به نام خدا

نظریه بازی‌ها

تمرین دوم نظری

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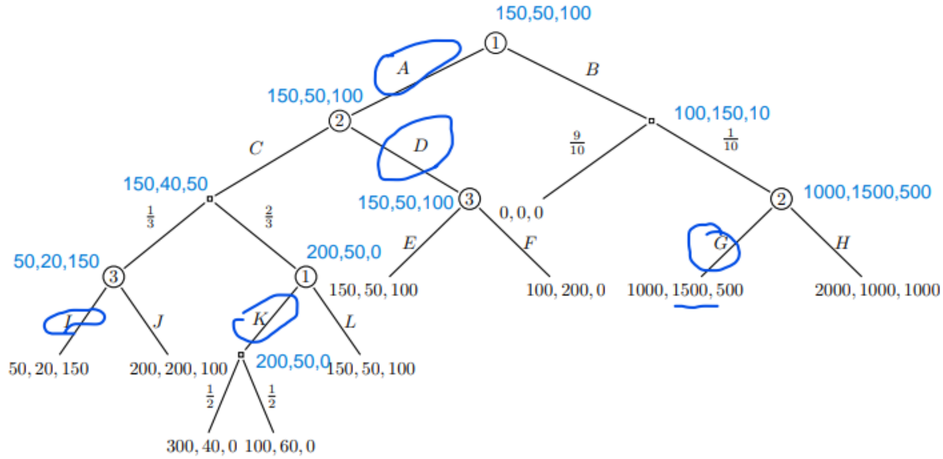
شماره دانشجویی :

۹۹۱۰۵۶۶۷

سوال ۱


150,50,100
1
A
B
150,50,100
2
100,150,10
9/10
1/10
C
D
150,40,50
150,50,100
3
0, 0, 0
2
1000,1500,500
1/3
2/3
E
F
G
H
50,20,150
3
200,50,0
1
150, 50, 100
100, 200, 0
1000, 1500, 500
2000, 1000, 1000
I
J
K
L
50, 20, 150
200, 200, 100
200,50,0
150, 50, 100
1/2
1/2
300, 40, 0
100, 60, 0

سوال ۲

For this question we consider the payoff for the players, if we want the strategy to converge to a nash equilibrium, we should declare the overall payoffs.
First off all, we should see the payoff of each player whenever in odd rounds, A,A is played and in even rounds, B,B is played.

$Payoff_{p1} = 0 * \beta^1 + 5 * \beta^2 + 0 * \beta^3 + \cdots = \sum_{i=0}^{\infty} \beta^{2i} * 5 = \frac{5\beta^2}{1-\beta^2}$

$Payoff_{p2} = 5 * \beta^1 + 0 * \beta^2 + 5 * \beta^3 + \cdots = \sum_{i=0}^{\infty} \beta^{2i-1} * 5 = \frac{1}{\beta} * \frac{5\beta^2}{1-\beta^2} = \frac{5\beta}{1-\beta^2}$

Now in order to get a nash equilibrium, no player should change their strategy. Now if a player wants deviate and change their strategy from A,A to A,B or from B,B to A,B and so forth , their utility would be as below :

$Payoff = 1 * \beta^1 + 1 * \beta^2 + 1 * \beta^3 + \cdots = \sum_{i=0}^{\infty} \beta^i = \frac{\beta}{1-\beta}$

$\frac{\beta}{1-\beta} < \frac{5\beta}{1-\beta^2} \xrightarrow{0<\beta<1} True$

$\frac{\beta}{1-\beta} < \frac{5\beta^2}{1-\beta^2} \rightarrow \frac{1}{4} < \beta < 1$

سوال ۳

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We know that player 2 would either choose (2,5) or (x,2) depending on player 1's choice, so:

$$\begin{cases} x < 2 \rightarrow & P1 : A, P2 : C \\ x > 2 \rightarrow & P1 : B, P2 : F \end{cases}$$

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We want to turn the game into a normal form game :

	CE	CF	DE	DF
A	2,5	2,5	1,0	1,0
B	1,0	x,2	1,0	x,2

$\xrightarrow{x=3}$

	CE	CF	DE	DF
A	2,5	2,5	1,0	1,0
B	1,0	3,2	1,0	3,2

Remove strictly dominated strategies :

	CE	CF	DE	DF
A	2,5	2,5	1,0	1,0
B	1,0	3,2	1,0	3,2

$\rightarrow$

	CE	CF	DF
A	2,5	2,5	1,0
B	1,0	3,2	3,2

We could show that no strategies are dominated by a mixed strategy.

$E(U(A, \{P_{CE}, P_{CF}, P_{DF}\})) = 2 * P_{CE} + 2 * P_{CF} + P_{DF}$

$E(U(B, \{P_{CE}, P_{CF}, P_{DF}\})) = 1 * P_{CE} + 3 * P_{CF} + 3P_{DF}$

$E(U(\{P_A, P_B\}, CE)) = 5 * P_A + 0 * P_B$

$E(U(\{P_A, P_B\}, CF)) = 5 * P_A + 2 * P_B$

$E(U(\{P_A, P_B\}, DF)) = 0 * P_A + 2 * P_B$

$2 * P_{CE} + 2 * P_{CF} + P_{DF} = 1 * P_{CE} + 3 * P_{CF} + 3P_{DF} \rightarrow$
$P_{CE} - P_{CF} - 2P_{DF} = 0$
$P_{CE} + P_{CF} + P_{DF} = 1$

$5 * P_A = 2 * P_B = 5 * P_A + 2 * P_B$

We could see the equation above is not right, hence we should remove CE and DF due to being weakly dominated.

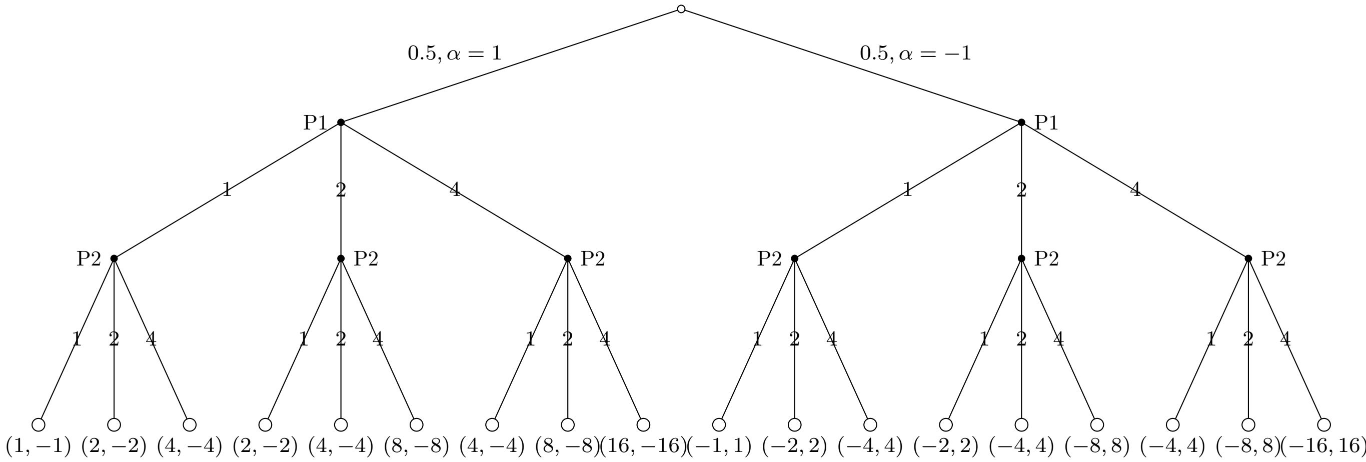
	CF
A	2,5
B	3,2

$\rightarrow$

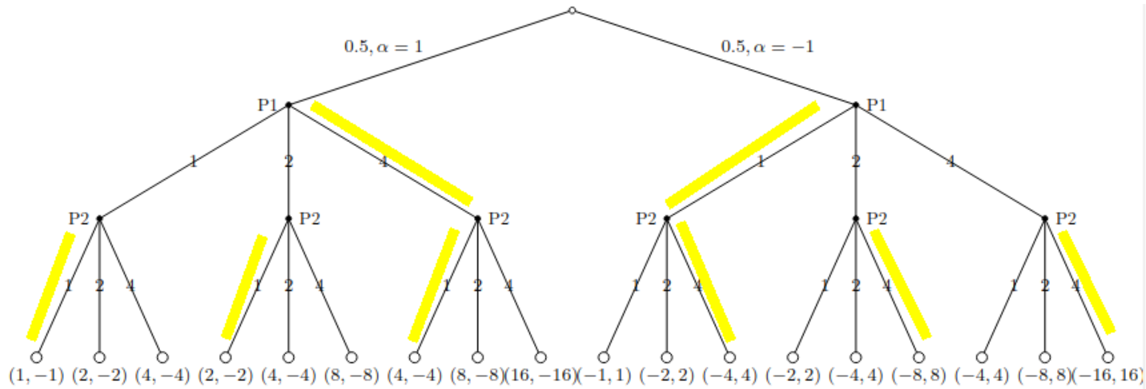
	CF
B	3,2

سوال ۴

If both players knew what α was, the question would be quite simple as below.



$$\text{if}: \begin{cases} \alpha = 1 \rightarrow & x:1, y:2 = x:2, y:1, , x:1, y:4 = x:2, y:2 = x:4, y:1, , x:2, y:4 = x:4, y:2 \\ \alpha = -1 \rightarrow & x:1, y:2 = x:2, y:1, , x:1, y:4 = x:2, y:2 = x:4, y:1, , x:2, y:4 = x:4, y:2 \end{cases}$$



As you could see the best strategies for each player if they each knew about α would be :

$$\begin{cases} \alpha = 1 \rightarrow & x:4, y:1 \\ \alpha = -1 \rightarrow & x:1, y:4 \end{cases}$$

Now if player 2 **doesn't know the value of** α, he would not know what branch player 1 picked but he know 2 facts, first of all player 1 would never choose 2 and second, player 2 will either choose from (4,-4) (8,-8) (16,-16) or (-1,1) (-2,2) (-4,4) so it would be rational for player 2 to pick the option with best mean value.

$$\text{player 2}: \begin{cases} 1 \rightarrow & (-4+1)/2 = -1.5 \\ 2 \rightarrow & (-8+2)/2 = -4 \\ 4 \rightarrow & (-16+4)/2 = -6 \end{cases}$$ So player 2 would be better off if he picked choice **1**

سوال ۵

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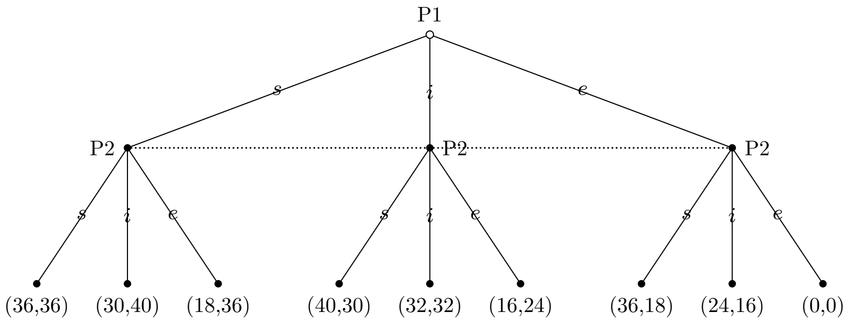
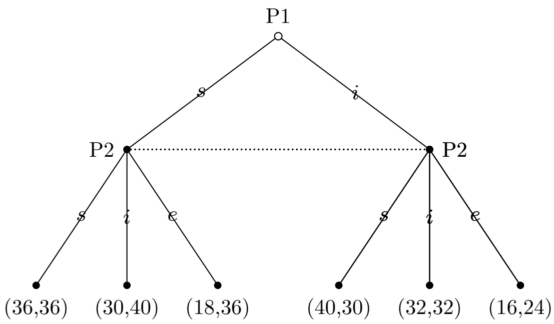
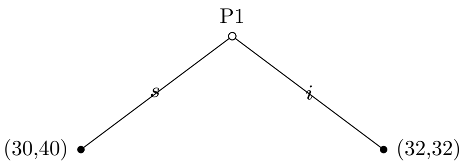
s = same, i = increase, e = extreamly increase



We know that player 1 doesn't know what player 2 plays but we know if player 1 chooses strategy e it doesn't matter because player 1's payoff will be less than strategy i.



We also know that either strategy that Player 1 chooses, player 2 will choose strategy i.



So player 1 would choose strategy i.
Nash equilibrium : i,i

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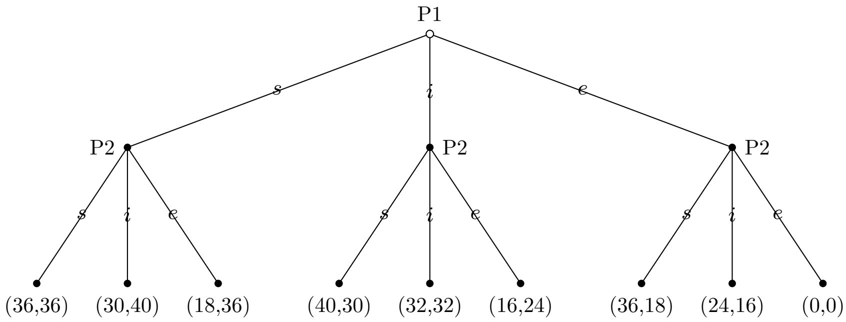
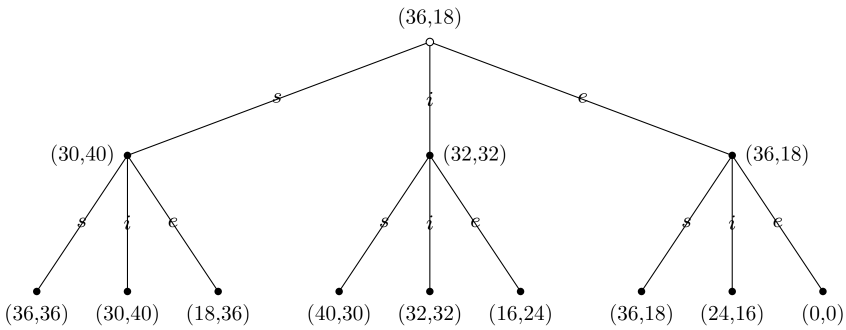
we just remove the imperfect information edge :

s = same, i = increase, e = extreamly increase



now we write the strategies.



strategy set for player 1 : (e), strategy set for player 2 : (i,s)

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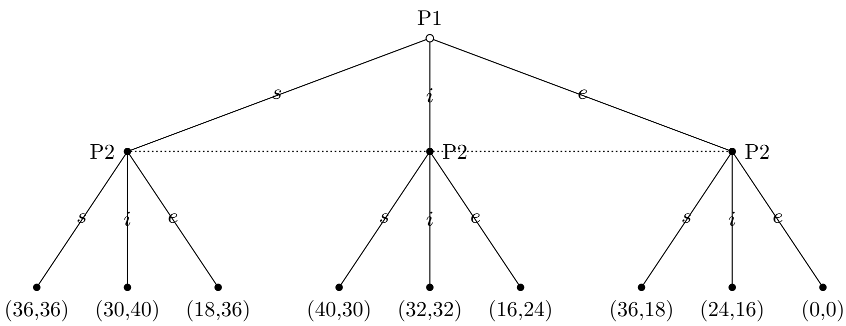
It is just like the first part because in the first part also player B didn't know player A's choice.

s = same, i = increase, e = extreamly increase



سوال ۶

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If each member of the board chooses its preference, clearly in the first level, Y would be removed and in the second level, X would be removed the only remaining option would be Z.

$$\text{Illustration}: \begin{cases} A: & X > Y > Z \rightarrow X > Z \\ B: & Z > Y > X \rightarrow Z > X \\ C: & Y > Z > X \rightarrow Z > X \end{cases}$$

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In this state, the members should be carefull because if they remove a choice it could impact the other groups.
In this state each member of A would know that group B and C's members have choice X as their least favorite option, hence X would not be chosen even if it makes it to the second level. So they would pick their second best option, Y knowing that it could possibly win in second level.
Members of B and C also know that their best options could be a potential winner in the second level so they would vote for their first option.

سوال ۷

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First we remove dominated strategies :
for player 1, C is definatly dominated by D and by itterative domination, D would also be the dominating strategy for player 2.
So the only strategy which is also the nash equilibrium is (D,D).

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Think of it as this way, they both have played for k-1 rounds using (C,C) as strategy, now player 1 chooses to deviate, and for n rounds after that would get 1 as reward.
Lets write the utility. Max utility for player 1 = U_{P1} and U'_{P1} = max utility after k+n+1 steps

In case of deviation : $U_{P1} = \sum_{i=1}^{k-1} \delta^{i-1} * 4 + 6 * \delta^k + \sum_{i=k+1}^{k+n+1} \delta^i + U'_{P1}$

In case of no deviation : $U_{P1} = \sum_{i=1}^{k-1} \delta^{i-1} * 4 + 4 * \delta^k + 4 * \sum_{i=k+1}^{k+n+1} \delta^i + U'_{P1}$

For the player to deviate, case of deviation should have more payoff than no deviation.
$\sum_{i=1}^{k-1} \delta^{i-1} * 4 + 6 * \delta^k + \sum_{i=k+1}^{k+n+1} \delta^i + U'_{P1} > \sum_{i=1}^{k-1} \delta^{i-1} * 4 + 4 * \delta^k + 4 * \sum_{i=k+1}^{k+n+1} \delta^i + U'_{P1} \rightarrow$
$2\delta^k > 3 * \sum_{i=k+1}^{k+n+1} \delta^i \xrightarrow{\sum_{i=k+1}^{k+n+1} \delta^i = \frac{\delta^{k+1}(1-\delta^n)}{1-\delta}} 2\delta^k > 3 * \frac{\delta^{k+1}(1-\delta^n)}{1-\delta} \rightarrow 2 > 3 * \frac{\delta(1-\delta^n)}{1-\delta} \xrightarrow{\delta=\frac{1}{2}}$
$2 > 3 * (1 - \frac{1}{2^n}) \rightarrow \frac{1}{2^n} > \frac{1}{3} \rightarrow 3 > 2^n \rightarrow n < \log_2^3 \xrightarrow{\text{n is natural}} n = 1$

So the only way deviation works for player 1 is when n is equal to 1, otherwise it would not be rational for the player to deviate.
So for $2 \leq n$ the strategy would be stable and the players would always play (C,C)