سوال ۱

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$$\begin{array}{ccc} & x_2 & y_2 \\ x_1 & 3, 7 & 6, 6 \\ y_1 & 2, 2 & 7, 3 \end{array}$$

(x1,x2) and (y1,y2) are the nash equilibriums.

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$$\begin{array}{ccc} & \cancel{x_2} & y_2 \\ x_1 & 7, \cancel{3} & 6, 6 \\ \cancel{y_1} & \cancel{2}, \cancel{2} & \cancel{3}, 7 \end{array}$$

(x1,y2) is the nash equilibrium.

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$$\begin{array}{cccc} & x_2 & y_2 & z_2 \\ x_1 & 0, 4 & 5, 6 & 8, 7 \\ y_1 & 2, 9 & 6, 5 & 5, 1 \end{array}$$

(x2,y1) and (x1,z2) are the nash equilibriums.

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$$\begin{array}{cccc} & x_2 & y_2 & z_2 \\ x_1 & 0, 0 & 5, 4 & 4, 5 \\ y_1 & 4, 5 & 0, 0 & 5, 4 \\ z_1 & 5, 4 & 4, 5 & 0, 0 \end{array}$$

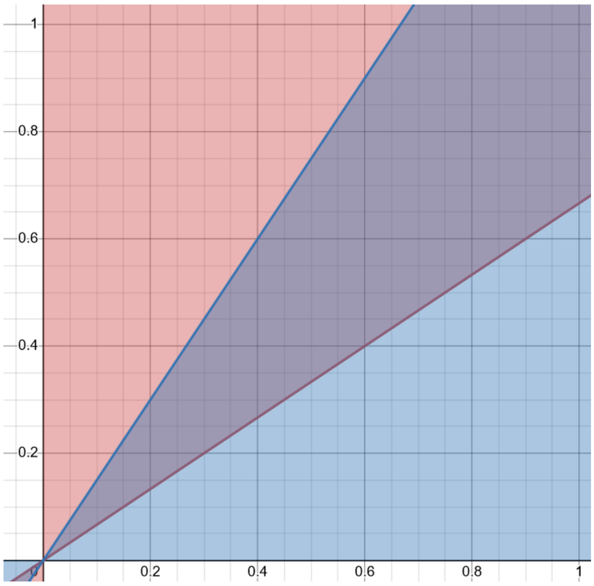
There is no pure nash equilibrium here.

سوال ۲

We could see that for player 2, if a2 >= 1, then the payoff would be less or equal to a2 = 0, so a2 >= 1 strategies are weakly dominated by a2 = 0 strategy, hence we should cut them off.
Now that we know that player 2 strategies are limited to only $0 \leq a_2 \leq 1$, we should find the payoffs :
If player 1 plays x_1 , the best possible move for player 2 would be $-\frac{a_1}{2} + \frac{1}{2}$ and if player 2 plays a_2, the best possible move for player 1 would be $\frac{a_2}{2}$.
If player 2 starts off with 1, player 1 would choose $\frac{1}{2}$ and if player 1 chooses $\frac{1}{2}$, player 2 would choose $\frac{1}{4}$ and so on ...
$a_2 = 1 \Rightarrow a_1 = \frac{1}{2} \Rightarrow a_2 = \frac{1}{4} \Rightarrow a_1 = \frac{1}{8} \Rightarrow ...$
If you get the limit of this equation you will see that $x_1 \rightarrow 0.2, x_2 \rightarrow 0.4$

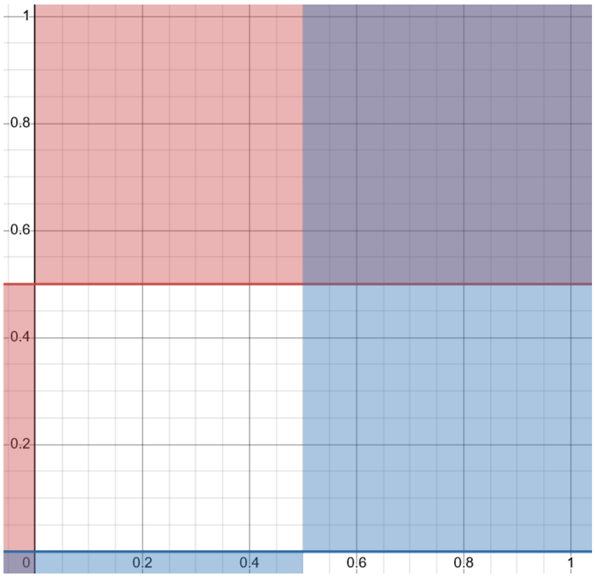
سوال ۳

We have two players, so the player space would be $N = \{p_1, p_2\}$
The action set for player i, Ai would be $a =$ interval of $(0, 1)$
For player i, the utility function would be $U_i = \frac{f(x_1,x_2)}{2} - c(x_i)$
We should now, find the inteval in which both players payoffs are equal or greater than zero

1. 

   As you could see, the intersection of the blue and red area is the place the game would take place(both payoffs are higher or equal to zero).
   If player 1, chooses 0 to work, then the utilities would be : $U_1 = 0$ and $U_2 = -x_2$ so then playet 2 would also choose 0 to work.
   So if player 1, plays x_1, player two would play argmax $\frac{3}{2}x_1x_2 - x_2^2$ which would be $x_2 = \frac{3x_1}{4}$ and player 1s payoff would be $\frac{9*x_1^2}{8} - x_1^2 = \frac{x_1^2}{8}$.
   We know that in interval of (0,1) player 1s payoff is strictly ascending so player 1s best choice would be to choose : 1
   So the best moves are when player 1 has move $\frac{3}{4} * x_2$ and player 2 has $\frac{3}{4} * x_1$ so this goes until both are 0 so nash equillibrium happens in point (0,0)

2.



As you could see, the intersection of the blue and red area is the place the game would take place(both payoffs are higher or equal to zero).
If player 1, plays x_1 then player 2 would play argmax $\frac{4}{2}x_1x_2 - x_2$ which would depend on x_1 :

$$\begin{cases} x_1 > \frac{1}{2} & x_2 = 1 \\ x_1 \leq \frac{1}{2} & x_2 = 0 \end{cases}$$

Then the payoff of player 1 would be :

$$\begin{cases} x_1 > \frac{1}{2} & x_1 \\ x_1 \leq \frac{1}{2} & -x_1 \end{cases}$$

We see that for player 1, it is best to choose 1 and also for player 2 it is best to choose 1 so the nash equillibrium would be in strategy (1,1)

سوال ۴

If we choose a profile in which a = (e,e,e,...), then we have nash equillibrium.
The proof is that $u_i = e$ and by mathamatical proof we show that for each player, utility of derivation of this profile is less that the original.
We assume that there exists one player j that chooses $e^{'}$:

$$\begin{cases} e^{'} < e \quad \rightarrow min_j e_j = e^{'} \rightarrow u_j = 2e^{'} - e^{'} < e \otimes \\ e^{'} > e \quad \rightarrow min_j e_j = e \rightarrow u_j = 2e - e^{'} < 2e - e \otimes \end{cases}$$

So we see that no other player would want to change the strategy, so a = (e,e,e,...) is a nash equillibrium.
Now we need to show that no other nash equillibrium points exist in this game.
We assume that there is a equillibrium $a = (e, e, e^{'}, e, ...)$ which at least one player has chosen a differenct strategy $e^{'}$ and the profile is also a nash equillibrium.

$$\begin{cases} e^{'} < e \quad \rightarrow min_j e_j = e^{'} \rightarrow u_j = 2e^{'} - e^{'} < e \\ e^{'} > e \quad \rightarrow min_j e_j = e \rightarrow u_j = 2e - e^{'} < 2e - e \end{cases} \Rightarrow \text{player with strategy } e^{'} \text{ will choose strategy e}$$

to get better utility hence the original point $a = (e, e, e^{'}, e, ...)$ was not a nash equillibrium.

سوال ۵

To describe a game we would need : Players, Action Set, Utilities
To simlify the question, we name Hamid : player 1 and Masoud : player 2
Players : $N = \{1, 2\}$
Action Set : $a_1 = (ACEG, ADEG, ADFG)$, $a_2 = (BCEH, BDFH, BDEH)$
Utilities : $u_1(a_1, a_2) = \text{cost for player 1}$, $u_2(a_1, a_2) = \text{cost for player 2}$

Now we draw the utility matrix :

$$\begin{array}{cccc} & BCEH & BDFH & BDEH \\ ACEG & -13, -10 & -11, -5 & -11, -7 \\ ADEG & -8, -8 & -8, -5 & -10, -9 \\ ADFG & -5, -8 & -6, -6 & -5, -7 \end{array}$$

And now to remove the dominated strategies :

$$\begin{array}{cccc} & \cancel{BCEH} & BDFH & \cancel{BDEH} \\ \cancel{ACEG} & -13, -10 & -11, -5 & -11, -7 \\ \cancel{ADEG} & -8, -8 & -8, -5 & -10, -9 \\ ADFG & -5, -8 & -6, -6 & -5, -7 \end{array}$$

$$\begin{array}{cc} & BDFG \\ ADFG & -6, -6 \end{array}$$

So the nash equilibrium would be : (ADFG,BDFG) ✓

We want the nash equilibrium to happen in (ACEG,BCEH) so it should not be dominated.
-10 should become more than -5 and -13 should become more than -5.
cost of multiple passes from CE < 2
cost of single pass from CE $<$ cost of multiple passes from CE < 2
In this case if our costs are integers and we dont have negative costs, then the only feasable solution would be CE = (0,1).
Now we draw our utility matrix once again :

$$\begin{array}{cccc} & BCEH & BDFH & BDEH \\ ACEG & -7, -4 & -6, -5 & -6, -7 \\ ADEG & -8, -3 & -8, -5 & -10, -9 \\ ADFG & -5, -3 & -6, -6 & -5, -7 \end{array}$$

As we could see, ACEG is weakly dominated by ADFG so point (ACEG,BCEH) could not be a nash equilibrium.