if we will remind ourselves that it was on purely metaphysical grounds that Newton refused to countenance Huyghens' wave theory of light. Whether, as in the one case, it leads us to valid conclusions, or, as in the other, to false ones, metaphysical reasoning is something to avoid. Einstein, I think, has avoided it about as thoroughly as anyone ever did.]* [111]

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V

THAT PARALLEL POSTULATE

Modern Geometric Methods; the Dividing Line Between Euclidean and Non-Euclidean; and the Significance of the Latter

BY THE EDITOR

The science of geometry has undergone a revolution of which the outsider is not informed, but which it is necessary to understand if we are to attain any comprehension of the geometric formulation of Einstein's results; and especially if we are to appreciate why it is proper and desirable to formulate these results geometrically at all. The classical geometer regarded his science from a narrow viewpoint, as the study of a certain set of observed phenomena —those of the space about us, considered as an entity in itself and divorced from everything in it. It is clear that some things about that space are not as they appear (optical illusions), and that other things about it are true but by no means apparent (the sum-of-squares property of a right triangle, the formulæ for surface and volume of a sphere, etc.). While many things about space *are* "obvious," these need in the one case disproof and in the other discovery and proof. With all their love of mental processes for their own sake, it is then not [112] surprising that the Greeks should have set themselves the task of proving by logical process the properties of space, which a less thoughtful folk would have regarded as a subject only for observational and experimental determination.

But, abstract or concrete, the logical structure must have a starting point; and it is fair to demand that this consist in a statement of the terms we are going to use and the meanings we are going to attach to them. In other words, the first thing on the program will be a definition, or more probably, several definitions.

Now the modern scientist has a somewhat iconoclastic viewpoint toward definitions, and especially toward the definition of his very most fundamental ideas.

We do not speak here in terms of dictionary definitions. These have for object the eminent necessity of explaining the meaning and use of a word to some one who has just met it for the first time. It is easy enough to do this, if the doer possesses a good command of the language. It is not even a matter of grave concern that the words used in the definition be themselves known to the reader; if they are not, he must make their acquaintance too. Dr. Johnson’s celebrated definition of a needle stands as perpetual evidence that when he cannot define a simple thing in terms of things still simpler, the lexicographer is forced to define it in terms of things more complex. Or we might demonstrate this by noting that the best dictionaries are driven to define such words as “and” and “but” by using such complex notions as are embodied in [113]
“connective,” “continuative,” “adversative,” and “particle.”

It is otherwise with the scientist who undertakes to lay down a definition as the basis of further procedure in building up the tissue of his science. Here a degree of rigorous logic is called for which would be as superfluous in the dictionary as the effort there to attain it would be out of place. The scientist, in building up a logical structure that will withstand every assault, must define everything, not in terms of something which he is more or less warranted in supposing his audience to know about, but actually in terms of things that have already been defined. This really means that he must explain what he is talking about in terms of simpler ideas and simpler things, which is precisely what the lexicographer does not have to worry about. This is why it is quite trivial to quote a dictionary definition of time or space or matter or force or motion in settlement of a controversy of scientific or semi-scientific nature.

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Terms We Cannot Define

But the scientist who attempts to carry out this ideal system of defining everything in terms of what precedes meets one obstacle which he cannot surmount directly. Even a layman can construct a passable definition of a complex thing like a parallelopiped, in terms of simpler concepts like point, line, plane and parallel. But who shall define point in terms of something simpler and something which precedes point in the formulation of geometry? The scientist is embarrassed, not in handling the complicated later parts of his work, but in the very beginning, in dealing with the simplest concepts with [114]
which he has to deal.

Suppose a dictionary were to be compiled with the definitions arranged in logical rather than alphabetical order: every word to be defined by the use only of words that have already been defined. The further back toward the beginning we push this project, the harder it gets. Obviously we can never define the first word, or the second, save as synonymous with the first. In fact we should need a dozen words, more or less, to start with—God-given words which we cannot define and shall not try to define, but of which we must *agree that we know* the significance. Then we have tools for further procedure; we can start with, say, the thirteenth word and define all the rest of the words in the language, in strictly logical fashion.

What we have said about definitions applies equally to statements of fact, of the sort which are going to constitute the body of our science. In the absence of simpler facts to cite as authority, we shall never be able to prove anything, however simple this may itself be; and in fact the simpler it be, the harder it is to find something simpler to underlie it. If we are to have a logical structure of any sort, we must begin by laying down certain terms which we shall not attempt to define, and certain statements which we shall not try to prove. Mathematics, physics, chemistry—in the large and in all their many minor fields—all these must start somewhere. Instead of deceiving ourselves as to the circumstances surrounding their start, we prefer to be quite frank in recognizing that they start where we decide to start them. If we don't like one [115]
set of undefined terms as the foundation, by all means let us try another. But always we must have such a set.

The classical geometer sensed the difficulty of defining his first terms. But he supposed that he had met it when he defined these in words free of technical significance. "A point is that which has position without size" seemed to him an adequate definition, because "position" and "size" are words of the ordinary language with which we may all be assumed familiar. But today we feel that "position" and "size" represent ideas that are not necessarily more fundamental than those of "line" and "point," and that such a definition begs the question. We get nowhere by replacing the undefined terms "point" and "line" and "plane," which really everybody understands, by other undefined terms which nobody understands any better.

In handling the facts that it was inconvenient to prove, the classical geometer came closer to modern practice. He laid down at the beginning a few statements which he called "axioms," and which he considered to be so self-evident that demonstration was superfluous. That the term "self-evident" left room for a vast amount of ambiguity appears to have escaped him altogether. His axioms were axioms solely because they were obviously true.

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Laying the Foundation

The modern geometer falls in with Euclid when he writes an elementary text,
satisfying the beginner's demand for apparent rigor by defining point and line [116]
in some fashion. But when he addresses to his peers an effort to clarify the foundations of geometry to a further degree of rigor and lucidity than has ever before been attained, he meets these difficulties from another quarter. In the first place he is always in search of the utmost possible generality, for he has found this to be his most effective tool, enabling him as it does to make a single general statement take the place and do the work of many particular statements. The classical geometer attained generality of a sort, for all his statements were of *any* point or line or plane. But the modern geometer, confronted with a relation that holds among points or between points and lines, at once goes to speculating whether there are not other elements among or between which it holds. The classical geometer isn't interested in this question

at all, because he is seeking the absolute truth about the points and lines and planes which he sees as the elements of space; to him it is actually an object so to circumscribe his statements that they may by no possibility refer to anything other than these elements. Whereas the modern geometer feels that his primary concern is with the fabric of logical propositions that he is building up, and not at all with the elements about which those propositions revolve.

It is of obvious value if the mathematician can lay down a proposition true of points, lines and planes. But he would much rather lay down a proposition true at once of these and of numerous other things; for such a proposition will group more phenomena under a single principle. He feels that on pure scientific grounds there is quite as much interest in any one set of elements to [117] which his proposition applies as there is in any other; that if any person is to confine his attention to the set that stands for the physicist's space, that person ought to be the physicist, not the geometer. If he has produced a tool which the physicist can use, the physicist is welcome to use it; but the geometer cannot understand why, on that ground, he should be asked to confine *his* attention to the materials on which the physicist employs that tool.

It will be alleged that points and lines and planes lie in the mathematician's domain, and that the other things to which his propositions may apply may not so lie—and especially that if he will not name them in advance he cannot expect that they will so lie. But the mathematician will not admit this. If mathematics is defined on narrow grounds as the science of number, even the point and line and plane may be excluded from its field. If any wider definition be sought—and of course one must be—there is just one definition that the mathematician will accept: Dr. Keyser's statement that "mathematics is the art or science of rigorous thinking."

The immediate concern of this science is the means of rigorous thinking—undefined terms and definitions, axioms and propositions. Its collateral concern is the things to which these may apply, the things which may be thought about rigorously—everything. But now the mathematician's domain is so vastly extended that it becomes more than ever important for him to attain the utmost generality in all his pronouncements. [118]

One barrier to such generalization is the very name "geometry," with the restricted significance which its derivation and long usage carry. The geometer therefore must have it distinctly understood that for him "geometry" means simply the process of deducing a set of propositions from a set of undefined primitive terms and axioms; and that when he speaks of "a geometry" he means some particular set of propositions so deduced, together with the axioms, etc., on which they are based. If you take a new set of axioms you get a new geometry.

The geometer will, if you insist, go on calling his undefined terms by the familiar names "point," "line," "plane." But you must distinctly understand that this is a concession to usage, and that you are not for a moment to restrict the application of his statements in any way. He would much prefer, however, to be allowed new names for his elements, to say "We start with three elements of different sorts, which we assume to exist, and to which we attach the names A, B and C—or if you prefer, primary, secondary and tertiary elements—or yet again, names possessing no intrinsic significance at all, such as ching, chang and chung." He will then lay down whatever statements he requires to serve the purposes of the ancient axioms, all of these referring to some one or more of his elements. Then he is ready for the serious business of proving that, *all his hypotheses being granted*, his elements A, B and C, or I, II and III, or ching, chang and chung, are subject to this and that and the other propositions. [119]

The objection will be urged that the mathematician who does all this usurps the place of the logician. A little reflection will show this not to be the case. The logician in fact occupies the same position with reference to the geometer that the geometer occupies with reference to the physicist, the chemist, the arithmetician, the engineer, or anybody else whose primary interest lies with some particular set of elements to which the geometer's system applies. The mathematician is the tool-maker of all science, but he does not make his own tools—these the logician supplies. The logician in turn never descends to the actual practice of rigorous thinking, save as he must necessarily do this in laying down the general procedures which govern rigorous thinking. He is interested in processes, not in their application. He tells us that if a proposition is true its converse may be true or false or ambiguous, but its contrapositive is always true, while its negative is always false. But he never, from a particular proposition "If A is B then C is D," draws the particular contrapositive inference "If C is not D then A is not B." That is the mathematician's business.

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The Rôle of Geometry

The mathematician is the quantity-production man of science. In his absence, the worker in each narrower field where the elements under discussion take particular concrete forms could work out, for himself, the propositions of the logical structure that applies to those elements. But it would then be found that the engineer had duplicated the work of the physicist, and so for many other cases; for the whole trend of modern science is toward showing that the same [120] background of principles lies at the root of all things. So the mathematician develops the fabric of propositions that follows from this, that and the other group of assumptions, and does this without in the least concerning himself as to the nature of the elements of which these propositions may be true. He knows only that they are true for any elements of which his assumptions are true, and that is all he needs to know. Whenever the worker in some particular field finds that a certain group of the geometer's assumptions *is* true for his elements, the geometry of those elements is ready at hand for him to use.

Now it is all right purposely to avoid knowing what it is that we are talking about, so that the names of these things shall constitute mere blank forms which may be filled in, when and if we wish, by the names of any things in the universe of which our "axioms" turn out to be true. But what about these axioms themselves? When we lay them down in ignorance of the identity of the elements to which they may eventually apply, they cannot by any possibility be "self-evident." We may, at pleasure, accept as self-evident a statement about points and lines and planes; or one about electrons, centimeters and seconds; or one about integers, fractions, and irrational numbers; or one about any other concrete thing or things whatever. But we cannot accept as self-evident a statement about chings, changs and chungs. So we must base our "axioms" on some other ground than this; and our modern geometer has his ground ready and waiting. He accepts his axioms on the [121] ground that it pleases him to do so. To avoid all suggestion that they are supposed to be self-evident, or even necessarily true, he drops the term "axiom" and substitutes for it the more color-less word "postulate." A postulate is merely something that we agreed to accept, for the time being, as a basis of further argument. If it turns out to be true, or if we can find circumstances under which and elements to which it applies, any conclusions which we deduce from it by trustworthy processes are valid within the same limitations. And the propositions which tell us that, *if* our postulates are true, such and such conclusions are true—they, too, are valid, but without any reservation at all!

Perhaps an illustration of just what this means will not be out of place. Let it be admitted, as a postulate, that $7 + 20$ is greater, by 1, than $7 + 19$. Let us then consider the statement: "If $7 + 19 = 65$, then $7 + 20 = 66$." We know—at least we are quite certain—that $7 + 19$ is not equal to 65, if by "7" and "19" and "65" we mean what you think we mean. We are equally sure, on the same grounds, that $7 + 20$ is not equal to 66. But, under the one assumption that we have permitted ourselves, it is unquestionable that *if* $7 + 19$ *were* equal to 65, then $7 + 20$ certainly *would be* equal to 66. So, while the conclusion of the proposition which I have put in quotation marks is altogether false, the proposition itself, under our assumption, is entirely true. I have taken an illustration designed to be striking rather than to possess scientific interest; I could just as easily have shown a true proposition leading to a false conclusion, but of such sort that it would be of decided scientific interest as [122] telling us one of the consequences of a certain assumption.

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What May We Take for Granted?

This is all very fine; but how does the geometer know what postulates to lay down? One is tempted to say that he is at liberty to postulate anything he pleases, and investigate the results; and that whether or not his postulate ever be realized, the propositions that he deduces from it, being true, are of scientific interest. Actually, however, it is not quite as simple as all that. If it were sufficient to make a single postulate it *would* be as simple as all that; but it turns out that this is not sufficient any more than it is sufficient to have a single undefined term. We must have several postulates; and they must be such, as a whole, that a geometry flows out of them. The requirements are three.

In the first place, the system of postulates must be "categorical" or complete—there must be enough of them, and they must cover enough ground, for the support of a complete system of geometry. In practice the test for this is direct. If we got to a point in the building up of a geometry where we could not prove whether a certain thing was one way always, or always the other way, or sometimes one way and sometimes the other, we should conclude that we needed an additional postulate covering this ground directly or indirectly. And we should make that postulate—because it is precisely the things that we can't [123] prove which, in practical work, we agree to assume. Even Euclid had to adopt this philosophy.

In the second place, the system of postulates must be consistent—no one or more of them may lead, individually or collectively, to consequences that contradict the results or any other or others. If in the course of building up a geometry we find we have proved two propositions that deny one another, we search out the implied contradiction in our postulates and remedy it.

Finally, the postulates ought to be independent. It should not be possible to prove any one of them as a consequence of the others. If this property fails, the geometry does not fail with it; but it is seriously disfigured by the superfluity of assumptions, and one of them should be eliminated. If we are to assume *anything* unnecessarily, we may as well assume the whole geometry and be done with it.

The geometer's business then is to draw up a set of postulates. This he may do on any basis whatever. They may be suggested to him by the behavior of points, lines and planes, or by some other concrete phenomena; they may with equal propriety be the product of an inventive imagination. On proceeding to deduce their consequences, he will discover and remedy any lack of categoricity or consistence or independence which his original system of postulates may have lacked. In the end he will have so large a body of propositions without contradiction or failure that he will conclude the propriety of his postulates to have been established, and the geometry based on them to be a valid one. [124]

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And What Is It All About?

Is this geometry ever realized? Strictly it is not the geometer's business to ask or answer this question. But research develops two viewpoints. There is always the man who indulges in the pursuit of facts for their sake alone, and equally the man who wants to see his new facts lead to something else. One great mathematician is quoted as enunciating a new theory of surpassing mathematical beauty with the climacteric remark "And, thank God, no one will ever be able to find any use for it." An equally distinguished contemporary, on being interrogated concerning possible applications for one of his most abstruse theorems, replied that he knew no present use for it; but that long experience had made him confident that the mathematician would never develop any tool, however remote from immediate utility, for which the delvers in other fields would not presently find some use.

If we wish, however, we may inquire with perfect propriety, from the side lines, whether a given geometry is ever realized. We may learn that so far as has yet been discovered there are no elements for which all its postulates are verified, and that there is therefore no realization known. On the other hand, we may more likely find that many different sets of elements are such that the postulates can be interpreted as applying to them, and that we therefore have numerous realizations of the geometry. As a human being the geometer may be interested in all this, but as a geometer it really makes little difference to him. [125]

When we look at space about us, we see it, for some reason grounded in the psychological history of the human race, as made up in the small of points, which go to make up lines, which in turn constitute planes. Or we can start at the other end and break space down first into planes, then into lines, finally into points. Our perceptions and conceptions of these points, lines and planes are very definite indeed; it seems indeed, as the Greeks thought, that certain things about them are self-evident. If we wish to take these self-evident properties of point, line and plane, and combine with them enough additional hair-splitting specifications to assure the modern geometer that we have really a categorical system of assumptions, we shall have the basis of a perfectly good system of geometry. This will be what we unavoidably think of as the absolute truth with regard to the space about us; but you mustn’t say so in the presence of the geometer. It will also be what we call the Euclidean geometry. It has been satisfactory in the last degree, because not only space, but pretty much every other system of two or three elements bearing any relations to one another can be made, by employing as a means of interpretation the Cartesian scheme of plotting, to fit into the framework of Euclidean geometry. But it is not the only thing in the world of conceptual possibilities, and it begins to appear that it may not even be the only thing in the world of cold hard fact that surrounds us. To see just how this is so we must return to Euclid, and survey the historical development of geometry from his day to the present time. [126]

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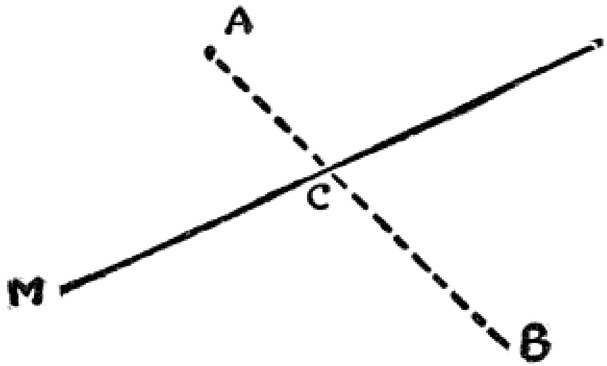
Euclid’s Geometry

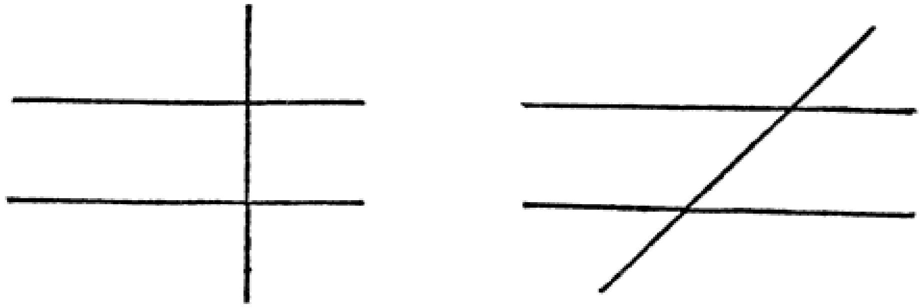
Point, line and plane Euclid attempts to define. Modern objection to these efforts was made clear above. Against Euclid’s specific performance we urge the further specific fault that his “definitions” are really assumptions bestowing certain properties upon points, lines and planes. These assumptions Euclid supplements in his axioms; and in the process of proving propositions he unconsciously supplements them still further. This is to be expected from one whose justification for laying down an axiom was the alleged obvious character of the statement made. If some things are too obvious to require demonstration, others may be admitted as too obvious to demand explicit statement at all.



Thus, if Euclid has two points A and B in a plane, on opposite sides of a line M, he will draw the line AB and without further formality speak of the point C *in which it intersects* M. That it does so intercept M, rather than in some way dodges it, is really an assumption as to the nature of lines and planes. Or again, [127] Euclid will speak of a point D on the line AB, between or outside the points A and B, without making the formal assumption necessary to insure that the line is "full" of points so that such a point as D must exist. That such assumptions as these are necessary follows from our previous remarks. If we think of our geometry as dealing with "chings," "changs," and "chungs," or with elements I, II and III, it is no longer in the least degree obvious that the simplest property in the world applies to these elements. If we wish any property to prevail we must state it explicitly.

With the postulates embodied in his definitions, those stated in his axioms, and those which he reads into his structure by his methods of proof, Euclid has a categorical set—enough to serve as foundation for a geometry. We may then climb into Euclid's shoes and take the next step with him. We follow him while he proves a number of things about intersecting lines and about triangles. To be sure, when he proves that two triangles are identically constituted by moving one of them over on top of the other, we may protest on the ground that the admission of motion, especially of motion thus imposed from without, into a geometry of things is not beyond dispute. If Euclid has caught our modern viewpoint, he will rejoin that if we have any doubts as to the admissibility of motion he will lay down a postulate admitting it, and we shall be silenced.

Having exhausted for the present the interest of intersecting lines, our guide now passes to a consideration of lines in the same plane that never meet. He [128] defines such lines as parallel. If we object that he should show the existence of a derived concept like this before laying down a definition that calls for it to exist, he can show that two lines drawn perpendicular to the same line never meet. He will execute this proof by a special sort of superposition, which requires that the plane be folded over on itself, through the third dimension of surrounding space, rather than merely slid along upon itself.



We remain quiet while Euclid demonstrates that if two lines are cut by *any* transversal in such a way as to make corresponding angles at the two intersections equal, the lines are parallel. It is then in order to investigate the converse: if the lines are parallel to begin with, are the angles equal?

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Axioms Made to Order

This sounds innocent enough; but in no way was Euclid able to devise a proof—or, for that matter, a disproof. So he took the only way out, and said that if the lines were parallel, obviously they extended in the same direction and made the angles equal. The thing was so obvious, he argued, that it was really [129] an axiom and he didn’t have to prove it; so he stated it as an axiom and proceeded. He didn’t state it in precisely the form I have used; he apparently cast about for the form in which it would appear most obvious, and found a statement that suited him better than this one, and that comes to the same thing. This statement tells us that if the transversal makes two corresponding angles *unequal*, the lines that it cuts are not parallel and do meet if sufficiently prolonged. But wisely enough, he did not transplant this axiom, once he had arrived at it, to the beginning of the book where the other axioms were grouped; he left it right where it was, following the proposition that if the angles were equal the lines were parallel. This of course was so that it might appeal back, for its claim to obviousness, to its demonstrated converse of the proposition.

Euclid must have been dissatisfied with this cutting of the Gordian knot; his successors were acutely so. For twenty centuries the parallel axiom was regarded as the one blemish in an otherwise perfect work; every respectable mathematician had his shot at removing the defect by “proving” the objectionable axiom. The procedure was always the same: expunge the parallel axiom, in its place write another more or less “obvious” assumption, and from this derive the parallel statement more or less directly. Thus if we may assume that the sum of the angles of a triangle is always exactly 180 degrees, or that there can be drawn only one line through a given point parallel to a given line, we can prove Euclid’s axiom. Sometimes the substitute assumption was openly [130] made and stated, as in the two instances cited; as often it was admitted into the demonstration implicitly, as when it is quietly assumed that we can draw a triangle similar to any given triangle and with area as great as we please, or when parallels are “defined” as everywhere equidistant. But such “proofs” never satisfied anyone other than the man who made them; the search went merrily on for a valid "proof" that should not in substance assume the thing to be proved.

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Locating the Discrepancy

Saccheri, an Italian Jesuit, would have struck bottom if he had had a little more imagination. He gave an exhaustive *reductio ad absurdum*, on the basis of the angle-sum theorem. This sum must be (a) greater than or (b) equal to or (c) less than 180 degrees. Saccheri showed that if one of these alternatives occurs in a single triangle, it must occur in every triangle. The first case gave little trouble; admitting the possibility of superposing in the special manner mentioned above, which he did implicitly, he showed that this "obtuse-angled hypothesis" contradicted itself. He pursued the "acute-angled hypothesis" for a long time before he satisfied himself that he had caught it, too, in an inconsistency. This left only the "right-angled hypothesis," proving the Euclidean angle-sum theory and through it the parallel postulate. But Saccheri was wrong: he had found no actual contradiction in the acute-angled hypothesis—for none exists therein.

The full facts were probably first known to Gauss, who had a finger in every [131] mathematical pie that had to do with the transition to modern times. They were first published by Lobatchewsky, the Russian, who anticipated the Hungarian John Bolyai by a narrow margin. All three worked independently of Saccheri, whose book, though theoretically available in Italian libraries, was actually lost to sight and had to be rediscovered in recent years.

Like Saccheri, Lobatchewsky investigated alternative possibilities. But he chose another point of attack: through a given point it must be possible to draw, in the same plane with a given line (a) no lines or (b) one line or (c) a plurality of lines, which shall not meet the given line. The word parallel is defined only in terms of the second of these hypotheses, so we avoid it here. These three cases correspond, respectively, to those of Saccheri.

The first case Lobatchewsky ruled out just as did Saccheri, but accepting consciously the proviso attached to its elimination; the third he could not rule out. He developed the consequences of this hypothesis as far as Euclid develops those of the second one, sketching in a full outline for a system of geometry and trigonometry based on a plurality of "non-cutters." This geometry constitutes a coherent whole, without a logical flaw.

This made it plain what was the matter with Euclid's parallel axiom. Nobody could prove it from his other assumptions *because it is not a consequence of these*. True or false, it is independent of them. Trinity Church is in New York, Faneuil Hall is in Boston, but Faneuil Hall is not in Boston *because* Trinity is in New York; and we could not prove that Faneuil Hall was in Boston if we [132] knew nothing about America save that Trinity is in New York. The mathematicians of 2,000 years had been pursuing, on a gigantic scale, a delusion of *post hoc, ergo propter hoc*.

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What the Postulate Really Does

Moreover, in the absence of an assumption covering the ground, we shall not know which of the alternatives (a), (b), (c) holds. But when one holds in a single case it holds permanently, as Saccheri and Lobatchewsky both showed. So we cannot proceed on this indefinite basis; we must know which one is to hold. Without the parallel postulate or a substitute therefor that shall tell us the same thing or tell us something different, we have not got a categorical set of assumptions—we cannot build a geometry at all. That is why Euclid had to have his parallel postulate before he could proceed. That is why his successors had to have an assumption equivalent to his.

The reason why it took so long for this to percolate into the understanding of the mathematicians was that they were thinking, not in terms of the modern geometry and about undefined elements; but in terms of the old geometry and about strictly defined and circumscribed elements. If we understand what is meant by Euclidean line and plane, of course the parallel postulate, to use the old geometer's word, is true—of course, to adopt the modern viewpoint, if we agree to employ an element to which that assumption applies, the assumption is realized. The very fact of accepting the "straight" line and the "flat" plane of [133] Euclid constitutes acceptance of his parallel postulate—the only thing that can separate his geometry from other geometries. But of course we can't prove it; the prior postulates which we would have to use in such an attempt apply where it does not apply, and hence it cannot possibly be consequences of.

To all this the classical Euclidean rejoins that we seem to have in mind elements of some sort to which, with one reservation, his postulates apply. He wants to know what these elements look like. We can, and must, produce them—else our talk about generality is mere drivel. But we must take care that the Euclidean geometer does not try to apply to our elements the notions of straightness and flatness which inhere in the parallel postulate. We cannot satisfy and defy that postulate at the same time. If we do not insist on this point, we shall find that we are reading non-Euclidean properties into Euclidean geometry, and interpreting the elements of the latter as straight lines that are not straight, flat planes that are not flat. It is not the mission of non-Euclidean geometry thus to deny the possibility of Euclidean geometry; it merely demands a place of equal honor.

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The Geometry of Surfaces

Let us ask the Euclidean geometer whether he can recognize his plane after we have crumpled it up like a piece of paper en route to the waste basket. He will hesitate only long enough to recall that in the special case of superposition he [134] has reserved for himself the privilege of deforming his own plane, and to realize that he can always iron his plane out smooth again after we are through with it. This emphasizes the true nature of the two-dimensionality which is the fundamental characteristic of the plane (and of other things, as we shall directly see). The plane is two-dimensional in points *not* because two sets of mutually perpendicular Euclidean straight lines can be drawn in it defining directions of north-south and east-west, but because a point in it can be located by means of two measures. The same statement may be made of anything whatever to which the term "surface" is applicable; anything, however crumpled or irregular it be, that possesses length and breadth without thickness. The surface of a sphere, of a cylinder, of an ellipsoid, of a cone, of a doughnut (mathematically known as a torus), of a gear wheel, of a French horn, all these possess two-dimensionality in points; on all of them we can draw lines and curves and derive a geometry of these figures. If we get away from the notion that geometry of two dimensions must deal with planes, and adopt in place of this idea the broader restriction that it shall deal with surfaces, we shall have the generalization which the Euclidean has demanded that we produce, and the one which in the hands of the modern geometer has shown results.

In this two-dimensional geometry of surfaces in general, that of the plane is merely one special case. Certain of the features met in that case are general. If we agree that we know what we mean by distance, we find that on every surface there is a shortest distance between two points, together with a series [135] of lines or curves along which such distances are taken. These lines or curves we call geodesics. On the plane the geodesic is the straight line. On surfaces in general the geodesic, whatever its particular and peculiar shape, plays the same rôle that is played by the straight line in the plane; it is the secondary element of the geometry, the surface itself and all other surfaces of its type are the tertiary elements. And it is a fact that we can take all the possible spheres, or all the possible French-horn surfaces, and conceive of space *as we know it* being broken down by analysis into these surfaces instead of into planes. The only reason we habitually decompose space into planes is because it comes natural to us to think that way. But geometric points, lines and surfaces must be recognized as abstractions without actual existence, for all of them lack one or more of the three dimensions which such existence implies. These figures exist in our minds but not in the external world about us. So any decomposition of space into geometric elements is a phenomenon of the mind only; it has no parallel and no significance in the external world, and is made in one way or in another purely at our pleasure. There isn't a true, honest-to-goodness geometrical plane in existence any more than there is an honest-to-goodness spherical surface: so on intrinsic grounds one decomposition is as reasonable as another.

Certain of the most fundamental postulates are obeyed by *all* surfaces. As we attempt to discriminate between surfaces of different types, and get, for instance, a geometry that shall be valid for spheres and ellipsoids but not for [136] conicoids in general, we must do so by bringing in additional postulates that embody the necessary restrictions. A characteristic shared by planes, spheres, and various other surfaces is that the geodesics can be freely slid along upon themselves and will coincide with themselves in all positions when thus slid; with a similar arrangement for the surface itself. But the plane stands almost unique among surfaces in that it does not force us to distinguish between its two sides; we can turn it over and still it will coincide with itself; and this property belongs also to the straight line. It does not belong to the sphere, or to the great circles which are the geodesics of spherical geometry; when we turn one of these over, through the three-dimensional space that surrounds it, we find that the curvature lies in the wrong way to make superposition possible. If we postulate that superposition be possible under such treatment, we throw out the sphere and spherical geometry; if we postulate that superposition be only by sliding the surface upon itself we admit that geometry—as Saccheri failed to see, as Lobatchewsky realized, and as Riemann showed at great length in rehabilitating the "obtuse-angled hypothesis." Lobatchewsky's acute-angled geometry is realized on a surface of the proper sort, which admits of unrestricted superposition; but it is not the sort of a surface that I care to discuss in an article of this scope.

Euclidean geometry is the natural and easy one, I suppose, because it makes it easy to stop with three dimensions. If we take a secondary element, a geodesic, which is "curved" in the Euclidean sense, we get a tertiary element, a surface, [137] which is likewise curved. Then unless we are to make an altogether abrupt and unreasonable break, we shall find that just as the curved geodesic generated a curved surface, the curved surface must give rise to a "curved space"; and just as the curved geodesic needed a second dimension to curve into, and the curved surface a third, so the curved three-space requires a fourth. Once started on this sort of thing, there doesn't really seem to be any end.

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Euclidean or Non-Euclidean

Nevertheless, we must face the possibility that the space we live in, or any other manifold of any sort whatever with which we deal on geometric principles, may turn out to be non-Euclidean. How shall we finally determine this? By measures—the Euclidean measures the angles of an actual triangle and finds the sum to be exactly 180 degrees; or he draws parallel lines of indefinite extent and finds them to be everywhere equally distant; and from these data he concludes that our space is really Euclidean. But he is not necessarily right.

We ask him to level off a plot of ground by means of a plumb line. Since the line always points to the earth's center, the "level" plot is actually a very small piece of a spherical surface. Any test conducted on this plot will exhibit the numerical characteristics of the Euclidean geometry; yet we know the geometry of this surface is Riemannian. The angle-sum is really greater than 180 degrees; lines that are everywhere equidistant are not both geodesics. [138]

The trouble, of course, is that on this plot we deal with so minute a fraction of the whole sphere that we cannot make measurements sufficiently refined to detect the departure from Euclidean standards. So it is altogether sensible for us to ask: "Is the universe of space about us really Euclidean in whatever of realized geometry it presents to us? Or is it really non-Euclidean, but so vast in size that we have never yet been able to extend our measures to a sufficiently large portion of it to make the divergence from the Euclidean standard discernible to us?"

This discussion is necessarily fragmentary, leaving out much that the writer would prefer to include. But it is hoped that it will nevertheless make it clear that when the contestants in the Einstein competition speak of a non-Euclidean universe as apparently having been revealed by Einstein, they mean simply that to Einstein has occurred a happy expedient for testing Euclideanism on a smaller scale than has heretofore been supposed possible. He has devised a new and ingenious sort of measure which, if his results be valid, enables us to operate in a smaller region while yet anticipating that any non-Euclidean characteristics of the manifold with which we deal will rise above the threshold of measurement. This does not mean that Euclidean lines and planes, as we picture them in our mind, are no longer non-Euclidean, but merely that these concepts do not quite so closely correspond with the external reality as we had supposed.

As to the precise character of the non-Euclideanism which is revealed, we may leave this to later chapters and to the competing essayists. We need only point [139] out here that it will not necessarily be restricted to the matter of parallelism. The parallel postulate is of extreme interest to us for two reasons; first because historically it was the means by which the possibilities and the importance of non-Euclidean geometry were forced upon our attention; and second because it happens to be the immediate ground of distinction between Euclidean geometry and two of the most interesting alternatives. But Euclidean geometry is characterized, not by a single postulate, but by a considerable number of postulates. We may attempt to omit any one of these so that its ground is not specifically covered at all, or to replace any one of them by a direct alternative. We might conceivably do away with the superposition postulate entirely, and demand that figures be proved equivalent, if at all, by some more drastic test. We might do away with the postulate, first properly formulated by Hilbert, on which our ideas of the property represented in the word "between" depend. We might do away with any single one of the Euclidean postulates, or with any combination of two or more of them. In some cases this would lead to a lack of categoricity and we should get no geometry at all; in most cases, provided we brought a proper degree of astuteness to the formulation of alternatives for the rejected postulates, we should get a perfectly good system of non-Euclidean geometry: one realized, if at all, by other elements than the Euclidean point, line and plane, and one whose elements behave toward one another differently from the Euclidean point, line and plane. [140]

Merely to add definiteness to this chapter, I annex here the statement that in the geometry which Einstein builds up as more nearly representing the true external world than does Euclid's, we shall dispense with Euclid's (implicit) assumption, underlying his (explicitly stated) superposition postulate, to the effect that the act of moving things about does not affect their lengths. We shall at the same time dispense with his parallel postulate. And we shall add a fourth dimension to his three—not, of course, anything in the nature of a fourth Euclidean straight line perpendicular, in Euclidean space, to three lines that are already perpendicular to each other, but something quite distinct from this, whose nature we shall see more exactly in the next chapter. If the present chapter has made it clear that it is proper for us to do this, and has prevented anyone from supposing that the results of doing it must be visualized in a Euclidean space of three dimensions or of any number of dimensions, it will have served its purpose. [141]

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VI

THE SPACE-TIME CONTINUUM

Minkowski's World of Events, and the Way It Fits Into Einstein's Structure

BY THE EDITOR, EXCEPT AS NOTED

Seeking a basis for the secure formulation of his results, and especially a means for expressing mathematically the facts of the dependence which he had found to exist between time and space, Einstein fell back upon the prior work of Minkowski. It may be stated right here that the idea of time as a fourth dimension is not particularly a new one. It has been a topic of abstract speculation for the best part of a century, even on the part of those whose notions of the fourth dimension were pretty closely tied down to the idea of a fourth dimension of Euclidean point-space, which would be marked by a fourth real line, perpendicular to the other three, and visible to us if we were only able to see it. Moreover, every mathematician, whether or not he be inclined to this sort of mental exercise, knows well that whenever time enters his equations at all, it does so on an absolutely equal footing with each of his space coordinates, so that as far as his algebra is concerned he could never distinguish between them. When the variables x, y, z, t come to the mathematician in connection with some physical investigation, he knows [142] before he starts that the first three represent the dimensions of Euclidean three-space and that the last stands for time. But if the algebraic expressions of such a problem were handed to him independently of all physical tie-up, he would never be able to tell, from them alone, whether one of the four variables represented time, or if so, which one to pick out for this distinction.

It was Minkowski who first formulated all this in a form susceptible of use in connection with the theory of relativity. His starting point lies in the distinction between the point and the event. Mr. Francis has brought this out rather well in his essay, being the only competitor to present the Euclidean geometry as a real predecessor of Newtonian science, rather than as a mere part of the Newtonian system. I think his point here is very well taken. As he says, Euclid looked into the world about him and saw it composed of points. Ignoring all dynamic considerations, he built up in his mind a static world of points, and constructed his geometry as a scientific machine for dealing with this world in which motion played no part. It could to be sure be introduced by the observer for his own purposes, but when so introduced it was specifically postulated to be a matter of no moment at all to the points or lines or figures that were moved. It was purely an observational device, intended for the observer's convenience, and in the bargain a mental device, calling for no physical action and the play of no force. So far as Euclid in his daily life was obliged to take cognizance of the fact that in the world of work-a-day realities motion existed, [143] he must, as a true Greek, have looked upon this as a most unfortunate deviation of the reality from his beautiful world of intellectual abstraction, and as something to be deplored and ignored. Even in their statuary the Greeks clung to this idea. A group of marvelous action, like the Laocoon, they held to be distinctly a second rate production, a prostitution of the noble art; their ideal was a figure like the majestic Zeus—not necessarily a mere bust, be it understood, but always a figure in repose without action. Their statuary stood for things, not for action, just as their geometry stood for points, not for events.

Galileo and Newton took a different viewpoint. They were interested in the world as it is, not as it ought to be; and if motion appears to be a fundamental part of that world, they were bound to include it in their scheme. This made it necessary for them to pay much more attention to the concept of time and its place in the world than did the Greeks. In the superposition process, and even when he allowed a curve to be generated by a moving point, the sole interest which Euclid had in the motion was the effect which was to be observed upon his static figures after its completion. In this effect the rate of the motion did not enter. So all questions of velocity and time are completely ignored, and we have in fact the curious spectacle of motion without time.

To Galileo and Newton, on the other hand, the time which it took a body to pass from one point of its path to another was of paramount importance. The motion itself was the object of their study, and they recognized the part played by velocity. But Galileo and Newton were still sufficiently under the influence [144]
of Euclid to fit the observed phenomena of motion, so far as they could, upon Euclid's static world of points. This they effected by falling in with the age-old procedure of regarding time and space as something entirely disassociated and distinct. The motion of an object—in theory, of a point—was to be recorded by observing its successive positions. With each of these positions a time was to be associated, marking the instant at which the point attained that position. But in the face of this association, space and time were to be maintained as entirely separate entities.

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The Four-Dimensional World of Events

This severe separation of time and space Minkowski has now questioned, with the statement that the elements of which the external world is composed, and which we observe, are not points at all, but are *events*. This calls for a revision of our whole habit of thought. It means that the perceptual world is four-dimensioned, not three-dimensioned as we have always supposed; and it means, at the very least, that the distinction between time and space is not so fundamental as we had supposed.

[This should not impress us as strange or incomprehensible. What do we mean when we say that a plane is two-dimensional? Simply that two coordinates, two numbers, must be given to specify the position of any point of the plane. Similarly for a point in the space of our accustomed concepts we must give three numbers to fix the position—as by giving the latitude and longitude of a [145]
point on the earth and its height above sea-level. So we say this space is three-dimensional. But a material body is not merely somewhere; it is somewhere now,][182] or *was* somewhere yesterday, or *will be* somewhere tomorrow. The statement of position for a material object is meaningless unless we at the same time specify the time at which it held that position. [If I am considering the life-history of an object on a moving train, I must give three space-

coordinates and one time-coordinate to fix each of its positions.][182] And each of its positions, with the time pertaining to that position, constitutes an event. The dynamic, ever-changing world about us, that shows the same aspect at no two different moments, is a world of events; and since four measures or coordinates are required to fix an event, we say this world of events is four-dimensional. If we wish to test out the soundness of this viewpoint, we may well do so by asking whether the naming of values for the four coordinates fixes the event uniquely, as the naming of three under the old system fixes the point uniquely.

Suppose we take some particular event as the one from which to measure, and agree upon the directions to be taken by our space axes, and make any convention about our time-axis which subsequent investigation may show to be necessary. Certainly then the act of measuring so many miles north, and so many west, and so many down, and so many seconds backward, brings us to a definite time and place—which is to say, to a definite event. Perhaps nothing "happened" there, in the sense in which we usually employ the word; but that [146] is no more serious than if we were to locate a point with reference to our familiar space coordinate system, and find it to lie in the empty void of interstellar space, with no material body occupying it. In this second case we still have a point, which requires, to insure its existence and location, three coordinates and nothing more; in the first case we still have an event, which requires for *its* existence and definition four coordinates and nothing more. It is not an event about which the headline writers are likely to get greatly excited; but what of that? It is there, ready and waiting to define any physical happening that falls upon it, just as the geometer's point is ready and waiting to define any physical body that chances to fall upon *it*.

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A Continuum of Points

It is now in order to introduce a word, which I shall have to confess the great majority of the essayists introduce, somewhat improperly, without explanation. But when I attempt to explain it, I realize quite well why they did this. They had to have it; and they didn't have space in their three thousand words to talk adequately about it and about anything else besides. The mathematician knows very well indeed what he means by a continuum; but it is far from easy to explain it in ordinary language. I think I may do best by talking first at some length about a straight line, and the points on it.

If the line contains only the points corresponding to the integral distances 1, 2, [147] 3, etc., from the starting point, it is obviously not continuous—there are gaps in it vastly more inclusive than the few (comparatively speaking) points that are present. If we extend the limitations so that the line includes all points corresponding to ordinary proper and improper fractions like $\frac{1}{4}$ and $\frac{17}{29}$ and $\frac{1633}{7}$—what the mathematician calls the rational numbers—we shall apparently fill in these gaps; and I think the layman's first impulse would be to say that the line is now continuous. Certainly we cannot stand now at one point on the line and name the "next" point, as we could a moment ago. There is no "next" rational number to $\frac{116}{125}$, for instance; $\frac{115}{124}$ comes before it and $\frac{117}{126}$ comes after it, but between it and either, or between it and any other rational number we might name, lie many others of the same sort. Yet in spite of the fact that the line containing all these rational points is now "dense" (the technical term for the property I have just indicated), it is still not continuous; for I can easily define numbers that are not contained in it—irrational numbers in infinite variety like $\sqrt{2}$; or, even worse, the number pi = 3.141592 … which defines the ratio of the circumference of a circle to the diameter, and many other numbers of similar sort.

If the line is to be continuous, there may be no holes in it at all; it must have a point corresponding to every number I can possibly name. Similarly for the plane, and for our three-space; if they are to be continuous, the one must contain a point for every possible pair of numbers x and y, and the other for every possible set of three numbers x, y and z, that I can name. There may be [148]
no holes in them at all.

A line is a continuum of points. A plane is a continuum of points. A three-space is a continuum of points. These three cases differ only in their dimensionality; it requires but one number to determine a point of the first continuum, two and three respectively in the second and third cases. But the essential feature is not that a continuum shall consist of points, or that we shall be able to visualize a pseudo-real existence for it of just the sort that we can visualize in the case of line, plane and point. The essential thing is merely that it shall be an aggregate of elements numerically determined in such a way as to leave no holes, but to be just as continuous as the real number system itself. Examples, however, aside from the three which I have used, are difficult to construct of such sort that the layman shall grasp them readily; so perhaps, fortified with the background of example already presented, I may venture first upon a general statement.

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The Continuum in General

Suppose we have a set of "elements" of some sort—any sort. Suppose that these elements possess one or more fundamental identifying characteristics, analogous to the coordinates of a point, and which, like these coordinates, are capable of being given numerical values. Suppose we find that no two elements of the set possess identically the same set of defining values. Suppose finally—and this is the critical test—that the elements of the set are such that, [149]
no matter what numerical values we may specify, it we *do* specify the proper number of defining magnitudes we define by these *an actual element of the set*, that corresponds to this particular collection of values. Our elements then share with the real number system the property of leaving no holes, of constituting a continuous succession in every dimension which they possess. We have then a continuum. Whatever its elements, whatever the character of their numerical identifiers, whatever the number n of these which stands for its dimension, there may be no holes or we have no continuum. There must be an element for every possible combination of n numbers we can name, and no two of these combinations may give the same element. Granted this condition, our elements constitute a continuum.

As I have remarked, it is not easy to cite examples of continua which shall mean anything to the person unaccustomed to the term. The totality of carbon-oxygen-nitrogen-hydrogen compounds suggested by one essayist as an example is not a continuum at all, for the set contains elements corresponding only to integer values of the numbers which tell us how many atoms of each substance occur in the molecule. We cannot have a compound containing $\sqrt{2}$ carbon atoms, or 3π oxygen atoms. Perhaps the most satisfactory of the continua, outside the three Euclidean space-continua already cited, [is the manifold of music notes. This is four-dimensional; each note has four distinctions—length, pitch, intensity, timbre—to distinguish it perfectly, to tell how long, how high, how loud, how rich.][263] We might have a little difficulty in reducing the characteristic of richness to numerical expression, but [150] presumably it could be done; and we should then be satisfied that every possible combination of four values, *l*, *p*, *i*, *t* for these four identifying characteristics would give us a musical effect, and one to be confused with no other.

There is in the physical world a vast quantity of continua of one sort or another. The music-note continuum brings attention to the fact that not all of these are such that their elements make their appeal to the visual sense. This remark is a pertinent one; for we are by every right of heritage an eye-minded race, and it is frequently necessary for us to be reminded that so far as the external world is concerned, the verdict of every other sense is entirely on a par with that of sight. The things which we really see, like matter, and the things which we abstract from these visual impressions, like space, are by no means all there is to the world.

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Euclidean and Non-Euclidean Continua

If we are dealing with a continuum of any sort whatever having one or two or three dimensions, we are able to represent it graphically by means of the line, the plane, or the three-space. The same set of numbers that defines an element of the given continuum likewise defines an element of the Euclidean continuum of the same dimensionality; so the one continuum corresponds to the other, element for element, and either may stand for the other. But if we have a continuum of four or more dimensions, this representation breaks down in the absence of a real, four-dimensional Euclidean point-space to serve as a [151] picture. This does not in the least detract from the reality of the continuum which we are thus prevented from representing graphically in the accustomed fashion.

The Euclidean representation, in fact, may in some cases be unfortunate—it may be so entirely without significance as to be actually misleading. For in the Euclidean continuum of points, be it line, plane or three-space, there are certain things which we ordinarily regard as secondary derived properties, but which possess a great deal of significance none the less.

In particular, in the Euclidean plane and in Euclidean three-space, there is the distance between two points. I have indicated, in the chapter on non-Euclidean geometry, that the parallel postulate of Euclid, which distinguishes his geometry from others, could be replaced by any one of numerous other postulates. Grant Euclid's postulate and you can prove any of these substitutes; grant any of the substitutes and you can prove Euclid's postulate. Now it happens that there is one of these substitutes to which modern analysis has given a position of considerable importance. It is merely our good old friend the Pythagorean theorem, that the square on the hypotenuse equals the sum of the squares on the sides; but it is dressed in new clothes for the present occasion.

Mr. Francis' discussion of this part of the subject, and especially his figure, ought to make it clear that this theorem can be considered as dealing with the distance between any two points. When we so consider it, and take it as the fundamental, defining postulate of Euclidean geometry which distinguishes [152] this geometry from others, we have a statement of considerable content. We have, first, that the characteristic property of Euclidean space is that the distance between two points is given by the square root of the sum of the squares of the coordinate-differences for these points—by the expression

$$D = \sqrt{(X-x)^2 + (Y-y)^2 + (Z-z)^2}\,,$$

where the large letters represent the coordinates of the one point and the small ones those of the other. We have more than this, however; we have that this distance is the same for all observers, no matter how different their values for the individual coordinates of the individual points. And we have, finally, as a direct result of looking upon the thing from this viewpoint, that the expression for *D* is an "invariant"; which simply means that every observer may use the same expression in calculating the value of *D* in terms of *his own* values for the coordinates involved. The distance between two points in our space is given numerically by the square root of the sum of the squares of *my* coordinate-differences for the two points involved; it is given equally by the square root of the sum of the squares of *your* coordinate-differences, or those of any other observer whatsoever. We have then a natural law—*the* fundamental natural law characterizing Euclidean space. If we wish to apply it to the Euclidean two-space (the plane) we have only to drop out the superfluous coordinate-difference; if we wish to see by analogy what would be the fundamental natural law for a four-dimensional Euclidean space, we have [153] only to introduce under the radical a fourth coordinate-difference for the fourth dimension.

If we were not able to attach any concrete meaning to the expression for *D* the value of all this would be materially lessened. Consider, for instance, the continuum of music notes. There is no distance between different notes. There is of course significance in talking about the difference in pitch, in intensity, in duration, in timbre, between two notes; but there is none in a mode of speech that implies a composite expression indicating how far one note escapes being identical with another *in all four respects at once*. The trouble, of course, is that the four dimensions of the music-note continuum are not measurable in terms of a common unit. If they were, we should expect to measure their combination more or less absolutely in terms of this same unit. We can make measurements in all three dimensions of Euclidean space with the same unit, with the same measuring rod in fact. [This presents a peculiarity of our three-space which is not possessed by all three-dimensional manifolds. Riemann has given another illustration in the system of all possible colors, composed of arbitrary proportions of the three primaries, red, green and violet. This system forms a three-dimensional continuum; but we cannot measure the "distance" or difference between two colors in terms of the difference between two others.][130]

Accordingly, in spite of the fact that the Euclidean three-space gives us a formal representation of the color continuum, and in spite of the fact that the hypothetical four-dimensional Euclidean space would perform a like office for [154] the music-note continuum, this representation would be without significance. We should not say that the geometry of these two manifolds is Euclidean. We should realize that *any* set of numerical elements can be plotted in a Euclidean space of the appropriate dimensionality; and that accordingly, before allowing such a plot to influence us to classify the geometry of the given manifold as Euclidean, we must pause long enough to ask whether the rest of the Euclidean system fits into the picture. If the square root of the sum of the squares of the coordinate-differences between two elements possesses significance in the given continuum, and if it is invariant between observers of that continuum who employ different bases of reference, then and only then may we allege the Euclidean character of the given continuum.

If under this test the given continuum fails of Euclideanism, it is in order to ask what type of geometry it does present. If it is of such character that the "distance" between two elements possesses significance, we should answer this question by investigating that distance in the hope of discovering a non-Euclidean expression for it which will be invariant. If it is not of such character, we should seek some other characteristic of single elements or groups of elements, of real physical significance and of such sort that the numerical expression for it would be invariant.

If the continuum with which we have to do is one in which the "distance" between two elements possesses significance, and if it turns out that the invariant expression for this distance is not the Pythagorean one, but one [155] indicating the non-Euclideanism of our continuum, we say that this continuum has a "curvature." This means that, if we interpret the elements of our continuum as points in space (which of course we may properly do) and if we then try to superpose this point-continuum upon a Euclidean continuum, it will not "go"; we shall be caught in some such absurdity as trying to force a sphere into coincidence with a plane. And of course if it won't go, the only possible reason is that it is curved or distorted, like the sphere, in such a way as to prevent its going. It is unfortunate that the visualizing of such curvature requires the visualizing of an additional dimension for the curved continuum to curve into; so that while we can picture a curved surface easily enough, we can't picture a curved three-space or four-space. But that is a barrier to visualization alone, and in no sense to understanding.

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Our World of Four Dimensions

It will be observed that we have now a much broader definition of non-Euclideanism than the one which served us for the investigation of Euclid's parallel postulate. If we may at pleasure accept this postulate or replace it by another and different one, we may presumably do the same for any other or any others of Euclid's postulates. The very statement that the distance between elements of the continuum shall possess significance, and shall be measurable by considering a path in the continuum which involves other elements, is an assumption. If we discard it altogether, or replace it by one postulating that [156] some other joint property of the elements than their distance be the center of interest, we get a non-Euclidean geometry. So for any other of Euclid's postulates; they are all necessary for a Euclidean system, and in the absence of any one of them we get a non-Euclidean system.

Now the four-dimensional time-space continuum of Minkowski is plainly of a sort which ought to make susceptible of measurement the separation between two of its events. We can pass from one element to another in this continuum —from one event to another—by traversing a path involving "successive" events. Our very lives consist in doing just this: we pass from the initial event of our career to the final event by traversing a path leading us from event to event, changing our time and space coordinates continuously and simultaneously in the process. And while we have not been in the habit of measuring anything except the space interval between two events and the time interval between two events, separately, I think it is clear enough that, considered as events, as elements in the world of four dimensions, there is a less separation between two events that occur in my office on the same day than between two which occur in my office a year apart; or between two events occurring 10 minutes apart when both take place in my office than when one takes place there and one in London or on Betelgeuse.

It is not at all unreasonable, *a priori*, then, to seek a numerical measure for the separation, in space-time of four-dimensions, of two events. If we find it, we shall doubtless be asked just what its subjective significance to us is. This must [157] be answered with some circumspection. It will presumably be something which we cannot observe with the visual sense alone, or it would have forced itself upon our attention thousands of years ago. It ought, I should think, to be something that we would sense by employing at the same time the visual sense and the sense of time-passage. In fact, I might very plausibly insist that, by my very remarks about it in the above paragraph, I have sensed it.

Minkowski, however, was not worried about this phase of the matter. He had only to identify the invariant expression for distance; sensing it could wait. He found, of course, that this expression was not the Euclidean expression for a four-dimensional interval. He had discarded several of the Euclidean assumptions and could not expect that the postulate governing the metric properties of Euclid's space would persist. Especially had he violated the Euclidean canons in discarding, with Einstein, the notion that nothing which may happen to a measuring rod in the way of uniform translation at high velocity can affect its measures. So he had to be prepared to find that his geometry was non-Euclidean; yet it is surprising to learn how slightly it deviates from that of Euclid. Without any extended discussion to support the statement, we may say that he found that when two observers measure the time- and the space-coordinates of two events, using the assumptions and therefore the methods of Einstein and hence subjecting themselves to the condition that their measures of the pure time-interval and of the pure space-interval between these events will not necessarily be the same, they will [158] discover that they both get the same value for the expression

$$S = \sqrt{(X-x)^2 + (Y-y)^2 + (Z-z)^2 - (CT-Ct)^2}\,.$$

If our acceptance of this as the numerical measure of the separation in space-time between the two events should lead to contradiction we could not so accept it. No contradiction arises however and we may therefore accept it. And at once the mathematician is ready with some interpretative remarks.

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The Curvature of Space-Time

The invariant expression for separation, it will be seen, is in the same form as that of the Euclidean four-dimensional invariant save for the minus sign before the time-difference (the appearance of the constant *C* in connection with the time coordinate *t* is merely an adjustment of units; see page 153). This tells us that not alone is the geometry of the time-space continuum non-Euclidean in its methods of measurement, but also in its results, to the extent that it possesses a curvature. It compares with the Euclidean four-dimensional continuum in much the same way that a spherical surface compares with a plane. As a matter of fact, a more illuminating analogy here would be that between the cylindrical surface and the plane, though neither is quite exact. To make this clear requires a little discussion of an elementary notion which we have not yet had to consider. [159]

Our three-dimensional existence often reduces, for all practical purposes, to a two-dimensional one. The objects and the events of a certain room may quite satisfactorily be defined by thinking of them, not as located in space, but as lying in the floor of the room. Mathematically the justification for this viewpoint is got by saying that we have elected to consider a slice of our three-dimensional world of the sort which we know as a plane. When we consider this plane and the points in it, we find that we have taken a cross-section of the three-dimensional world. A line in that world is now reduced, for us, to a single point—the point where it cuts our plane; a plane is reduced to a line—the line where it cuts our plane; the three-dimensional world itself is reduced to our plane itself. Everything three-dimensional falls down into its shadow in our plane, losing in the process that one of the three dimensions which is not present in our plane.

For simplicity's sake it is usual to take a cross-section of space parallel to one of our coordinate axes. We think of our three dimensions as extending in the directions of those axes; and it is easier to take a horizontal or vertical section which shall simply wipe out one of these dimensions than to take an oblique section which shall wipe out a dimension that consists partly of our original length, and partly of our original width, and partly of our original height.

If we have a four-dimensional manifold to begin with, we may equally shake out one of the four dimensions, one of the four coordinates, and consider the three-dimensional result of this process as a cross-section of the original four-dimensional continuum. [160] And where, in cross-sectioning a three-dimensioned world, we have but three choices of a coordinate to eliminate, in cross-sectioning a world of four dimensions we have four choices. By dropping out either the x, or the y, or the z, or the t, we get a three-dimensioned cross-section.

Now our accustomed three-dimensional space is strictly Euclidean. When we cross-section it, we get a Euclidean plane no matter what the direction in which we make the cut. Likewise the Euclidean plane is wholly Euclidean, because when we cross-section it in any direction whatever we get a Euclidean line. A cylindrical surface, on the other hand, is neither wholly Euclidean nor wholly non-Euclidean in this matter of cross-sectioning. If we take a section in one direction we get a Euclidean line and if we take a section in the other direction we get a circle (if the cylindrical surface be a circular one). And of course if we take an oblique section of any sort, it is neither line nor circle, but a compromise between the two—the significant thing being that it is still not a Euclidean line.

The space-time continuum presents an analogous situation. When we cross-section it by dropping out any one of the three space dimensions, we get a three-dimensional complex in which the distance formula is still non-Euclidean, retaining the minus sign before the time-difference and therefore retaining the geometric character of its parent. But if we take our cross-section in such a way as to eliminate the time coordinate, this peculiarity disappears. The signs in the invariant expression are then all plus, and the cross-section is [161]
in fact our familiar Euclidean three-space.

If we set up a surface geometry on a sphere, we find that the elimination of one dimension leaves us with a line-geometry that is still non-Euclidean since it pertains to the great circles of the sphere rather than to Euclidean straight lines. In shaking Minkowski's continuum down into a three-dimensional one by eliminating any one of his coordinates, if we eliminate either the x, the y or the z, we have left a three-dimensional geometry in which the disturbing minus sign still occurs in the distance-formula, and which is therefore still non-Euclidean. If we omit the t, this does not occur. We see, then, that the time dimension is the disturbing factor, the one which gives to space-time its non-Euclidean character so far as the possession of curvature is concerned. And we see that this curvature is not the same in all directions, and in one direction is actually zero—whence the attempted analogy with a cylinder instead of with a sphere.

Many writers on relativity try to give the space-time continuum an appeal to our reason and a character of inevitableness by insisting on the lack of any fundamental distinction between space and time. The very expression for the space-time invariant denies this. Time *is* distinguishable from space. The three dimensions of space are quite indistinguishable—we can interchange them without affecting the formula, we can drop one out and never know which is gone. But the very formula singles out time as distinct from space, as inherently different in some way. It is not so inherently different as we have always supposed; it is not sufficiently different to offer any obstacle to our [162]
thinking in terms of the four-dimensional continuum. But while we can group space and time together in this way, [this does not mean at all that space and time cease to differ. A cook may combine meat with potatoes and call the product hash, but meat and potatoes do not thereby become identical.][223]

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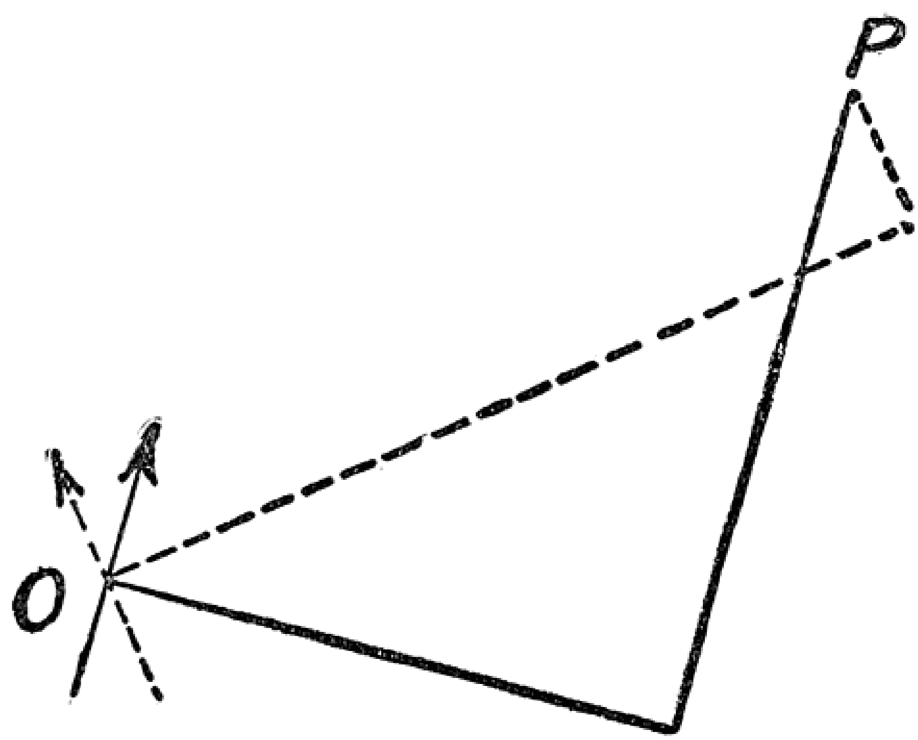
The Question of Visualization

To the layman there is a great temptation to say that while, mathematically speaking, the space-time continuum may be a great simplification, it does not really represent the external world. To be sure, you can’t see the space-time continuum in precisely the same way that you can the three-dimensional space continuum, but this is only because Einstein finds the time dimension to be not quite freely interchangeable with the space dimension. Yet you do perceive this space-time continuum, in the manner appropriate for its perception; and it would be just as sensible to throw out the space continuum itself on the ground that perception of the two is not of exactly the same sort, as to throw out the space-time continuum on this ground. With appropriate conventions, either may stand as the mental picture of the external world; it is for us to choose which is the more convenient and useful image. Einstein tells us that his image is the better, and tells us why.

Before we look into this, we must let him tell us something more about the geometry of his continuum. What he tells us is, in its essentials, just this. The [163] observer in a pure space continuum of three dimensions finds that as he changes his position, his right-and-left, his backward-and-forward, and his up-and-down are not fixed directions inherent in nature, but are fully interchangeable. The observers, in the adjoined sketch, whose verticals are as indicated by the arrows, find very different vertical and horizontal components for the distance between the points *O* and *P*; a similar situation would prevail if we used all three space directions. The statement analogous to this for Einstein’s four-dimensional continuum of space and time combined



is that, as observers change their *relative motion*, their time axes take slightly different directions, so that what is purely space or purely time for the one becomes space with a small component in the time direction, or time with a small component in the space direction, for the other. This it will be seen explains fully why observers in relative motion can differ about space and time [164] measurements. We should not be at all surprised if the two observers of the figure reported different values for horizontals and verticals; we should realize that what was vertical for one had become partly horizontal for the other. It is just so, says Einstein, with his observers of time and space who are in relative motion to one another; what one sees as space the other sees as partly time, because their time axes do not run quite parallel.

The natural question here, of course, is "Well, where are their time axes?" If you know what to look for, of course, you ought to be able to perceive them in just the way you perceive ordinary time intervals—with the reservation that they are imaginary, after all, just like your space axes, and that you must only expect to see them in imagination. If you look for a fourth axis in Euclidean three-space to represent your time axis, you will of course not find it. But you will by all means agree with me that your time runs in a definite direction; and this it is that defines your time axis. Einstein adds that if you and I are in relative motion, my time does not run in quite the same direction as yours.

How shall we prove it? Well, how would we prove it if he told us that our space axes did not run in precisely the same direction? Of course we could not proceed through direct measures upon the axes themselves; we know these are imaginary. What we should do would be to strike out, each of us, a very long line indeed in what seemed the true horizontal direction; and we should hope that if we made them long enough, and measured them accurately enough, we [165] should be able to detect any divergence that might exist. This is precisely what we must do with our time axes if we wish to verify Einstein's statement that they are not precisely parallel; and what better evidence could we demand of the truth of this statement than the evidence already presented—that when we measure our respective time components between two events, we get different results?

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What It All Leads To

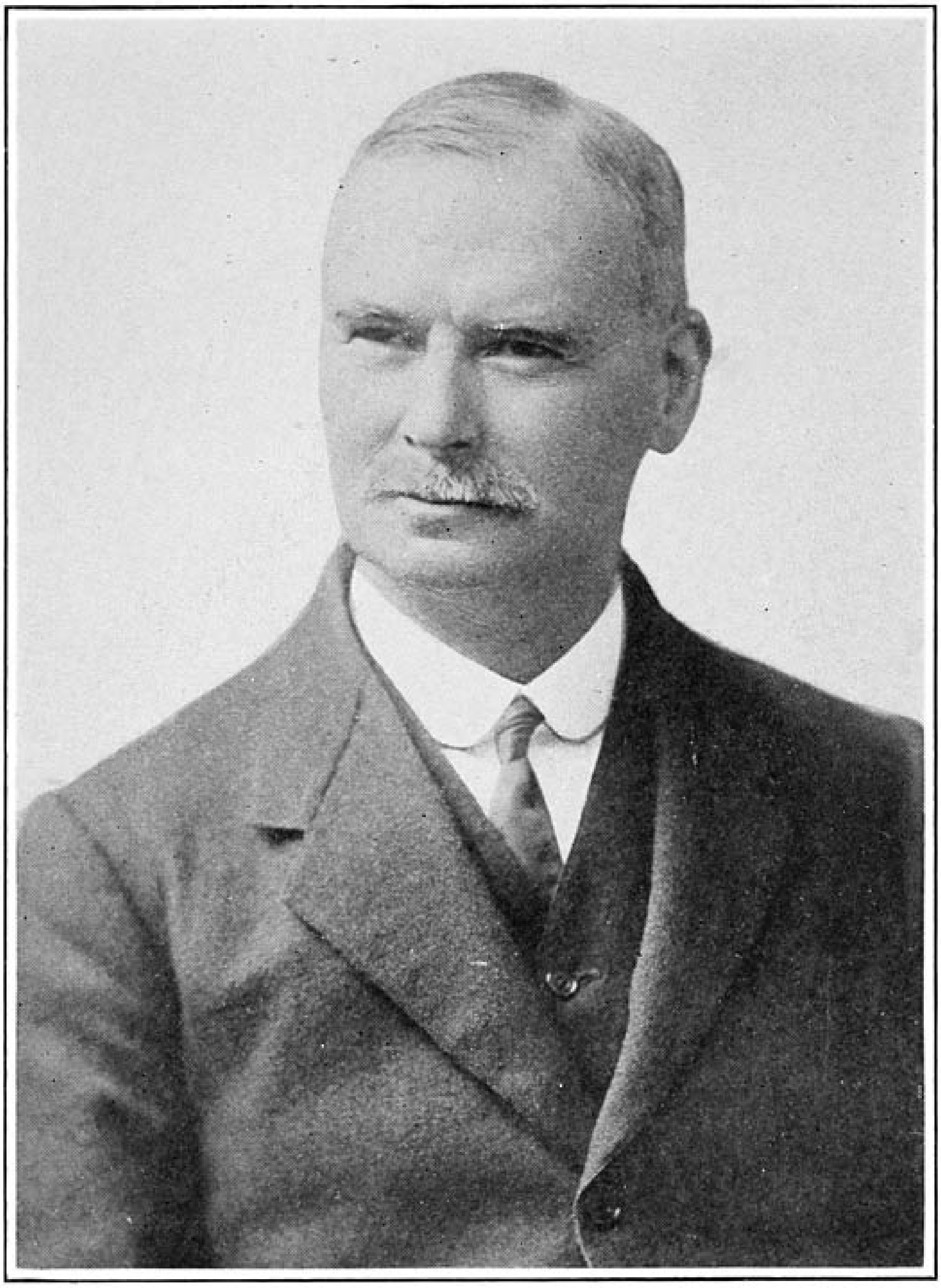
The preceding chapters have been compiled and written with a view to putting the reader in a state of mind and in a state of informedness which shall enable him to derive profit from the reading of the actual competing essays which make up the balance of the book. For this purpose it has been profitable to take up in detail the preliminaries of the Special Theory of Relativity, and to allow the General Theory to go by default, in spite of the fact that it is the latter which constitutes Einstein's contribution of importance to science. The reason for this is precisely the same as that for taking up Euclidean geometry and mastering it before proceeding to the study of Newtonian mechanics. The fundamental ideas of the two theories, while by no means identical, are in general terms the same; and the conditions surrounding their application to the Special Theory are so very much simpler than those which confront us when we apply them to the more general case, that this may be taken as the controlling factor in a popular presentation. We cannot omit the General [166] Theory from consideration, of course; but we can omit it from our preliminary discussion, and leave its development to the complete essays which follow, and which in almost every case give it the larger half of their space which its larger content demands. In the process of the slow and difficult preparation of the lay mind for the assimilation of an altogether new set of fundamental ideas, it is altogether desirable to give the Special Theory, with its simpler applications of these ideas, a place out of proportion to its importance in Einstein's completed structure; and this we have therefore done.

The Special Theory, postulating the relativity of uniform motion and deducing the consequences of that relativity, is often referred to as a "special case" of the General Theory, in which this restriction of uniformity is removed. This is not strictly speaking correct. The General Theory, when we have formulated it, will call our attention to something which we really knew all the time, but to which we chose not to give heed—that in the regions of space to which we have access, uniform motion does not exist. All bodies in these regions are under the gravitational influence of the other bodies therein, and this influence leads to accelerated motion. Nothing in our universe can possibly travel at uniform velocity; the interference of the rest of the bodies in the universe prevents this.

Obviously, we ought not to apply the term "special case" to a case that never occurs. Nevertheless, this case is of extreme value to us in our mental processes. Many of the motions with which we are concerned are so nearly at [167]
constant velocity that we find it convenient to treat them as though they were uniform, either ignoring the resulting error or correcting for it at the end of our work. In many other cases we are able to learn what actually occurs under accelerated motion by considering what *would have occurred* under uniform motion were such a thing possible. Science is full of complications which we unravel in this fashion. The physicist deals with gas pressures by assuming temperatures to be constant, though he knows temperature never is constant; and in turn he deals with temperatures by assuming pressures to be constant. After this, he is able to predict what will happen when, as in nature, pressures and temperatures are varying simultaneously. By using as a channel of attack the artificially simple case that never occurs, we get a grip on the complex case that gives us a true picture of the phenomenon. And because in actual nature we can come as close as we please to this artificial case, by supposing the variable factor to approach constancy, so when we assume it to be absolutely constant we speak of the result as the *limiting* case. This situation does not occur, but is the limiting case for those that do occur.

When, in the matter of motion, we abandon the artificial, limiting case of uniform velocity and look into the general, natural one of unrestricted motion, we find that the structure which we have built up to deal with the limiting case provides us with many of the necessary ideas and viewpoints. This is what we expect—in it lies the value of the limiting case. We shall see that the relativity of time and space, established for the limiting case, holds good in the general [168]
one. We shall see that the idea of the four-dimensional space-time continuum as representing the external world persists, forming the whole background of the General Theory much more definitely than in the Special Theory. Incidentally we shall see that the greater generality of the case under consideration will demand a greater degree of generality in the geometry of this continuum, a non-Euclideanism of a much more whole-hearted type than that of the Special Theory. But all the revisions of fundamental concepts which we have been at such pains to make for the sake of the Special Theory will

remain with us in the General. With this we may consider our preliminary background as established, and give our attention to the essayists, who will try to take us more deeply into the subject than we have yet gone, without losing us in its intricacies.



Lyndon Bolton,
Winner of the Einstein Prize Essay Contest

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VII

RELATIVITY

The Winning Essay in the Contest for the Eugene Higgins $5,000 Prize

BY LYNDON BOLTON
SENIOR EXAMINER, BRITISH PATENT OFFICE LONDON

The reader is probably acquainted with the method of specifying positions of points in a plane by their distances from two mutually perpendicular lines, or if the points are in space by their distances from three mutually perpendicular planes like adjacent sides of a flat-sided box. The method is in fact in common use for exhibiting relations between quantities by graphs or diagrams. These sets of axes, as they are called, together with any scales used for measuring, must be supposed rigid, otherwise the events or points which they are used to specify are indefinite. The lengths which locate any point with reference to a set of axes are called its coordinates.

When such systems are used for physical purposes, they must be supplemented by clocks to enable the times at which events occur to be determined. The clocks must be synchronized, and must go at the same rate, but it must suffice here to state that this is possible without indicating how these conditions can [170]
be attained. A system of axes with its clocks will hereinafter be called a Frame of Reference, and every observer will be supposed to be provided with such a frame partaking of his motion. All the objects which partake of an observer's motion will be called his *system*.

It is a question whether among all possible frames of reference any one frame or class of frames is more suited than another for the mathematical statement of physical laws. This is for experience to decide, and a Principle of Relativity is a statement embodying the answer.

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The Mechanical Principle of Relativity

It has been ascertained that all such frames are equally suitable for the mathematical statement of general *mechanical* laws, provided that their motion is rectilinear and uniform and without rotation. This fact is comprehended in the general statement that *all unaccelerated frames of reference are equivalent for the statement of the general laws of mechanics*. This is the mechanical principle of relativity.

It is well recognized however that the laws of dynamics as hitherto stated involve the assumptions that the lengths of rigid bodies are unaffected by the motion of the frame of reference, and that measured times are likewise unaffected; that is to say that any length measured on his own system by either of two relatively moving observers appears the same to both observers, or that lengths of objects and rates of clocks do not alter whatever the motion relative to an observer. These assumptions seem so obvious that it is scarcely perceived [171] that they are assumptions at all. Yet this is the case, and as a matter of fact they are both untrue.

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The Special Principle of Relativity

Although all unaccelerated frames of reference are equivalent for the purposes of mechanical laws, this is not the case for physical laws generally as long as the above suppositions are adhered to. Electromagnetic laws *do* alter their form according to the motion of the frame of reference; that is to say, if these suppositions are true, electromagnetic agencies act in different ways according to the motion of the system in which they occur. There is nothing *a priori* impossible in this, but it does not agree with experiment. The motion of each locality on the earth is continually changing from hour to hour but no corresponding changes occur in electromagnetic actions. It has however been ascertained that on discarding these suppositions the difficulty disappears, and electromagnetic laws retain their form under all circumstances of unaccelerated motion. According to the theory of relativity, the correct view which replaces these suppositions is deducible from the following postulates:

(1) By no experiment conducted on his own system can an observer detect the unaccelerated motion of his system.
(2) The measure of the velocity of light *in vacuo* is unaffected by relative motion between the observer and the source of light.

Both these postulates are well established by experiment. The first may be illustrated by the familiar difficulty of determining whether a slowly moving train one happens to be sitting in, or an adjacent one, is in motion. The passenger has either to wait for bumps (that is, accelerations) or else he has to look out at some adjacent object which he knows to be fixed, such as a building (that is, he has to perform an experiment on something outside his system), before he can decide. [172]

The second postulate is an obvious consequence of the wave theory of light. Just as waves in water, once started by a ship, travel through the water with a velocity independent of the ship, so waves in space travel onward with a speed bearing no relation to that of the body which originated them. The statement however is based on experiment, and can be proved independently of any theory of light.

It is not difficult to deduce from these postulates certain remarkable conclusions relating to the systems of two observers, A and B, in relative motion, among them the following:

(1) Objects on B's system appear to A to be shorter in the direction of relative motion than they appear to B.
(2) This opinion is reciprocal. B thinks that A's measurements on A's system are too great.
(3) Similarly for times: each observer thinks that the other's clocks have a slower rate than his own, so that B's durations of time appear shorter to B than to A, and conversely.

(4) Events which appear simultaneous to A do not in general appear so to B, and conversely. [173]
(5) Lengths at right angles to the direction of motion are unaffected.
(6) These effects vary with the ratio of the relative velocity to that of light. The greater the relative velocity, the greater the effects. They vanish if there is no relative velocity.
(7) For ordinary velocities the effects are so small as to escape notice. The remarkable point however is their occurrence rather than their magnitude.
(8) The observers similarly form different estimates of the velocities of bodies on each other's systems. The velocity of light however appears the same to all observers.

Taking into account these revised views of lengths and times the mechanical principle of relativity may be extended to physical laws generally as follows: *All unaccelerated frames of reference are equivalent for the statement of the general laws of physics.* In this form the statement is called the Special, or Restricted, Principle of Relativity, because it is restricted to unaccelerated frames of reference. Naturally the laws of classical mechanics now require some modification, since the suppositions of unalterable lengths and times no longer apply.

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The Four Dimensional Continuum

Lengths and times therefore have not the absolute character formerly attributed to them. As they present themselves to us they are relations between the object and the observer which change as their motion relative to him changes. Time can no longer be regarded as something independent of position and motion, [174] and the question is what is the reality? The only possible answer is that objects must be regarded as existing in four dimensions, three of these being the ordinary ones of length, breadth and thickness, and the fourth, time. The term "space" is applicable only by analogy to such a region; it has been called a "continuum," and the analogue of a point in ordinary three-dimensional space has been appropriately called an "event." By "dimension" must be understood merely one of four independent quantities which locate an event in this continuum. In the nature of the case any clear mental picture of such a continuum is impossible; mankind does not possess the requisite faculties. In this respect the mathematician enjoys a great advantage. Not that he can picture the thing mentally any better than other people, but his symbols enable him to abstract the relevant properties from it and to express them in a form suitable for exact treatment without the necessity of picturing anything, or troubling whether or not the properties are those on which others rely for their conceptions.

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Gravitation and Acceleration

The limitation of statements of general law to uniformly moving systems is hardly satisfactory. The very concept of general law is opposed to the notion of limitation. But the difficulties of formulating a law so that the statement of it shall hold good for all observers, whose systems may be moving with different and possibly variable accelerations, are very great. Accelerations imply forces which might be expected to upset the formulation of any general dynamical [175] principles, and besides, the behavior of measuring rods and clocks would be so erratic as to render unmeaning such terms as rigidity and measured time, and therefore to preclude the use of rigid scales, or of a rigid frame of reference which is the basis of the foregoing investigation.

The following example taken from Einstein will make this clear, and also indicate a way out of the difficulty. A rotating system is chosen, but since rotation is only a particular case of acceleration it will serve as an example of the method of treating accelerated systems generally. Moreover, as it will be seen, the attribution of acceleration to the system is simply a piece of scaffolding which can be discarded when the general theory has been further developed.

Let us note the experiences of an observer on a rotating disk which is isolated so that the observer has no direct means of perceiving the rotation. He will therefore refer all the occurrences on the disk to a frame of reference fixed with respect to it, and partaking of its motion.

He will notice as he walks about on the disk that he himself and all the objects on it, whatever their constitution or state, are acted upon by a force directed away from a certain point upon it and increasing with the distance from that point. This point is actually the center of rotation, though the observer does not recognize it as such. The space on the disk in fact presents the characteristic properties of a gravitational field. The force differs from gravity as we know it by the fact that it is directed away from instead of toward a center, and it obeys [176] a different law of distance; but this does not affect the characteristic properties that it acts on all bodies alike, and cannot be screened from one body by the interposition of another. An observer aware of the rotation of the disk would say that the force was centrifugal force; that is, the force due to inertia which a body always exerts when it is accelerated.

Next suppose the observer to stand at the point of the disk where he feels no force, and to watch someone else comparing, by repeated applications of a small measuring rod, the circumference of a circle having its center at that point, with its diameter. The measuring rod when laid along the circumference is moving lengthwise relatively to the observer, and is therefore subject to contraction by his reckoning. When laid radially to measure the diameter this contraction does not occur. The rod will therefore require a greater proportional number of applications to the circumference than to the diameter, and the number representing the ratio of the circumference of the circle to the diameter thus measured will therefore be greater than 3.14159+, which is its normal value. Moreover the relative velocity decreases as the center is approached, so that the contraction of the measuring rod is less when applied to a smaller circle; and the ratio of the circumference to the diameter, while still greater than the normal, will be nearer to it than before, and the smaller the circle the less the difference from the normal. For circles whose centers are not at the point of zero force the confusion is still greater, since the velocities relative to the observer of points on them now change from point to point. The [177] whole scheme of geometry as we know it is thus disorganized. Rigidity becomes an unmeaning term since the standards by which alone rigidity can be tested are themselves subject to alteration. These facts are expressed by the statement that the observer's measured space is non-Euclidean; that is to say, in the region under consideration measurements do not conform to the system of Euclid.

The same confusion arises in regard to clocks. No two clocks will in general go at the same rate, and the same clock will alter its rate when moved about.

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The General Principle of Relativity

The region therefore requires a space-time geometry of its own, and be it noted that with this special geometry is associated a definite gravitational field, and if the gravitational field ceases to exist, for example if the disk were brought to rest, all the irregularities of measurement disappear, and the geometry of the region becomes Euclidean. This particular case illustrates the following propositions which form the basis of this part of the theory of relativity:

(1) Associated with every gravitational field is a system of geometry, that is, a structure of measured space peculiar to that field.
(2) Inertial mass and gravitational mass are one and the same.
(3) Since in such regions ordinary methods of measurement fail, owing to the indefiniteness of the standards, the systems of [178]
geometry must be independent of any particular measurements.
(4) The geometry of space in which no gravitational field exists in Euclidean.[1]

The connection between a gravitational field and its appropriate geometry suggested by a case in which acceleration was their common cause is thus assumed to exist from whatever cause the gravitational field arises. This of course is pure hypothesis, to be tested by experimental trial of the results derived therefrom.

Gravitational fields arise in the presence of matter. Matter is therefore presumed to be accompanied by a special geometry, as though it imparted some peculiar kink or twist to space which renders the methods of Euclid inapplicable, or rather we should say that the geometry of Euclid is the particular form which the more general geometry assumes when matter is either absent or so remote as to have no influence. The dropping of the notion of acceleration is after all not a very violent change in point of view, since under any circumstances the observer is supposed to be unaware of the acceleration. All that he is aware of is that a gravitational field and his geometry coexist.

The prospect of constructing a system of geometry which does not depend upon measurement may not at first sight seem hopeful. Nevertheless this has
been done. The system consists in defining points not by their distances from [179]
lines or planes (for this would involve measurement) but by assigning to them arbitrary numbers which serve as labels bearing no relation to measured

distances, very much as a house is located in a town by its number and street. If this labeling be done systematically, regard being had to the condition that the label-numbers of points which are close together should differ from one another by infinitesimal amounts only, it has been found that a system of geometry can actually be worked out. Perhaps this will appear less artificial when the fact is called to mind that even when standards of length are available no more can be done to render lengths of objects amenable to calculation than to assign numbers to them, and this is precisely what is done in the present case. This system of labeling goes by the name of "Gaussian coordinates" after the mathematician Gauss who proposed it.

It is in terms of Gaussian coordinates that physical laws must be formulated if they are to have their widest generality, and the general principle of relativity is that *all Gaussian systems are equivalent for the statement of general physical laws*. For this purpose the labeling process is applied not to ordinary space but to the four dimensional space-time continuum. The concept is somewhat difficult and it may easily be aggravated into impossibility by anyone who thinks that he is expected to visualize it. Fortunately this is not necessary; it is merely one of these irrelevancies to which those who are unaccustomed to think in symbols are liable.

It will now be seen that among physical laws the law of gravitation stands pre- [180]
eminent, for it is gravitating matter which determines the geometry, and the geometry determines the form of every other law. The connection between the geometry and gravitation is the law of gravitation. This law has been worked out, with the result that Newton's law of the inverse square is found to be approximate only, but so closely approximate as to account for nearly all the motions of the heavenly bodies within the limits of observation. It has already been seen that departure from the Euclidean system is intensified by rapidity of motion, and the movements of these bodies are usually too slow for this departure to be manifest. In the case of the planet Mercury the motion is sufficiently rapid, and an irregularity in its motion which long puzzled astronomers has been explained by the more general law.

Another deduction is that light is subject to gravitation. This has given rise to two predictions, one of which has been verified. The verification of the other is as yet uncertain, though the extreme difficulty of the necessary observations may account for this.

Since light is subject to gravitation it follows that the constancy of the velocity of light assumed in the earlier part of this paper does not obtain in a gravitational field. There is really no inconsistency. The velocity of light is constant in the absence of gravitation, a condition which unaccelerated motion implies. The special principle of relativity is therefore a limiting case of the general principle. [181]

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1 It will be noted that Mr. Bolton pronounces the geometry of *space* to be Euclidean in the absence of gravitational fields, not that of space-time. This is in accord with what was pointed out on page 161.—Editor. ↑

VIII

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THE NEW CONCEPTS OF TIME AND SPACE

The Essay in Behalf of Which the Greatest Number of Dissenting Opinions Have Been Recorded

BY MONTGOMERY FRANCIS NEW YORK

We have all had experiences, on trains and boats, illustrating our inability to tell, without looking off to some external body, whether we are at rest or moving uniformly; and when we do so look, to tell, without reference to the ground or some other point external to both systems, whether ours or the other be the seat of motion. Uniform motion must be relative, because we find nowhere in the universe a body in the unique state of absolute rest from which alone absolute motion might be measured.

True, the wave theory of light with its homogeneous space-filling ether seemed to provide a reference standard for the concept of absolute motion, and for its measurement by experiment with light rays. But when Michelson and Morley looked for this absolute motion they found no trace of it. To the physicist, observational student of the external world, nothing exists save observationally; what he can never observe is not there. So: I. *By no means* [182] *whatever may we regard uniform straight-line motion as other than relative.*

As a further direct consequence of the Michelson-Morley experiment we have: II. *Light in a vacuum presents the same velocity,* $C = 186,330$ *miles per second, to all observers whatever their velocity of relative motion.* In addition to being experimentally established, this is necessary to support I, for if light will distinguish between our velocities, its medium is necessarily a universal standard for absolute motion. But it is contrary to common sense to suppose that if I pass you at 100 miles per hour, the same light impulse can pass us both at the same speed, *C*. We feel, instinctively, that space and time are not so constituted as to make this possible. But the fact has been repeatedly demonstrated. And when common sense and fundamental concepts clash with facts, it is not the facts that must yield. We have survived such crises, notably one where we had to change the fundamental concept of up-and-down; if another one is here, says Einstein, let us meet it.

This the Special Theory of Relativity does. It accepts Postulates I and II above; their consequences it deduces and interprets. For extensive demonstration of these I lack space, and this has been satisfactorily done by others so it is not my chief duty; but clearly they will be startling. For the very ray of light which refuses to recognize our relative motion is the medium through which I must observe your system and you mine.

It turns out that I get different values for lengths and time intervals *in your system* than you get, and vice versa. And we are both right! For me to accept [183] your "correction" were for me to admit that you are at absolute rest and I in absolute motion, that your measure of light velocity is right and mine wrong: admissions barred by the postulates. We have nothing to correct; we can only recognize the reason for the discrepancy; and knowing our relative velocity, each can calculate from his own results what the other's will be. We find, of course, that at ordinary velocities the discrepancy is many times too small for detection; but at relative velocities at all comparable with that of light it rises above the observational horizon.

To inquire the "true" length is meaningless. Chicago is east of Denver, west of Pittsburgh, south of Milwaukee; we do not consider this contradictory, or demand the "true" direction of Chicago. Einstein finds that the concept of length, between points in space or events in time, does not as we had supposed represent an intrinsic property of the points or the events. Like direction, it is merely a relation between these and the observer—a relation whose value changes with the observer's velocity relative to the object. If our ideas of the part played in the world by time and space do not permit us to believe this, we must alter these ideas. Let us see how we may do this.

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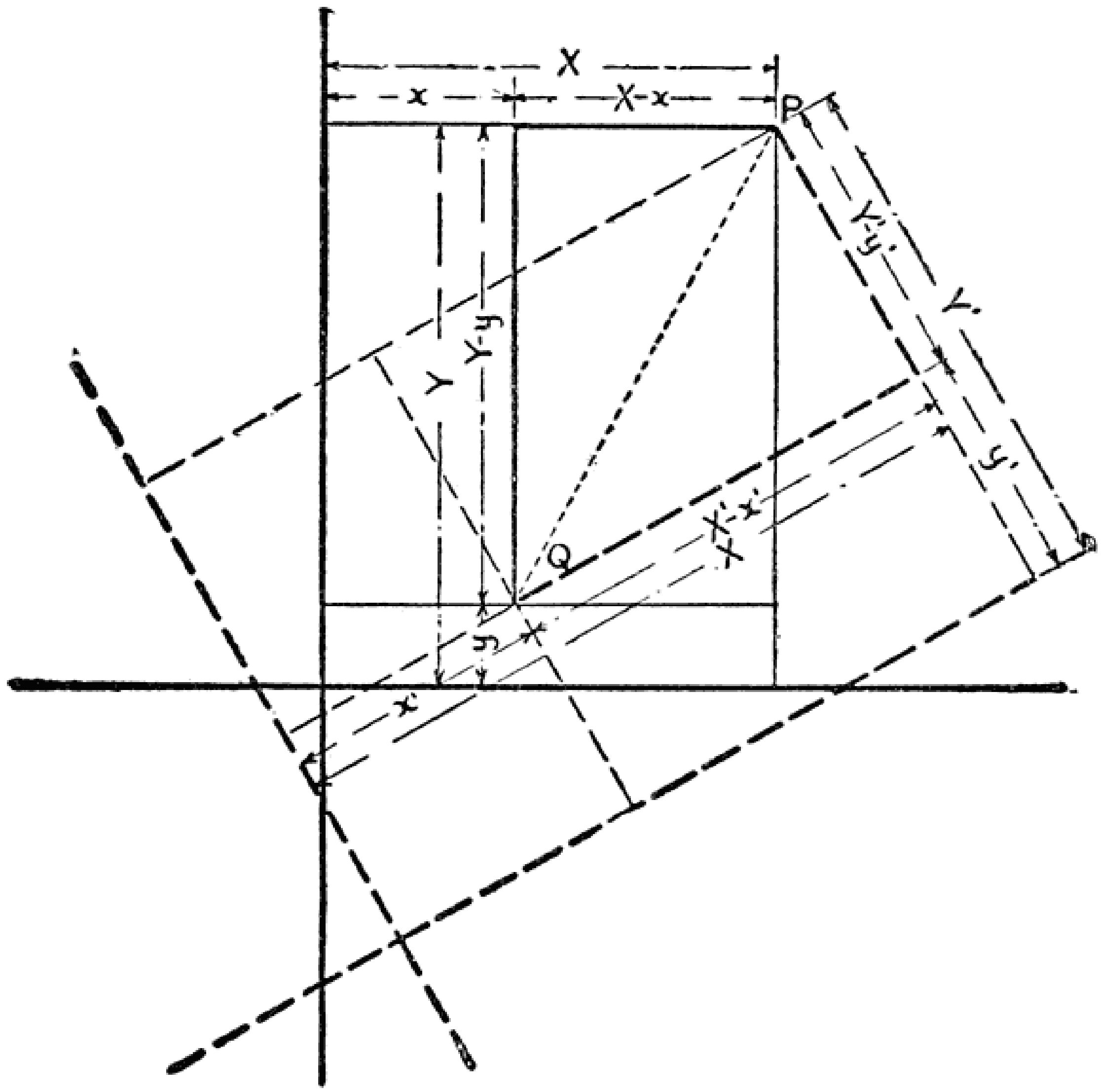
A World of Points

To deal with points in a plane the mathematician draws two perpendicular lines, and locates any point, as *P*, by measuring its distances, *X* and *Y*, from [184]
these "coordinate axes." The directions of his axes acquire for him a peculiar significance, standing out above other directions; he is apt to measure the distances $X - x$ and $Y - y$ between the points *P* and *Q* in these directions, instead of measuring the single distance *PQ*. We do the same thing when we say that the railroad station is five blocks north and two east.



The mathematician visualizes himself as an observer, located on his coordinate framework. For another observer on another framework, the horizontal and vertical distances $X' - x'$ and $Y' - y'$ between *P* and *Q* are different. But for both, the distance from *P* direct to *Q* is the same. In each case the right triangle tells us that: [185]

$$PQ = \sqrt{(X-x)^2 + (Y-y)^2} = \sqrt{(X'-x')^2 + (Y'-y')^2}$$

Imagine an observer so dominated by his coordinate system that he knows no way of relating *P* with *Q* save by their horizontal and vertical separation. His whole scheme of things would be shattered by the suggestion that other observers on other reference frames find different horizontal and vertical components. We have to show him the line *PQ*. We have to convince him that this length is the absolute property enjoyed by his pair of points; that horizontals and verticals are merely relations between the points and the observer, result of the observer's having analyzed the distance *PQ* into two components; that different observers effect this decomposition differently; that this seems not to make sense to him only because of his erroneous concept of a fundamental difference between verticals and horizontals.

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The Four-Dimensional World of Events

We too have created a distinction in our minds corresponding to no sufficient reality. Our minds seize on time as inherently separable from space. We see the world made up of things in a continuum of three space dimensions; to make this dead world live there runs through it a one-dimensional time continuum, [186]
imposed from without, unrelated.

But did you ever observe anything suggesting the presence of time in the absence of space, or vice versa? No; these vessels of the universe always occur together. Association of the space dimensions into a manifold from which time is excluded is purely a phenomenon of the mind. The space continuum cannot begin to exist until the time dimension is supplied, nor can time exist without a place to exist in.

The external world that we observe is composed, not of points, but of events. If a point lacks position in time it does not exist; give it this position and it becomes an event. This world of events is four-dimensional—which means nothing more terrifying than that you must make four measures to locate an event. It does not mean, at all, that you must visualize four mutually perpendicular lines in your accustomed three-space or in a four-space analogous to it. If this world of four dimensions seems to lack reality you will be able to exhibit no better reality for your old ideas. Time belongs, without question; and not as an afterthought, but as part of the world of events.

To locate an event we use four measures: X, Y and Z for space, T for time. Using the same reference frame for time and space, we locate a second event by the measures x, y, z, t. Minkowski showed that the quantity

$$\sqrt{(X-x)^2+(Y-y)^2+(Z-z)^2-(CT-Ct)^2}$$

is the same for all observers, no matter how different their x's, y's, z's and t's; [187]
just as in the plane the quantity

$$\sqrt{(X-x)^2+(Y-y)^2}$$

is the same for all observers, no matter how different their x's and y's.

Such a quantity, having the same value for *all* observers, is absolute. In the plane it represents the true, absolute distance between the points—their intrinsic property. In dealing with events it represents the true, absolute "interval," *in time and space together* between the events. It is not space, nor time, but a combination of the two. We have always broken it down into separate space and time components. In this we are as naive as the plane observer who could not visualize the distance PQ until it was split into separate horizontals and verticals. He understood with difficulty that another observer, employing a different reference frame because in different position, would make the decomposition differently. We understand with difficulty that another observer, employing a different reference frame because *in uniform motion relative to us*, will decompose the "interval" between events into time and space components different from ours. Time and space are relative to the observer; only the interval representing space-time is absolute. So common sense stands reconciled to the Special Theory of Relativity.

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Successive Steps Toward Generality

Is then our laboriously acquired geometry of points in a three-dimensional space to go into the discard? By no means. Jeans, investigating the equilibrium [188] of gaseous masses, found the general case too difficult for direct attack. So he considered the case where the masses involved are homogeneous and incompressible. This never occurs; but it throws such light on the general case as to point the way toward attack on it.

Euclidean geometry excludes motion, save that engineered by the observer; and then the time is immaterial. Time does not enter at all; the three space dimensions suffice. This simple case never occurs where matter exists; but its conclusions are of value in dealing with more general cases.

When we look into a world alleged to be that of Euclid and find motion, we may retain the Euclidean concept of what constitutes the world and invent a machinery to account for the motion; or we may abandon the Euclidean world, as inadequate, in favor of a more general one. We have adopted the second alternative.

Newton's laws tells us that a body free to move will do so, proceeding in a straight line at uniform velocity until interfered with. We do not ask, nor does the theory tell us, whence comes the initial motion. There is no machinery to produce it; it is an inherent property of Newton's world—assured by the superposition of the time continuum upon Euclid's world to make Newton's, accepted without question along with that world itself.

But Newton saw that his world of uniform motion, like Euclid's, was never realized. In the neighborhood of one particle a second is interfered with, forced to give up its uniform motion and acquire a constant acceleration. This Newton [189] explained by employing the first of the alternatives mentioned above. He tells us that in connection with all matter there exists a force which acts on other matter in a certain way. He does not display the actual machinery through which this "force" works, because he could not discover any machinery; he had to stop with his brilliant generalization of the observed facts. And all his successors have failed to detect the slightest trace of a machinery of gravitation.

Einstein asks whether this is not because the machinery is absent—because gravitation, like position in Euclid's world and motion in Newton's, is a fundamental property of the world in which it occurs. His point of attack here lay in precise formulation of certain familiar facts that had never been adequately appreciated. These facts indicate that even accelerated motion is relative, in spite of its apparently real and absolute effects.

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Gravitation and Acceleration

An observer in a closed compartment, moving with constant acceleration through empty space, finds that the "bottom" of his cage catches up with objects that he releases; that it presses on his feet to give him the sensation of weight, etc. It displays all the effects that he would expect if it were at rest in a gravitational field. On the other hand, if it were falling freely under gravitational influence, its occupant would sense no weight, objects released would not leave his hand, the reaction from his every motion would change his [190] every position in his cage, and he could equally well assume himself at rest in a region of space free from gravitational action. Accelerated motion may *always* be interpreted, by the observer on the system, as ordinary force effects on his moving system, or as gravitational effects on his system at rest.

An alternative statement of the Special Theory is that the observed phenomena of uniform motion may equally be accounted for by supposing the object in motion and the observer with his reference frame at rest, or vice versa. We may similarly state the General Theory: The observed phenomena of uniformly *accelerated* motion may in every case be explained on a basis of stationary observer and accelerated objective, or of stationary objective with the observer and his reference system in accelerated motion. Gravitation is one of these phenomena. It follows that if the observer enjoy properly accelerated axes (in time-space, of course), the absolute character of the world about him must be such as to present to him the phenomenon of gravitation. It remains only to identify the sort of world, of which gravitation *as it is observed* would be a fundamental characteristic.

Euclid's and Newton's systems stand as first and second approximations to that world. The Special Relativity Theory constitutes a correction of Newton, presumably because it is a third approximation. We must seek in it those features which we may most hopefully carry along, into the still more general case.

Newton's system retained the geometry of Euclid. But Minkowski's invariant [191] expression tells us that Einstein has had to abandon this; for in Euclidean geometry of four dimensions the invariant takes the form:

$$\sqrt{(X-x)^2+(Y-y)^2+(Z-z)^2+(T-t)^2}\,,$$

analogous to that of two and three dimensions. It is not the presence of the constant *C* in Minkowski's formula that counts; this is merely an adjustment so that we may measure space in miles and time in the unit that corresponds to a mile. It is the minus sign where Euclidean geometry demands a plus that makes Minkowski's continuum non-Euclidean.

The editor has told us what this statement means. I think he has made it clear that when we speak of the geometry of the four-dimensional world, we must not read into this term the restrictions surrounding the kind of geometry we are best acquainted with—that of the three-dimensional Euclidean continuum. So I need only point out that if we are to make a fourth (and we hope, final) approximation to the reality, its geometry must preserve the generality attained by that of the third step, if it goes no further.

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Einstein's Time-Space World

Einstein accordingly examined the possible non-Euclidean geometries of four dimensions, in search of one displaying fundamental characteristics which, interpreted in terms of space-time, would lead to the observed facts of gravitation. The mathematics of this investigation is that part of his work which, we are told, but twelve men can follow; so we may only outline his conclusions. [192]

If we assume that in the neighborhood of matter the world of space-time is non-Euclidean, and that its curvature or distortion or non-Euclideanism is of a certain type already known to mathematicians; that the curvature of this world in the neighborhood of matter increases with the mass, and decreases as the distance from the matter increases; and that every particle of matter that is not interfered with travels through space-time in the most direct path possible *in that continuum*; then the observed facts of gravitation are accounted for as an inherent geometric property of this space-time world. We usually say that the presence of matter distorts this world, and that this distortion gives the track of particles through the region affected its non-uniform character.

Gravitation then is not a force at all; it is the fundamental nature of things. A body free to move through the world must follow some definite path. Euclid says it will stand still; Newton that it will traverse a straight line in three-space at uniform time-rate; Einstein that it will move in a "geodesic" through time-space—in every-day language, that it will fall.

The numerical consequences of Einstein's theory are, within the limits of observation, the same as those of Newton's for all bodies save one—Mercury. This planet shows a small deviation from the path predicted by Newton's law; Einstein's theory gives its motion exactly. Again, when modern research showed that light must be affected by gravitation, Einstein's theory, because of the extreme velocity of light, deviates from Newton's, where the speed is less a [193] determining factor; and observations of starlight deflected by the sun during the eclipse were in much better accord with Einstein's theory than Newton's. Moreover, the Special Theory predicts that mass is an observational variable like length and duration. Radioactive emanations have a velocity high enough to give appreciable results here, and the prediction is verified, tending to support the general theory by supporting its limiting case.

We like always to unify our science; and seldom, after effecting a unification, are we forced to give it up. Einstein for the first time brings mechanical, electromagnetic and gravitational phenomena within one structure. This is one reason why physicists are so open minded toward his theory—they want it to be true.

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The Layman's Last Doubt

The final answer to any series of questions is inevitably "because the world is so constructed." The things we are content to leave on that basis are those to which we are accustomed, and which we therefore think we understand; those for which this explanation leaves us unsatisfied are those which are new and unfamiliar. Newton told us that the world of three-dimensional space with one-dimensional time superposed was so constructed that bodies left to themselves would go on forever in a straight line at constant speed. We think we understand this, but our understanding consists merely of the unspoken query, "Why, of course; what is there to prevent?" The Greeks, an intelligent people, [194] looked at this differently; they would have met Newton with the unanimous demand "Why so; what is there to keep them going?" So if, in seeking an explanation of anything, we come sooner than we had expected to the finality "Because the world is so constructed," let us not feel that we have been cheated. [195]

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IX

THE PRINCIPLE OF RELATIVITY

A Statement of What it is All About, in Ideas of One Syllable

BY HUGH ELLIOT
CHISLEHURST, KENT, ENGLAND

The invariance of the laws of nature was one of the most popular themes of nineteenth century philosophy. For it was not till last century that general acceptance was accorded to the doctrine of the "Uniformity of Law," adumbrated in ancient times by Epicurus and Lucretius. It is now a cardinal axiom of science that the same cause in the same conditions is always followed by the same effect. There exists in nature no indeterminate element; all things are governed by fixed laws, and the discovery of these laws is the main business of science.

> It is necessary to guard against reading into this statement an erroneous idea of the content of a "law of nature." Such a law is of course not an enactment of any sort; and it is not even to be thought of as an actual explanation of the how and why of the phenomena with which it has to do. It really is nothing but an expression of our belief in the pronouncement of the preceding paragraph, that like conditions do produce like results. It is a prediction based on past [196] experience, and is of value merely in that past experience leads us to credit its accuracy. The composite essay beginning on page 19 discusses this question of the reality of natural laws, and should be consulted in connection with the present contribution.—Editor.

This great philosophic principle was derived of course from the study of natural science; *i.e.*, from observations and experiments conducted upon the *earth*. Their comprehensiveness is therefore limited by the fact that the observer is always in a state of rest, or nearly so, as compared with the earth. All observers upon the earth are moving through space at the same velocity; and it was possible to argue that the uniformity of law might only hold good, when experiments were conducted at this velocity. An observer moving at very different velocity might discover that the laws of nature under these new conditions were somewhat different.

Such a view could indeed never be very plausible, for motion is only a relative conception. Imagine a universe consisting of infinite "empty" space, in which there is poised a single material body. How shall we determine whether this body is at rest, or whether it is moving at high or low velocity through space? It is never getting nearer to anything or farther from anything, since there is no other body for it to get nearer to or farther from. If we say it is moving at a uniform velocity of a thousand miles a second, our statement really has no significance. We have no more reason for affirming that it is in motion than we have for affirming that it is at rest. In short, there is no such thing as absolute [197] motion; the conception of motion only arises when there are two or more bodies changing their position relatively to one another. This is what is meant by the relativity of motion. It seemed therefore improbable that the laws of nature would be different if the observer were moving at high velocity; for the movement of the observer is not an absolute quantity, but merely a statement of his relation to other bodies, and if there *were* no other bodies, the statement itself would be meaningless.

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The Behavior of Light

Now among the established laws of nature is that which specifies the velocity of light moving through a vacuum. If the laws of nature are invariable, this velocity will always be the same. But consider what would happen under the following circumstances: Suppose that we are at rest, and that an observer on another body flies past us at 150,000 miles a second. Suppose that at the moment he passes, a piece of flint projecting from him grazes a piece of steel projecting from us, giving rise to a spark; and that we both thereupon set about to measure the velocity of the light so produced. After one second, we should find that the light had traveled about 186,000 miles away, and since during this second the other observer had traveled 150,000 miles, we should infer that the light traveling in his direction was only about 36,000 miles ahead of him. We should also infer that he would find this out by his experiment, and that he would estimate the velocity of light as only 36,000 miles a second in his own [198] direction, and 336,000 miles a second in the opposite direction. But if this is so, then that law of nature which specifies the velocity of light is quite different for him and for us: the laws of nature must be dependent upon the observer's motion—a conclusion which appears incompatible with the idea of the relativity of motion.

And it so happens that it is also contradictory to experimental conclusions. Experiments undertaken to settle the point show that each observer finds the same velocity for the light of the spark; and after one second, each observer finds that the light has traveled 186,000 miles from himself. But how is it possible that when it has traveled 186,000 miles in the same direction as the other observer who himself has moved 150,000 miles meanwhile, he should still think it 186,000 miles ahead of him? That is the initial paradox; and since there has been no room for error in the experiments, we are forced to conclude that there was something wrong in the assumptions and preconceptions with which we started.

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Space and Time

There can in fact be only one interpretation. If we each find that the light has moved the same number of miles in the same number of seconds, then we must be meaning something different when we speak of miles and seconds. We are speaking in different languages. Some subsidence has occurred in the foundations of our systems of measurement. We are each referring to one and the same objective fact; but since we describe it quite differently, and at first [199] sight incompatibly, some profound alteration must have occurred in our perceptions—all unsuspected by ourselves. It has been shown precisely what this alteration is. A body moving at high velocity must become flattened in the direction of its motion; all its measuring apparatus, when turned in that direction, is shortened, so that no hint of the flattening can be obtained from it. Furthermore, the standards of time are lengthened out, and clocks go slower. The extent of this alteration in standards of space and time is stated in the equations of the so-called Lorentz transformation.

> Objection might be urged to the above paragraph on the ground that the connection of the observer with the variability of measured lengths and times is not sufficiently indicated, and that this variability therefore might be taken as an intrinsic property of the observed body—which of course it is not.—Editor.

We are accustomed to describe space as being of three dimensions, and time as being of one dimension. As a matter of fact, both space and time are "ideas," and not immediate sense-perceptions. We perceive matter; we then *infer* a universal continuum filled by it, which we call space. If we had no knowledge of matter, we should have no conception of space. Similarly in the case of time: we *perceive* one event following another, and we then invent a continuum which we call time, as an abstraction based on the sequence of events. We do not see space, and we do not see time. They are not real things, in the sense that matter is real, and that events are real. They are products of imagination: useful enough in common life, but misleading when we try to [200] look on the universe as a whole, free from the artificial divisions and landmarks which we introduce into it for practical convenience. Hence it is perhaps not so surprising after all that in certain highly transcendental investigations, these artificial divisions should cease to be a convenience, and become a hindrance.

Take for instance our conception of time. It differs from our conception of space in that it has only one dimension. In space, there is a right and left, an up and down, a before and after. But in time there is only before and after. Why should there be this limitation of the time-factor? Merely because that is the verdict of all our human experience. But is our human experience based on a sufficiently broad foundation to enable us to say that, under all conditions and in all parts of the universe, there can be only one time-direction? May not our belief in the uniformity of time be due to the uniformity of the motion of all observers on the earth? Such in fact is the postulate of relativity. We now believe that, at velocities very different from our own, the standard of time would also be different from ours. From our point of view, that different standard of time would not be confined to the single direction fore and aft, as we know it, but would also have in it an element of what we might call right and left. True, it would still be of only one dimension, but its direction would differ from the direction of our time. It would still run like a thread through the universe, but not in the direction which we call straight forward. It would have a slant in it, and the angle of the slant depends upon the velocity of motion. It [201] does not follow that because we are all traveling in the same direction down the stream of time, therefore that stream can only flow in the direction which we know. "Before" and "after" are expressions which, like right and left, depend upon our personal situation. If we were differently situated, if to be precise we were moving at very high velocity, we should, so to speak, be facing in a new direction and "before" and "after" would imply a different direction of progress from that with which we are now familiar.

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The World of Reality

But, after all, the objective universe is the same old universe however fast we are moving about in it, and whatever way we are facing. These details merely determine the way we divide it up into space and time. The universe is not affected by any arbitrary lines which we draw through it for our personal convenience. For practical purposes, we ascribe to it four dimensions, three in space and one in time. Clearly if the time direction is altered, all dimensions both of space and time must have different readings. If, for instance, the time direction slopes away to the left, as compared with ours, then space measurements to right and left must be correspondingly altered. An analogy will simplify the matter.

Suppose we desire to reach a point ten miles off in a roughly northeasterly direction. We might do so by walking six miles due east and then eight miles due north. We should then be precisely ten miles from where we started. But [202] suppose our compass were out of order, so that its north pole pointed somewhat to the west of north. Then in order to get to our destination, we might have to walk seven miles in the direction which we thought was east, and a little more than seven miles in the direction which we thought was north. We should then reach the same point as before. Both observers have walked according to their lights, first due east and then due north, and both have reached the same point: the one observer is certain that the finishing point is six miles east of the starting-point, while the other is sure it is seven miles.

Now we on the earth are all using a compass which points in the same direction as regards time. But other observers, on bodies moving with very different velocity, have a compass in which the time-direction is displaced as compared with ours. Hence our judgments of distances will not be alike. In our analogy, the northerly direction corresponds to time, and the easterly direction to space; and so long as we use the same compass we do not differ in our measurements of distances. But for any one who has a different notion of the time-direction, not only time intervals but space distances will be judged differently.

In short, the universe is regarded as a space-time continuum of four dimensions. A "point" in space-time is called an "event"—that which occurs at a specified moment and at a specified place. The distance between two points in space-time is called their "interval." All observers will agree as to the magnitude of any interval, since it is a property of the objective universe; but [203] they will disagree as to its composition in space and time separately. In short, space and time are *relative* conceptions; their relativity is a necessary consequence of the relativity of motion. The paradox named at the outset is overcome; for the two observers measuring the velocity of the light produced as they passed one another, were using different units of space and time. And hence emerges triumphant the *Special Principle of Relativity*, which states that the laws of nature are the same for all observers, whether they are in a state of rest or of uniform motion in a straight line.

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Accelerated Motion

Uniform motion in a straight line is however a very special kind of motion. Our experience in ordinary life is of motions that are neither uniform nor in a straight line; both speed and direction of motion are altering. The moving body is then said to undergo "acceleration": which means either that its speed is increasing or diminishing, or that its direction of motion is changing, or both. If we revert to our former supposition of a universe in which there is only a single body in "empty" space, we clearly cannot say whether it has acceleration any more than whether it is moving, there being no outside standard of comparison; and the General Principle of Relativity asserts the invariance of the laws of nature for all states of motion of the observer. In this case, however, a difference might be detected by an observer on the moving [204] body itself. It would be manifested to him as the action of a force; such for instance as we feel when a train in which we are traveling is increasing or reducing speed, or when, without changing speed, it is rounding a corner. The force dies away as soon as the velocity becomes uniform. Thus acceleration reveals itself to us under the guise of action by a force. Force and acceleration go together, and we may either say that the acceleration is due to the force, or the impression of force to the acceleration.

Now when we are traveling with accelerated motion, we have quite a different idea of what constitutes a straight line from that which we had when at rest or in uniform motion. If we are moving at uniform velocity in an airplane and drop a stone to the earth it will appear to us in the airplane to fall in a straight line downward, while to an observer on the earth it will appear to describe a parabola. This is due to the fact that the stone gathers speed as it falls; it is subject to the acceleration associated with gravity. Acceleration obliterates the fundamental difference between a straight and curved line. Unless we know what is the absolute motion of the stone, and the two observers, we cannot say whether the line is "really" a straight or a curved line. Since absolute motion is an illegitimate conception, it follows that there is no such thing as "really" straight or "really" curved. These are only appearances set up as a consequence of our relative motions with respect to the bodies concerned. If there were no such thing as acceleration—if the stone fell to the earth at uniform velocity—then an observer on the earth or anywhere else would agree [205] that it fell in a straight line; and straight lines would always be straight lines.

Under these circumstances, Euclidean geometry would be absolutely true. But if we are in a state of acceleration, then what we think are straight lines are "really" curved lines, and Euclidean geometry, based on the assumption that its lines are straight, must founder when tested by more accurate measurements. And in point of fact we are in a state of acceleration: for we are being acted upon by a force—namely, the force of gravitation. Wherever there is matter, there is gravitation; wherever there is gravitation there is acceleration; wherever there is acceleration Euclidean geometry is inaccurate. Hence in the space surrounding matter a different geometry holds the field; and bodies in general move through such space in curved lines.

Different parts of space are thus characterized by different geometrical properties. All bodies in the universe proceed on their established courses through space and time. But when they come to distorted geometrical areas, their paths naturally seem to us different from when they were moving through less disturbed regions. They exhibit the difference by acquiring an acceleration; and we explain the acceleration by alleging the existence of a force, which we call the force of gravitation. But their motions can in fact be perfectly predicted if we know the geometry of the space through which they are traveling. The predictions so based have in fact proved more accurate than those based on the law of gravitation. [206]

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X

SPACE, TIME AND GRAVITATION

An Outline of Einstein's Theory of General Relativity

BY W. DE SITTER
PROFESSOR OF ASTRONOMY IN THE UNIVERSITY OF LEYDEN

> "Henceforth space by itself and time by itself shall sink to mere shadows, and only a union of the two shall preserve reality."

The prophecy contained in the above-quoted words, spoken by Minkowski at the meeting of German "Naturforscher und Aerzte" at Cologne in 1908, has, however, only been completely fulfilled by Einstein's "Allgemeine Relativitäts-theorie" of 1915, which incorporated gravitation into the union. In the following pages an attempt is made to set forth, without using any technical language, the leading ideas of that theory: I will confine myself to the theory as published by Einstein in November, 1915, which forms a consistent whole, complete in itself; and I will not refer to later developments, which are still more or less tentative, and not necessary for the understanding of the theory. The mathematics used by Einstein is the so-called Absolute Differential Calculus. It is not more difficult or recondite than that used in other branches of theoretical physics, but it is somewhat unfamiliar to most of us, because it is [207] not generally taught in the regular university courses. I will, however, in this essay abstain from using any mathematics at all, at least, I will not be using it openly. It is of course unavoidable to use at least the results of the mathematical reasoning, if not the reasoning itself; but so long as they are not put into formulas they will, it is hoped, not look so formidable to the reader.

Referring to the quoted words of Minkowski, we may ask what is meant by "reality." Physical science, like common sense, takes for granted that there *is* a reality behind the phenomena, which is independent of the person by whom, and the particular methods by which it is observed, and which is also there when it is not observed. Strictly speaking, all talk about what is not observed is metaphysics. Nevertheless the physicist unhesitatingly believes that his laws are general, and that the phenomena continue to happen according to them when nobody is looking. And since it would be impossible to prove that they did not, he is fully entitled to his belief. The observed phenomena are the effects of the action of this reality, of which we assume the existence, on the observer's senses—or apparatus, which are extended and refined sense-organs. The laws governing the phenomena therefore must convey some information regarding this reality. We shall never by any means be able to know anything else about it but just these laws. To all intents and purposes the laws *are* the reality, if we eliminate from them all that refers to the observer alone. What refers to the reality is called "absolute," and what involves reference to the [208] observer "relative." The elimination of the relative is one of the things the theory of relativity has set out to do.

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The External World and its Geometry

To describe the phenomena and derive laws from them, we locate them in space and time. To do this we use geometry. Here it is that the part contributed by the observer comes in. There are an infinite number of geometries, and *a priori* there seems to be no reason to choose one rather than the other. Taking geometry of two dimensions as an example, we can draw figures on a piece of paper, and discuss their properties, and we can also do so on the shell of an egg. But we cannot draw *the same* figures on the egg as on the paper. The ones will be distorted as compared with the others: the two surfaces have a different geometry. Similarly it is not possible to draw an accurate map of the earth on a sheet of paper, because the earth is spherical and its representation on the flat paper is always more or less distorted. The earth requires spherical geometry, which differs from the flat, or Euclidean, geometry of the paper.

Up to a few years ago Euclidean (*i.e.* flat) geometry of three dimensions had been exclusively used in physical theories. Why? Because it is the true one, is the one answer generally given. Now a statement about facts can be true or false, but a mathematical discipline is neither true nor false; it can only be correct—*i.e.* consistent in itself—or incorrect, and of course it always is [209] correct. The assertion that a certain geometry is the "true" one can thus only mean, that it is the geometry of "true" space, and this again, if it is to have any meaning at all, can only mean that it corresponds to the physical "reality." Leaving aside the question whether this reality has any geometry at all, we are confronted with the more immediately practical consideration how we shall verify the asserted correspondence. There is no other way than by comparing the conclusions derived from the laws based upon our geometry, with observations. It thus appears that the only justification for the use of the Euclidean geometry is its success in enabling us to "draw an accurate map" of the world. As soon as any other geometry is found to be more successful, that other must be used in physical theories, and we may, if we like, call it the "true" one.

Accurate observations always consist of *measures*, determining the position of material bodies in space. But the positions change, and for a complete description we also require measures of time. An important remark must be made here. Nobody has ever measured a pure space-distance, nor a pure lapse of time. The only thing that can be measured is the distance from a body at a certain point of space and a certain moment of time, to a body (either the same or another) at another point and another time. We can even go further and say that time cannot be measured at all. We profess to measure it by clocks. But a clock really measures space, and we derive the time from its space-measures by a fixed rule. This rule depends on the laws of motion of the mechanism of [210] the clock. Thus finally time is defined by these laws. This is so, whether as a "clock" we use an ordinary chronometer, or the rotating earth, or an atom emitting light-waves, or anything else that may be suggested. The physical laws, of course, must be so adjusted that all these devices give the same time. About the reality of time, if it has any, we know nothing. All we know about time is that we want it. We cannot adequately describe nature with the three space-coordinates alone, we require a fourth one, which we call time. We might thus say with some reason that the physical world has four dimensions. But so long as it was found possible adequately to describe all known phenomena by a space of three dimensions and an independent time, the statement did not convey any very important information. Only after it had been found out that the space-coordinates and the time are not independent, did it acquire a real meaning.

As is well known the observation by which this was found out is the famous experiment of Michelson and Morley. It led to the "special" theory of relativity, which is the one referred to by Minkowski in 1908. In it a geometry of four dimensions is used, not a mere combination of a three-dimensional space and a one-dimensional time, but a continuum of truly fourfold order. This time-space is not Euclidean, since the time-component and the three space-components are not on the same footing, but its fundamental formula has a great resemblance to that of Euclidean geometry. We may call it "pseudo-Euclidean." [211]

This theory, which we need not explain here, was very satisfactory so far as the laws of electromagnetism, and especially the propagation of light, were concerned, but it did not include gravitation, and mechanics generally. We then had this curious state of affairs, that physicists actually believed in two different "realities." When they were thinking of light they believed in Minkowski's time-space; when they were thinking of gravitation they believed in the old Euclidean space and independent time. This, of course, could not last. Attempts were made so to alter Newton's law of gravitation that it would fit into the four-dimensional world of the special relativity-theory, but these only succeeded in making the law, which had been a model of simplicity, extremely complicated, and, what was worse, it became ambiguous.

It is Einstein's great merit to have perceived that gravitation is of such fundamental importance, that it must not be fitted into a ready-made theory, but must be woven into the space-time geometry from the beginning. And that he not only saw the necessity of doing this, but actually did it.

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Gravitation and its Place in the Universe

To see the necessity we must go back to Newton's system of mechanics. Newton did two things (amongst others). He canonised Galileo's system of mechanics into his famous "laws of motion," the most important of which is the law of *inertia*, which says that: [212]

> *a body, that is not interfered with, moves in a straight line with constant velocity*.

The velocity, of course, can be nil, and the body at rest. This is a perfectly general law, the same for all material bodies, whatever their physical or chemical status. Newton took good care exactly to define what he meant by uniform motion in a straight line, and for this purpose he introduced the absolute Euclidean space and absolute time as an essential part of his system of laws at the very beginning of his great work. The other thing Newton did was to formulate the law of gravitation. Gravitation was in his system considered as an interference with the free, or inertial, motion of bodies, and accordingly required a law of its own.

But gravitation has this in common with inertia, and in this it differs from all other interferences, that it is perfectly general. All material bodies are equally subjected to it, whatever their physical or chemical status may be. But there is more. Gravitation and inertia are actually indistinguishable from each other, and are measured by the same number: the "mass". This was already remarked by Newton himself, and from his point of view it was a most wonderful accidental coincidence. If an apple falls from the tree, that which makes it fall is its weight, which is the gravitational attraction by the earth, diminished by the centrifugal force due to the earth's rotation and the apple's inertia. In Newton's system the gravitational attraction is a "real" force, whereas the centrifugal force is only "fictitious". But the one is as real as the other. The most refined experiments, already begun by Newton himself, have not [213] succeeded in distinguishing between them. Their identity is actually one of the best established facts in experimental physics. From this identity of “fictitious,” or inertial, and “real,” or gravitational, forces it follows that locally a gravitational field can be artificially created or destroyed. Thus inside a closed room which is falling freely, say a lift of which the cable has been broken, bodies have no weight: a balance could be in equilibrium with different weights in the two scales.

Having thus come to the conclusion that gravitation is *not* an interference, but is identical with inertia, we are tempted to restate the law of motion, so as to include both, thus:

> *Bodies which are not interfered with*—do not move in straight lines, but—*fall*.

Now this is exactly what Einstein did. Only the “falling” of course requires a precise mathematical definition (like the uniform motion in a straight line), and the whole gist of his theory is the finding of that definition. In our earthly experience the falling never lasts long, very soon something—the floor of the room, or the earth itself—interferes. But in free space bodies go on falling forever. The motion of the planets is, in fact, adequately described as falling, since it consists in nothing else but obeying Newton’s law of gravitation together with his law of inertia. A body very far removed from all other matter is not subjected to gravitation, consequently it falls with constant velocity in a straight line according to the law of inertia. The problem was thus to find a mathematical definition of “falling,” which would embrace the uniform straight-line motion very far from all matter as well as the complex paths of [214] the planets around the sun, and of an apple or a cannon-ball on earth.

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Gravitation and Space-Time

For the definition of the uniform rectilinear motion of pure inertia Newton’s Euclidean space and independent time were sufficient. For the much more complicated falling under the influence of gravitation and inertia together, evidently a more complicated geometry would be needed. Minkowski’s pseudo-Euclidean time-space also was insufficient. Einstein accordingly introduced a general non-Euclidean four-dimensional time-space, and enunciated his law of motion thus:

> *Bodies which are not interfered with move in geodesics.*

A geodesic in curved space is exactly the same thing as a straight line in flat space. We only call it by its technical name, because the name “straight line” would remind us too much of the old Euclidean space. If the curvature gets very small, or zero, the geodesic becomes very nearly, or exactly, a straight line.

The problem has now become to assign to time-space such curvatures that the geodesics will exactly represent the tracks of falling bodies. Space of two dimensions can just be flat, like a sheet of paper, or curved, like an egg. But in geometry of four dimensions there are several steps from perfect flatness, or “pseudo-flatness,” to complete curvature. Now the law governing the curvature of Einstein’s time-space, *i.e.*, the law of gravitation, is simply that *it can never, outside matter, be curved more than just one step beyond perfect (pseudo-)flatness*. [215]

Since I have promised not to use any mathematics I can hardly convey to the reader an adequate idea of the difficulty of the problem, nor do justice to the elegance and beauty of the solution. It is, in fact, little short of miraculous that this solution, which was only adopted by Einstein because it was the simplest he could find, does so exactly coincide in all its effects with Newton’s law. Thus the remarkably accurate experimental verification of this law can at once be transferred to the new law. In only one instance do the two laws differ so much that the difference can be observed, and in this case the observations confirm the new law exactly. This is the well known case of the motion of the perihelion of Mercury, whose disagreement with Newton’s law had puzzled astronomers for more than half a century.

Since Einstein’s time-space includes Minkowski’s as a particular case, it can do all that the other was designed to do for electro-magnetism and light. But it does more. The track of a pulse of light is also a geodesic, and time-space being curved in the neighborhood of matter, rays of light are no longer straight lines. A ray of light from a star, passing near the sun, will be bent round, and the star consequently will be seen in a different direction from where it would be seen if the sun had not been so nearly in the way. This has been verified by the observations of the eclipse of the sun of 1919 of May 29. [216]

There is one other new phenomenon predicted by the theory, which falls within the reach of observation with our present means. Gravitation chiefly affects the time-component of the four-dimensional continuum, in such a way that natural clocks appear to run slower in a strong gravitational field than in a weak one. Thus, if we make the hypothesis—which, though extremely probable, is still a hypothesis—that an atom emitting or absorbing light-waves is a natural clock, and the further hypothesis—still very probable, though less so than the former—that there is nothing to interfere with its perfect running, then an atom on the sun will give off light-waves of smaller frequency than a similar atom in a terrestrial laboratory emits. Opinions as yet differ as to whether this is confirmed or contradicted by observations.

* * *

The great strength and the charm of Einstein’s theory do however not lie in verified predictions, nor in the explanation of small outstanding discrepancies, but in the complete attainment of its original aim: the identification of gravitation and inertia, and in the wide range of formerly apparently unconnected subjects which it embraces, and the broad view of nature which it affords.

* * *

Outside matter, as has been explained, the law of gravitation restricts the curvature of time-space. Inside continuous matter the curvature can be of any arbitrary kind or amount; the law of gravitation then connects this curvature with measurable properties of the matter, such as density, velocity, stress, etc. Thus these properties define the curvature, or, if preferred, the curvature [217] defines the properties of matter, *i.e.* matter itself.

From these definitions the laws of conservation of energy, and of conservation of momentum, can be deduced by a purely mathematical process. Thus these laws, which at one time used to be considered as the most fundamental ones of mechanics, now appear as simple corollaries from the law of gravitation. It must be pointed out that such things as length, velocity, energy, momentum, are not absolute, but relative, *i.e.* they are not attributes of the physical reality, but relations between this reality and the observer. Consequently the laws of conservation are not laws of the real world, like the law of gravitation, but of the observed phenomena. There is, however one law which, already before the days of relativity, had come to be considered as the most fundamental of all, *viz*: the principle of least action. Now action is absolute. Accordingly this principle retains its central position in Einstein's theory. It is even more fundamental than the law of gravitation, since both this law, and the law of motion, can be derived from it. The principle of least action, so far as we can see at present, appears to be *the* law of the real world. [218]

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XI

THE PRINCIPLE OF GENERAL RELATIVITY

How Einstein, to a Degree Never Before Equalled, Isolates the External Reality from the Observer's Contribution

BY E. T. BELL
UNIVERSITY OF WASHINGTON
SEATTLE

Einstein's general relativity is of such vast compass, being coextensive with the realm of physical events, that in any brief account a strict selection from its numerous aspects is prescribed. The old, restricted principle being contained in the general, we shall treat the latter, its close relations with gravitation, and the significance of both for our knowledge of space and time. The essence of Einstein's generalization is its final disentanglement of that part of any physical event which is contributed by the observer from that which is inherent in the nature of things and independent of all observers.

The argument turns upon the fact that an observer must describe any event with reference to some framework from which he makes measurements of time and distance. Thus, suppose that at nine o'clock a ball is tossed across the room. At one second past nine the ball occupies a definite position which we can specify by giving the three distances from the centre of the ball to the [219] north and west walls and the floor. In this way, refining our measurements, we can give a precise description of the entire motion of the ball. Our final description will consist of innumerable separate statements, each of which contains four numbers corresponding to four measurements, and of these one will be for time and three for distances at the time indicated.

Imagine now that a man in an automobile looks in and observes the moving ball. Suppose he records the motion. To do so, he must refer to a timepiece and some body of reference. Say he selects his wrist-watch, the floor of his auto and two sides meeting in a corner. Fancy that just as he begins his series of observations his auto starts bucking and the main-spring of his watch breaks, so that he must measure "seconds" by the crazy running-down of his watch, and distances with reference to the sides of his erratic auto. Despite these handicaps he completes a set of observations, each of which consists of a time measured by his mad watch and three distances reckoned from the sides of his bucking machine. Let us assume him to have been so absorbed in his experiment that he noticed neither the disorders of his watch nor the motion of his auto. He gives us his sets of measurements. We remark that his seconds are only small fractions of ours, also his norths and wests are badly mixed. If we interpret his sets in terms of our stationary walls and sober clock we find the curious paradox that the ball zigzagged across the room like an intoxicated bee. He obstinately argues that we know no more than he about how the ball actually moved. For we got a smooth description, he asserts, by choosing an [220] artificially simple reference framework, having no necessary relations whatever to the ball. The crooked path plotted from his observations proves, he declares, that the ball was subject to varying forces of which we in the room suspected nothing. He contends that our room was being jarred by a system of forces which exactly compensated and smoothed out the real jaggedness of path observed by himself. But if we know all about his watch and auto we can easily apply necessary corrections to his measurements, and, fitting the corrected set to our reference-framework of walls and clock, recover our own smooth description.

For consistency we must carry our readjustments farther. The path mapped from our measurements is a curve. Perhaps the curvature was introduced by some peculiarity of our reference framework? Possibly our own room is being accelerated upward, so that it makes the ball's true path—whatever that may be—appear curved downward, just as the autoist's zigzags made the path he mapped appear jagged. Tradition attributes the downward curving to the tug of gravity. This force we say accelerates the ball downward, producing the curved path. Is this the only possible explanation? Let us see.

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Gravitation and Acceleration

Imagine a man in a room out of which he cannot see. He notices that when he releases anything it falls to the floor with a constant acceleration. Further he observes that all his objects, independently of their chemical and physical [221] properties, are affected in precisely the same way. Now, he previously has experimented with magnets, and has remarked that they attract certain bodies in essentially the same way that the things which he drops are "attracted" to whatever is beneath the floor. Having explained magnetic attraction in terms of "forces," he makes his first hypothesis: (A) He and his room are in a strong "field of force," which he designates gravitational. This force pulls all things downward with a constant acceleration. Here he notes a singular distinction between magnetic and gravitational "forces": magnets attract only a few kinds of matter, notably iron; the novel "force," if indeed a force at all, acts similarly upon all kinds of matter. He makes another hypothesis: (B) His room and he are being accelerated upward.

* * *

Either (A) or (B) describes the facts perfectly. By no experiment can he discriminate between them. So he takes the great step, and formulates the *Equivalence Hypothesis*:

> *A gravitational field of force is precisely equivalent in its effects to an artificial field of force introduced by accelerating the framework of reference, so that in any small region it is impossible to distinguish between them by any experiment whatever.*

Next reconsidering his magnetic "forces," he extends the equivalence hypothesis to cover all manifestations of force: The effects attributed to forces of any kind whatever can be described equally well by saying that our reference frameworks are accelerated; and moreover there is possible no experiment which will discriminate between the descriptions. [222]

If the accelerations are null, the frameworks are at rest or in *uniform* motion relatively to one another. This special case is the "restricted" principle of relativity, which asserts that it is impossible experimentally to detect a uniform motion through the ether. Being thus superfluous for descriptions of natural phenomena, the ether may be abandoned, at least temporarily. The older physics sought this absolute ether framework to which all motions could be unambiguously referred, and failed to find it. The most exacting experiments, notably that of Michelson-Morley, revealed no trace of the earth's supposed motion through the ether. Fitzgerald accounted for the failure by assuming that such motion would remain undetected if every moving body contracted by an amount depending upon its velocity in the direction of motion. The contraction for ordinary velocities is imperceptible. Only when as in the case of the beta particles, the velocity is an appreciable fraction of the velocity of light, is the contraction revealed. This contraction follows immediately from Einstein's generalization constructed upon the equivalence hypothesis and the restricted relativity principle. We shall see that the contraction inevitably follows from the actual geometry of the universe.[1]

Let us return for a moment to the moving ball. Four measures, three of distances and one of time, are required in specifying its position with reference to some framework at each point and at each instant. All of these measures can [223]
be summed up in one compendious statement—the equations of motion showed how in changing from our room to his accelerated auto we found a new summary, "transformed equations," which seemed to indicate that the ball had traversed a strong, variable field of force. Is there then in the chaos of observational disagreements anything which is independent of all observers? There is, but it is hidden at the very heart of nature.

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Paths Through the World of Four Dimensions

To exhibit this, we must recall a familiar proposition of geometry: the square on the longest side of a right-angled triangle is equal to the sum of the squares on the other two sides. It has long been known that from this alone all the metrical properties of Euclidean space—the space in which for 2,000 years we have imagined we were living—can be deduced. Metrical properties are those depending upon measurement. Now, in the geometry of any space, Euclidean or not, there is a single proposition of a similar sort which tells us how to find the most direct distance between any two points that are very close together. This small distance is expressed in terms of the two sets of distance measurements by which the end-points are located, just as two neighboring positions of our ball were located by two sets of four measurements each. We say by analogy that two consecutive positions of the ball are separated by a [224] small *interval* of time-space. From the formula for the very small interval of time-space we can calculate mathematically all the metrical properties of the time and space in which measurements for the ball's motion must be made. So in any geometry mathematical analysis predicts infallibly the truth about all facts depending upon measurements from the simple formula of the interval between neighboring points. Thus, on a sphere the sum of the angles of any triangle formed by arcs of great circles exceeds 180°, and this follows from the formula for the shortest ("geodesic") distance between neighboring points on the spherical surface.

We saw that it takes four measurements, one for time and three for distances, to fix an elementary event, *viz.*, the position of the centre of our ball at any instant. A system of all possible such sets of four measurements each, constitutes what mathematicians call a four-dimensional space. The study of the four-dimensional time-space geometry, once its shortest-distance proposition is known, reveals all those relations in nature which can be ascertained by measurements, that is, experimentally. We have then to find this indispensable proposition.

Imagine the path taken by a particle moving solely under the influence of gravitation. This being the simplest possible motion of an actual particle in the real world, it is natural to guess that its path will be such that the particle moves from one point of time-space to another by the most direct route. This in fact is verified by forming the equations of the free particle's motion, which turn out to be precisely those that specify a geodesic (most direct line) joining the two points. On the (two-dimensional) surface of a sphere such a line is the [225] position taken by a string stretched between two points on the surface, and this is the *shortest* distance on the surface between them. But in the time-space geometry we find a remarkable distinction: the interval between any two points of the path taken is the *longest* possible, and between any two points *there is only one longest path*. Translated into ordinary space and time this merely asserts that the *time* taken between any two points on the natural path is the longest possible.

Recall now that when the line-formula for any kind of space is known *all the metrical properties* of that space are completely determined, and combine with this what we have just found, namely, the equations of motion of a particle subject only to gravitation are the same equations as those which fix the line-formula for the four-dimensional time-space. Since gravitation alone determines the motion of the particle, and since this motion is completely described by the very equations which fix all the metrical properties of time-space, it follows that the metrical (experimentally determinable) properties of time-space are equivalent to those of gravitation, in the sense that each set of properties implies the other.

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The Universe of Space-Time

We have found the thing in nature which is independent of all observers, and it turns out to be the very structure of time-space itself. The motion of the free particle obviously is a thing unconditioned by accidents of observation; the [226] particle under the influence of gravitation alone must go a way of its own. And if some observer in an artificial field of force produced by the acceleration of his reference framework describes the path as knotted, he merely is foisting eccentricities of his own motion upon the direct path of the particle. The conclusion is rational, for we believe that time-space exists independently of any man's way of perceiving it.

Incidentally note that this space is that of the physical world. For only by measurements of distances and times can we become aware of our extension in time and space. If beyond this time-space geometry of measurements there is some "absolute geometry," science can have no concern with it, for never can it be revealed by the one exploring device we possess—measurement.

We have followed a single particle. Let us now form a picture of several. Any event can be analyzed into a multitude of coincidences in time-space. For consider two moving particles—say electrons. If they collide they both are in very approximately one place at the same time. We imagine the path of an electron through time-space plotted by a line (in four-dimensional space), which will deviate from a "most direct" (geodesic) path if the electron is subjected to forces. This is the "world-line" of the electron. If the world lines of several electrons intersect at one point in time-space, the intersection pictures the fact of their coincidence somewhere and somewhen; for all their world-lines having a time-space point in common, at some instant they must have been in collision. Each point of a world-line pictures the position at a [227] certain place at a certain time; and it is the intersections of world-lines which correspond to physical events. Of what lies between the intersections we have no experimental knowledge.

Imagine the world-lines of all the electrons in the universe threading time-space like threads in a jelly. The intersections of the tangle are a complete history of all physical events. Now distort the jelly. Clearly the mutual *order* of the intersections will be unchanged, but the distances between them will be shortened or lengthened. To a distortion of the jelly corresponds a special choice (by some observer) of a reference framework for describing the order of events. He cannot change the natural sequence of events. Again we have found something which is independent of all observers.

We can now recapitulate our conclusions and state the principle of relativity in its most general form.

(1) Observers describe events by measures of times and distances made with regard to their frameworks of reference.

(2) The complete history of any event is summarized in a set of equations giving the positions of all the particles involved at every instant.

(3) Two possibilities arise. (A) Either these equations are the same in form for all space-time reference frameworks, persisting formally unchanged for all shifts of the reference scheme; or (B), they subsist only when some special framework is used, altering their form as they are referred to different frameworks. If (B) holds, we naturally assume that the equations, and the phenomena which they profess to represent, owe their existence to some [228] peculiarity of the reference framework. They do not, therefore, describe anything which is inherent in the nature of things, but merely some idiosyncrasy of the observer's way of regarding nature. If (A) holds, then obviously the equations describe some real relation in nature which is independent of all possible ways of observing and recording it.

(4) In its most general form the principle of relativity states that those relations, and those alone, which persist unchanged in form for all possible space-time reference frameworks are the inherent laws of nature.

To find such relations Einstein has applied a mathematical method of great power—the calculus of tensors—with extraordinary success. This calculus threshes out the laws of nature, separating the observer's eccentricities from what is independent of him, with the superb efficiency of a modern harvester. The residue is a physical geometry—or geometrical physics—of time-space, in which it appears that the times and spaces contributed by the several observers' reference frameworks are shadows of their own contrivings; while the real, enduring universe is a fourfold order of time and space indissolubly bound together. One observer separates this time-space into his own "time" and "space" in one way, determined by his path through the world of events; another, moving relatively to the first, separates it differently, and what for one is time shades into space for another.

This time-space geometry is non-Euclidean. It is "warped" (curved), the amount of warping at any place being determined by the intensity of the [229] gravitational field there. Thus again gravitation is rooted in the nature of things. In this sense it is not a force, but a property of space. Wherever there is matter there is a gravitational field, and hence a warping of space. Conversely, as long ago imagined by Clifford, wherever there is a warping of space, there is matter; and matter is resolved ultimately into wrinkles in time-space.

To visualize a warped space, consider a simple analogy. A man walks away from a polished globe; his image recedes into the mirror-space, shortening and thinning as it goes, and thinning (in the direction of motion) faster than it shortens. Everything around him experiences a like effect. If he tries to discover this by a footrule it automatically shortens faster as he turns it into the horizontal position, so his purpose eludes him. The mirror-space is warped in the direction of the image's motion. So is our own. For all bodies, as evidenced by the Fitzgerald contraction, shorten in the direction of motion. And just as the image can never penetrate the mirror-space a greater distance than half its radius, so probably time-space is curved in such a way that our universe, like the surface of a sphere, is finite in extent, but unbounded. [230]

1 The author here comes perilously close to ascribing to this "contraction" the sort of physical reality which it does not possess. See page 96.—Editor. ↑

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XII

FORCE VS. GEOMETRY

How Einstein Has Substituted the Second for the First in Connection With the Cause of Gravitation

BY SAUL DUSHMAN
GENERAL ELECTRIC LABORATORIES SCHENECTADY, N. Y.

The theory of relativity represents a most strikingly original conception of time and space, which was suggested by Einstein in order to correlate with all our past experience certain observations made in recent years. It is therefore extremely comprehensive in its scope; it demands from us a radical revision in our notions of time and space; it throws new light on the nature of mass and energy, and finally, it furnishes a totally new conception of the old problem of gravitation.

The starting point of the theory is the familiar observation that motion is always relative: that is, to define the motion of any object we must always use some point of reference. Thus we speak of the velocity of a train as 40 miles per hour with respect to the earth's surface, but would find it impossible to determine its absolute speed, or motion in space, since we know of no star whose position can be spoken of as absolutely fixed. These and similar [231] considerations have led to the conclusion, pointed out by Newton and others, that it is impossible by any mechanical experiments on the earth to measure its velocity in space.

However, the results of observations on the phenomena of light and electricity led to the revival of the same problem under another form. As well known, there was evolved from these discoveries, the theory that light and electrical energy are of the same nature, and are in each case manifestations of wave-disturbances propagated through a hypothetical medium, the *ether*, with a velocity of 186,000 miles per second.

The problem therefore arose as to whether the earth and all stellar bodies move through this ether. In that case it ought to be possible to measure the velocity of the earth with respect to this medium, and under these conditions we could speak, in a sense, of absolute motion.

A large number of experiments has been tried with this end in view. The most famous of these, and the one which stimulated the subsequent development of the theory of relativity, was that carried out by Michelson and Morley in 1887. To understand the significance of this experiment we shall refer briefly to an analogous observation which is quite familiar.

Does it take longer to swim to a point 1 mile up a stream and back or to a point 1 mile across stream and back? The experienced swimmer will answer that the up-and-down journey takes longer. If we assume that the swimmer has a speed of 5 miles an hour in still water and that the current is 3 miles an hour, we find [232] that, while it requires five-eighths hour to make the up-and-down journey, it takes only one-half hour for the trip across stream and back. The ratio between the times required for the two journeys is thus five-fourths, and if this is written in the form

$$\frac{1}{\sqrt{1-(\frac{3}{5})^2}}$$

it shows how the result depends upon the square of the ratio of the speeds of the swimmer and the current.

Now the earth is moving in its orbit about the sun with a velocity of 18 miles per second. If the earth moves through the ether and a light-beam passes from one mirror to another and back again, the time taken for this journey ought to be longer when the light-path is in the direction of the earth's motion than when it is at right angles to this direction. For we can consider the light as a swimmer having a speed of 186,000 miles per second and travelling in a stream whose current is 18 miles per second.

When Michelson and Morley tried the experiment they could not observe any difference in the velocity of light in the two directions. The experiment has since been repeated under various conditions, but always with negative results.

Einstein's contribution to science consists in interpreting this result as being in accord with Newton's ideas on mechanical relativity in that it demonstrates the impossibility of measuring absolute motion, not only by mechanical, but also by optical or electrical experiments. Consequently the velocity of light must be [233] regarded as constant and independent of the motion of either source or observer.

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The Relativity of Uniform Motion

Let us consider some of the consequences which follow from this principle. An observer travelling with say one-half the velocity of light in the same direction as a ray of light would find that the latter has the usual velocity of 186,000 miles per second. Similarly an observer travelling in the opposite direction to that of the light-ray, with one-half the velocity of light, would obtain the same result.

Einstein has shown that these conclusions can be valid only if the units of time and space used by the two observers depend upon their relative motions. A careful calculation shows that the unit of length used by either observer appears to the other observer contracted when placed in the direction of their relative motion (but not, when placed at right angles to this direction), and the unit of time used by either observer appears to the other too great. Moreover, the ratio of the units of length or of time varies with the square of the relative speed of the two observers, according to a relation which is similar to that mentioned above for the swimmer in the current. This relation shows that as the relative speed approaches that of light the discrepancy between the units increases.

Thus, for an observer moving past our earth with a velocity which is nine-tenths that of light, a meter stick on the earth would be 44 centimeters as measured by him, while a second on our clocks would be about two and a half [234] seconds as marked by his clock. Similarly, what he calls a meter length would, for us, be only 44 centimeters and he would appear to us to be living about two and a half times slower than we are. Each observer is perfectly consistent in his measurements of time and space as long as he confines his observations to his own system, but when he tries to make observations on another system moving past his, he finds that the results which he obtains do not agree with those obtained by the other observer.

It is not surprising that in accordance with this conclusion it also follows that the mass of a body must increase with its velocity. For low velocities the increase is so small that we cannot ever hope to measure it, but as the velocity of light is approached the difference becomes more and more appreciable and a body having the velocity of light would possess infinite mass, which simply means that such a velocity cannot be attained by any material object. This conclusion has been experimentally confirmed by observations on the mass of the extremely small negatively charged particles which are emitted by radioactive elements. Some of these particles are ejected with velocities which are over nine-tenths that of light, and measurements show that the increase in mass is in accord with this theory.

The relativity theory also throws new light on the nature of mass itself. According to this view, mass and energy are equivalent. The absolute destruction of 1 gram of any substance, if possible, would yield an amount of energy which is one hundred million times as much as that obtained by [235] burning the same mass of coal. Conversely, energy changes are accompanied by changes in mass. The latter are ordinarily so inappreciably small as to escape our most refined methods of measurements, but in the case of the radioactive elements we actually observe this phenomenon. From this standpoint, also, the laws of conservation of energy and of mass are shown to be intimately related.

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Universal Relativity

So far we have dealt with what has been designated as the special theory of relativity. This, as we have seen, applies to uniform motion only. In extending the theory to include non-uniform or accelerated motion, Einstein has at the same time deduced a law of gravitation which is much more general than that of Newton.

A body falling towards the earth increases in velocity as it falls. The motion is said to be accelerated. We ascribe this increase in velocity to a gravitational force exerted by the earth on all objects. As shown by Newton, this force acts between all particles of matter in the universe, and varies inversely as the square of the distance, and directly as the product of the masses.

Of course, we have had a number of theories of gravitation, and none of them have proven successful. Einstein, however, was the first one to suggest a conception of gravitation which has proven extremely significant. He points out that a gravitational force is non-existent for a person falling freely with the [236] acceleration due to gravity. For this person there is no sensation of weight, and if he were in a closed box which is also falling with the same acceleration, he would be unable to decide as to whether his system were falling or situated in interplanetary space where there is no gravitational field. Furthermore, if he were to carry out any optical or electrical experiments in this box he would observe the same results as an experimenter on the earth. A ray of light would travel in a straight line so far as this observer can perceive, while an external observer would, of course, judge differently.

Einstein shows that this is equally true for all kinds of acceleration including that due to rotation. In the case of a rotating body there exists a centrifugal force which tends to make objects on the surface fly outwards, but for an external observer this force does not exist any more than gravity exists for the observer falling freely.

Thus we can draw the general conclusion that a gravitational field or any other field of force may be eliminated by choosing an observer moving with the proper acceleration. For this observer, however, the laws of optics and electricity must be just as valid as for an observer on the earth.

In postulating this equivalence hypothesis Einstein merely makes use of the very familiar observation that, independently of the nature of the material, all bodies possess the same acceleration in a given field of force.

The problem which Einstein now sets out to solve is that of determining the law which shall describe the motion of any system in a field of force in such a [237] general manner as to leave unaltered the fundamental relations of electricity and optics.

In connection with the solution of this problem he finds it necessary to discard the limitations placed on us by ordinary or Euclidean geometry. In this manner geometrical concepts as well as those of force are completely robbed of all notions of absoluteness, and the goal of a general theory of relativity is attained.

The Geometry of Gravitation

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Let us consider a circular disc rotating with a uniform peripheral speed. According to the deductions from the "special theory" of relativity, an observer situated near the edge of this disc, but not rotating with it, will observe that units of length measured along the circumference of the disc are contracted. On the other hand, measurements along the diameter, which is at right angles to the direction of motion of the circumference, will show no contraction whatever, and, consequently the observer will find that the ratio of circumference to diameter has not the well known value 3.14159 … but exceeds this value, the difference being greater and greater as the peripheral speed approaches that of light. That is, the laws of ordinary geometry no longer hold true.

However, we know other cases in which the ordinary or Euclidean geometry is not applicable. Thus suppose that on the surface of a sphere we describe a series of concentric circles. Since the surface is curved, we are not surprised at finding that the circumference of any one of these circles is less than 3.14159 [238] … times the distance across the circle as measured on the surface of the sphere. What this means, therefore, is that we cannot use Euclidean geometry to describe measurements on the surface of a sphere, and every schoolboy knows this from comparing Mercator's projection of the earth's surface with the actual representation on a globe.

When we come to think of it, the reason we realize all this is because our sense of three dimensions enables us to differentiate flat surfaces from those that are curved. Let us, however, imagine a two-dimensional being living on the surface of a large sphere. So long as his measurements are confined to relatively small areas he will find it possible to describe all his measurements in terms of Euclidean geometry. As, however, his area of operation increases he will begin to observe greater and greater discrepancies. Being unfamiliar with the existence of such a three-dimensional object as a sphere, and therefore not realizing that he is on the surface of one, our intelligent two-dimensional being will conclude that the disturbance in his geometry is due to the action of a force, and by means of plausible assumptions on the "law" of this force he will reconcile his observations with the laws of plane geometry.

Now since an acceleration in a gravitational field is identical with that due to centrifugal force produced by rotation, we concluded that the geometry in a gravitational field must also be non-Euclidean. That is, space in the neighborhood of matter is distorted or curved. The curvature of space bears the [239] same relation to three dimensions that the curvature of a spherical surface bears to two dimensions, and that is why we do not perceive it, any more than the intelligent two-dimensional being would be aware of the distortion of his space (or surface). Furthermore, like this being, we have assumed the existence of a gravitational force to account for discrepancies in our geometrical measurements.

The identification in this manner of gravitational effects with geometrical curvature of space enables Einstein to derive a general law for the path of any particle in a gravitational field, with respect both to space and to time. Furthermore, the law expresses this motion in terms which are independent of the relative motion and position of the observer, and satisfies the condition that the fundamental laws of physics be equally valid for all observers. The solution of the problem involved the use of a new kind of higher calculus, elaborated by two Italian mathematicians, Ricci and Levi-Civita. The result is a law of motion which is extremely general in its validity.

For low velocities it approximates to Newton's solution, and in the absence of a gravitational field it leads to the same conclusions as the special theory of relativity. There are three deductions from this law which have aroused a great deal of interest, and the confirmation of two of these by actual observation must be regarded as striking proof of Einstein's theory. [240]

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XIII

AN INTRODUCTION TO RELATIVITY

A Treatment in Which the Mathematical Connections of Einstein's Work are Brought Out More Strongly and More Successfully Than Usual in a Popular Explanation

BY HAROLD T. DAVIS, UNIVERSITY OF WISCONSIN, Madison, Wis.

One of the first questions which appears in philosophy is this: What is the great reality that underlies space and time and the phenomena of the physical universe? Kant, the philosopher, dismissed it as a subjective problem, affirming that space and time are "a priori" concepts beyond which we can say no more.

Then the world came upon some startling facts. In 1905 a paper appeared by Professor Albert Einstein which asserted that the explanation of certain remarkable discoveries in physics gave us a new conception of this strange four-dimensional manifold in which we live. Thus, the great difference between the space and time of philosophy and the new knowledge is the objective reality of the latter. It rests upon an amazing sequence of physical facts, and the generalized theory, which appeared several years later, founded as it is upon the abstruse differential calculus of Riemann, Christoffel, Ricci and Levi-Civita, emerges from its maze of formulas with the prediction of real [241] phenomena to be sought for the in the world of facts.

We shall, therefore, approach the subject from this objective point of view. Let us go to the realm of actual physical events and see how the ideas of relativity gradually unfolded themselves from the first crude wonderings of science to the stately researches that first discovered the great ocean of ether and then penetrated in such a marvelous manner into some of its most mysterious properties.

The Electromagnetic Theory of Light

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Suppose that we go out on a summer night and look into the dark depths of the sky. A thousand bright specks are flashing there, blue, red, yellow against the dark velvet of space. And as we look we must all be impressed by the fact that such remote objects as the stars can be known to us at all. How is it that light, that curious thing which falls upon the optic nerve and transmits its pictures to the brain, can ever reach us through the black regions of interstellar space? That is the question which has for its answer the electromagnetic theory of light.

The first theory to be advanced was Newton's "corpuscular" theory which supposed that the stars are sending off into space little pellets of matter so infinitesimally small that they can move at the rate of 186,000 miles a second without injuring even so delicate a thing as the eye when they strike against it.

But in 1801, when Thomas Young made the very important discovery of interference, this had to give way to the wave theory, first proposed by Huyghens in the 17th century. The first great deduction from this, of course, [242] was the "luminiferous ether," because a wave without some medium for its propagation was quite unthinkable. Certain peculiar properties of the ether were at once evident, since we deduce that it must fill all space and at the same time be so extremely tenuous that it will not retard to any noticeable degree the motion through it of material bodies like the planets.

But how light was propagated through the ether still remained a perplexing problem and various theories were proposed, most prominent among them being the "elastic solid" theory which tried to ascribe to ether the properties of an elastic body. This theory, however, laid itself open to serious objection on the ground that no longitudinal waves had been detected in the ether, so that it began to appear that further insight into the nature of light had to be sought for in another direction.

This was soon forthcoming for in 1864 a new theory was proposed by James Clerk Maxwell which seemed to solve all of the difficulties. Maxwell had been working with the facts derived from a study of electrical and magnetic phenomena and had shown that electromagnetic disturbances were propagated through the ether at a velocity identical with that of light. This, of course, might have been merely a strange coincidence, but Maxwell went further and demonstrated the interesting fact that an oscillating electric charge should give rise to a wave that would behave in a manner identical with all of the known properties of a light wave. One particularly impressive assertion was that these waves, consisting of an alternating electric field accompanied by an alternating [243] magnetic field at right angles to it, and hence called electromagnetic waves, would advance in a direction perpendicular to the alternating fields. This satisfied the first essential property of light rays, *i.e.*, that they must be transverse waves, and the ease with which it explained all of the fundamental phenomena of optics and predicted a most striking interrelation between the electrical and optical properties of material bodies, gave it at once a prominent place among the various theories.

The electromagnetic theory, however, had to wait until 1888 for verification when Heinrich Hertz, in a series of brilliant experiments, succeeded in producing electromagnetic waves in the laboratory and in showing that they possessed all of the properties predicted by Maxwell. These waves moved with the velocity of light: they could be reflected, refracted, and polarized: they exhibited the phenomenon of interference and, in short, could not be distinguished from light waves except for their difference in wave length.

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The Michelson-Morley Experiment

With the final establishment of the electromagnetic theory of light as a fact of physics, we have at last endowed the ether with an actual substantiality. The "empty void" is no longer empty, but a great ocean of ether through which the planets and the suns turn without ever being aware that it is there.

In 1881 A. A. Michelson undertook an experiment, originally suggested by Maxwell, to determine the relative motion of our earth to the ether ocean and six years later he repeated it with the assistance of E. W. Morley. The experiment is now known as the Michelson-Morley experiment and since it is the great physical fact upon which the theory of relativity rests, it will be well for us to examine it in detail. [244]

Since we can scarcely think that our earth is privileged in the universe and that it is at rest with respect to this great ether ocean that fills space, we propose to discover how fast we are actually moving. But the startling fact is that the experiment devised for this purpose failed to detect any motion whatever of the earth relative to the ether.[1]

The explanation of this very curious fact was given by both H. A. Lorentz and G. F. Fitzgerald in what is now widely known under the name of the "contraction hypothesis." It is nothing more nor less than this:

Every solid body undergoes a slight change in dimensions, of the order of (v^2/c^2), *when it moves with a velocity v through the ether.*

The reason why the experiment failed, then, was not because the earth was not moving through the ether, but because the instruments with which the experiment was being conducted had shrunk just enough to negative the effect that was being looked for.[2] [245]

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The Lorentz Transformation

We can not at this point forebear introducing a little mathematics to further emphasize the theory and the very logical nature of this contraction hypothesis.

Let us suppose that we were on a world that was absolutely motionless with respect to the ether and were looking at a ray of light. The magnetic and electric fields which form the ray can be described by means of four mathematical expressions which have come to bear the name of "Maxwell's field equations." Now suppose that we ask ourselves the question: How must these equations be changed so that they will apply to a ray of light which is being observed by people on a world that is moving with a velocity v through the ether?

The answer is immediate. From the Michelson-Morley experiment we know that we can not tell how fast or how slowly we are moving with respect to the ether. This means that no matter what world we may be upon, the form of the Maxwell field equations will always be the same, even though the second set of axes (or frame of reference) may be moving with high velocity with respect to the first.

Starting from this hypothesis (called in technical language the covariance of the equations with respect to a transformation of coordinates), Lorentz found that the transformation which leaves the field equations unchanged in form was the following:

$$x' = k(x - vt), y' = y, z' = z, t' = k(t - vx/c)$$

where *k* is as on page 92. [246]

And what, now, can be deduced from these very simple looking equations? In the first place we see that the space of x', y', z', t' is not our ordinary concept of space at all, but a space in which time is all tangled up with length. To put it more concretely, we may deduce from them the interesting fact that whenever an aviator moves with respect to our earth, his shape changes, and if he were to compare his watch with one on the earth, he would find that his time had changed also. A sphere would flatten into an ellipse, a meter stick would shorten up, a watch would slow down and all because, as H. Minkowski has shown us from these very equations, we are really living in a physical world quite different from the world of Euclid's geometry in which we are accustomed to think we live.

A variety of objections has very naturally been made to this rather radical hypothesis in an attempt to discredit the entire theory, but it is easily seen that any result obtained through the field equations must necessarily be in conformity with the theory of contraction, since this theory is only the physical interpretation of that transformation which leaves the field equations unaltered. Indeed, it is even possible to postulate the Lorentz transformation together with the assumption that each element of charge is a center of uniformly diverging tubes of strain and derive the Maxwell field equations from this, which shows from another point of view the truly fundamental nature of the transformation.

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The First Theory of Relativity

The whole question of the ether had arrived at this very interesting point when Professor Einstein in 1905 stated the theory of relativity. He had noticed that the equations of dynamics as formulated by Newton did not admit the Lorentz transformation, but only the simple Galilean transformation: [247]

$$x' = x - vt, y' = y, z' = z; t' = t\,.$$

Here, indeed, was a curious situation. Two physical principles, that of dynamics and that of electromagnetism, were coexistent and yet each one admitted a different transformation when the system of reference was transferred to axes moving with constant velocity with respect to the ether.

Now the electromagnetic equations and their transformation had been shown to be in accord with experimental fact, whereas it had long been felt that Newton's equations were only a first approximation to the truth. For example, the elliptic orbit of a planet had been observed by Leverrier to exhibit a disquieting tendency to rotate in the direction of motion. This precession, which in the case of Mercury was as large as 43″ per century, could not be accounted for in any way by the ordinary Newtonian laws and was, consequently, a very celebrated case of discordance in gravitational astronomy.

With this example clearly before him, Einstein took the great step and said that the laws of dynamics and all other physical laws had to be remade so that they, also, admit the Lorentz transformation. That is to say,

The laws of physical phenomena, or rather the mathematical expressions for these laws, are covariant (unchanged in form) when we apply the Lorentz transformation to them. [248]

The deductions from the Michelson-Morley experiment now seem to have reached their ultimate conclusion.

One discordant fact in this new theory remained, however. That same precession of the perihelion of Mercury which had first lead Einstein to his theory remained unsettled. When the new approximations were applied to the formula of orbital motion, a precession was, indeed, obtained, but the computed value fell considerably below that of the observed 43″ per century.

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The Inclusion of Gravitation

With the idea of investigating the problem from the very bottom, Einstein now undertook a broader and more daring point of view. In the first place he said that there is no apparent reason in the great scheme of world events why any one special system of coordinates should be fundamental to the description of phenomena, just as in the special theory a ray of light would appear the same whether viewed from a fixed system or a system moving with constant velocity with respect to the ether. This makes the very broad assumption that no matter what system of coordinates we may use, the mathematical expressions for the laws of nature must be the same. In Einstein's own words, then, the first principle of this more general theory of relativity must be the following:

"*The general laws of nature are expressed through equations which hold for all systems of coordinates, that is, they are covariant with respect to arbitrary substitutions.*"[3] [249]

But this was not enough to include gravitation so Einstein next formulated what he was pleased to call his "equivalence hypothesis." This is best illustrated by an example. Suppose that we are mounting in an elevator and wish to investigate the world of events from our moving platform. We mount more and more rapidly, that is with constant acceleration, and we appear to be in a strong gravitational field due to our own inertia. Suppose, on the other hand, that the elevator descends with an acceleration equal to that of gravity. We would now feel certain that we were in empty space because our own relative acceleration has entirely destroyed that of the earth's gravitational field and all objects placed upon scales in an elevator would apparently be without weight.

Applying this idea, then, Einstein decided to do away with gravitation entirely by referring all events in a gravitational field to a new set of axes which should move with constant acceleration with respect to the first. In other words we are going to deal with a system moving with uniform acceleration with respect to the ether, just as we considered a system moving with uniform velocity in the special theory.

The next step in the construction of this complicated theory is to reduce these two hypotheses to the language of mathematics and this was accomplished by [250]
Einstein with the help of M. Grossmann by means of the theory of tensors.

On account of the very great intricacy of the details, we must content ourselves with the mere statement that this really involved the generalization of the famous expressions known as Laplace's and Poisson's equations, on the explicit assumption that these two equations would still describe the gravitational field when we are content to use a first approximation to the truth. The set of ten differential equations which Einstein got as a result of his generalization he called his field equations of gravitation.[4] [251]

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1 Dr. Davis went rather fully into the algebra of the Michelson-Morley experiment. But Dr. Russell has covered the same ground in a form somewhat more advantageous from the typographical viewpoint, and the point is not one which it is profitable to discuss twice; so we eliminate this part of Dr. Davis' text.—Editor. ↑

2 This statement is objectionable, as explained in Chapter IV.—Editor. ↑

3 A. Einstein: Die Grundlage der allgemeinen Relativitätstheorie. Ann. d. Physik. 4, vol. 49, page 776. ↑

4 At this point we have again used the blue pencil on Dr. Davis' text, his discussion of the three observational tests of the General Theory adding nothing to Dr. Pickering's.—The Editor. ↑

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XIV

NEW CONCEPTS FOR OLD

What the World Looks Like After Einstein Has Had His Way With It

BY JOHN G. McHARDY, COMMANDER R.N., LONDON

“The new-created world, which fame in heaven
Long had foretold, a fabric wonderful,
Of absolute perfection.”

Einstein’s Theory of Relativity has led to determining a key law of nature—the law of gravitation—which is also the basic law of mechanics. Thus it embraces a whole realm of physics, and promises, through the researches of Professor Weyl, to embrace another realm—electro-dynamics. Its limitations are not yet reached, for Einstein has already postulated therefrom a theory of a finite, yet unbounded, universe. This essay, however, is mainly concerned with mechanics, and electrical forces are not considered.

To have synthesised Newton’s two great principles—his law of motion and law of gravitation—interpreting in the process the empirical law of equality of gravitational and inertial mass, is alone an immense achievement; but Einstein’s researches have opened up a new world to the physicist and philosopher which is of greater importance. He has given us a vision of the [252] immaterial world, a geometrical or mathematical vision, which is more satisfying than the “ether” conceptions hitherto presented. The fabric of his vision is not baseless. It is this fabric we shall consider, touching on certain aspects of the Einstein theory in the endeavor to present an image in miniature of his edifice of thought and to show the firmness of its foundations. That they are well and truly laid was demonstrated by the verification, from observations made during the solar eclipse in 1919, of Einstein’s prediction of the displacement of a wave of light in a gravitational field, showing light to have the property of weight.

The physical world is shown by Einstein to be a world of “relations.” Underlying it there is an absolute world of which physical phenomena are the manifestation. “Give me matter and motion,” says Descartes, “and I will construct the world.” “Give me a world in which there are ordered relations,” says the Relativist, “and I will show you the behavior of matter therein” (mechanics). We first view this underlying world as an abstraction, abstracting energy (“bound” as in matter and electrons, “free” as in light), and its attribute force. This abstraction we will call the “World-Frame.” Later, we will study the underlying world in connection with energy, and will call this absolute world the “World-Fabric.” The connection between the geometrical character of the World-Frame and the geometrical characters of the World-Fabric is the key to the law of gravitation. [253]

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The World-Frame

This is our conception of a world, if such were possible, entirely free from the influence of energy. We may conceive of it as an amorphous immaterial something containing “point-events” (a point-event being an instant of time at a point in space—a conception, not a definition). These point-events have a fourfold order and definite relation in this Frame, *i.e.* they can be specified by four variables or coordinates in reference to some base called a reference system, with respect to which they are forward or backward, right or left, above or below, *sooner or later*. This shows the World-Frame to be four-dimensional. Thus an aggregate of point-events (or an "event," which implies limited extension in space and limited duration in time)[1] would have what we familiarly describe as length, breadth, height and time. To express these metrical properties most simply we must choose a four-dimensional reference system having a particular form—rectilinear axes (Cartesian coordinates), and a particular motion—uniform and rectilinear, *i.e.* unaccelerated, and non-rotating with respect to the path of a light ray. We call this an inertial system because Newton's Law of Inertia holds for such a system alone. This system [254] indicates how observers partition the World-Frame into space and time. It restricts observers to uniform rectilinear motion, and observations to bodies and light-pulses in such motion. Thus gravitational and other forces are discounted, and we obtain World-Frame conditions notwithstanding the fact that observers are in the presence of energy.

Now the separation between point-events which have a definite relation to each other must be *absolute*. The separation between two points in a plane is defined by the unique distance between them (the straight line joining them). Between point-events the analogue of this unique distance, which we call the "separation-interval" (to indicate its time-like and space-like nature), is also unique. Its unique and absolute character give it great importance as thereby it is the same for all observers regardless of their reference system.

If, in place of the rather cumbersome expression $X - x$ to indicate the difference between the *x*-coordinates of two points, we employ the more compact expression *dx*; if for the benefit of readers who have a little algebra but no analysis we state explicitly that this expression is a single symbol for a single quantity, and has nothing to do with any product of two quantities *d* and *x*; and if we extend this notation to all our coordinates: then it is clear from previous essays that the distance *S* between two points in a plane referred to a rectilinear system *OX*, *OY*, is given by the simple equation $S^2 = (dx)^2 + (dy)^2$. Einstein and Minkowski show that the value for the [255] separation interval Ω, the analogue of *S*, referred to an inertial system is given by the equation

$$\Omega^2 = (dx)^2 + (dy)^2 + (dz)^2 - (dt)^2 \,,$$

which is seen to be a modified extension to four dimensions of the equation for *S*. We must measure *t* in the same units as *x*, *y*, *z*. By taking the constant velocity of light (300,000 kilometres per second) as unit velocity, we can measure in length or time indiscriminately.[2]

We will analyse briefly this equation as it epitomizes the Special Theory of Relativity. If the World-Frame had been Euclidean the equation would have been

$$\Omega^2 = (dx)^2 + (dy)^2 + (dz)^2 + (dt)^2$$

but this would not satisfy the "transformation equations" which resulted from the Special Theory. These transformation equations arose directly from a reconciliation between two observed facts; (a) the observed *agreement* of *all* natural phenomena with the "Restricted Principle of Relativity"—a principle which shows that absolute rectilinear motion cannot be established—(as regards mechanics this was recognized by Newton; the Michelson-Morley and other experiments showed this principle also applied to optical and electro- [256] dynamical phenomena); and (b) the observed *disagreement* of optical and electro-dynamical phenomena (notably the constancy of light velocity) with the laws of dynamics as given by classical mechanics, *e.g.*, in regard to the compounding of relative velocities. Einstein effected this reconciliation by detecting a flaw in classical mechanics. He showed that by regarding space and time measurements as relative to the observer—not absolute as Newton defined them—there was nothing incompatible between the Principle of Relativity and the laws of dynamics so modified. Newton's definitions were founded on conception. Einstein's recognition of the relativity of space and time is based on observation.

Equation (1) shows that the geometry of the World-Frame referred to an inertial system is semi-Euclidean (hyperbolic), and that space and time measurements are relative to the observer's inertial reference system. The equation shows that the World-Frame has a certain geometrical character which we distinguish as four-dimensional "flatness." It is everywhere alike (homaloidal). Its flat character is shown by the straight line nature of the separation-interval and of the system to which it is most simply referred.

Thus we have found two absolute features in the World-Frame—(1) Its geometrical character—"*flatness*"; (2) The *separation-interval*—which can be expressed in terms of measurable variables called space and time partitions, this partitioning being dependent on the observer's motion.

We are now in a position to explore the World-Fabric. Already we see that, [257] studied under inertial conditions (free of force), it agrees with the World-Frame.

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The World-Fabric

The General Theory of relativity is largely concerned with the investigation of the World-Fabric. Consider the World-Frame to be disturbed. We may regard this disturbance, which manifests itself in physical phenomena, as energy, or more correctly "action."

When energy is thwarted in its natural flow, force is manifested, with which are associated non-uniform motions such as accelerations and rotations. This disturbed World-Frame we distinguish as the World-Fabric. It is found to have various non-Euclidean characters differing from the simple "flat" character of the World-Frame according to the degree of disturbance (action) in the region. Disturbance gives the fabric a geometrical character of "curvature"; the more considerable the disturbance, the greater the curvature. Thus an empty region (not containing energy, but under its influence) has less curvature than a region in which free energy abounds.

Our problem, after showing the relativity of force (especially gravitational force), is to determine the law underlying the fabric's geometrical character; to ascertain how the degree of curvature is related to the energy influencing a region, and how the curvature of one region is linked by differential equations to that of neighboring regions. Such a law will be seen to be the law of gravitation. [258]

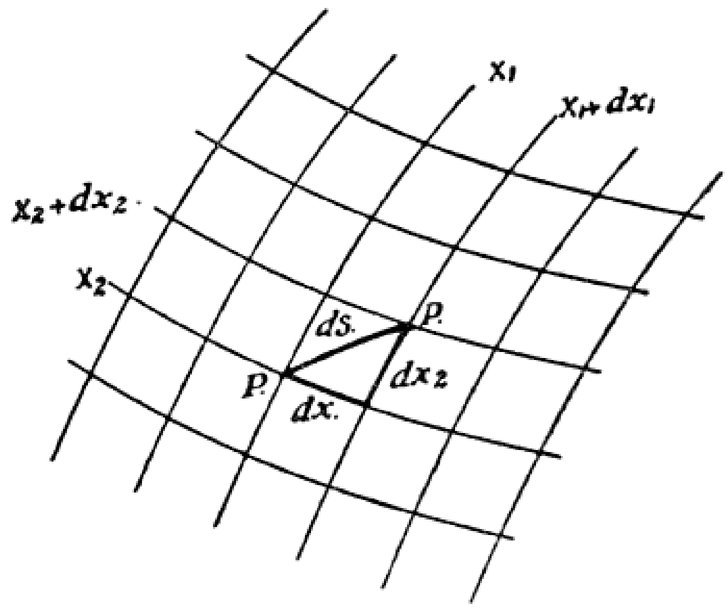
We study the World-Fabric by considering tracks on which material particles and light-pulses progress; we find such tracks regulated and defined by the Fabric's curvature, and not, as hitherto supposed, by attractive force inherent in matter. As a track is measurable by summing the separation-intervals between near-by point-events on it, all observers will agree which is the unique track between two distant point-events. Einstein postulates that freely progressing bodies will follow unique tracks, which are therefore called natural tracks (geodesics).

If material bodies are prevented from following natural tracks by contact with matter or other causes, the phenomenon of gravitational force is manifested relative to them. Whenever the natural flow of energy is interrupted force is born. For example, when the piston interrupts the flow of steam, or golf ball flow of club, force results—the interruption is mutual, and the force relative to both. Likewise when the earth interrupts the natural track of a particle (or observer) gravitational force is manifested relative to both.

So long as a body moves freely no force is appreciated by it. A falling aviator (neglecting air resistance) will not appreciate any gravitational force. He follows a natural track, thereby freeing himself from the force experienced in contact with matter. He acquires an accelerating motion with respect to an inertial system. By acquiring a particular accelerating motion an observer can annul *any* force experienced in any small region where the field of force can be considered constant.

Thus Einstein, interpreting the equality of gravitational and inertial mass, [259] showed that the same quality manifests itself according to circumstances as "weight" or as inertia, and that all force is purely relative and may be treated as one phenomenon (an interruption in energy flow). This "Principle of Equivalence" shows that small portions of the World-Fabric, observed from a freely moving particle (free of force), could be treated as small portions of the World-Frame.[3]

If such observations were practicable, we could determine the Fabric curvature by referring point-event measurements to equation (1). We cannot observe from unique tracks but we can observe them from our restrained situation. Their importance is now apparent, because, by tracing them over a region, we are tracing something absolute in the Fabric—its geometrical character. We study this curvature by exploring separation-intervals on the tracks of freely moving bodies, relating these separation-intervals to actual measurements in terms of space and time components depending on the observer's reference system. The law of curvature must be the law of gravitation. To illustrate the lines on which Einstein proceeded to survey the World-Fabric from the earth we will consider a similar but more simple problem—the survey of the sea-surface curvature from an airship. We study this curvature by exploring small distances on the tracks of ships (which we must suppose can only move uniformly on unique tracks—arcs of great circles), relating such distances to [260] actual measurements in terms of length and breadth components depending on the observer's reference system. This two-dimensional surface problem can be extended to the four-dimensional Fabric one.



We consider the surface to be covered by two arbitrarily drawn intersecting series of curves: curves in one series not intersecting each other, *vide* figure. This Gaussian system of coordinates is appropriate only when the smaller the surface considered, the more nearly it approximates to Euclidean conditions. It admits of defining any point on the surface by two numbers indicating the curves intersecting at that point. *P* is defined by x_1, x_2. P_1 (very near *P*) is defined by $x_1 + dx_1, x_2 + dx_2$. The equation for the minute distance *s* between two adjacent points in such a system is given by the general formula

$$s^2 = g_{11} dx_1{}^2 + g_{12} dx_1 dx_2 + g_{22} dx_2{}^2 \,.$$

The *g*'s may be constants or functions of x_1, x_2. *Their value is dependent on the observer's reference system and on the geometrical character of the surface observed.* The curves being arbitrary, the formula is appropriate for any reference system, or even if the observer does not know exactly what his reference system is. (The Fabric observer does not know what his space and time partitioning actually is because he is in a gravitational field). It is the *g*'s which disclose the geometry of an observer's partitions, and their values also contain a reflection of the character of the region observed. [261]

We find *s* by direct exploration with a moving ship (Ω is found by direct exploration with a freely moving particle); dx_1, dx_2 are the observed length and breadth measurement differences which we have to relate to *s*. By making sufficient observations in a small area and referring them to the general formula we can find the values of the *g*'s for the observer's particular reference system. Different values for *g*'s will be found if the observer changes his reference system, but there is a limitation to the values so obtainable owing to the part played by the surface itself, *which is diffidently expressing its intrinsic geometrical character in the g's in each observation*.

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Einstein's Results

Thus we approach the absolute character of the surface through the relative nature of the observer's reference system. There is a relationship common to all values of the g's that belong to the same curvature. This relationship is [262] expressed by a differential equation. It is this equation of curvature that the airship's observer must find. Einstein's problem was similar, but he was concerned with four dimensions, which entailed a general formula with ten g's, and he had to find a set of differential equations of the second order to determine the law of Fabric curvature. He divided the Fabric into regions: I. World-Frame—beyond influence of energy. II. Empty region—free of energy, but under its influence. III. Region containing free energy only. Each region has a characteristic curvature. By means of an absolute differential calculus—a wonderful mathematical scaffolding erected by Riemann, Christoffel and others—involving the theory of tensors, he succeeded in finding such a set of equations. He kept the following points in view: (1) The equations must not only give the character of region II, but must satisfy the special case of region I; (2) They must be independent of any partitioning system, because the General Theory of Relativity demands that a law of nature be in a form appropriate for all observers whatever their position and motion; (3) They must be concerned with energy which is conserved, not mass which the Special Theory showed dependent on velocity. This set of differential equations which shows how the curvature of the Fabric at any point links to the curvature at neighboring points is the law of gravitation, a law which has been severely tested by the practical observation of the solar eclipse already referred to. At a first approximation these equations degenerate into Newton's Law. At a second approximation they account for the motion of the perihelion [263] of Mercury, which had hitherto baffled astronomers. All the laws of mechanics are deducible from this law of World-Fabric curvature, *i.e.* conservation of energy (which includes conservation of mass since we re-define mass as energy) and conservation of momentum (re-defined by a relativist). It must be noted that this law and the General Theory show that the velocity of light is not absolutely constant, but, like everything else, a light-pulse is affected by the Fabric curvature in a gravitational field. In conclusion we will contrast some conspicuous differences in the old world view of classical mechanics and the new view presented by Einstein.

1. A three-dimensional ether medium with variously conceived properties which communicated the supposed inherent attractive force in matter in some unexplained way, and transmitted electromagnetic waves, has been replaced by a four-dimensional external World-Fabric, the geometrical character of which controls the motion of matter (energy) and accounts for all mechanical laws.

2. After separating the observer's subjective share in definitions from nature's share in the things defined, space, time, and force, hitherto regarded as absolute, have been shown to be purely relative and dependent on the observer's track. Mass has also proved to be relative to velocity unless re-defined as energy. As classical mechanics bases all definitions on space, time, and mass units, the relativity of such defined quantities is now apparent.

3. Newton's laws of motion, his law of gravitation, and the laws of [264] conservation, hitherto regarded as unrelated, are now synthesised in a basic

law of mechanics.

Einstein has not disturbed the electric theory of matter, and both the old and new physics have in common the "Principle of Least Action." We obtain a glimpse of this principle in the unique tracks pursued by freely moving bodies, which may be regarded as tracks of least effort, force only being manifested as an expression of the Fabric's resentment when bodies depart from these natural tracks. Einstein has approached nearer to the truth in regard to the laws underlying nature, and, as always, this means a simplification. His theory, which entails a readjustment of such fundamental conceptions as space and time, opens up fresh fields to scientific investigation and to philosophic thought. It reveals a bridge uniting the domains of physics and philosophy, and it heralds a new era in the history of science. [265]

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1 Commander McHardy uses the term "event" in a sense somewhat different from that seen in a majority of the essays. He reserves for the four-dimensional element—the instant of time at a point in space—the name "point-event"; and the term "event" he applies to a collection of these forming, together, an observable whole. An actual physical happening, like a railroad wreck or a laboratory experiment, it will be realized is of the latter sort, occupying an appreciable region of space rather than a single point, and an appreciable interval of time rather than a single second. To the element, the "point-event" of Commander McHardy's essay, this bears the same relation that the geometer's solid bears to his point. This comment is in no sense to be taken as criticism of Commander McHardy's terminology, which rather appeals to us; we make it merely to guard against confusion in the reader's mind.—Editor. ↑

2 This paragraph is the result of an editorial revision of the author's text, designed to retain the substance of his presentation, while tying up what he has to say more definitely with the preceding essays, and eliminating the distinction between finite and infinitesimal intervals, which we believe to be out of place in an essay of this character. We will not apologize to our mathematical readers for having used finite and differential notation in the same equation, in violation of mathematical convention.—Editor. ↑

3 Although gravitational force in a small region can be imitated or annulled by accelerating motion, there remains the disturbing influence of gravitational matter already referred to and expressed in the fabric curvature. It is this that defines how unique tracks run, or rather, how bodies progress.—Author. ↑

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XV

THE NEW WORLD

A Universe in Which Geometry Takes the Place of Physics, and Curvature That of Force

BY GEORGE FREDERICK HEMENS, M.C., B.SC., LONDON

It is familiar knowledge that the line, the surface and ordinary Euclidean space are to be regarded as spaces of one, two and three dimensions respectively and readers of this journal are aware that a hypothetical space of four dimensions has been closely investigated. The most convenient space to study is the surface or two-space, since we can regard it as embedded in a three-space. If a surface is curved it is generally impossible to draw a straight line on it, for as we see clearly, the "straightest" line is changing its direction at every point. To describe this property accurately it is necessary to ascribe to each point a magnitude which expresses what happens to the direction of a short line in the region when displaced a short distance parallel to itself. This is called *the direction-defining magnitude*. Different sets of values of this magnitude relate to surfaces of different curvatures.

A second fundamental property has recently been pointed out. There is inherent in every part of a space a measure of length peculiar to that particular region and which in general varies from region to region. To describe this [266]
variation accurately it is necessary to ascribe to each point another magnitude called *the length-defining magnitude*, which expresses the change from each point to the next of the unit of length. These two magnitudes define the surface completely.

Similarly, a space of any number of dimensions is defined completely by a similar pair of magnitudes. A space is the "field" of such a magnitude-pair and the nature of these magnitudes defines the dimensions of the space. The four-space usually described is the Euclidean member of an infinity of four-spaces.

When we look into a mirror we see a space differing from ordinary space in that right and left are interchanged and this is described mathematically by saying that if we locate points as usual by specifying three distances X_1, X_2, X_3 of the point from three mutually perpendicular planes, then a point X_1, X_2, X_3, in actual space corresponds with a point X_1, X_2, $-X_3$ in the mirrored space: in other words the mirrored space is derived from the real space by multiplying the X_3 coordinates by -1. If we were to multiply by $\sqrt{-1}$ instead of -1 we should derive a different space; in this case, however, we have no mirror to show us what it looks like. Such a space is said to have one negative dimension and it has the peculiar property that in the figure derived from the right triangle of ordinary space the square of the "hypotenuse" equals the difference and not the sum of the squares of the other two sides, so that the length of a line may sometimes have to be represented by the square-root of a [267]
negative number, a "complex" number.

In considering what at first sight may appear to be fantastic statements made by this theory, it must be borne in mind that all our knowledge of the external universe comes through our sense-impressions, and our most confident statements about external things are really of the nature of inferences from these sense-impressions and, being inferences, liable to be wrong. So that if the theory says that a stone lying on the ground is not a simple three-dimensional object, and that its substance is not the same as its substance a moment before, the matter is one for due consideration and not immediate disbelief.

The idea that the universe extends in time as well as in space is not new, and fiction-writers have familiarized us with wonderful machines in which travellers journey in time and are present at various stages of the world's history. This conception of the universe, to which the name "space-time" is usually applied, is adopted by the new theory and assigned the status of a physical reality.

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The World Geometry

The fundamental creed of the new theory is that the space-time universe constitutes a true four-dimensional space of one negative dimension, this dimension being *time*. The variations from point to point of the direction-defining and length-defining magnitudes generate the geometrical properties of curvature, etc., and these are cognised by the human mind as physical phenomena: our sense-impressions are nothing more nor less than perceptions of the geometry of a fourspace. So instead of inferring from our sense- [268] impressions the existence of matter, motion and the like as we are accustomed to do, we should with equal justice infer the existence of a geometrical fourspace. Thus it becomes necessary to prepare a dictionary in which the familiar things of our world are identified with those geometrical properties of the four-space which really constitute them, and in so doing parts of our geometrical knowledge assume the guise of new physical knowledge.

Through the fourspace our consciousness travels, cognising a changing three-dimensional section of it as it goes and thus giving rise to *time*. It becomes aware that the fourspace is pleated or folded along lines all running roughly in the same direction, and possibly because this is the easiest direction to follow, it travels along the lines. The direction of this motion is the negative dimension. Thus consciousness is always aware of the nearly constant forms of the cross-sections of the pleats along which it travels. These unvarying forms constitute matter: matter is the form of a section through a uniform pleat of the fourspace—a three-dimensional aspect of a four-dimensional curvature; so that in strict accuracy we should say that a stone is the shape or form of a changing section of a four-dimensional object, the complete object being a long fold in the fourspace. The physical interpretation of this conservation of form of the cross-section is that matter is conserved. It is thus seen that the conscious mind, by following these pleats, has so determined time that the law of the conservation of matter must hold. The mathematical treatment of the subject [269] makes it clear that practically all other physical laws similarly follow as a direct result of this choice of time. The type of order prevailing in the physical universe, the laws of gravitation, heat, motion and the rest are *not* directly imposed by some external power, but are apparently chosen by mind itself.

In the neighborhood of these pleats the fourspace is still curved, but to a smaller degree. This we cognise as energy or as a field of force. Thus energy is seen to be the same kind of thing as matter and would therefore be expected to have weight. This was experimentally demonstrated in 1919 when light was in effect actually weighed. Conversely, matter consists of energy; and it is calculated that one liter of water contains sufficient energy to develop a million horsepower for about four years. It is now believed that the sun's energy is derived from the disintegration of the matter of which it is made.

The method of establishing these identifications will be clear from the following: We already knew that matter is made up of electrons and that radiant energy is electromagnetic and before the advent of this theory it was regarded as certain that practically all observed physical phenomena except gravitation were manifestations of the electromagnetic field. The new theory has confirmed this belief. It is found that the gravitational and electromagnetic conditions of the universe are completely defined if to each point of space-time a gravitational and an electric potential are ascribed. These are magnitudes of the same nature as the direction-defining and length-defining magnitudes which must necessarily be associated with every point of space-time if it is a [270] true "space," and they are therefore identified with these. By performing ordinary mathematical operations on these magnitudes statements of fact clothed in mathematical form are obtained, which are to be interpreted on the one hand as physical laws and on the other as geometrical properties of the fourspace. Nearly all our physical laws are derivable mathematically in this way, so that an extensive identification is effected which has been fruitful of results.

It has been mentioned that a slight curvature is sometimes cognised as force and as this identification appeared originally as a postulate its history is interesting.

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The Genesis of the Theory

An experiment by Michelson and Morley (1887), on which the whole theory is based, made it appear that if a man measures the velocity at which light passes him he will get the same result whether he is stationary, rushing to meet the light, or moving in the same direction as the light. The solution was provided by Einstein in 1905. He suggested that since we know the results of these determinations ought not to agree, something must have happened to the clocks and measuring-rods used in measuring the velocity so that the standards of length and time were not the same in the three cases, the alterations being exactly such as to make the velocity of light constant. This solution is universally accepted as true and is the fundamental postulate. Thus the length of a stick and the rate at which time passes will change as the velocity of the [271] person observing these things changes. If a man measured the length of an aeroplane going past him at 161,000 miles per second it would measure only half the length observed when stationary. If the aeroplane were going with the velocity of light, its length would vanish though its breadth and height would be unaltered. Similarly, if of two twin brothers one were continually moving with reference to the other their ages would gradually diverge, for time would go at different rates for the two. If one moved with the velocity of light, time would stand still for him while for the other it would go on as usual. To get actually younger it would be necessary to move quicker than light which is believed to be impossible. The velocity of light is assumed to be the greatest velocity occurring in nature.

Evidently then if the distance in space and the interval in time separating two given events, such as the firing of a gun and the bursting of the shell, are measured by two observers in uniform relative motion, their estimates will not agree. Consider now the simple problem of measuring the distance between

two points on an ordinary drawing-board. If we draw two perpendicular axes, we can define this distance by specifying the lengths of the projections on the two axes of the line joining the points. If we choose two different axes the projections will not be the same but will define the same length. Similarly, in a Euclidean four-space the distance between two points will be defined by the projections on the four axes, but if these axes be rotated slightly, the projections will be different, but will define the same length. Now, returning to [272]
the two observers just mentioned, it was noticed by Minkowski in 1908 that if the space measurements between the two events are split into the usual three components, and if the time measurements are multiplied by $\sqrt{-1}$, the difference between the two sets of measurements is exactly the same as would have occurred had these two events been points in a Euclidean fourspace, and two different observations made of their distance apart using two sets of axes inclined to each other. The velocity of light is made equal to 1 in this calculation by a suitable choice of units. This discovery threw a vivid light on the problem of space-time, showing that it is probably a true four-space of one negative dimension, a simple derivative of the much-discussed and now familiar Euclidean four-space.

Although this discovery gave a tremendous impetus to the progress of the theory, it is probable that it holds a deeper significance not yet revealed. It is probably a statement of the “stuff” of which the four-space is made, and perhaps also of how it is made; but the problem remains unsolved.

It thus becomes plain that our two observers are merely looking at the same thing from different viewpoints. Each has just as much right as the other to regard himself as being at rest in ordinary space (this is the postulate of the relativity of uniform motion) and to regard his time direction as a straight line in the four-space. The difference is merely that the two time axes are inclined to each other. If, however, one were moving with an acceleration with reference to the other his path in the four-space will appear curved to the other, [273]
though he himself, since he regards it as his time axis, will still assume it to be straight. If there is a body moving in what one observer sees to be a straight line, the other will, of course, in general see it as curved, and following the usual custom, since this body, without apparent reason, deviates from the straight path, will say there must be some force acting on it. Thus the curvature of his time axis, due to his accelerated motion, makes it appear that there is round him a field of force, which causes freely moving bodies to deviate from the straight path. Now if space-time is itself inherently curved it is not generally possible for any line in it to be straight any more than it is possible for any line on the surface of a sphere to be straight. Hence, all axes must be curved, and all observers, whatever their states of motion, must experience fields of force which are of the same nature as those due to motion only. The extra force experienced when a lift begins to rise is an example of force due to pure motion: gravitation is the similar force due to an inherent curvature of the four-space, and it was the postulate that these forces were similar that made possible Einstein’s solution of the general problem of gravitation.

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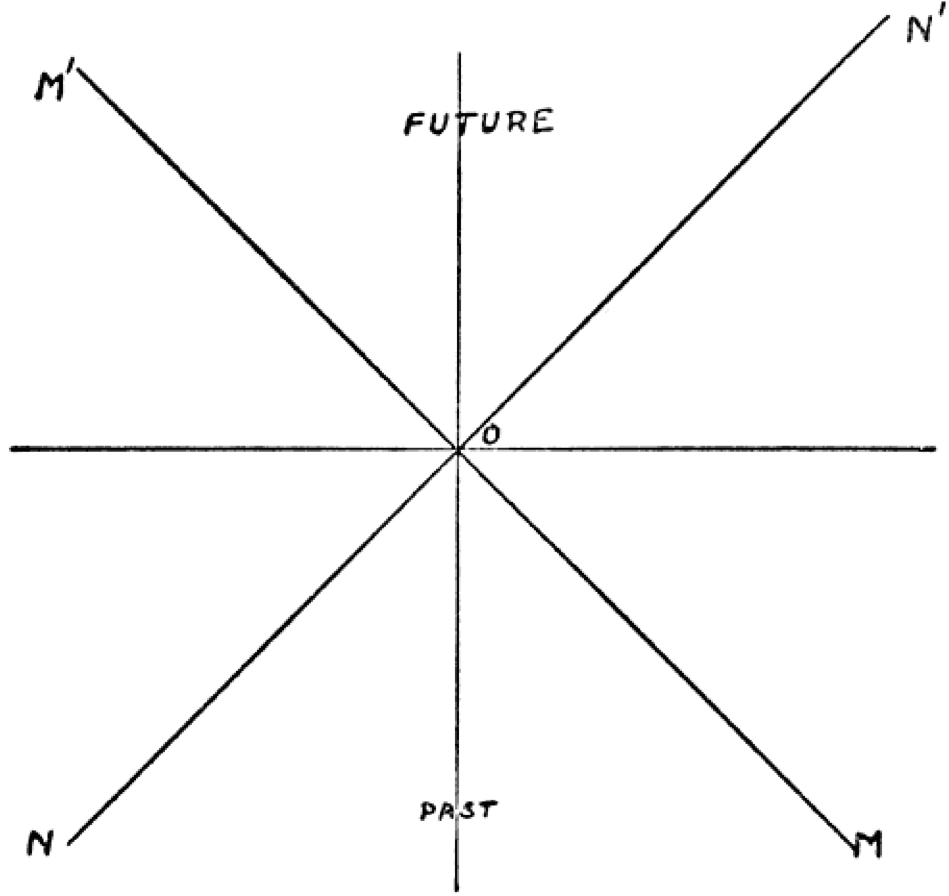
The Time Diagram

The correlation of time with its geometrical analogue is of absorbing interest. Representing velocity by the common method of plotting a curve showing positions at various times and marking distances horizontally and times vertically, the velocity of light being 1, *MM′* and *NN′* will both represent this [274] velocity. Since this is assumed to be the greatest velocity occurring in nature, all other possible velocities



are represented by lines falling within the upper and lower V's. Now this diagram correctly represents two dimensions of Minkowski's Euclidean four-space so, transmuting to real but flat four-space by multiplying times by $\sqrt{-1}$, it is seen that there is a region outside which no effect can be propagated from *O* since that would involve the existence of a velocity greater than that of light. This region represents the future of *O*. Similarly, *O* can only be affected by events within the region derived from the downward-opening V, which therefore represents the past of *O*. The region between the two represents events which may be either simultaneous with *O* or not, according to the [275] velocity of the observer at *O*. Thus in this theory an event dictated by free-will, could affect points in its "future" region, but not in any other, which agrees with experience and shows that the theory is *not essentially "determinist."* If "free-will" is really free, the future is not yet determined, and the fourspace must be in some way formed by the will as time progresses.

The trains of thought inspired by Einstein's postulates have already carried us to a pinnacle of knowledge unprecedented in the history of man. On every hand, as we look out upon the universe from our new and lofty standpoint, unexpected and enthralling vistas open up before us, and we find ourselves confronting nature with an insight such as no man has ever before dared aspire to.

It is completely unthinkable that this theory can ever be swept aside. Apart from experimental verifications which, in point of fact, lend it the strongest support, no one could work through the theory without feeling that here, in truth, the inner workings of the universe were laid bare before him. The harmony with nature is far too complete for any doubt to arise of its truth. [276]

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XVI

THE QUEST OF THE ABSOLUTE

Modern Developments in Theoretical Physics, and the Climax Supplied by Einstein

BY DR. FRANCIS D. MURNAGHAN,
JOHNS HOPKINS UNIVERSITY
BALTIMORE

We shall discuss the more important aspects of the theory popularly known as the "Einstein Theory of Gravitation" and shall try to show clearly that this theory is a natural outcome of ideas long held by physicists in general. These ideas are:

(a) The impossibility of "action at a distance;" in other words we find an instinctive repugnance to admit that one body can affect another, remote from it, instantaneously and without the existence of an intervening medium.

(b) The independence of natural, *i.e.*, physical, laws of their mathematical mode of expression. Thus, when an equation is written down as the expression of a physical law it must be satisfied, no matter what units we choose in order to measure the quantities occurring in the equation. As our physics teacher used to say "the expression of the law must have in every term *the same dimensions*." More than this the choice of the quantities used to express the law—if there be a choice open—must have no effect on its correctness. As we were told—"all physical laws are capable of expression as relations between vectors or else as relations between magnitudes of the same dimensions." We shall hope to make this clearer in its proper place in the essay, as its obvious generalization is Einstein's cardinal principle of relativity. [277]

The measurements which an experimental physicist makes are always the expression of a coincidence of two points in space at the same time. If we ask such an experimenter what he means by a *point in space* he tells us that, for him, the term has no meaning until he has a material body with reference to which he can locate the point by measurements; in general it requires *three* measurements and he expresses this by saying that space has *three* dimensions.

He measures his distance, as a rule, parallel to three mutually perpendicular lines fixed in the material body—a Cartesian reference-frame so-called. So that a "point in space" is equivalent to a given material reference-frame and three numbers or *coordinates*. If, for any reason, we prefer to use a new material reference-frame the coordinates or measurements will change and, if we know the relative positions of the two material reference-frames, there is a definite relation between the two sets of three coordinates which is termed a transformation of coordinates. But which particular material reference-frame shall we use? The first choice would, we think, be that attached to the earth. But, even yet, we are in doubt as there are numberless Cartesian frameworks [278] attached to the earth (as to any material body) and it is here that our idea (b) begins to function. We say it must be immaterial which of these Cartesian frames we use. In each frame a *vector* has three components and when we change from one frame to another the components change in such a way that if two vectors have their three components equal in one framework they will be equal in any other attached to the *same* material system. So our idea (b), which says that our physical equations must be vector equations, is equivalent to saying that the choice of the framework attached to any given material body can have no effect on the mode of expression of a natural law.

Shall we carry over our idea (b) to answer the next question: "To which material body shall we attach our framework?" To this question Newton gave one answer and Einstein another. We shall first consider Newton's position and then we may hope to see clearly where the new theory diverges from the *classical* or *Newtonian* mechanics. Newton's answer was that there is a particular material frame with reference to which the laws of mechanics have a remarkably simple form commonly known as "Newton's laws of motion" and so it is preferable to use this framework which is called an *absolute* frame.

What is the essential peculiarity of an absolute frame? Newton was essentially an empiricist of Bacon's school and he observed the following facts. Let us suppose we have a framework of reference attached to the *earth*. Then a small particle of matter under the gravitational influence of surrounding bodies, [279] including the earth, takes on a certain acceleration A_1. Now suppose the surrounding bodies removed (since we cannot remove the earth we shall have to view the experiment as an abstraction), and another set introduced; the particle, being again at its original position, will begin to move with an acceleration A_2. If both sets of surrounding bodies are present simultaneously the particle begins to move with an acceleration which is *approximately but not quite* the sum of A_1 and A_2. Newton postulated there there is a certain absolute reference frame in which the approximation would be an equality; and so the acceleration, relative to the material frame, furnishes a convenient measure of the effect of the surrounding bodies—which effect we call their *gravitational force*. Notice that if the effect of the surrounding bodies is small the acceleration is small and so we obtain as a limiting case, *Newton's law of inertia* which says that a body subject to no forces has no acceleration; a law which, as Poincaré justly observed, can never be subjected to experimental justification. The natural questions then arise: which is the absolute and privileged reference-frame and how must the simple laws be modified when we use a frame more convenient for us—one attached to the earth let us say? The absolute frame is one attached to the fixed stars; and to the absolute or real force defined as above, we must add certain terms, usually called centrifugal forces. These are referred to as *fictitious* forces because, as it is explained, they are due to the motion of the reference-frame with respect to the absolute frame and in no way depend on the distribution of the surrounding bodies. [280] Gravitational force and centrifugal forces have in common the remarkable property that they depend in no way on the material of the attracted body nor on its chemical state; they act on all matter and are in this way different from other forces met with in nature, such as magnetic or electric forces. Further Newton found that he could predict the facts of observation accurately on the hypothesis that two small particles of matter attracted each other, in the direction of the line joining them, with a force varying inversely as the square of the distance between them. This law is an "action at a distance" law and so is opposed to the idea (a).

We have tacitly supposed that the space in which we make our measurements is that made familiar to us by the study of Euclid's elements. The characteristic property of this space is that stated by the theorem of Pythagoras that the distance between two points is found by extracting the square root of the sum of the squares of the differences of the Cartesian coordinates of the two points. Mathematicians have long recognized the possibility of other types of space and Einstein has followed their lead. He *abandons the empiricist method* and when asked what he means by a point in space replies that to him a point in space is equivalent to four numbers *how obtained it is unnecessary to know a priori*; in certain special cases they may be the three Cartesian coordinates of the experimenter (measured with reference to a definite material framework) together with the time. Accordingly he says his *space is of four dimensions*. Between any two "points" we may insert a sequence of sets of four numbers, [281] varying continuously from the first set to the second, thus forming what we call a curve joining the two points. Now we define the "length" of this curve in a manner which involves all the points on it and stipulate that this length has a physical reality, *i.e.*, according to our idea (b) its value is independent of the particular choice of coordinates we make in describing the space. Among all the joining curves there will be one with the property of having the smallest length; this is called a *geodesic* and corresponds to the straight line in Euclidean space. We must now, for lack of an *a priori* description of the actual significance of our coordinates, extend the idea of vector so that we may speak of the components of a vector no matter what our coordinates may actually signify. In this way are introduced what are known as *tensors*; if two tensors are equal, *i.e.*, have all their components equal, in any one set of coordinates they are equal in any other and the fundamental demand of the new physics is that *all physical equations which are not merely the expression of equality of magnitudes must state the equality of tensors*. In this way no one system of coordinates is privileged above any other and the laws of physics are expressed in a form independent of the actual coordinates chosen; they are written, as we may say, in an absolute form.

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The Gravitational Hypothesis

Einstein flatly denies Newton's hypothesis that there is an absolute system (and, indeed, many others before him had found it difficult to admit that so [282] insignificant a part of the universe as our fixed star system should have such a privileged position as that accorded to it in the Newtonian Mechanics). In any system, he says, we have no reason to distinguish between the so-called *real* gravitational force and the so-called *fictitious* centrifugal forces—if we wish so to express it gravitational force is fictitious force.[1] A particle moving in the neighborhood of material bodies moves according to a law of inertia—a physical law expressible, therefore, in a manner quite independent of the choice of coordinates. The law of inertia is *that a particle left to itself moves along the geodesics or shortest lines in the space*. If the particle is remote from other bodies the space has the Euclidean character and we have Newton's law of inertia; otherwise the particle is in a space of a non-Euclidean character (the space being always the four-dimensional space) and the path of the particle is along a geodesic in that space. Einstein, in order to make the theory more concrete, makes a certain stipulation as to the nature of the gravitational space which stipulation is expressed, as are all physical laws, by means of a tensor equation—and this is sometimes called his law of gravitation.

Perhaps it will be well, in exemplification, to explain why light rays, which pass close to the sun, should be bent according to the new theory. It is *assumed* that light rays travel along certain geodesics known as minimal geodesics. The sun has an intense gravitational field near it—or, as we now say, the departure [283] of the four-dimensional space from the Euclidean is very marked for points near the sun—but for points so remote as the earth this departure is so small as to be negligible. Hence the form of the geodesics near the sun is different from that near the earth. *If the space surrounding the sun were Euclidean the actual paths of the light rays* would appear different from geodesics or straight-lines. Hence Einstein speaks of the curvature of the light rays due to the gravitational field of the sun; but we must not be misled by a phrase. Light always travels along geodesics (or straight lines—the only definition we have of a *straight line* is that it is a geodesic); but, owing to the "distortion" of the space they traverse, due to the sun, these geodesics reach us with a direction different from that they would have if they did not pass through the markedly non-Euclidean space near the sun.

The consideration of the fundamental four-dimensional space as being non-Euclidean where matter is present gives a possibility of an answer to the world old question: Is space finite or infinite? Is time eternal or finite? The fascinating possibility arises that the space may be like the two-dimensional surface of a sphere which to a limited experience seems infinite in extent and flat or Euclidean in character. A new Columbus now asks us to consider other possibilities in which we should have a finite universe—finite not only as to space measurement but as to time (for the space may be such that all of the four coordinates of its points are bounded in magnitude). However, although Einstein speaks of the possibility of a finite universe, we do not, personally, think his argument convincing. Points on a sphere may be located by the [284] Cartesian coordinates of their stereographic projections on the equatorial plane and these coordinates, which might well be those actually measured, are not bounded.

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The Special Relativity Theory

In our account of the Einstein theory we have not followed its historical order of development for two reasons. Firstly, the earlier Special Relativity Theory properly belongs to a school of thought diametrically opposed to that furnishing the "General Theory of Relativity" and, secondly, the latter cannot be obtained from the former by the process of generalization as commonly understood. Einstein, when proposing the earlier theory, adopted the position of the empiricist so that to him the phrase, *a point in space*, had no meaning without a material framework of reference in which to measure space distances. When he came to investigate what is meant by time and when he asked the question "what is meant by the statement that two remote events are simultaneous?" it became evident that some mode of communication between the two places is necessary; the mode adopted was that by means of light-signals. The fundamental hypothesis was then made that the velocity of such signals is independent of the velocity of their source (some hypothesis is necessary if we wish to compare the time associated with events, when one material reference-system is used, and the corresponding time when another in motion relative to the first is adopted). It develops that time and space [285] measurements are inextricably interwoven; there is no such thing as *the* length of a body or *the* duration of an event but rather these are *relative* to the reference-system.[2] Minkowski introduced the idea of the space of events—of four dimensions—but this space was supposed Euclidean like the three-dimensional space of his predecessors. To Einstein belongs the credit of taking from this representation a purely formal mathematical character and of insisting that the "real" space—whose distances have a physical significance—is the four-dimensional space. But we cannot insist too strongly on the fact that in the gravitational space of the general theory there is no postulate of the constancy of velocity of a light-signal and accordingly no method of assigning a time to events corresponding to that adopted in the special theory. In this latter theory attention was confined to material systems moving with uniform velocity with respect to each other and it developed that the velocity of light was the ultimate velocity faster than which no system could move—a result surprising and *a priori* rather repugnant. It is merely a consequence of our mode of comparing times of events; if some other method—thought transference, let us say—were possible the velocity of this would be the "limiting velocity."

In conclusion we should remark that the postulated equivalence of "gravitational" and "centrifugal" forces demands that anything possessed of inertia will be acted upon by a gravitational field and this leads to a possible [286] identification of matter and *energy*. Further our guiding idea (a) will prompt us to say, following the example of Faraday in his electrical researches, that the geodesics of a gravitational space have a physical existence as distinct from a mere mathematical one. The four-dimensional space we may call the *ether*, and so restore this bearer of physical forces to the position it lost when, as a three-dimensional idea in the Special Relativity Theory, it had to bear an identical relation to a multitude of relatively moving material systems. The reason for our seemingly paradoxical title for an essay on Relativity will be clear when it is remembered that in the new theory we consider those space-time properties which are *absolute* or devoid of reference to any particular material reference-frame. Nevertheless, although the general characteristics of the theory are thus described, without reference to experiment, when the theory is to be tested it is necessary to state what the four coordinates discussed actually are—how they are determined by measurement. It is our opinion that much remains to be done to place this portion of the subject on a satisfactory basis. For example, in the derivation of the nature of the gravitational space, surrounding a single attracting body, most of the accounts use Cartesian coordinates as *if the space were Euclidean* and step from these to polar coordinates by the formulæ familiar in Euclidean geometry. But these details are, perhaps, like matters of elegance, if we shall be allowed to give Einstein's quotation from Boltzmann, to be left to the "tailor and the cobbler." [287]

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1 Not all gravitational fields may be transformed away by a proper choice of coordinates. If this were so, the space, whose nature is independent of any choice of coordinates, would always be Euclidean.—AUTHOR. ↑

2 Thus when it it said that a body contracts or that a clock runs slow when it is put in motion no actual physical change is implied. The judgment of different observers—one at rest with respect to the body and one not—are different.—AUTHOR. ↑

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XVII

THE PHYSICAL SIDE OF RELATIVITY

THE IMMEDIATE CONTACTS BETWEEN EINSTEIN'S THEORIES AND CURRENT PHYSICS AND ASTRONOMY

BY PROFESSOR WILLIAM H. PICKERING
HARVARD COLLEGE OBSERVATORY,
MANDEVILLE, JAMAICA

The Theory of Relativity will be treated first from the physical side, leaving the three astronomical tests to which it has been put to be discussed later. There is one astronomical fact however that must be mentioned in this connection, and this is the discovery of the aberration of light by Bradley in 1726. It is found that every star in the heavens apparently describes a small annual ellipse, whose major axis is 41″ in length. This Bradley showed to be due to a combination of the velocity of the earth in its orbit, and the velocity of light; and it is so explained in all the elementary text-books on astronomy. It implies a stationary ether through which the earth is moving. The importance of this statement will appear presently.

The subject is usually illustrated by supposing a man to go out in a rainstorm carrying a vertical tube. If the rain is falling vertically, and the man stands still, the sides of the tube will not be wet, save by an occasional drop, but if the tube is moved, it must then be inclined forward in order to keep it dry. The angle of inclination, which corresponds to aberration, will depend on the relative velocity of the tube, corresponding to the earth, and the rain drops which correspond to the waves of light. [288]

If three lines are dropped upon a point in space, each line being perpendicular to the plane containing the other two, we have what is known as a system of coordinates. Einstein’s original theory of relativity, which he now designates as the “special theory,” depends on two principles. The first is that “Every law of nature which holds good with respect to a coordinate system *K* must also hold good for any other system *K′*, provided that *K* and *K′* are in uniform movement of translation.” The second principle is that “Light in a vacuum has a definite and constant velocity, independent of the velocity of its source.”

These two sentences may be considered as authoritative, being quoted in Einstein’s own words.[1] The first of these principles need not greatly surprise us. The second is not well expressed, because it is ambiguous. He does not say how the first “velocity” is measured, whether relatively to the ether or relatively to the observer. In fact this is the very gist of the whole matter, as we shall presently see. In the case of sound the velocity is constant with regard to the medium, the air, in the case of light it is supposed to be constant with regard to the observer. It reaches him with a constant velocity, no matter how [289]
he moves.

In order to understand this statement clearly let us consider the appended tabular diagram. On a calm day imagine a source of sound at *S* in line *a*. This may be either a gun or a bell. Imagine an observer 1,100 feet distant, located at *O*. The velocity of sound in air is 1,100 feet per second. This velocity we will take as unity, as indicated in the third column, and the velocity with which the sound reaches the observer is also 1, as shown in the fourth. It will reach him in a unit interval of 1 second, as shown in the fifth. If the bell is struck, it will give its normal pitch or frequency, which we will also call unity, in the sixth column.

Now imagine case *b* where the observer is on a train advancing toward *S*. When he is 1,100 feet distant, the gun is fired, but as he is advancing toward it, he hears it at *O* in rather less than a second, as shown in the fifth column. The velocity of the sound with regard to him is rather more than unity, as shown in the fourth column. If the bell is sounded, the pitch, that is the frequency, is raised, because he receives more sound waves per second than before.

In case *c* the observer is stationary, but the source of sound is receding. At a distance of 1,100 feet the gun is fired, and the observer hears it after an interval of just one second, as in case *a*. The velocities with regard to the observer and through the medium are also unity. If the bell is struck the pitch is lowered, since he receives fewer sound waves per second, the reverse of case
b. [290]

	Source		Velocity in Medium	Velocity to Observer	Interval	Frequency	Observer	
				Air				
a		S	1	1	1	1		O
b		S	1	1 +	1 -	1 +	O	
c	S		1	1	1	1 -		O
d	S		1	1 +	1 -	1	O	
				Ether				
A		S	1	1	1	1		O
B		S	1 -	1	1	1 +	O	
C	S		1	1	1	1 -		O

	Velocity				
Source	in Medium	to Observer	Interval	Frequency	Observer
D S	1 -	1	1	1	O

In case *d* imagine the source and the observer 1,100 feet apart, and advancing on the same train. When the gun is fired, the velocity of the sound waves will be greater with regard to the observer, and he will hear the sound in less than a second, as in case *b*. When the bell is struck it will have the normal pitch, the same as in case *a*.

We find therefore that for sound the velocity with regard to the medium is always unity, while the velocity with regard to the observer, and the interval elapsed, depend only on the motion of the observer himself, and are independent of the motion of the source. The frequency of the vibrations, on the other hand, depends only on the relative motion of the observer and the source, but is independent of their common motion in any direction. Further, it makes no difference whether the source and the observer are moving on a train, or whether they are stationary, and a uniform wind is blowing past them.

In the case of light waves we shall find a very different state of affairs, although the rules for frequency are the same as they are for sound. In case *A* we have the normal conditions, where both the source and observers are [291] stationary. In case *B* we have a representation of the Michelson-Morley experiment as supplemented by that of Majorana,[2] where the source is stationary and the observer advances. Unlike the case of sound, the interval elapsed, as shown by the experiment, is now the same as in case *A*, and since the distance to the observer is less, the velocity of light with respect to the ether must also be less than unity. Since the observer is advancing against the light, this will permit the velocity of light with regard to the observer to remain unity, in conformity with the second principle of relativity. Compare with case *b* for sound. As Jeans expresses it, "The velocity of light in all directions is the same, whatever the motion of the observer."[3] That is to say it appears to be the same to him, however he moves.

Case *C* represents Einstein's statement, as confirmed by Majorana's experiment. It does not differ from case *c* for sound. Case *D* is more complex, but accepting the statement above that the velocity is constant with regard to the observer, we see that the velocity through the medium must be less, and that the interval elapsed will be constant, as in case *B*. Could we use the brighter stars and planets as sources of light, several of these cases could be further tested.

This brings us at once to statements that contradict our common sense. For instance, Jeans says "no matter what the velocity of the observer is, the light surface, as observed by that observer, is invariably a sphere having that observer as center."[3] That is to say the light surface, or wave front, is a contracting, not an expanding, sphere. This, if confirmed, would go a long way [292] toward making our universe a subjective rather than an objective phenomenon. Again imagine a flash of light, such as an explosion, to occur when an observer is in a given position. It makes no difference how the observer may move while the light is approaching him, whether several miles forward or backward, the light will reach him in exactly the same time, as is shown by Michelson's experiment. Or if two observers are at the same spot when the explosion occurs, and one moves forward, and the other backward, they will both see the explosion at exactly the same instant.

This sounds ridiculous, but not only is it what Jeans says, but it is the logical interpretation of Einstein's second principle, if Einstein means by velocity, velocity with regard to the observer. If he means velocity with regard to the medium, then the case is exactly the same as that of sound in air, and Michelson's experiment as well as the Maxwell-Lorentz theory of light are contradicted. This theory is now universally accepted, and Michelson's experiment has been carefully repeated by other observers, and fully confirmed. This is the very heart of the relativity question.

If we state the matter objectively it comes to this. The velocity of light with regard to the ether is a variable quantity, depending merely on where the observer chooses to go. As Eddington well says, "these relations to the ether have no effect on the phenomena and can be disregarded—a step which appears to divest the ether of the last remnants of substantiality."[4] [293]

The only way of avoiding this apparent absurdity seems to be to consider that the ether moves with the earth. Michelson's result would then be fully explained. Of course this can only be true for a few miles above the earth's surface. Beyond that the ether must either be stationary or move with the sun. The velocity of light with regard to the ether would then be a constant, just as the velocity of sound is constant with regard to the air. This would contradict Einstein's second principle as it is generally understood. The trouble with this suggestion is that it fails to account for aberration, which, as already explained, appears to require that the earth should be moving through the ether. To meet this emergency would involve some modification of the undulatory theory of light, which apparently would not be impossible, but has not yet been made.

In 1915 Einstein brought out an extension of his first principle. This he calls the "general theory of relativity." It states that in our choice of coordinate systems we "should not be limited in any way so far as their state of motion is concerned."[1] This leads to the three astronomical consequences mentioned later in this paper, two of which have been more or less confirmed, and the third practically contradicted as far as quantitative measures are concerned.[5]

As is well known the kinetic energy of a moving body may be expressed as $e = 1/2mv^2$, but if the body is charged electrically, the fraction becomes $1/2(m + m')v^2$, where m' is a quantity dependent on the square of the electrical charge. That is to say, we have the normal mass of the body, and also what we may call its electrical mass. If when in this condition a portion of the mass is electrical, the question at once occurs to us, why may not the whole mass be electrical, in other words, a form of energy? Although this has not been satisfactorily proved hitherto, yet such is the general belief among physicists. As Einstein puts it "inert mass is nothing else than latent energy."[1] The same idea is sometimes expressed as "the mass of ordinary matter is due to the electromagnetic energy of its ultimate particles, and electromagnetic energy wherever found must possess mass, *i.e.*, *inertia*."[6] If that is so, since a ray of light on the undulatory theory is a form of electromagnetic energy, it too must possess mass. Since all mass with which we are familiar is subject to the attraction of gravitation, it seemed likely that a ray of light would be bent out of its course in passing near the sun, and this as we have seen was proved to be true at the recent solar eclipse.

That portion of the mass of a body due to its electrical charge can be readily shown experimentally to vary with the velocity of the body. Einstein has shown the same to be true of the normal mass, as is illustrated in the advance of the perihelion of the orbit of Mercury. He has also pointed out that gravitation, inertia and centrifugal force are all closely related, and obey similar laws. Thus if we rise from the earth with accelerated velocity, we apparently increase our weight. Again if the velocity of rotation of the earth on its axis should be increased, our weight would be diminished. These facts are [295] suggestive when we come to consider the ultimate cause of gravitation.

Another fact which must be rather startling to the older school of scientists is that momentum is no longer simply *mv*, mass times velocity, but that the velocity of light *c*, comes into the question, and the formula for momentum now assumes the form of

$$\frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

For ordinary velocities this correction is extremely small, but it has been shown to be necessary, both theoretically and experimentally, when dealing with the high velocities with which we are now familiar.

The theory of relativity is so widespread in its application that several other theories have become more or less intimately combined with it, for which Einstein is in no way responsible. One of these is known as the Fitzgerald-Lorentz theory, that all bodies are subject to a contraction in the direction of their motions through space. This was first suggested in order to explain the Michelson-Morley experiment, but has proved inadequate to do so, particularly when the observer is receding from the source. This contraction is expressed by the same factor used in the denominator of the revised expression for momentum, given above. Again the quantity *c* is so enormous, that even for large bodies at planetary velocities the contraction amounts to very little. Thus the earth moving at a speed of eighteen miles per second in its orbit, is flattened only 1/200,000,000, or 2.5 inches. On the other hand for high [296] velocities of many thousand miles per second, such as we have become familiar with in the case of the radioactive substances, the flattening is a very considerable fraction of the diameter of the moving body, one-half or more, and in the case of the corpuscles of light, if that theory were adopted, this flattening becomes equal to the diameter, and their thickness is reduced to zero.

When we view Einstein's theories from the astronomical standpoint, the earliest fact bearing on relativity that we need consider was the discovery of aberration, by Bradley, in 1726, as seen above. In 1872 Airy observed the star γ Draconis through a telescope filled with water. Since the velocity of light is less in water than in air, we should naturally expect to find the aberration appreciably increased. It was found, on the other hand, however, to be unaffected.

In 1887 the results of the famous Michelson-Morley experiment were published.[7] In this experiment the velocity of light was measured in various directions with regard to the motion of the earth in its orbit. If the ether were stationary, and the earth moving through it, different velocities should be obtained in different directions. Such was not the case however, and the experiment indicated that the ether moved with the earth. It thus flatly contradicted the conclusions founded on aberration.

Einstein's Special Theory of Relativity, of 1905, as we have seen, resolves this contradiction. But as we shall presently see, it is the General Theory, of 1915, that leads to astronomical applications of broad scope. It indicates, for instance, that there is no essential difference between gravitation and inertia. [297] This idea may be crudely illustrated by our feelings of increased weight when an elevator starts rapidly upwards. A man while falling freely in space ceases to feel the pull of gravitation.

But we must not as yet conceive of the theory of relativity as a universally accepted and unquestioned truth of science. Eddington is its leading English exponent, and he is supported by such men as Jeans, Larmor, and Jeffreys. On the other hand, the theory has been severely criticised by Lodge, Fowler, Silberstein, and Sampson. Few American scientists have expressed any opinions in print on the subject, and the recent eclipse observations, to which we shall refer later, are to be repeated with more suitable instruments for verification in 1922, in the hope of obtaining more accurate and accordant results.[8]

An appurtenance of the Einstein theories which bears much the same relation to them as does the Lorentz-Fitzgerald contraction, mentioned above, is the idea, first clearly stated by Minkowski, that time is a kind of space—a fourth dimension. This the reader will doubtless find to be the most difficult portion of the theory to picture in his own mind. It is entirely unsupported by experiment or observation, necessarily so, and is based wholly on mathematical and philosophical conceptions. Our distinction between space and time seems to be that the direction in which we progress without effort is time; the other directions, in which we have to make an exertion to move ourselves, or in which we are carried, are space. How many dimensions empty space may have, we really have no means of knowing, because we can neither [298] see nor feel it. Matter we know has three, length, breadth, and thickness, also that it lies remote from us in three corresponding directions. These facts may have given us the erroneous impression that space too has only three dimensions. Now it is claimed that time is a fourth, and that there are also others.

In order to illustrate this, Eddington asks us to imagine a movie film taken of a man or of any moving object. Let the separate pictures be cut apart and piled on one another. This would form a sort of pictorial history of the individual for a brief interval in his life, in the form of a cube. If we attempt to pick it up, it falls apart, thus clearly showing the difference between time and space. But suppose it now all glued together in one solid cube, so that it is no easier to cut a section in one direction than in another. That is Minkowski's idea of space and time, and further, that the direction in which we should cut it depends merely on the velocity with which we are moving through space. I should cut it parallel to the films, but a man on a rapidly moving star, in order to separate it into space and time, would cut it in an inclined direction. That is a thing which may be true, but it is one which we believe no mortal man can clearly picture to himself.

On the other hand Turner has recently made a very interesting point,[9] namely, that the fourth dimension as actually treated by the mathematicians is not time itself, but time multiplied by a constant—the velocity of light.[10] Without affecting the astronomical proofs of relativity at all, this simplifies our conceptions enormously. In ordinary everyday life time and space cannot be [299] identical, any more than a yard can be identical with a quart. On what is known to physicists as the centimeter-gram-second system, distance is represented by l, mass by m, and time by t. Velocity is then distance divided by time, l/t, or as we say in English units, so many feet per second, and the fourth dimension may be expressed as time multiplied by velocity, $t \times l/t = l$. That is to say, it is simply distance, just like the other three dimensions. To say that time is the fourth dimension from this point of view, appears to us just as ridiculous as it would be to attempt to measure the velocity of a train in quarts. It is quite correct, however, although unusual, to speak of a given train as moving at a speed of 10 quarts per square inch per second, $l^3/l^2t = l/t$. This would be equivalent to a velocity of 33 miles per hour.

If I wish to give a complete dimensional description of myself in my four dimensions, I must give my length, my breadth, and my thickness, ever since I came into being, and also the course I have traversed through space since that time. This latter distance will be expressed in terms of a unit whose length is 186,000 miles, the distance traversed by light in one second. The distance which I travel through space annually is enormous, and very complex as to direction. It involves not merely my own motions as I cross the room, or take a train or steamer, but also those due to the rotation of the earth on its axis, its revolution round the sun, and the motion of the latter through the heavens. In general I travel, or in other words increase my length in the fourth dimension, by over 4,000 units a year. The fourth dimension accordingly, if this view is [300] accepted, is simply a distance like the other three, and perfectly easy to understand.

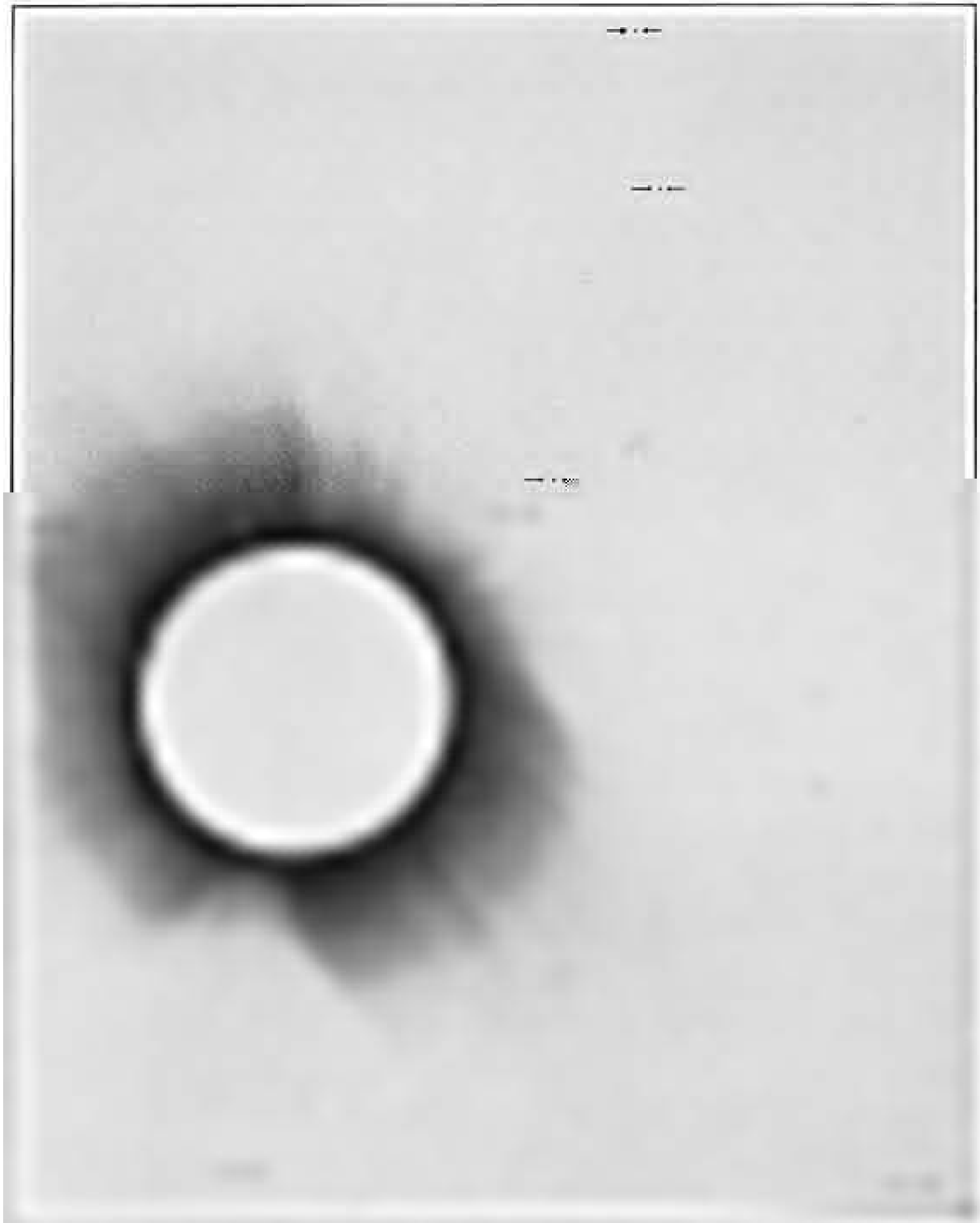
We now come to the three actual tests by which the theory has been tried. The planets as is well known revolve about the sun in ellipses, with the sun in one of the foci. That is to say, the sun is not in the center, but a little on one side of it. The end of the ellipse where the planet comes nearest to the sun is called the perihelion, and here the planet is moving most rapidly. The other end is called the aphelion, and here the motion is slowest. According to Newton's theory of gravitation, if a spherical sun possesses a single planet or companion, its orbit will be permanently fixed in space unless perturbed by some other body. If a second planet exist, it will cause the perihelion of the first slowly to advance. According to Einstein the mass of a planet depends in part on its velocity. It will therefore be less at aphelion where it is moving slowly than at perihelion where it is moving rapidly, consequently in addition to the Newtonian attraction we have another one which increases as we approach the sun. The effect of this will be to cause the perihelion of the orbit to advance, whether there is a second planet or not.

Among the larger planets Mercury has the most eccentric orbit, and it also moves most rapidly, so that it is particularly well adapted to test the relativity theory. The observed advance of its perihelion is 574″ per century, instead of the theoretical figure 532″, due to the other planets—a difference of 42″.[11] This has long been a puzzling discrepancy between observation and the law of gravitation. Prior to Einstein, attempts were made to eliminate it by assuming a [301] certain oblateness of the solar disk. If the equatorial diameter exceeded the polar by only 0″.5 the whole advance would be accounted for, but not only has this ellipticity failed of detection, but if it existed, it should produce a very noticeable and inadmissible change in the inclination of Mercury's orbit, amounting to about 3″ per century, as has been demonstrated by both Herzer and Newcomb.[12]

Einstein from computations alone, without introducing any new constants or hypotheses whatever, showed, if the theory of relativity be accepted, that the sun should produce an acceleration of 43″ per century, thus entirely accounting for the observed discrepancy, far within the limits of accuracy of the observations. The only other planet whose orbit has a large eccentricity, and that is suitable for investigation, is the planet Mars. Here the discrepancy between observation and theory is very slight, only 4″, and a portion of that may be due to the attraction of the asteroids. This deviation is so slight that it may well be due entirely to accidental errors of observation, but however that may be, Einstein's theory reduces it to 2″.7.

This all seems very satisfactory and complete, but the trouble with it is that the coincidence for Mercury is rather too good. It is based on the assumption that the sun is a perfect sphere, and that the density of its surface is uniform from the equator to the poles. This would doubtless be true if the sun did not revolve on its axis. In point of fact it does revolve, in a period in general of about 26 days. Consequently an object on its equator must experience a certain amount [302] of centrifugal force. Therefore if its surface were of uniform density the shape of the sun would be an oblate spheroid.

It can be readily shown that the theoretical excess of the equatorial over the polar diameter, due to the centrifugal force, should amount to only 0″.04, an amount which could hardly be detected by observation, and might readily be concealed by a slight excess of equatorial over polar density. Any reasonable excess of density at the center would diminish this result but slightly. The molecular weight of the central material[13] is probably about 2. This computed equatorial excess is one-twelfth of the amount necessary to cause the observed advance, and should therefore cause an advance of the perihelion of about 3″.5 per century, reducing the difference between the observed advance and that caused by gravitation to 38″.5. According to Einstein the advance due to relativity should be, as we saw, 43″, a discrepancy of 4″.5 per century, or 10 per cent. Jeffreys has remarked that any discrepancy such as 10″ "would be fatal to a theory such as Einstein's, which contains no arbitrary constituent capable of adjustment to suit empirical facts."[14] It must be pointed out here however, that so far as known, this small correction to the motion of Mercury's perihelion has not previously been suggested, so that there has been no opportunity hitherto for its criticism by others.



One of the eclipse photographs

The arrows pointing to the star-images have been inserted by hand; and the star-images themselves have had to be materially strengthened in order to make them show in the engraving at all.

Photograph submitted by Dr. Alexander McAdie, Harvard University, by courtesy of the Royal Observatory, Greenwich.

It was due largely to the success with Mercury that it was decided to put the relativity theory to another test. According to the Newtonian theory, as stated [303] by Newton himself, corpuscles as well as planets have mass, and must therefore be attracted by the sun. According to Einstein, owing to their high velocity, this attraction must be twice as great as it would be according to the theory of gravitation. If the ray of light proceeding from a star were to pass nearly tangent to the sun’s limb it should be deflected 0″.87 according to Newton. According to the theory of relativity it should be deflected 1″.75.

Stars of course cannot usually be observed near the sun. It is therefore necessary to take advantage of a total solar eclipse, when the sun is completely hidden by the moon, in order to secure these observations.

Two expeditions, one to Africa, and one to South America, observed successfully the total eclipse of May 29, 1919. The former was located on the Island of Principe in the Gulf of Guinea. The latter was located at Sobral, Brazil. Their equipment and results are shown in the following table, where the successive columns give the location, the aperture in inches of the telescopes employed, their focus in feet, the number of plates secured, the number of stars measured, their mean deduced deflection from their true positions by the attraction of the sun, and the deviations from the theoretical results.[15] In the first and last line of the table shown herewith, this

Location	Aperture	Focus	Plates	Stars	Defl.	Dev.
Principe	13	11	2	5	1″.60	-0″.15
Sobral	13	11	19	12	0 .93	(+0 .06)
,,	4	19	8	7	1 .98	+0 .23

deviation is taken from Einstein's computed value of 1″.75. In the second line the difference shown is from the value required by the Newtonian theory, 0″.87. The results obtained with this telescope were rejected however, although they were much the most numerous, because it was found that for some reason, supposed to be the heating of the mirror by the sun before the eclipse, the star images were slightly out of focus, and were therefore considered unreliable. The results with the two other telescopes were not very accordant, but the 4-inch had the longer focus, secured the greater number of plates, and showed the greater number of stars. The results obtained with it therefore appear to have been the more reliable. They differ from Einstein's prediction by 13 per cent. In future expeditions to test this question, the mirror in front of the telescope will be eliminated. [304]

We now come to the final test which has been applied to Einstein's theory. Einstein showed that in the intense gravitational field of the sun, the theory of relativity required that all of the spectrum lines should be shifted slightly toward the red end. The shift however is exceedingly small, and can only be detected and measured with the most powerful modern instruments. Moreover only certain lines can be used, because owing to varying pressure in the solar atmosphere, which affects many lines, as well as to rapid motion in the line of sight, which may affect all of them, still larger displacements are liable to occur.

According to the theory of relativity the displacement of the lines should be $+0.0080A$. St. John at Mt. Wilson found a displacement for the cyanogen lines of only $+0.0018A$.[16] Evershed at Kodaikanal found +0.0060 at the north pole of the sun, and +0.0080 at the south pole. These latter values however were only for the stronger lines. The weaker lines give much smaller shifts, as do those of calcium and magnesium.[17] According to Einstein all lines should give nearly the same shift, an amount proportional to the wave length. It therefore appears that we must conclude by saying that Einstein's theory of relativity has been partially, but not completely, verified. [305]

The reference numbers in the above text have nothing to do with the numbers used in other parts of this volume to acknowledge the work of the various contestants; they refer to Dr. Pickering's sources, as follows:

[1] *Journ. Brit. Astron. Assoc.*, 1919, 30, 76.

[2] *Comptes Rendus*, 165, 424, and 167, 71.

[3] *Monthly Notices R. A. S.*, 1919, 80, 104.

[4] *Monthly Notices R. A. S.*, 1917, 77, 379.

[5] *Astro-Physical Journal*, 1917, 46, 249. *Journ. Brit. Astron. Assoc.*, 1920, 30, 276.

[6] *Monthly Notices, R. A. S.*, 1917, 77, 377.

[7] *Amer. Journ. Sci.*, 34, 333.

[8] *Monthly Notices, R. A. S.*, 1920, 80, 628.

[9] *The Observatory* 1920, April. From an Oxford Note Book.

[10] *Monthly Notices, R. A. S.*, 1917. 78, 3 *De Sitter*, 1919, 80, 121, *Jeans*, 80, 146 *Jeffreys*.

[11] "Gravitation and the Principle of Relativity," Eddington. Royal Institution of Great Britain, 1918.

[12] *Journ. Brit. Astron. Assoc.*, 1920, 30, 125.

[13] "The Interior of a Star," Eddington. *Scientia*, 1918, 23, 15.

[14] *Monthly Notices, R. A. S.*, 1919, 80, 138.

[15] *Monthly Notices. R. A. S.*, 1920, 80, 415. *Journ. Brit. Astron. Assoc.*, 1919, 30, 46.

[16] *Astro-Physical Journ.*, 1917, 46, 249.

[17] *Journ. Brit. Astron. Assoc.*, 1920, 30, 276. [306]

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XVIII

THE PRACTICAL SIGNIFICANCE OF RELATIVITY

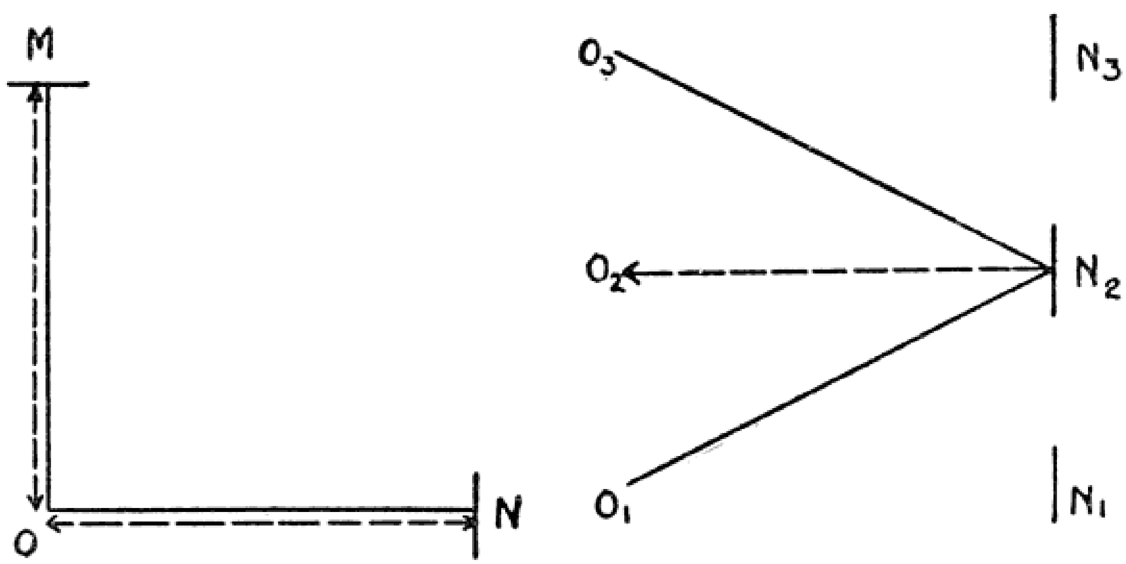
THE BEST DISCUSSION OF THE SPECIAL THEORY AMONG ALL THE COMPETING ESSAYS

BY PROFESSOR HENRY NORRIS RUSSELL, PRINCETON UNIVERSITY

Can a small child catch a baseball moving sixty miles an hour without getting hurt? We should probably answer "No"—but suppose that the boy and his father were sitting side by side in an express train, and the ball was tossed lightly from one to the other. Then there would be no trouble about it, whether the train was standing still, or going at full speed. Only the *relative* motion of ball and boy would count.

This every-day experience is a good illustration of the much discussed Principle of Relativity, in its simplest form. If there were no jolting, the motion of the train, *straight ahead at a uniform speed, would have no effect at all upon the relative motions of objects inside it*, nor on the forces required to produce or change these motions. Indeed, the motion of the earth in its orbit, which is free from all jar, but a thousand times faster, does not influence even the most delicate apparatus. We are quite unconscious of it, and would not know that the earth was moving, if we could not see other bodies outside it. [307]
This sort of relativity has been recognized for more than two centuries and lies at the bottom of all our ordinary dynamical reasoning, upon which both science and engineering are based.

But there are other things in nature besides moving bodies,—above all, light, which is intimately related to electricity and magnetism, and can travel through empty space, between the stars. It moves at the enormous speed of 186,000 miles per second, and behaves exactly like a series of vibrations or "waves." We naturally think of it as travelling through some medium, and call this thing, which carries the light, the "ether."



Can we tell whether we are moving through this ether, even though all parts of our apparatus move together, and at the same rate? Suppose that we have two mirrors, *M* and *N*, at equal distances, *d*, from a point *O*, but in directions at right angles to one another, and send out a flash of light from *O*. If everything is at rest, the reflected flashes will evidently come back to *O* at the same instant, and the elapsed time will be $2d/c$ seconds if *c* is the velocity of light. [308]

But suppose that *O*, *M*, and *N* are fastened to a rigid frame work, and all moving in the direction OM, with velocity *V*. The light which goes from *O* toward *M*, at the speed *c*, will overtake it with the difference of their speeds, , taking seconds to reach *M*. On the way back, *O* will be advancing to meet it, and the return trip will occupy seconds. The elapsed time for the round trip comes out seconds, which is *longer* than when the system was at rest—the loss of time in the "stern chase" exceeding the saving on the return.

The light which is reflected from N has a different history. When it starts, *O* and *N* have certain positions in the ether, and . By the time it reaches the mirror, this is at , and *O* is at , and when it returns, it finds *O* at . The distances for the outward and inward journeys are now equal, but (as is obvious from the figure), each of them is greater than *d*, or , and the time for the round trip will be correspondingly increased. A simple calculation shows that it is .

The increase above the time required when the system was at rest is less in this case than the preceding. Hence, if the apparatus is moving through the ether, the flashes reflected from *M* and *N* will not return at the same instant.

For such velocities as are attainable—even the 18 miles per second of the earth in its orbit—the difference is less than a hundred-millionth of the elapsed time. [309] Nevertheless, Michelson and Morley tried to detect it in their famous experiment.

A beam of light was allowed to fall obliquely upon a clear glass mirror (placed at *O* in the diagram) which reflected part of it toward the mirror, *M*, and let the rest pass through to the mirror *N*. By reuniting the beams after their round trips, it was possible to tell whether one had gained upon the other by even a small fraction of the time of vibration of a single light wave. The apparatus was so sensitive that the predicted difference, though amounting to less than a millionth part of a billionth of a second, could easily have been measured; but they actually found *no difference at all*—though the earth is certainly in motion.

Other optical experiments, more intricate, and even more delicate, were attempted, with the same object of detecting the motion of the earth through the ether; and they all failed.

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The Special Theory and Its Surprising Consequences

It was upon these facts that Einstein based his original, or "special" theory of Relativity. He assumed boldly that *the universe is so constituted that uniform straight-ahead motion of an observer and all his apparatus will not produce any difference whatever in the result of any physical process or experiment of any kind*. Granting this, it follows that if all objects in the visible universe were moving uniformly together in any direction, no matter how fast, we could not find this out at all. We cannot determine whether the universe, as a whole, is at [310] rest or in motion, and may as well make one guess as another. Only the *relative* motions of its parts can be detected or studied.

This seems simple and easy enough to understand. But the consequences which follow from it are extraordinary, and at first acquaintance seem almost absurd.

In the first place, if an observer measures the velocity of light, he must always get the same result, no matter how fast he and his apparatus are moving, or in what direction (so long as the motion is uniform and rectilinear). This sounds harmless; but let us go back to the Michelson-Morley experiment where the light came back in exactly the same time from the two mirrors. If the observer supposes himself to be at rest, he will say that the distances OM and were equal. But if he fancies that the whole universe is moving in the direction OM, he will conclude that *M* is nearer to *O* than *N* is—for if they were equidistant, the round-trip would take longer in the first case, as we have proved. If once more he fancies that the universe is moving in the direction , he will conclude that N is nearer to *O* than *M* is. *His answer to the question which of the two distances, OM or , is the greater will therefore depend on his assumption about the motion of the universe as a whole.*

Similar complications arise in the measurement of time. Suppose that we have two observers, *A* and *B*, provided with clocks which run with perfect uniformity, and mirrors to reflect light signals to one another. At noon exactly by his clock, *A* sends a flash of light towards *B*. *B* sees it come in at 12:01 by [311] *his* clock. The flash reflected from *B*'s mirror reaches *A* at 12:02 by *A*'s clock. They communicate these observations to one another.

If *A* and *B* regard themselves as being at rest, they will agree that the light took as long to go out as it did to come back, and therefore that it reached *B* at just 12:01 by *A*'s clock, and that the two clocks are synchronized. But they may, if they please, suppose that they (and the whole universe) are moving in the direction from *A* towards *B*, with half the speed of light. They will then say that the light had a "stern chase" to reach *B*, and took three times as long to go out as to come back. This means that it got to *B* at 1½ minutes past noon by *A*'s clock, and that *B*'s clock is slow compared with *A*'s. If they should assume that they were moving with the same speed in the opposite direction, they would conclude that *B*'s clock is half a minute fast.

Hence *their answer to the question whether two events at different places happen at the same time, or at different times, will depend on their assumption about the motion of the universe as a whole*.

Once more, let us suppose that *A* and *B*, with their clocks and mirrors, are in relative motion, with half the speed of light, and pass one another at noon by both clocks. At 12:02 by *A*'s clock, he sends a flash of light, which reaches *B* at 12:04 by his clock, is reflected, and gets back to *A*'s clock at 12:06. They signal these results to each other, and sit down to work them out. *A* thinks that he is at rest, and *B* moving. He therefore concludes that the light had the same distance to go out as to return to him and took two seconds each way, reaching [312] *B* at 12:04 by *A*'s clock, and that the two clocks, which agreed then, as well as at noon, are running at the same rate.

B, on the contrary, thinks that *he* is at rest and *A* in motion. He then concludes that *A* was much nearer when he sent out the flash than when he got it back, and that the light had three times as far to travel on the return journey. This means that it was 12:03 by *A*'s clock at the instant when the light reached *B* and *B*'s clock read 12:04. Hence *A*'s clock is running slow, compared with *B*'s.

Hence *the answer to the question whether two intervals of time, measured by observers who are in motion relative to one another, are of the same or of different durations, depends upon their assumptions about the motion of the universe as a whole*.

Now we must remember that one assumption about the motion of the universe as a whole is exactly as good—or bad—as another. No possible experiment can distinguish between them. Hence on the Principle of Relativity, we have left no *absolute* measurement of time or space. Whether two distances in different directions are to be called equal or not—whether two events in different places are to be called simultaneous or not—depends on our arbitrary choice of such an assumption, or "frame of reference." All the various schemes of measurement corresponding to these assumptions will, when applied to any imaginable experiment, predict exactly the same phenomena. But, in certain important cases, these predictions differ from those of the old familiar theory, and, *every time that such experiments have been tried, the result has agreed with the new theory, and not with the old*. [313]

We are therefore driven to accept the theory of relativity, strange as it is, as being more nearly "true to nature" than our older ideas. Fortunately, the difference between the results of the two become important only when we assume that the whole visible universe is moving together much faster than any of its parts are moving relatively to one another. Unless we make such an unwarranted assumption, the differences are so small that it takes the most ingenious and precise experiments to reveal them.

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The Generalization

Not content with all this, Einstein proceeded, a few years ago, to develop a "general" theory of relativity, which includes the effects of gravitation.

To make this idea clear, let us imagine two observers, each, with his measuring instruments, in a large and perfectly impervious box, which forms his "closed system."

The first observer, with his box and its contents, alone in space, is entirely at rest.

The second observer, with his box and its contents, is, it may be imagined, near the earth or the sun or some star, and falling freely under the influence of its gravitation.

This second box and its contents, including the observer, will then fall under the gravitational force, that is, get up an ever-increasing speed, but at exactly the same rate, so that there will be no tendency for their relative positions to be altered. [314]

According to Newton's principles, this will make not the slightest difference in the motions of the physical objects comprising the system or their attractions on one another, so that no dynamical experiment can distinguish between the condition of the freely falling observer in the second box and the observer at rest in the first.

But once more the question arises: What could be done by an optical experiment?

Einstein assumed that the principle of relativity still applied in this case, so that it would be impossible to distinguish between the conditions of the observers in the two boxes by any optical experiment.

It can easily be seen that it follows from this new generalized relativity that light cannot travel in a straight line in a gravitational field.

Imagine that the first observer sets up three slits, all in a straight line. A ray of light which passes through the first and second will obviously pass exactly through the third.

Suppose the observer in the freely falling system attempts the same experiment, having his slits *P*, *Q*, *R*, equally spaced, and placing them at right angles to the direction in which he is falling. When the light passes through *P*, the slits will be in certain position (Figure). By the time it reaches *Q*, [315]
they will have fallen to a lower level, , and when it reaches *R*, they will be still lower, . The times which the light takes to move from *P* to *Q* and *Q* to *R* will be the same: but, since the system is falling ever faster and faster the distance will be greater than . Hence, if the light which has passed through *P* and *Q* moves in a straight line, it will strike above *R*, as is illustrated by the straight line in the figure. But, on Einstein's assumption, the light must go through the third slit, as it would do in the system at rest, and *must therefore move in a curved line*, like the curved line in the figure, *and bend downward in the direction of the gravitational force*.

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The Tests

Calculation shows that the deviation of light by the moon or planets would be too small to detect. But for a ray which had passed near the sun, the deflection comes out 1.7″, which the modern astronomer regards as a large quantity, easy to measure. Observations to test this can be made only at a total eclipse, when we can photograph stars near the sun, on a nearly dark sky. A very fine chance came in May, 1919, and two English expeditions were sent to Brazil and the African coast. These photographs were measured with extreme care, and they show that the stars actually appear to be shifted, in almost exactly the way predicted by Einstein's theory.

Another consequence of "general relativity" is that Newton's law of gravitation needs a minute correction. This is so small that there is but a single [316] case in which it can be tested. On Newton's theory, the line joining the sun to the nearest point upon a planet's orbit (its perihelion) should remain fixed in direction, (barring certain effects of the attraction of the other planets, which can be allowed for). On Einstein's theory it should move slowly forward. It has been known for years that the perihelion of Mercury was actually moving forward, and all explanations had failed. But Einstein's theory not only predicts the direction of the motion, but exactly the observed amount.

Einstein also predicts that the lines of any element in the solar spectrum should be slightly shifted towards the red, as compared with those produced in our laboratories. Different observers have investigated this, and so far they disagree. The trouble is that there are several other influences which may shift the lines, such as pressure in the sun's atmosphere, motion of currents on the sun's surface, etc., and it is very hard to disentangle this Gordian knot. At present, the results of these observations can neither be counted for or against the theory, while those in the other two cases are decisively favorable.

The mathematical expression of this general relativity is intricate and difficult. Mathematicians—who are used to conceptions which are unfamiliar, if not incomprehensible, to most of us—find that these expressions may be described (to the trained student) in terms of space of four dimensions and of the non-Euclidean geometry. We therefore hear such phrases as "time as a sort of fourth dimension," "curvature of space" and others. But these are simply [317] attempts—not altogether successful—to put mathematical relationships into ordinary language, instead of algebraic equations.

More important to the general reader are the physical bearings of the new theory, and these are far easier to understand.

Various assumptions which we may make about the motion of the universe as a whole, though they do not influence the observed facts of nature, will lead us to different ways of interpreting our observations as measurements of space and time.

Theoretically, one of these assumptions is as good as any other. Hence we no longer believe in *absolute* space and time. This is of great interest philosophically. Practically, it is unimportant, for, unless our choice of an assumption is very wild, our conclusions and measurements will agree substantially with those which are already familiar.

Finally, the "general" relativity shows that gravitation and electro-magnetic phenomena—(including light) do not form two independent sides of nature, as we once supposed, but influence one another (though slightly) and are parts of one greater whole. [318]

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XIX

EINSTEIN'S THEORY OF RELATIVITY

A Simple Explanation of His Postulates and Their Consequences

BY T. ROYDS, KODAIKANAL OBSERVATORY, INDIA

Einstein's theory of relativity seeks to represent to us the world as it really is instead of the world of appearances which may be deceiving us. When I was in town last week to buy 5 yards of calico I watched the draper very carefully as he measured the cloth to make sure I was not cheated. Yet experiment can demonstrate, and Einstein's theory can explain, that the draper's yardstick became longer or shorter according to the direction in which it was held. The length of the yardstick did not appear to me to change simply because everything else in the same direction, the store, the draper, the cloth, the retina of my eye, changed length in the same ratio. Einstein's theory points out not only this, but every case where appearances are deceptive, and tries to show us the world of reality.

Einstein's theory is based on the principle of relativity and before we try to follow his reasoning we must spend a little time in understanding what he means by "*relativity*" and in grasping how the idea arises. Suppose I wish *to* [319] *define my motion* as I travel along in an automobile. I may be moving at the rate of 25 miles an hour relative to objects fixed on the roadside, but relative to a fellow-passenger I am not moving at all; relative to the sun I am moving with a speed of 18½ miles per second in an elliptical orbit, and again relative to the stars I am moving in the direction of the star *Vega* at a speed of 12 miles per second. Thus motion can only be defined relative to some object or point of reference. Now this is not satisfactory to the exact scientist. Scientists are not content with knowing, for example, that the temperature of boiling water is +100° C. relative to the temperature of freezing; they have set out to determine absolute temperatures and have found that water boils at 373° C. above absolute zero. Why should I not, therefore, determine the absolute motion of the automobile, not its motion relative to the road, earth, sun or stars, but relative to absolute rest?

Michelson and Morley set out in their famous experiment to measure the absolute velocity of their laboratory, which was, of course, fixed on the earth. The experiment consisted of timing two rays of light over two equal tracks at right angles to each other. When one track was situated in the direction of the earth's motion they expected to get the same result as when two scullers of equal prowess are racing in a river, one up and down the stream and the other across and back; the winner will be the sculler rowing across the stream, as [320] working out an example will convince. Even if the earth had been stationary at the time of one experiment, the earth's motion round the sun would have been reversed 6 months later and would then have given double the effect. They found, however, that the two rays of light arrived always an exact dead heat. All experimenters who have tried since have arrived at the same result and found it impossible to detect absolute motion.

The principle of relativity has its foundation in fact on these failures to detect absolute motion. This principle states that the only motion we can ever know about is *relative* motion. If we devise an experiment which ought to reveal absolute motion, nature will enter into a conspiracy to defeat us. In the Michelson and Morley experiment the conspiracy was that the track in the direction of the earth's absolute motion should contract its length by just so much as would allow the ray of light along it to arrive up to time.

We see, therefore, that according to the principle of relativity motion must always remain a relative term, in much the same way as vertical and horizontal, right and left, are relative terms having only meaning when referred to some observer. We do not expect to find an absolute vertical and are wise enough not to attempt it; in seeking to find absolute motion physicists were not so wise and only found themselves baffled.

The principle that all motion is relative now requires to be worked out to all its consequences, as has been done by Einstein, and we have his theory of [321] relativity. Einstein conceives a world of four dimensions built up of the three dimensions of space, namely up and down, backwards and forwards, right and left, with time as the fourth dimension. This is an unusual conception to most of us, so let us simplify it into something which we can more easily picture but which will still allow us to grasp Einstein's ideas. Let us confine ourselves for the present to events which happen on this sheet of paper, *i.e.*, to space of two dimensions only and take time as our third dimension at right angles to the plane of the paper. We have thus built up a three dimensional world of space-time which is every bit as useful to us as a four dimensional representation so long as we only need study objects moving over the sheet of paper.

Suppose a fly is crawling over this sheet of paper and let us make a movie record of it. If we cut up the strip of movie film into the individual pictures and cement them together one above another in their proper order, we shall build up a solid block of film which will be a model of our simplified world of space-time and in which there will be a series of dots representing the motion of the fly over the paper. Just as I can state the exact position of an object in my room by defining its height above the floor, its distance from the north wall and its distance from the east wall, so we can reduce the positions of the dots to figures for use in calculations by measuring their distances from the three faces intersecting in the lines *OX*, *OY*, and *OT*, where *OXAYTBCD* represents [322] the block of film. The mathematician would call the three lines *OX*, *OY*, *OT* the coordinate axes. Measuring all the dots in this way we shall obtain the motion of the fly relative to the coordinate axes *OX*, *OY*, *OT*. If we add a block *OTDYEFGH* of plain film we can use *EX*, *EH*, *EF* as coordinate axes and again obtain the motion of the fly relative to these new axes; or we

can add block after block so as to keep the axes moving. We can conceive of other changes of axes. The operator making the movie record might have taken the fly for the hero of the piece and moved the camera about so as to keep the fly more or less central in the picture; or he might, by turning the handle first fast and then slow and by moving the camera, have made the fly appear to be doing stunts. Moving the camera would change the axes of x and y, and turning the handle at different speeds would change the axis of time. Again, we might [323]
change the axes by pushing the block out of shape or by distorting it into a state of strain. Whatever change of axes we make, any dot in the block of film will signify a coincidence of the fly with a certain point of the paper at a certain time, and the series of dots will, in every case, be a representation of the motion of the fly. Maybe the representation will be a distorted one, but who is to say which is the absolutely undistorted representation? The principle of relativity which we laid down before says that no one set of coordinates will give the absolute motion of the fly, so that one set is as good as another. The principle that all motion is relative means, therefore, that no matter how we change our coordinates of space-time, the laws of motion which we deduce must be the same for all changes.

To use an analogy, the sculptured head of Shakespeare on my table may appear to have hollow cheeks when I admit light from the east window only, or to have sunken eyes with light from the skylight in the roof, but the true shape of the head remains the same in all lights.

Hence, if with reference to two consecutive dots in our block of film a mathematical quantity can be found which will not change no matter how we changes our axes of coordinates, that quantity must be an expression of the true law of motion of the fly between the two points of the paper and the two times represented by these two dots. Einstein has worked out such a quantity remaining constant for all changes of coordinates of the four dimensional [324]
world of space-time.

In passing we may notice a feature of Einstein's world of space-time which we shall doubtless find it difficult to conceive, namely, that there is no essential difference between a time and a distance in space. Since one set of coordinates is as good as another, we can transform time into space and space into time according as we choose our axes. For example if we change *OX*, *OT*, the axes of *x* and time in Fig. 2, into *OX′*, *OT′* by a simple rotation, the new time represented by *OT′* consists partly of *OA* in the old time and partly of *OB* in the old *x* direction. Referring to our block of movie film again, it means that although I might separate the block into space and time by slicing it into the original pictures, I can just as readily slice it in any direction I choose and still get individual pictures representing the motion of the fly but with, of course, new time and space. So whilst I may be believe that a liner has travelled 3,000 miles in 4 days, an observer on a star who knows nothing of my particular axes in space-time may say, with equal truth, that it went 2,000 miles in 7 days. Thus, time and space are not two separate identities in Einstein's view; there only exists a world of four dimensions which we can split up into time and space as we choose.

Let us see now how Einstein explains gravitation. When a body is not acted on by any forces (except gravitation) the quantity which remains constant for all changes of coordinates implies that the body will follow that path in the space of an outside observer which takes the least time. It is an observed fact that one [325] body attracts another by gravitation; that is, the path of one body is bent from its course by the presence of another. Now we can bend the path of the fly in our block of film by straining the block in some way. Suppose, therefore, that I strain the world so as to bend the path of a body exactly as the gravitation due to some other body bends it; *i.e.*, by a change of coordinates I have obtained the same effect as that produced by gravitation. Einstein's theory, therefore, explains gravitation as a distortion of the world of space-time due to the presence of matter. Suppose first that a body is moving with no other bodies near; according to Einstein it will take the path in space which requires the least time, *i.e.*, a straight line as agrees with our experience. If now the world be strained by the presence of another body or by a change of coordinates it will still pursue the path of least time, but this path is now distorted from the straight line, just as in a similar way the path on a globe requiring the least time to travel follows a great circle. So, on Einstein's view of gravitation, the earth moves in an elliptical path around the sun not because a force is acting on it, but because the world of space-time is so distorted by the presence of the sun that the path of least time through space is the elliptical path observed. There is, therefore, no need to introduce any idea of "force" of gravitation. Einstein's theory explains gravitation only in the sense that he has explained it away as a force of nature and makes it a property of space-time, namely, a distortion not different from an appropriate change of coordinates. He does [326] not, however, explain how or why a body can distort space-time. It is noteworthy that whilst the law of gravitation and the law of uniform motion in a straight line when no force is acting were separate and independent laws under Newton, Einstein finds one explanation for both under the principle of relativity.[1] [327]

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1 The balance of Dr. Royds' essay is given to a discussion of the phenomena of Mercury's perihelial advance, the deflection of light under the gravitational field of the sun, and the shift in spectral lines, in connection with which alone Einstein's theory makes predictions which are sufficiently at variance with those of Newtonian science to be of value in checking up the theory

observationally. In the interest of space conservation and in the presence of Dr. Pickering's very complete discussion of these matters we omit Dr. Royds' statement.—Editor. ↑

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XX

EINSTEIN'S THEORY OF GRAVITATION

The Discussion of the General Theory and Its Most Important Application, from the Essay by

PROF. W. F. G. SWANN, UNIVERSITY OF MINNESOTA, MINNEAPOLIS

Newton's great discovery regarding the motion of the planets consisted in his showing that these could all be summed up in the following statement: consider any planet in its relation to all particles in the universe. Write down, for the planet, in the line joining it to any particle, an acceleration proportional to the mass of the particle and to the inverse square of its distance from the planet. Then calculate the planet's resultant acceleration by combining all the accelerations thus obtained.

We have here purposely avoided the use of the word "force," for Newton's law is complete as a practical statement of fact without it; and this word adds nothing to the law by way of enhancing its power in actual use. Nevertheless, the fact that the acceleration is made up as it were of non-interfering contributions from each particle in the line joining it to the planet strongly suggests to the mind something of the nature of an elastic pull for which the particle is responsible, and to which the planet's departure from a straight-line [328] motion is due. The mind likes to think of the elastic; ever since the time of Newton people have sought to devise some mechanism by which these pulls might be visualized as responsible for the phenomena in the same way as one pictures an elastic thread as controlling the motion of a stone which swings around at its end.

This search has been always without success; and now Einstein has found a rather different law which fits the facts better than Newton's law. It is of such a type that it does not lend itself conveniently to expression in terms of force; the mind would gain nothing by trying to picture such forces as are necessary. It compensates for this, however, in being capable of visualization in terms of what is ultimately a much simpler concept.

In order to appreciate the fundamental ideas involved, suppose for a moment that gravitation could be annihilated, completely, and suppose I find myself upon this earth in empty space. You shall be seated at some point in space and shall watch my doings. If I am in the condition of mind of the people of the reign of King Henry VIII, I shall believe that the earth does not rotate. If I let go a stone, there being no gravity, I shall find that it flies away from me with an acceleration. You will know, however, that the stone really moves in a straight line with constant velocity, and that the apparent acceleration which I perceive is due to the earth's rotation. If I have argued that acceleration is due to force, I shall say that the earth repels the stone, and shall try to find the law governing the variation of this force with distance. I may go farther, and try to [329] imagine some reason for the force, some pushing action transmitted from the earth to the stone through a surrounding medium; and, you will pity me for all this wasted labor, and particularly for my attempt to find a mechanism to account for the force, since you know that if I would only accept your measurements all would appear so simple.

Let us probe this matter a little farther, however, from the stand-point of myself. I must believe in the reality of the force, since I have to be tied to my chair to prevent my departure from the earth. I might wonder how this field of force would affect the propagation of light, chemical action and so forth. For, even though I had discovered that, by using your measures, I could transform away the apparent effects of my field of force as far as concerned its power to hurl stones about, I could still regard this as a mathematical accident, and believe that the force was really there. Although I might *suspect* that the same transformation of view-point that would annul the field's effect as regards the stones would also annul its effect as regards light, etc., I should not be sure of this, as you would be; and my conscience would hardly allow me to do more than look upon the assumption of complete *equivalence* between the apparent field and a change in the system of measurement as a hypothesis. I should be strongly tempted to make the hypothesis, however.

Now the question raised by Einstein is whether the force of gravity, which we experience as a very real thing, may be put upon a footing which is in some [330] way analogous to that of the obviously fictitious centrifugal force cited above: whether gravitation may be regarded as a figment of our imagination engendered by the way in which we measure things. He found that it could be so regarded. He went still farther, and in his *Principle of Equivalence*, he postulated that the apparent effects of gravitation *in all phenomena* could be attributed to the same change in the system of our measurements that would account for the *ordinary* phenomena of gravitation. On the basis of this hypothesis he was able to deduce for subsequent experimental verification, the effects of gravitation on light. He did not limit himself to such simple changes in our measurements as were sufficient to serve the purpose of the problem of centrifugal force cited above; but, emboldened by the assumptions, in the older theory of relativity, of change in standards of length and time on account of motion, he went even farther than this, and considered the possibility of change of our measures due to mere proximity to matter.

His problem amounted to an attempt to find some way in which it was possible to conceive our scales and clocks as altered, relatively to some more fundamental set, so as to allow of the planetary motions being uniform and rectilinear with respect to these fundamental measures, although they appear as they do to us. If we allow our imaginations perfect freedom as to how the scales may be altered, we shall not balk at assuming alterations varying in any way we please, Einstein does, however, introduce restrictions for reasons [331] which we will now discern.

If we imagine our whole universe, with its observers, planetary orbits, instruments, and everything else, embedded in a jelly, and then distort the jelly and contents in any way, the numbers at which our planetary orbits (or rather their telescopic images) intersect our scales will be unaltered. Moreover, we could vary, in any manner, the times at which all objects (including the clock hands) occupied their distorted positions, and the hand of some clock near the point where the planetary image crossed the scale would record for this occurrence the same dial reading as before. An inhabitant of this distorted universe would be absolutely unconscious of the change. Now the *General Theory of Relativity* which expresses itself in slightly varied forms, amounts to satisfying a certain philosophical craving of the mind, by asserting that the laws of nature which control our universe ought to be such that another universe like the above, whose inhabitants would be unconscious of their change, would also satisfy these laws, not merely from the standpoint of its own inhabitants, but also from the standpoint of *our* measurements. In other words, this second universe ought to appear possible to us as well as to its inhabitants.

Einstein decides to make his theory conform to this philosophical desire, and this greatly limits the modifications of clocks and scales which he permits himself for the purpose of representing gravitation. Further, if we express the
alterations of the measures as functions of proximity to matter, velocity and so [332]
forth, our expressions for these alterations will include, as a particular case, that where matter is absent, although the scales and observer may still remain. Our alteration of the scales and clocks with velocity must thus revert, for this case, to that corresponding to the older theory of relativity, in order to avoid predicting that two observers, in uniform motion relative to each other in empty space, will measure different values for the velocity of light. In this way, the velocity of light comes to play a part in expressing the alterations of the measures.

Even with these restrictions, Einstein was able to do the equivalent of finding an alteration of scales and clocks in the presence of matter which would account for our finding that the planetary motions take place very nearly in accordance with Newton's law. The new law has accounted with surprising accuracy for certain astronomical irregularities for which Newton's law failed to account, and has predicted at least one previously unknown phenomenon which was immediately verified.

In conclusion, it may be of interest to state how the new law describes the motion of a particle in the vicinity of a body like the earth. The law amounts to stating that, if we measure a short distance, radially as regards the earth's center, we must allow for the peculiarity of our units by dividing by

where *r* is the distance from the earth's center, *m* the mass of the earth, *c* the [333]
velocity of light, and *G* the Newtonian gravitational constant. Tangential measurements require no correction, but intervals of time as measured by our clocks must be multiplied, for each particular place, by the above factor. Then, in terms of the corrected measures so obtained, the particle will be found to describe a straight line with constant velocity although, in terms of our actual
measures, it appears to fall with an acceleration. [334]

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XXI

THE EQUIVALENCE HYPOTHESIS

The Discussion of This, With Its Difficulties and the Manner in Which Einstein Has Resolved Them, from the Essay by

PROF. E. N. DA C. ANDRADE, ORDNANCE COLLEGE, WOOLWICH, ENGLAND

Having shown that, of several systems all moving with reference to one another with uniform motion, no one is entitled to any preference over the others, and having deduced the laws for such systems, Einstein was confronted with a difficulty which had long been felt. A body rotating, which is a special case of an accelerated body, can be distinguished from one at rest, without looking outside it, by the existence of the so-called centrifugal forces.

This circumstance, which gives certain bodies an absolute or preferential motion, is unpalatable to the relativist; he would like there to be no difference as regards forces[1] between the case when the earth rotates with reference to outside bodies (the stars) considered as fixed, and the case when the earth is considered fixed and all outside bodies rotate around it. This point cannot be investigated by direct experiment; we can spin a top but we cannot keep a top at rest and spin the world round it, to see if the forces are same. [335]

In considering the problem of how to devise laws which should make all rotations relative, Einstein conceived the brilliant yet simple idea that gravitation could be brought into the scheme as an acceleration effect, since both ordinary accelerational forces and gravitational forces are proportional to the same thing, the mass of a body. The impossibility of separating the two kinds of effect can be easily seen by considering the starting of an elevator. When the elevator is quickly accelerated upwards we feel a downward pull, just as if the gravitational pull had been increased, and if the acceleration continued to be uniform, bodies tested with a spring balance would all weigh more in the elevator than they did on firm ground. In a similar way the whole of the gravitational pull may be considered to be an accelerational effect, the difficulty being to devise laws of motion which will give the effects that we find by actual observation.

But it is obvious that we cannot, by ordinary mechanics, consider the earth as being accelerated in all directions, which we should have to do, apparently, to account for the fact that the gravitational pull is always toward the center. [It is obvious that we cannot explain gravitation by assuming that the earth's surface is continually moving outward with an accelerated velocity.][227] So Einstein found that, as long as we treat the problem by Euclid's geometry, we cannot reach a satisfactory solution. But he found that to the four-dimensional space made up of the three ordinary dimensions of space, together with the time-dimension which we have already mentioned in discussing the special theory, may be attributed a peculiar geometry, the nature of which departs more and more from Euclidean geometry as we approach a gravitational body, and the [336] net result of which is to make possible the universal correspondence of gravitation and acceleration.

This modification of the geometry of space is often spoken of as the “curvature of space,” an expression which is puzzling, especially as the space which is “curved” is four-dimensional time-space. But we can get an idea of what is meant by considering figures, triangles say, drawn on the surface, of a sphere. These triangles, although drawn on a surface, will not have the same properties as triangles drawn on flat paper—their three angles will not together equal right angles. They will be non-Euclidean. This is only a rough analogy, but we can see that the curvature of the surface causes a departure from Euclidean geometry for plane figures, and consequently the departure from Euclidean laws extended to four dimensions may be referred to as caused by “curvature of space.”

It is difficult to imagine a lump of matter affecting the geometry of the space round it. Once more we must use a rough illustration. Imagine a very hot body, and that, knowing nothing of its properties, we have to measure up the space round it with metal measuring-rods. The nearer we are to the body, the longer the rods will become, owing to the expansion of the metal. When we measure out a square, one side of which is nearer the body than the opposite side, its [337]
angles will not be right angles. If we knew nothing of the laws of heat we should say that the body had made the space round it non-Euclidean.

Einstein found, then, that by taking the properties of space, as given by measurement, to be modified in the neighborhood of masses of matter, he could devise general laws according to which gravitational effects would be produced, and there would be no absolute rotation. All forces will be the same whether a body rotates with everything outside it fixed, or the body is fixed, and everything rotates round it. *All* motion is then relative, and the theory is one of “general relativity.” The velocity of light is, however, no longer constant, and its path is not a straight line, if it is passing near gravitating matter. This does not contradict the special theory, which did not allow for gravitation. Rather, the special theory is a particular case to which the generalised theory reduces when there is no matter about, just as the Newtonian dynamics is a special case of the special theory, which we obtain when all velocities are small compared to that of light. [338]

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1 There is, on any view, no difference as regards observation of *position* only.—Author. ↑

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XXII

THE GENERAL THEORY

Fragments of Particular Merit on This Phase of the Subject

BY VARIOUS CONTRIBUTORS

When Dorothy was carried by the cyclone from her home in Kansas to the land of Oz, together with her uncle's house and her little dog Toto, she neglected to lower the trap door over the hole in the floor which formerly led to the cyclone cellar and Toto stepped through. Dorothy rushed to the opening expecting to see him dashed onto the rocks below but found him floating just below the floor. She drew him back into the room and closed the trap.

The author of the chronicle of Dorothy's adventures explains that the same force which held up the house held up Toto but this explanation is not necessary. Dorothy was now floating through space and house and dog were subject to the same forces of gravitation which gave them identical motions. Dorothy must have pushed the dog down onto the floor and in doing so must herself have floated to the ceiling whence she might have pushed herself back to the floor. In fact gravitation was apparently suspended and Dorothy was in a position to have tried certain experiments which Einstein has never tried [339]
because he was never in Dorothy's unique position.][188]

* * *

The Principle of Equivalence, of which Einstein's suspended cage experiment is the usual illustration, and upon which the generalized theory of relativity is built, is thus stated by Prof. Eddington: "A gravitational field of force is precisely equivalent to an artificial field of force, so that in any small region it is impossible, by any conceivable experiment, to distinguish between them. In other words, force is purely relative."

This may be otherwise stated by going back to our idea of a four-dimensional world, the points of which represent the positions and times of events. If we mark in such a space-time the successive positions of an object we get a line, or curve, which represents the whole history of the object, inasmuch as it shows us the position of the object at every time. The reader may imagine that all events happen in one plane, so that only two perpendicular dimensions are needed to fix positions in space, with a third perpendicular dimension for time. He may then conceive, if he may not picture, an analogous process for four-dimensional space-time. These lines, "tracks of objects through space-time," were called by Minkowski "world-lines." We may now say that all the events we observe are the intersections of world-lines. The temperature at noon was 70°. This means that if I plot the world-line of the top of the mercury column and the world-line of a certain mark on the glass they intersect in a certain point of space-time. All that we know are intersections of these world-lines. [340]
Suppose now we have a large number of them drawn in our four-dimensional world, satisfying all known intersections, and let us suppose the whole embedded in a jelly. We may distort this jelly in any way, changing our coordinates as we please, but we shall neither destroy nor create intersections of world-lines. It may be proved that a change from one system of reference, to which observations are referred, to any other system, moving in any way with respect to the first system, may be pictured as a distortion of the four-dimensional jelly. The laws of nature, therefore, being laws that describe intersections, must be expressible in a form independent of the reference system chosen.

From these postulates, Einstein was able to show such a formulation possible. His law may be stated very simply:—All bodies move through space-time in the straightest possible tracks.

The fact that an easy non-mathematical explanation can not be given, of how this law is reached, or of just why the straightest track of Mercury through space-time will give us an ellipse in space after we have split space-time up into space and time, is no valid objection to the theory. Newton's law that bodies attract with a force proportional to their masses and inversely proportional to the square of the distance is simple, but no one has ever given an easy non-mathematical proof of how that law requires the path of Mercury to be an ellipse, with the sun at a focus, instead of some other curve.][182]

* * *

One of the grave difficulties we have in gaining a satisfactory comprehension of Einstein's conceptions, is that they do not readily relate themselves to our modes of geometrical thought. Within limits we may choose our own geometry, but it may be at the cost of unwieldy complication. If we think with Newton in terms of Euclidean geometry and consider the earth as revolving around the sun, the motions of our solar system can be stated in comparatively simple terms. If, on the other hand, we should persist in stating them, as Ptolemy would have done, from the earth as a relatively stationary center, our formulas will become complicated beyond ready comprehension. For this reason it is much simpler in applying the theory of relativity, and in considering and describing what actually happens in the physical universe, to use geometrical conceptions to which the actual conditions can be easily related. We find such an instrument in non-Euclidean geometry, wherein space will appear as though it were projected from a slightly concave mirror. It is in this sense that some speak of space as curved. The analogy is so suggestive it tempts one to linger over it. Unless there were material objects within the range of the mirror, its conformation would be immaterial; the thought of the space which the mirror, as it were, circumscribes, is dependent upon the presence of such material objects. The lines of light and of all other movement will not be quite "straight" from the view-point of Euclidean geometry. A line drawn in a universe of such a nature must inevitably return upon itself. Nothing therefore, can ever pass out of this unlimitedly great but yet finite cosmos. But even now, since our imaginary mirror is only very slightly concave, it follows that for limited regions like the earth or even the solar system, our conception of geometry may well be rectilinear and Euclidean. Newton's law of gravitation will be quite accurate with only a theoretical modification drawn from the theory of relativity.][82] [341] [342]

* * *

The way in which a curvature of space might appear to us as a force is made plainer by an example. Suppose that in a certain room a marble dropped anywhere on the floor always rolled to the center of the room; suppose the same thing happened to a baseball, a billiard ball, and a tennis ball. These results could be explained in two ways; we might assume that a mysterious force of attraction existed at the center of the floor, which affected all kinds of balls alike; or we might assume that the floor was curved. We naturally prefer the latter explanation. But when we find that in the neighborhood of a large material body all other bodies move toward it in exactly the same manner, regardless of their nature or their condition, we are accustomed to postulate a mysterious attractive force (gravitation); Einstein, on the contrary, adopts the other alternative, that the space around the body is curved.][223]

* * *

In the ordinary "analytical geometry," the position and motion of all the points considered is referred to a *rigid* "body" or "frame of reference." This usually consists of an imaginary room of suitable size. The position of any point is [343] then given by three numbers, *i.e.*, its distances from one side wall and from the back wall and its height above the floor. These three numbers can only give *one* point, every *other* point having at least one number different. In four-dimensional geometry a fourth wall may be vaguely imagined as perpendicular to all three walls, and a fourth number added, giving the distance of the "point" from this wall also. Since "rigid" bodies do not exist in gravitational fields the "frame of reference" must be "non-rigid." The frame of reference in the Gaussian system need not be rigid, it can be of any shape and moving in any manner, in fact a kind of jelly. A "point" or "event" in the four dimensioned world is still given by four numbers but these numbers do not represent distances from anywhere; all that is necessary is that no two events shall have exactly the same four numbers to represent them, and that two events which are very close together shall be represented by numbers which differ only slightly from one another. This system assumes so little that it will be seen to be very wide in its scope; although to the ordinary mind, what is gained in scope seems more than that lost in concreteness. This does not concern the mathematician, however, and by using this system he gains his object, proving that the general laws of nature remain the same when expressed in *any* Gaussian coordinate system whatever.][220]

* * *

Einstein enunciates a general principle that it is possible to find a transformation of coordinate axes which is exactly equivalent to any force, and in particular one which is equivalent to the force of gravitation. That is by [344] concentrating our attention on the transformation which is a purely mathematical operation we can afford to neglect the force completely. To get a better idea of this principle of equivalence as it is called, let us consider a relatively simple example (which actually has nothing to do with gravitation, but which will serve to make our notions clearer.) A person on the earth unconsciously refers all his experiences, *i.e.*, the motions of the objects around him to a set of axes fixed in the earth on which he stands. However, we know that the earth is rotating about its axis, and his axes of reference are also rotating with respect to the space about him. From the point of view of general relativity it is exactly because we do refer motions on the surface of the earth to axes rotating with the earth that we experience the so-called centrifugal force of the earth's rotation, with which everyone is familiar. If we could find it *convenient* to transform from moving axes to fixed axes, the force would vanish, since it is exactly equivalent to the transformation from one set of axes to the other. However, we find it unnatural to refer daily experiences to axes that are not placed where we happen to be, and so we prefer to take the force and rotating axes instead of no force and fixed axes.][272]

* * *

We seem to have a direct experience of force in our muscular sensations. By pushing or pulling we can set bodies in motion. It is natural to assume, that something similar occurs, when Nature set bodies in motion. But is this not a relic of animism? The savage and the ancients peopled all the woods and skies [345] with Gods and demons, who carries on the activities of nature by their own bodily efforts. Today we have dispossessed the demons, but the *ghost of a muscular pull* still holds the planets in place.][141]

* * *

The general theory is an extension of the special theory which enables the law of gravitation to be deduced. Not in Newton's form, it is true, but in a better form, that is, one that accounts for two important facts otherwise not explained. But it is a far more general theory that indicated above. It is a complete study of the relations between laws expressed by means of any four coordinates (of which three space and one time is a special case), and the same laws expressed in the four coordinates of a system having any motion whatever with respect to the first system. By restricting this general study in accordance with certain postulates about the nature of the universe we live in, we arrive at a number of conclusions which fit more closely with observed facts that the conclusions drawn from Newton's theory.][221]

Colophon

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External References

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Corrections

The following corrections have been applied to the text:

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vii	is made is made	is made	8
viii	btween	between	1
xiii	Chiselhurst	Chislehurst	2
28	attriuting	attributing	1
38	axis	axes	1
46, 327	[*Not in source*]	,	1
48	[*Not in source*]	”	1
59	assums	assumes	1
61	II	III	1
74	follows	follow	1

74	parellel	parallel	1
96	inadmissable	inadmissible	1
110	apposite	opposite	1
114	synonomous	synonymous	1
123	indepedent	independent	1
125	Descartean	Cartesian	6
126	propostions	propositions	1
127	’	”	1
144	incomphehensible	incomprehensible	1
151	hypothenuse	hypotenuse	1
153	dimension	dimensions	1
156	Betelguese	Betelgeuse	2
169, 282, 282	coördinates	coordinates	1 / 0
170	[*Not in source*]	.	1
181	Michelsen	Michelson	1
182	Michelsen-Morley	Michelson-Morley	1
185	that:	[*Deleted*]	5
205	gometry	geometry	1
206	and	und	1
213	distroyed	destroyed	1
219	wells	walls	1
228	indisolubly	indissolubly	1
231	signficance	significance	1
240	amazinging	amazing	3
244	dimsensions	dimensions	1
249	Relativitatstheorie	Relativitätstheorie	1 / 0
254	oberservers	observers	2
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262	or	of	1
279	Poincairé	Poincaré	1
291	Jean	Jeans	1
298	methematicians	mathematicians	1
305, 305, 305	Asso.	Assoc.	1
305	Astro.	Astron.	1
312	axactly	exactly	1
315	.	,	1
319	acrross	across	1
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327	BY	[*Deleted*]	3
329	waste	wasted	1

330	analagous	analogous	1
335	rotatons	rotations	1
340	imbedded	embedded	1
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345	line	live	1