به نام خدا

نظریه بازی‌ها

تمرین سوم نظری

استاد :

مرضیه نیلی‌پور

نویسنده :

محمدهومان کشوری

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سوال ۱

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action a^* evolutionary stable iff (a^*, a^*) is NE and for every action, b, that is a best response to a^*, $u(a^*, b) > u(b, b)$.
Now lets see, the only strategy which could be evolutionary stable in our case would be the third strategy because (a^*, a^*) need to be a NE and the only place where both strategies by player I and player II are the same and NE is (1,1).
Lets check the second condition, $u(3rd, b) > u(b, b)$, now we could see that the best response to the 3rd action is only the 3rd action so the second condition is also satisfied and we could say that **the 3rd strategy is evolutionary stable**.
Now for the mixed strategy, a mixed strategy, α is evolutionary stable if $(\alpha.\alpha)$ is a NE and for every strategy,β that is a best response to α, $u(\alpha, \beta) > u(\beta, \beta)$.
Lets find a mixed strategy in our game.
$U_{II}(1st, \{p_1, p_2, p_3\}) = 3p_2$

$U_{II}(2nd, \{p_1, p_2, p_3\}) = p_1$

$U_{II}(3rd, \{p_1, p_2, p_3\}) = p_3$

$3p_2 = p_1 = p_3, p_1 + p_2 + p_3 = 1 \rightarrow p_1 = p_3 = \frac{3}{7}, p_2 = \frac{1}{7}$

Game is symmetric so $U_{II} = U_I$.
Now we see that our mixed strategy is a NE so lets see the second condition.
$U(\beta, \beta) = U(\{p_1, p_2, p_3\}, \{p_1, p_2, p_3\}) = p_2p_1 + 3p_1p_2 + p_3^2 = p_3^2 + 4p_1p_2$

$U(\alpha, \beta) = U(\{\frac{3}{7}, \frac{1}{7}, \frac{3}{7}\}, \{p_1, p_2, p_3\}) = \frac{p_1}{7} + \frac{9p_2}{7} + \frac{p_3}{7} = \frac{1+8p_2}{7}$

Now : $\frac{1+8p_2}{7} > p_3^2 + 4p_1p_2$, if for example $p_1 \approx 0.339811, 0 \leq p_2 \leq 0.4438, p_3 \approx 0.0714286(9.24264 - 14p_2)$, then the inequality would not hold, hence the mixed strategy is **not an ESS**.

سوال ۲

This means that there exists an element on row i column i which is **bigger than all other elements on row i** and since the game is **symmetric**, this element is also **greater than all elements on column i** so we could say that this is the best response for both player 1 and player 2 and we know that in a symmetric game, if (a_i, a_i) is a **strict NE**, then it is also evolutionary stable strategy.
So to sum things up, we showed that since this game is symmetric and by the questions hypothesis, a_{ii} is a strict NE and we know that strict NE in symmetric games where both strategies are the same is an ESS.

سوال ۳

	x	y	z
x	$-\infty, -\infty$	$3, 1$	$0, 2$
y	$1, 3$	$0, 0$	$1, -\infty$
z	$2, 0$	$-\infty, 1$	$0, 0$

As you could see, there is no pure NE.
Now to find mixed NEs:
$U_1(x, P) = -\infty * p_1 + 3 * p_2$

$U_1(y, P) = p_1 + p_3$

$U_1(z, P) = 2p_1 - \infty * p_2$

We see that ∞ is making a problem in our equations and we cant solve them, simply because infinity is considered unknown and that is why in an infinity utility game, there is a possibility which we will not find any equilibriums.

Now for the second part think as if we remove the profiles with $-\infty$ and randomize between the other strategies,
each player now has expected of $6/6 = 1$ payoff.
We could also remove the profiles in which both players get 0,0 so that each players payoff would be $6/4 = 1.5$ and no player would deviate.
Consider this scenario that there is a third party which could draw (y,x) (z,x) (x,y) (x,z) and then recommend a strategy to each player, we know that the players won't deviate from the recommended strategy.

سوال ۴

First we will see for what range of x, the game would have pure ESS.
Action a would be ESS iff (a,a) is NE and for every b that is best respones to a, u(a,b) > u(b,b).

	i	ii
i	x, x	$0.5x, 0.5$
ii	$0.5, 0.5x$	$1, 1$

1. i is ESS
   (x,x) is NE $\rightarrow$ $0.5 < x \rightarrow ESS$
   x = 0.5 $\rightarrow$ u(i,i) = u(ii,i) $\rightarrow$ u(i,ii) < u(ii,ii) $\rightarrow$ not ESS.

2. ii is ESS
   (1,1) is NE $\rightarrow$ 0.5x < 1 $\rightarrow$ x < 2 $\rightarrow$ ESS
   x = 2 $\rightarrow$ u(ii,ii) = u(i,ii) $\rightarrow$ u(ii,i) < u(i,i) $\rightarrow$ not ESS.

Now if either x > 0.5 or 2 > x then the game would have ESS.

سوال ۵

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We know that P is the matrix of probabilities which hold the probabilities for mixed nash equilibrium, so $RP = cI \xrightarrow{RR^{-1}=I} P = cR^{-1}$
We know by the property of matrix inversion, a matrix could only have at best, one inverse so we know that P is unique.

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This equilibrium exists if R is inversable.

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The strategy is best response for both players due to A being symmetic, so this strategy would always be an ESS.

سوال ۶

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If $V - c > \alpha V \rightarrow (protect, protect) = strictNE \rightarrow protect = ESS$
We also know that in this case, the $(leave, leave)$ option would be strictly dominated so it could not be an NE, hence $leave$ is not ESS in this case.
If $\alpha V - c < 0 \rightarrow (leave, leave) = strictNE \rightarrow ESS$
We also know that in this case, the $(protect, protect)$ option would be strictly dominated so it could not be an NE, hence $protect$ is not ESS in this case.
We proved that in the first and second case, there would only exist one ESS.

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First lets see protect, we could see that $(protect, protect)$ is a strict NE because :
$V - c > (1-\alpha)V - c > \alpha V - c \xrightarrow{c > \alpha V} V - c > \alpha V \rightarrow protect = ESS$
Also leave is a strict NE because :
$\alpha V < c \rightarrow \alpha V - c < 0 \rightarrow leave = ESS$

So we saw that protect and leave are both ESS because they are **both strict NEs**.

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In this state we could see that neither (protect,protect) or (leave,leave) is not a strict NE so they could not be ESS, hence we need to find the mixed NE and then find if its a mixed ESS.
$U_1(protect, P) = p_1 * (V - c) + (1 - p_1)(\alpha V - c)$
$U_1(leave, P) = p_1 * \alpha V$

$p_1 \alpha V = p_1 * (V - c) + (1 - p_1)(\alpha V - c) \rightarrow p_1 = \frac{\alpha V - c}{(2\alpha - 1)V}$

Now lets see if its also ESS, to do this we should check that for every mixed strategy, β u(NE,β) $>$ u(β,β) :

$\frac{\alpha V - c}{(2\alpha-1)V} * p * (V - c) + (1 - \frac{\alpha V - c}{(2\alpha-1)V}) * p * \alpha V + \frac{\alpha V - c}{(2\alpha-1)V} * (1 - p) * (\alpha V - c) >$
$p^2 * (V - c) + p * (1 - p) * (\alpha V - c) + p * (1 - p) * \alpha V$
$\rightarrow True \rightarrow ESS$