به نام خدا

هوش مصنوعی

تمرین پنجم

استاد :

مهدی سمیعی

نویسنده :

محمدهومان کشوری

شماره دانشجویی :

۹۹۱۰۵۶۶۷

Shoutout to Hirbod Behnam for helping me out on the questions.

سوال ۱

.۱

$P[p_1|p_2,-p_3] = \frac{\sum_{p_4} P[p_1,p_2,-p_3,p_4]}{P[p_2,-p_3]} = Z * \sum_{p_4} P[p_1]P[p_2|p_1]P[-p_3|p_2]P[p_4|p_2]$

$= Z * P[p_1]P[p_2|p_1]P[-p_3|p_2]P[+p_4|p_2] + Z * P[p_1]P[p_2|p_1]P[-p_3|p_2]P[-p_4|p_2]$

$= Z * 0.4 * 0.8 * (1-0.2) * 0.8 + Z * 0.4 * 0.8 * (1-0.2) * 0.2 = Z * 0.256$

$Z = \frac{1}{\sum_{p_1,p_4} P[p_1,p_2,-p_3,p_4]} \rightarrow$
$\sum_{p_1,p_4} P[p_1,p_2,-p_3,p_4] = P[+p_1]P[p_2|+p_1]P[-p_3|p_2]P[+p_4|p_2] + P[+p_1]P[p_2|+p_1]P[-p_3|p_2]P[-p_4|p_2] + P[-p_1]P[p_2|-p_1]P[-p_3|p_2]P[+p_4|p_2] + P[-p_1]P[p_2|-p_1]P[-p_3|p_2]P[-p_4|p_2]$

$= 0.4*0.8*0.8*0.8 + 0.4*0.8*0.8*0.2 + 0.6*0.5*0.8*0.8 + 0.6*0.5*0.8*0.2 = P[p_2,-p_3] = 0.496$

$\rightarrow P[p_1|p_2,-p_3] = \frac{0.256}{0.496} \simeq 0.52$

$P[p_2|-p_3] = \frac{P[p_2,-p_3]}{P[-p_3]} = \frac{0.496}{P[-p_3]} \rightarrow P[-p_3] = \sum_{p_1,p_2,p_4} P[p_1,p_2,-p_3,p_4] = P[+p_1]P[+p_2|+p_1]P[-p_3|+p_2] + P[+p_1]P[-p_2|+p_1]P[-p_3|-p_2] + P[-p_1]P[+p_2|-p_1]P[-p_3|+p_2] + P[-p_1]P[-p_2|-p_1]P[-p_3|-p_2] = 0.4*0.8*0.8 + 0.4*0.2*0.7 + 0.6*0.5*0.8 + 0.6*0.5*0.7 = P[-p_3] = 0.762$

$\rightarrow P[-p_3] = 0.762$

$\rightarrow P[p_2|-p_3] = \frac{0.496}{0.762} = 0.65$

.۲

$P[-p_3] = \sum_{p_1,p_2} P[p_1,p_2,-p_3] = \sum_{p_1,p_2} P[p_1]P[p_2|p_1]p[-p_3|p_2] =$

$\sum_{p_2} P[-p_3|p_2] * \sum_{p_1} P[p_1]P[p_2|p_1]$

$\sum_{p_1} P[p_1]P[p_2|p_1] = P[p_2] \rightarrow$

$P[+p_1,+p_2] = 0.32, P[+p_1,-p_2] = 0.08, P[-p_1,+p_2] = 0.30, P[-p_1,-p_2] = 0.30$

$\xrightarrow{\text{factor out } p_1} +p_2 = 0.62, -p_2 = 0.38$

$\sum_{p_2} P[-p_3|p_2]P[p_2]$

$P[+p_3,+p_2] = 0.124, P[-p_3,+p_2] = 0.496, P[+p_3,-p_2] = 0.114, P[-p_3,-p_2] = 0.266$

$\xrightarrow{\text{factor out } p_2} P[-p_3] = 0.762, P[+p_3] = 0.238$

$\rightarrow P[p_2|-p_3] = \frac{P[p_2,-p_3]}{P[-p_3]} = \frac{0.496}{0.762} = 0.65$

سوال ۲

.۱

$P[X_6|Y_2, Y_3, Y_4, Y_5, Y_6, X_1, ..., X_5] = \sum_{X_5} P[X_6|Y_2, ..., Y_6, X_5] \cdots$

We would eliminate 6 variables so we would have a factor of size 2^6

.۲

If we first eliminate $X_2 \rightarrow X_5$ we would have a factor size of 2^2

.۳

If we want to find $P[x_5|Y_5, ..., Y_{10}] \rightarrow x_9, x_{10}, x_4, x_7, x_8, x_3, x_6, x_2, x_1$
This sequence would lead to a factor size of 4

سوال ۳

.۱

In Rejection Sampling we would discard the samples that have the wrong evidence.

In this example if we get either +r or -f, the evidence is wrong, hence we should discard our sample.
In the samples below, the rejected samples are marked with an X
+r +e +w -m -f $\rightarrow X$
-r -e +w -m -f $\rightarrow X$
-r +e -w +m -f $\rightarrow X$
+r -e -w +m -f $\rightarrow X$
-r -e -w -m +f $\rightarrow \checkmark$
-r +e -w -m +f $\rightarrow \checkmark$
-r -e +w -m +f $\rightarrow \checkmark$
+r -e +w +m -f $\rightarrow X$
-r -e -w +m +f $\rightarrow \checkmark$
+r -e -w +m +f $\rightarrow X$
-r +e +w -m +f $\rightarrow \checkmark$
-r +e -w -m +f $\rightarrow \checkmark$

.۲

-r -e +w +m +f $\rightarrow w_1 = 0.6 * 0.15 = 0.09$
-r -e -w +m +f $\rightarrow w_2 = 0.6 * 0.75 = 0.45$
-r -e +w -m +f $\rightarrow w_3 = 0.6 * 0.15 = 0.09$
-r +e -w -m +f $\rightarrow w_4 = 0.6 * 0.75 = 0.45$
$\sum weights = 1.08 \rightarrow \frac{0.09+0.45}{1.08} = P = 0.5$

.۳

$$P[+e|-r,+f,+m,-w] = \frac{P[+e,-r,+f,+m,-w]}{P[-r,+f,+m,-w]} = \frac{P[+e,-r,+f,+m,-w]}{\sum_e P[e,-r,+f,+m,-w]} =$$

$$\frac{P[-r]P[-w|-r]P[+e|-r]P[+m|+e,-w]P[+f|-w]}{\sum_e P[-r]P[-w|-r]P[e|-r]P[+m|e,-w]P[+f|-w]} =$$

$$\frac{P[-r]P[-w|-r]P[+e|-r]P[+m|+e,-w]P[+f|-w]}{P[-r]P[-w|-r]P[+f|-w]\sum_e P[e|-r]P[+m|e,-w]} = \frac{P[+e|-r]P[+m|+e,-w]}{\sum_e P[e|-r]P[+m|e,-w]}$$

$$= \frac{0.6*0.45}{0.6*0.45+0.4*0.9} = 0.43$$

.۴

$$P[+m|-r,+f] = \frac{P[-r,+f,+m]}{P[-r,+f]} = \frac{\sum_{e,w} P[-r,+f,+m,e,w]}{\sum_{m,e,w} P[m,-r,+f,e,w]} =$$

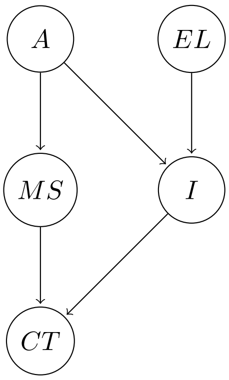
$$= \frac{\sum_{e,w} P[-r]P[w|-r]P[e|-r]P[+m|e,w]P[+f|w]}{\sum_{e,w,m} P[-r]P[w|-r]P[e|-r]P[m|e,w]P[+f|w]} = \frac{\sum_{e,w} P[w|-r]P[e|-r]P[+m|e,w]P[+f|w]}{\sum_{e,w,m} P[w|-r]P[e|-r]P[m|e,w]P[+f|w]}$$

سوال ۴

$U \perp V \to True$
$U \perp V|W \to False$:Common Effect
$U \perp V|Y \to False$:Common Effect
$U \perp Z|W \to False$:D-seperation
$U \perp Z|V, Y \to False$:D-seperation
$U \perp Z|X, W \to True$
$W \perp X|Z \to False$:D-seperation
$V \perp Z|X \to True$

سوال ۵

.۱



.۲

$$P[MS=0|CT=0,I=0] = \frac{P[MS=0,CT=0,I=0]}{P[CT=0,I=0]} = \frac{\sum_{A,EL} P[A,EL,MS=0,CT=0,I=0]}{\sum_{A,EL,MS} P[A,EL,MS,CT=0,I=0]}$$

$$= \frac{\sum_{A,EL} P[A]P[EL]P[MS=0|AL]P[I=0|A,EL]P[CT=0|MS=0,I=0]}{\sum_{A,EL,MS} P[A]P[EL]P[MS|AL]P[I=0|A,EL]P[CT=0|MS,I=0]}$$

سوال ۶

.۱

$P[B] \rightarrow states : 2$
$P[A|B] \rightarrow states : 4$
$P[C|B] \rightarrow states : 4$
$P[D|B] \rightarrow states : 4$
$P[E|B, C] \rightarrow states : 8$
$P[F|D, C, E] \rightarrow states : 16$

.۲

Each of the edges in the graph show a **conditional relationship**, meaning that if an edge goes from A to B, A would be the evidence for P[B|A]. The edges **don't necessarily** show dependancy because the **knowledge of other nodes** could lead to d-seperation of two nodes of an edge.

.۳

We could for example draw the edge from node B to F or turn the graph into a full graph.

.۴

If a bayesian network is a complete graph, we could conclude that it each of the variables of the network is related to other variables(nodes) and all nodes have direct casual relationship with each other. In a complete graph, the complexity of network analasys is rather high.
If a bayesian network is an empty graph, it means that there are no relations between the nodes of the bayesian network and all nodes are **conditionally independant of each other**.