Miniresearch Project

Decays of the Higgs boson

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Contents

1. Introduction

The Higgs mechanism is one of the cornerstones of the standard model as it gives masses to gauge bosons and fermions in a gauge invariant way. As a side product of this mechanism an additional scalar, neutral, massive particle has to be added to the spectrum of elementary particles: the Higgs boson. The very recent, possible discovery of the Higgs boson at 125 GeV at ATLAS as well as CMS [Aad12; Cha12] may be once again a confirmation of the theoretical concept of local gauge invariance, similar to the discovery of the W and Z boson. In order to analyze properties of the Higgs boson, it is necessary to calculate the decay width into the various particles of the standard modell (SM) and compare them. This is summarized in the so called branching ratios, which give the probability that the Higgs boson decays in a certain decay channel at a given Higgs mass.

To examine the possible decay channels, we will start to follow the path in the final project of Ref. [PS95]. This means that at first, we calculate the possible decays into pairs of fermions and W and Z bosons at tree level and then the one loop order decay into gluons. Finally the obtained branching ratios will be discussed and compared to the literature. We assume that the reader has knowledge about the standard model as it is summarized in the first chapter of Ref. [Djo08], for example, and only sketch some aspects which are important for the project in Section 1.1. Because we want to calculate the correction to our tree level results using effective masses, we briefly review some techniques of the renormalization group in Section 1.2. We work in natural units $\hbar = c = 1$ with the metric with signature $(+ - - -)$ and omit as many indices as possible.

1.1. Spontaneous symmetry breaking in the GWS model

The Glashow-Weinberg-Salam (GWS) electroweak theory is a Young-Mills theory based on the symmetry group $SU(2)_L \times U(1)_Y$ [Djo08]. Combined with the $SU(3)_C$ quantum chromodynamics (QCD) gauge theory, it is usually called the SM and describes three of the basic forces of nature: the electromagnetic, weak and strong force. The indices L, Y and C denote left-handed, hypercharge and color, respectively. The corresponding gauge fields are W_μ, B_μ and G_μ. To obtain gauge invariant objects, we need the covariant derivative D_μ instead of the partial derivative ∂_μ:

$$\mathrm{D}_\mu = \partial_\mu - \mathrm{i}g_s T_a G^a_\mu - \mathrm{i}g_2 T_a W^a_\mu - \mathrm{i}g_1 \frac{Y}{2} B_\mu \tag{1}$$

which leads through the massless Dirac equation $\bar{\psi}_f \, \mathrm{i}\not{\mathrm{D}} \, \psi_f = 0$ to couplings between fermion and gauge fields, where ψ_f stands for any type of fermion. The particle spectrum of the SM consists of left-handed leptons L, right-handed leptons e_{R_i} as well as left-handed quarks Q and right-handed quarks u_R, d_R:

$$L_i = \begin{pmatrix} \nu_{e_i} \\ {e_i}^- \end{pmatrix}_L, \quad e_{R_i} = \begin{pmatrix} {e_i}^- \end{pmatrix}_R, \qquad i \in \{1, 2, 3\}, \quad e_i \in \{e, \mu, \tau\}, \tag{2}$$

$$Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad u_{R_i} = \begin{pmatrix} u_i \end{pmatrix}_R, \quad d_{R_i} = \begin{pmatrix} d_i \end{pmatrix}_R, \quad u_i \in \{u, c, t\}, \quad d_i \in \{d, s, b\} \tag{3}$$

We can see that the left-handed fermions are doublets while the right-handed are singlets. Summing up the kinetic terms for leptons and quarks and adding the traces of contracted field strengths $F_{\mu\nu} = \mathrm{i}\,[\mathrm{D}_\mu, \mathrm{D}_\nu]$ for each of the three gauge fields defined in Eq. (1), we obtain the action of the SM without masses. This Lagrangian is invariant under local gauge transformations. In the electroweak sector, i.e. whithout quarks and gluons, these can be written as

[Djo08]

$$L(x) \to L'(x) = \mathrm{e}^{\mathrm{i}(T^a\alpha_a(x)+Y\beta(x))}L(x)\ ,\ e_R(x) \to e'_R(x) = \mathrm{e}^{\mathrm{i}Y\beta(x)}e_R(x) \quad \text{and} \tag{4a}$$

$$\boldsymbol{W}_\mu(x) \to \boldsymbol{W}_\mu(x) - \frac{1}{g_2}\partial_\mu\boldsymbol{\alpha}(x) - \boldsymbol{\alpha}(x) \times \boldsymbol{W}_\mu(x)\ ,\ B_\mu(x) \to B_\mu(x) - \frac{1}{g_1}\partial_\mu\beta(x), \tag{4b}$$

where the fact that there are three generators T^a for the $SU(2)$ is abused in a vector notation and the cross product follows from the adjoint representation of the gauge bosons. We cannot add the mass term $-m_f\bar{\psi}_f\psi_f = -m_f(\bar{\psi}_{f,R}\psi_{f,L} + \bar{\psi}_{f,L}\psi_{f,R})$ to our Lagrangian because the left-handed and right-handed fermions transform differently as seen in Eq. (4a). Furthermore Eq. (4b) tells us that mass terms for the gauge bosons like $\frac{1}{2}\,m_V^2 W_\mu W^\mu$ are no gauge invariant objects. In order to take the experimental fact into account that fermions and $W^\pm, Z$ bosons are massive, we employ the Higgs mechanism. Let Φ be an complex $SU(2)$ doublet of scalar, colorless fields ϕ:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad Y_\phi = +1\ . \tag{5}$$

The most general form of Langrangian for this field, consistent with $SU(2) \times U(1)$ gauge invariance, Lorentz invariance, and renormalizability is [Wei05]:

$$\mathcal{L}_\phi = (\mathrm{D}^\mu\Phi)^\dagger(\mathrm{D}_\mu\Phi) - V(\Phi) \quad \text{with} \quad V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2\ . \tag{6}$$

For $\lambda > 0$ and $\mu^2 < 0$ the potential has a minimum at

$$\left\langle 0 \middle| \phi^\dagger\phi \middle| 0 \right\rangle \equiv v^2 = -\mu^2/\lambda\ , \tag{7}$$

which is called a nonzero vacuum expectation value (vev). Since Φ has four degrees of freedom, the minima lie on a 3-sphere with radius v. However, we have to expand around *one* of the stable minima in order to get fields with zero vev again. The fact that this ground state of the system is not symmetric under the symmetry group of $\mathcal{L}_\phi$ is called spontaneous symmetry breaking (SSB). In fact, the Goldstone theorem states that for each broken generator of the original symmetry group a massless scalar Goldstone boson will appear [PS95]. The physical idea behind these Goldstone bosons is that they run along the surface where the potential does not change. In a theory with *local* gauge invariance these bosons can disappear and give masses to gauge fields. By writing Φ in spherical coordinates and expanding around the vev

$$\Phi = \begin{pmatrix} \phi_2(x) + \mathrm{i}\phi_1(x) \\ \frac{1}{\sqrt{2}}(v + H(x)) - \mathrm{i}\phi_3(x) \end{pmatrix} \overset{1.\text{order}}{=} \mathrm{e}^{\mathrm{i}\phi_a(x)\sigma^a(x)\,\sqrt{2}/v} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix}\ , \tag{8}$$

we see that the phase, i.e. the Goldstone bosons, can be absorbed in an appropriate gauge which is called the *unitary gauge* where $\phi^+ = 0$ and $\phi^0 = \frac{1}{\sqrt{2}}(v + H(x))$. We can identify the Higgs field H with radial oscillations around the vev. Note that it is neutral, following from $I_3 = -\frac{1}{2}$ and the definition of the electric charge via the Gell-Mann-Nishijima relation: $Q = I_3 + \frac{1}{2}Y = 0$ [BDJ01]. So this choice of vacuum breaks $SU(2)_L \times U(1)_Y$ but leaves $U(1)_{EM}$ invariant as one can see directly in a matrix representation of Q and the infinitesimal change $Q\Phi = 0$. This leads to massive $W^\pm_\mu$ and Z_μ bosons and a massless photon A_μ defined by

$$W^\pm = \frac{1}{\sqrt{2}}\left(W^1 \pm \mathrm{i}W^2\right), \quad Z = \frac{g_2 W^3 - g_1 B}{\sqrt{g_2^2 + g_1^2}}, \quad A = \frac{g_1 W^3 + g_2 B}{\sqrt{g_2^2 + g_1^2}}, \tag{9}$$

if we simply expand Eq. (6):

$$\begin{aligned}\mathcal{L}_\phi = &\frac{1}{4}(v+H)^2 g_2^2 W^-_\mu W^{+\mu} + \frac{1}{8}(v+H)^2\left(g_2 W^3 - g_1 B\right)_\mu \left(g_2 W^3 - g_1 B\right)^\mu \\ &+ \frac{1}{2}\partial_\mu H \partial^\mu H - \mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4}H^4 \,.\end{aligned} \tag{10}$$

Now we can read off that the Higgs boson has the mass $m_H = \sqrt{2}\mu$ and $m_W = \frac{g_2}{2}v$, $m_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}v$. This Lagrangian also contains the coupling of the Higgs to the other gauge bosons which we use later on. In addition, one obtains masses for fermions by plugging in Φ in the Yukawa Lagrangian which has to be included as well as possible gauge invariant term:

$$\mathcal{L}_Y = -\lambda_e \bar{L}\Phi e_R - \lambda_d \bar{Q}\Phi d_R - \lambda_u \bar{Q}\tilde{\Phi} u_R + h.c. \tag{11}$$

Here, we used $\tilde{\Phi} = \mathrm{i}\tau_2 \Phi^*$. This leads for example for the electron to

$$\mathcal{L}_Y = -\underbrace{\frac{\lambda_e v}{\sqrt{2}}}_{m_e}(1 + H/v)\bar{e}_L e_R + \ldots\,, \tag{12}$$

containing the mass term which we could not add due to gauge invariance in the beginning. Here we see why the coupling of the Higgs boson to fermions is always proportional to the mass of the fermion.

In Section 2.4, we need the coupling of the Higgs to Goldstone bosons which did not appear in the unitary gauge above but gave mass to the gauge bosons. So let us pretend we have not done the gauge transformation and plug in the linear expansion of Eq. (8) into the potential in Eq. (6). With $\Phi^\dagger\Phi = 2\phi^+\phi^- + \frac{1}{2}(v+H)^2 + \left(\phi^3\right)^2$, we can see that the interaction terms between Higgs and Goldstone bosons are in the λ term and read:

$$\left(\frac{m_H^2}{2v^2}\right) vH\left(2\phi^+\phi^- + \left(\phi^3\right)^2\right) \subset \mathcal{L}_\phi \tag{13}$$

With an appropriate tool to derive vertices of Feynman diagrams, e.g. functional derivatives in the path integral formalism $\frac{\delta}{\mathrm{i}\delta\phi^3}$, we get an additional factor of two in the coupling of H to $\phi^3\phi^3$ in Eq. (13). Now we can read off directly the Feynman rules from the above equation.

1.2. Renormalization group and running masses

Let $G^{(n)}(x_1,\dots,x_n;g_0,M_0) = \langle 0\,|\,\mathrm{T}\phi(x_1)\dots\phi(x_n)\,|0\rangle$ be an arbitrary n-Point Green's function of a massless theory, renormalized at an energy scale M_0. Changing the point of renormalization scale should not change the physics and therefore not the S matrix elements which follow from the Green's functions [BDJ01]. In perturbative renormalization schemes we usually change $g_0 \to g$ and $\phi \to \sqrt{Z}\phi$. Anticipating these changes, this leads to [Ohl12]

$$G^{(n)}(x_1,\dots,x_n;g,M)\;Z^{n/2}(M_0,M) = G^{(n)}(x_1,\dots,x_n;g_0,M_0) \tag{14a}$$

$$\Rightarrow M\frac{\mathrm{d}}{\mathrm{d}M}\left(G^{(n)}(x_1,\dots,x_n;g,M)\;Z^{n/2}(M_0,M)\right) = 0 \tag{14b}$$

$$\Rightarrow \left(M\frac{\partial}{\partial M} + \underbrace{M\frac{\mathrm{d}g(M)}{\mathrm{d}M}}_{\equiv\beta(g)}\frac{\partial}{\partial g} + n\underbrace{\frac{1}{2Z(M_0,M)}\frac{\mathrm{d}Z(M_0,M)}{\mathrm{d}M}}_{\equiv\gamma_\phi(g)}\right)G^{(n)} = 0\,, \tag{14c}$$

where we simply used the chain and product rule. Eq. (14c) is called the Callan-Symanzik-Equation. This type of partial differential equation can be solved using the method of characteristics. With $G(g,\phi,M) \to G(\bar{g}(t),\bar{\phi}(t),t)$, the variable substitution $t = \log\frac{Q^2}{M^2}$ and $\partial_t = -2M\partial_M$, the Callan-Symanzik equation implicitely requires the renormalization group equation

$$\frac{\mathrm{d}}{\mathrm{d}\log Q/M}\bar{g} = \beta(\bar{g})\;, \quad \text{with the initial condition} \quad \bar{g}(M) = g\;. \tag{15}$$

Now we want to expand this formalism to handle massive theories like $\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$. The mass m actually looks quite like a coupling and we should expect that it will run like a coupling if we change the energy scale of our processes. Considering this we get an additional term

$$\left(M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \gamma_m(g)m\frac{\partial}{\partial m} + \gamma_\phi(g)n\right)G^{(n)} = 0, \qquad \gamma_m = \frac{M}{m}\frac{\mathrm{d}m}{\mathrm{d}M}\;. \tag{16}$$

If we look at Eq. (16), we see that β and $m\gamma_m$ should be treated on an equal footing. The running mass $\bar{m}(\mu)$ will be in fact calculated with [PS95]

$$\frac{\mathrm{d}}{\mathrm{d}\log Q/M}\bar{m} = \gamma_m(\bar{g})\bar{m}\;, \qquad \bar{m}(M) = m \tag{17}$$

In QCD with three colors and n_f approximately massless quarks, the β function can be calculated in one loop order as [PS95]

$$\beta(g) = -\frac{b_0 g^3}{(4\pi)^2} \quad \text{with} \quad b_0 = 11 - \frac{2}{3} n_f \; . \tag{18}$$

In this case, the solution to Eq. (15) is easy to find:

$$\alpha_S(Q) = \frac{\alpha_S}{1 + \frac{b_0 \alpha_S}{2\pi} \log \frac{Q}{M}} \; , \tag{19}$$

using $\alpha_S(Q) = \bar{g}^2/(4\pi)$ and the initial condition $\alpha_S = g^2/(4\pi)$. An alternative way to write this is to define a mass scale Λ by

$$1 \equiv g^2 \frac{b_0}{8\pi^2} \log \frac{M}{\Lambda} = -\frac{\alpha_S b_0}{2\pi} \log \frac{\Lambda}{M} \; . \tag{20}$$

Then we can expand the fraction in Eq. (19) and get rid off M:

$$\alpha_S(Q) = \frac{2\pi}{b_0 \log Q/\Lambda} \; . \tag{21}$$

Note, that now Λ is just the scale at which α_S becomes strong and in QCD, experimental measurements give $\Lambda \approx 200\,\text{MeV}$ [PS95].

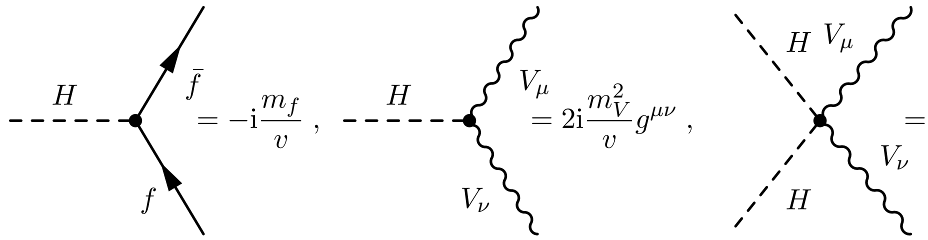
2. Higgs decays at tree level

In this section, we will discuss the decays which are possible at tree level, i.e. without loops. The bare result for the decay into quarks will be improved by using the running mass. The decay into vector bosons W, Z can be checked in the high-energy limit (large m_H) by means of the Goldstone Boson Equivalence Theorem.

2.1. $H \to \bar{f} f$

We start with the simplest decay of a Higgs boson H to two fermions $f\bar{f}$ which may be leptons or quarks. The couplings of H to fermions and gauge bosons are basically given by Eq. (12)

and Eq. (10)



$$= -\mathrm{i}\frac{m_f}{v}\ , \qquad = 2\mathrm{i}\frac{m_V^2}{v}g^{\mu\nu}\ , \qquad = 2\mathrm{i}\frac{m_V^2}{v^2}g^{\mu\nu}\ , \tag{22}$$

where $V \equiv Z^0$ or $W^\pm$. The sign of the Hff coupling has been taken from Ref. [PS95]. As noted before, the coupling is proportional to the mass of the involved particles $(v+H)$, therefore we expect that the decay of H to heavy particles will dominate if kinematically possible. The polarized matrix element $\tilde{\mathcal{M}}$ for $H \to f\bar{f}$ follows directly from the first diagram in Eq. (22):

$$\tilde{\mathcal{M}} = \bar{u}(p_1)\left(-\mathrm{i}\frac{m_f}{v}\right)v(p_2)\ . \tag{23}$$

Here p_1 and p_2 are the momenta of the outgoing fermion and antifermion, respectively. For the unpolarized matrix element $\mathcal{M}$ we have to sum over the possible spin directions which can be simplified using the identity

$$\sum_{s_3} u^\pm(\boldsymbol{p}, s_3)\bar{u}^\pm(\boldsymbol{p}, s_3) = \left(\not{p} \pm m\right)\ , \qquad u^+ \equiv u, \quad u^- \equiv v \tag{24}$$

and obtain

$$|\mathcal{M}|^2 = \sum_{\text{spins}} \left|\tilde{\mathcal{M}}\right|^2 \tag{25a}$$

$$= \sum_{\text{spins}} \bar{u}(p_1)\underbrace{v(p_2)\bar{v}(p_2)}u(p_1)\left(\frac{m_f}{v}\right)^2 \tag{25b}$$

$$= \mathrm{Tr}\left[(\not{p}_2 - m_f)(\not{p}_1 + m_f)\right]\left(\frac{m_f}{v}\right)^2 \tag{25c}$$

$$= \left(p_1 p_2 - m_f^2\right)\mathrm{Tr}\left[\mathbb{1}\right]\left(\frac{m_f}{v}\right)^2\ . \tag{25d}$$

Here we used that the trace of an odd number of γ matrices is zero and $\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\right] = g_{\mu\nu}\mathrm{Tr}\left[\mathbb{1}\right]$, due to cyclic property of the trace and the defining relation $\left[\gamma_\mu, \gamma_\nu\right]_+ = 2g_{\mu\nu}\mathbb{1}$. To obtain the total decay width Γ, we use

$$\mathrm{d}\Gamma = \frac{1}{2E(p)}\left|\mathcal{M}\right|^2 \mathrm{dLIPS} \quad \text{with} \quad \mathrm{dLIPS} = (2\pi)^4\delta^4(p - \sum_i p_i')\prod_i \mathrm{D}p_i' \tag{26}$$

and $\mathrm{D}p_i = \frac{\mathrm{d}^3 p_i}{2E(p_i)(2\pi)^3}$. For our case of $1 \to 2$ body decay, we can simplify this by going into the center of mass system (cms) of the Higgs boson, where $s = m_H^2 = E(p)^2$, $p = (m_H, \mathbf{0})$, $p_{1,2} = (E(p_{1,2}), \pm\boldsymbol{p})$ and $E(p_{1,2}) = \sqrt{m_f^2 + \boldsymbol{p}^2}$. This yields

$$\mathrm{d}\Gamma = \frac{1}{2m_H} |\mathcal{M}|^2 \left(\frac{1}{(2\pi)^2} \frac{1}{4} \frac{\mathrm{d}^3 p_1}{E(p_1)} \frac{\mathrm{d}^3 p_2}{E(p_2)} \delta(m_H - 2E(p_{1,2}))\delta^3(\boldsymbol{p} - \boldsymbol{p}) \right) \tag{27a}$$

$$= \frac{|\mathcal{M}|^2}{32\pi^2 m_H} \delta(m_H - 2E(p_{1,2})) \frac{\mathrm{d}^3 p_1}{E(p_{1,2})^2} \qquad \text{using} \quad \mathrm{d}^3 p_1 = \boldsymbol{p}^2 d\,|\boldsymbol{p}|\,\mathrm{d}\Omega \tag{27b}$$

$$= \frac{|\mathcal{M}|^2}{32\pi^2 m_H} \delta(m_H - 2E(p_{1,2})) \frac{|\boldsymbol{p}|\,\mathrm{d}\Omega \mathrm{d}E(p_{1,2})}{E(p_{1,2})} \qquad \text{since} \quad \frac{\mathrm{d}E(p_{1,2})}{\mathrm{d}\,|\boldsymbol{p}|} = \frac{|\boldsymbol{p}|}{E(p_{1,2})} \tag{27c}$$

$$= \frac{|\mathcal{M}|^2}{32\pi^2 m_H^2} |\boldsymbol{p}|\,\mathrm{d}\Omega\,. \tag{27d}$$

Now we can plug in Eq. (25d), use $p_1 p_2 = m_f^2 + 2\boldsymbol{p}^2$ and obtain

$$\mathrm{d}\Gamma = \frac{1}{32\pi^2 m_H^2} 2\,|\boldsymbol{p}|^3\, 4 \left(\frac{m_f}{v}\right)^2 \mathrm{d}\Omega\,. \tag{28}$$

Due to energy conservation $|\boldsymbol{p}| = \sqrt{\frac{m_H^2}{4} - m_f^2}$ and we can carry out the trivial integration over the sphere:

$$\Gamma = \frac{m_f^2 m_H}{8\pi v^2} \left(1 - 4\frac{m_f^2}{m_H^2}\right)^{3/2} \qquad \text{with} \quad v = \frac{2m_W \mathrm{s}_W}{e}, \quad \alpha = \frac{e^2}{4\pi} \tag{29a}$$

$$= \frac{m_f^2 m_H \alpha}{8 m_W^2 \mathrm{s}_W^2} \left(1 - 4\frac{m_f^2}{m_H^2}\right)^{3/2}, \tag{29b}$$

using the shorthand notation $\mathrm{s}_W \equiv \sin\theta_W$. Since quarks can occur in three colors, we have to multiply the result with an additional factor N_f which is 3 (1) for quarks (leptons) due to the sum.

2.2. Improvements using effective masses

To calculate the effective mass $\bar{m}(Q)$, we need the anomalous mass dimension γ_m. In one loop order, it can be calculated as [PS95]

$$\gamma_m = -8\frac{g^2}{(4\pi)^2}\,. \tag{30}$$

Now we have to keep in mind that the coupling constant itself is a function of the momentum scale and is given by Eq. (21). Plugging this in Eq. (17), we have to solve

$$\frac{\mathrm{d}}{\mathrm{d}\log Q/M}\bar{m} = -\frac{8}{b_0 \log Q^2/\Lambda^2}\bar{m} \; . \tag{31}$$

After seperating the variables this equation can be integrated. A convenient way of writing the resulting constant, which makes the satisfaction of the initial condition obvious, is

$$\bar{m}(Q^2) = \left(\frac{\log M^2/\Lambda^2}{\log Q^2/\Lambda^2}\right)^{4/b_0} m \; , \tag{32}$$

or in terms of Eq. (21)

$$\bar{m}(Q^2) = \left(\frac{\alpha_S(Q^2)}{\alpha_S(M^2)}\right)^{4/b_0} m \; . \tag{33}$$

We use this expression for the effective masses to improve the bare results we obtained above. In our process Q^2 corresponds to m_H^2. Using the masses of Ref. [al12]

$$\begin{aligned} &\bar{m}_u(2\,\mathrm{GeV}) = 2.3\,\mathrm{MeV} && \bar{m}_d(2\,\mathrm{GeV}) = 4.8\,\mathrm{MeV} && \bar{m}_s(2\,\mathrm{GeV}) = 95\,\mathrm{MeV} \\ &\bar{m}_c(m_c) = 1.275\,\mathrm{GeV} && \bar{m}_b(m_b) = 4.18\,\mathrm{GeV} && \bar{m}_t(m_t) = 173.5\,\mathrm{GeV} \; , \end{aligned} \tag{34}$$

as well as $\alpha_S(m_Z) = 0.1184$, we start with five active flavors at 5 GeV. Lower values for m_H definitely violate vacuum stability and are highly excluded by LEP searches as discussed in Section 3.6. This saves us some work in the matching & running process. We calculate the running mass, according to Eq. (33), up to m_t and then use $\alpha_S(m_t^2)$ and $\bar{m}(m_t^2)$ as new starting point, which is called matching. Then we continue with six flavors which leads to a different exponent for the masses as well as the slope in Eq. (21). However this effect can not really be seen in Fig. 1 since the change amounts only $\big(b_0(n_f = 5) - b_0(n_f = 6)\big)/b_0(n_f = 5) = 9\,\%$. Overall, we see significant changes in the decay width compared to the calculation using only constant masses.

2.3. $H \to W^+W^-, Z^0Z^0$

We continue with the second graph of Section 2.1. Note that this process can only occur as real process if $m_H > 2m_W$. With the comparison of data and prediction for these processes, a heavy Higgs boson may be excluded. The polarized matrix element reads

$$\tilde{\mathcal{M}} = \epsilon^*_{\lambda_1}(p_1)_\mu \left(2\mathrm{i}\frac{m_V^2}{v} g^{\mu\nu}\right) \epsilon^*_{\lambda_2}(p_2)_\nu \equiv 2\mathrm{i}\frac{m_V^2}{v}\epsilon_2^{*\mu}\epsilon^*_{1,\mu} \; . \tag{35}$$

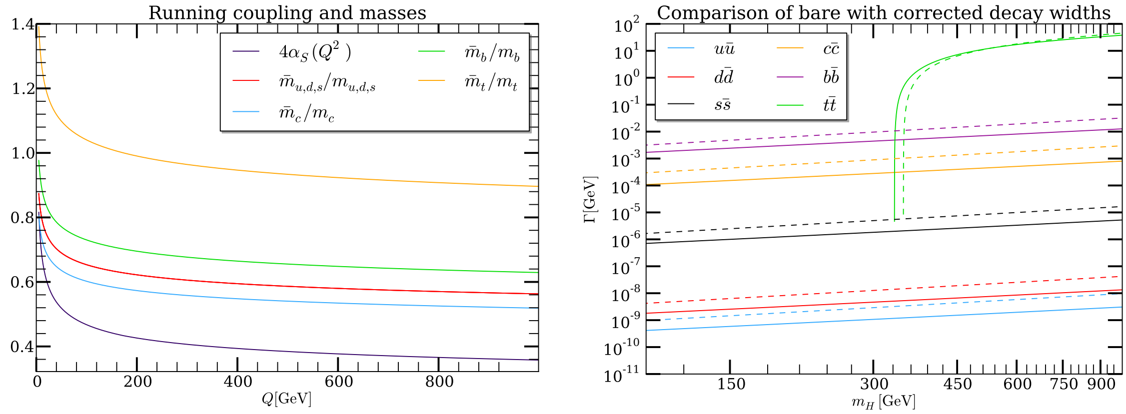


Figure 1: In the left panel, the ratio of the running masses to the bare masses is shown and we see that the running masses are a factor $\sim 1.5 - 2$ smaller in the intermediate mass range of the Higgs boson. In addition the strong coupling constant α_S is scaled with a factor of four in this figure for a convenient representation. On the right panel we plot the total decay width. The corrected (bare) decay widths are drawn with solid (dashed) lines. Note that above the threshold $2m_t$ the quark masses do not vary very much. This is why the top decay width receives only moderate corrections.

Using the polarization sum for massive gauge bosons

$$\sum \epsilon^{\mu}(q)\epsilon^{\nu *}(q) = -\left(g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{m_V^2}\right) , \tag{36}$$

we obtain

$$|\mathcal{M}|^2 = \sum 4\frac{m_V^4}{v^2}\epsilon_2^{\mu}\epsilon_2^{\nu *}\epsilon_{1,\mu}\epsilon_{1,\nu}^{*} \tag{37a}$$

$$= 4\frac{m_V^4}{v^2}\left(4 - \frac{p_1^2 + p_2^2}{m_V^2} + \frac{(p_1 p_2)^2}{m_V^4}\right) \tag{37b}$$

$$= 4\frac{m_V^4}{v^2}\left(3 + 4\left(\frac{\boldsymbol{p}^2}{m_V^2} + \frac{\boldsymbol{p}^4}{m_V^4}\right)\right) \tag{37c}$$

$$= 4\frac{m_V^4}{v^2}\left(3 + \frac{m_H^2}{m_V^2}\left(\frac{m_H^2}{4m_V^2} - 1\right)\right) , \tag{37d}$$

where we used the same kinematics as in Section 2.1. For the decay width, we use again Eq. (27d):

$$\Gamma = \frac{|\mathcal{M}|^2}{32\pi^2 m_H^2}\frac{m_H}{2}\sqrt{1 - 4\frac{m_V^2}{m_H^2}}\,4\pi \tag{38a}$$

$$= \frac{m_H^3}{16\pi v^2}\sqrt{1 - 4\frac{m_V^2}{m_H^2}}\left(1 - 4\frac{m_V^2}{m_H^2} + 12\frac{m_V^4}{m_H^4}\right) . \tag{38b}$$

Due to the fact that Z^0Z^0 are identical particles, we have to multiply this result by a factor N_V which is 1 ($1/2$) for W (Z) bosons. We can see that for $m_H \gg m_V$ the decay width is proportional to m_H^3. We will use this high energy limit as consistency check of our result in the next subsection.

2.4. Goldstone Boson Equivalence Theorem

In the rest frame of the gauge boson, we can define transversal and longitudinal polarizations vectors as [Djo08]

$$\epsilon^\mu_{T1} = (0,1,0,0), \qquad \epsilon^\mu_{T2} = (0,0,1,0), \qquad \epsilon^\mu_L = (0,0,0,1). \tag{39}$$

A boost to $p^\mu = (E(\boldsymbol{p}), 0, 0, |\boldsymbol{p}|)$ along z does not change the transversal polarizations while the longitudinal one becomes

$$\epsilon^\mu_L = \left(|\boldsymbol{p}|, 0, 0, E(\boldsymbol{p})\right) \frac{1}{m_V} \xrightarrow{E \gg m_V} \frac{p^\mu}{m_V}\,. \tag{40}$$

Note that for all p^μ and m_V the condition $\epsilon \cdot p = 0$ is fullfilled because the high-energy limit coincides with a massless case $p^2 = 0$. This polarization is proportional to the gauge boson momentum and therefore the longitudinal amplitudes will dominate at very high energies which corresponds to large m_H in our decay. Furthermore, we could have used the longitudinal part of Eq. (36), i.e. $q^\mu q^\nu / m_V^2$, and obtained the same result as Eq. (38b) in the limit $m_V/m_H \to 0$. This motivates to use the Goldstone Boson Equivalence Theorem [PS95; Lan12] which states that the matrix element involving longitudinally polarized vector bosons may be replaced by one with goldstone bosons, i.e. $W_L^\pm \to \phi^\pm$ and $Z_L^0 \to \phi^3$, up to a phase and a correction of $\mathcal{O}(m_V/E)$. This is illustrated in Fig. 2. In our simple case the phase plays no role and may be ignored.

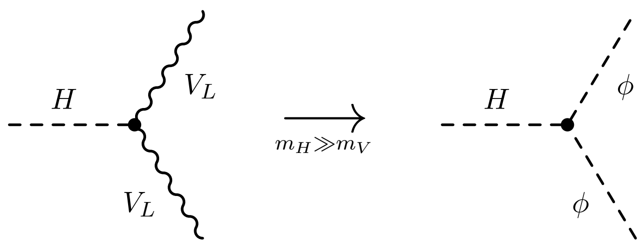


Figure 2: The Goldstone Boson Equivalence Theorem allows us to check our result in the high energy limit. The calculations become very easy because there are now only scalar particles involved.

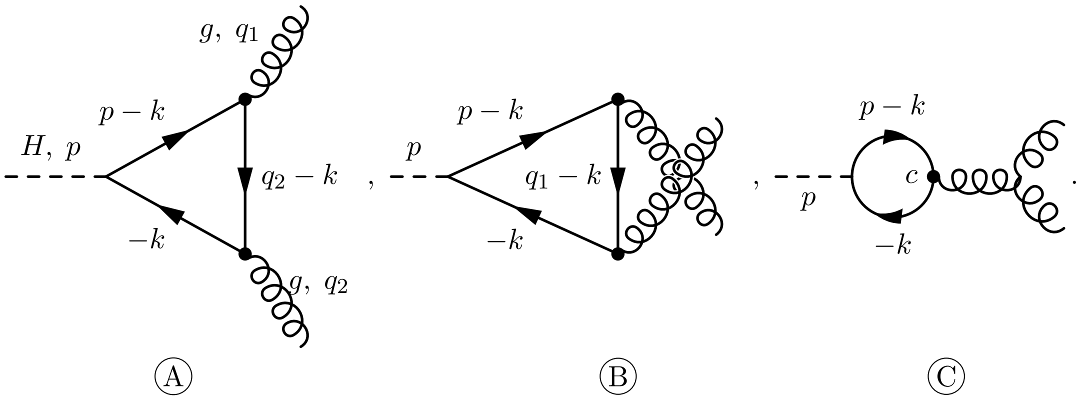


Figure 3: Feynman graphs of processes contributing to $H \to gg$ at one-loop level. In the text, it is shown that of these three possible topologies, the first two give the same contribution while the third is zero.

$\boldsymbol{H \to \phi^+\phi^-}$ **:** Since there are only three scalar particles involved at tree level, the amplitude is given by the vertex shown in Eq. (13):

$$|\mathcal{M}|^2 = \frac{m_H^4}{v^2} \tag{41}$$

In the high energy limit ${}^{m_V}/_{m_H} \to 0$, this is exactly the same expression as Eq. (37d) and leads therefore to the same decay rate like in Eq. (38b).

$\boldsymbol{H \to \phi^3\phi^3}$ **:** We already have noticed that $\phi^3\phi^3$ and $\phi^+\phi^-$ share the same coupling to H and therefore we obtain Eq. (41) again. This leads to the same decay rate as $H \to Z^0Z^0$ in the high energy limit because for $\phi^3\phi^3$ the factor $\frac{1}{2}$ for the phase space of identical particles has to be included as well. Overall we can say that both results of Section 2.3 are confirmed by the result using Goldstone bosons.

3. $H \to gg$

In this section we want to calculate the decay of the Higgs boson into gluons. This can only occur at 1-loop order, since the gluons are massless, which is thus the leading order of this process.

3.1. The diagrams

The possible diagrams at 1-loop order are shown in Fig. 3. The external particles in the crossed diagram (B) are the same as in (A). In fact, the second diagram can also be seen as the first but with a counterclockwise fermion loop, if one rotates the diagram while keeping the external particles fixed. $q_{1/2}$ is accompanied by the color index a/b and the polarization vectors $\epsilon^*_{1/2}$. The loop leads to traces over the γ matrices and the generators of the $SU(3)$ T^a as well as an integral over k. All three diagrams contain exactly one closed fermion loop, the corresponding $(-)$ factor is consistently dropped. Without further effort, we can calculate the matrix element for an arbitrary loop quark with mass m_f. Later when we square the amplitude, we will discuss which quarks should be considered. For diagram (A), we obtain the amplitude

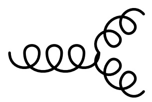
$$\tilde{\mathcal{M}}_A = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \operatorname{Tr}\left[\not{\epsilon}^*_1 \mathrm{i} g_s \frac{\mathrm{i}}{\not{p} - \not{k} - m_f}\left(-\frac{\mathrm{i} m_f}{v}\right)\frac{\mathrm{i}}{-\not{k} - m_f}\not{\epsilon}^*_2 \mathrm{i} g_s \frac{\mathrm{i}}{\not{q}_2 - \not{k} - m_f}\right] \operatorname{Tr}\left[T^a T^b\right] . \tag{42}$$

Due to cyclicity of the trace, the color factor stays the same for (B). But the gluons $\epsilon^*_{1/2}$ and their impulses $q_{1/2}$ in the propagators exchange their places compared to Eq. (42). Therefore we obtain

$$\tilde{\mathcal{M}}_B = \tilde{\mathcal{M}}_A\big|_{\epsilon_1 \leftrightarrow \epsilon_2,\, q_1 \leftrightarrow q_2}. \tag{43}$$

For diagram (C) the amplitude reads

$$\mathcal{M}_C = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \operatorname{Tr}\left[\left(-\frac{\mathrm{i} m_f}{v}\right)\frac{\mathrm{i}}{-\not{k} - m_f}\mathrm{i} g_s \gamma_\mu \underbrace{T^c_{ii}}_{=0} \frac{\mathrm{i}}{\not{p} - \not{k} - m_f}\right]\Big(\quad \Big)^{\mu}_{abc},$$



$$= 0 \tag{44}$$

because the generators of the $SU(N)$ have trace zero. Similar to diagram (A) the quark loop leads to a trace over T^c since the fermion propagators are diagonal in the color indices. Therefore, a loop coupled to exactly one vector boson will always be zero in the SM. Of course, one could also have argued from the beginning that (C) is zero since it violates color conservation. Continuing with the kinematics which are slightly different than in Section 2.1 because the gluons are massless, i.e. $p = (m_H, \mathbf{0})$, $q_{1/2} = (|\boldsymbol{p}|, \pm\boldsymbol{p})$, with $p = q_1 + q_2$ we have $|\boldsymbol{p}| = m_H/2$ and $p \cdot q_i = m_H^2/2 = q_1 \cdot q_2$. The generators are normalized such that $\operatorname{Tr}\left[T^a T^b\right] = \frac{1}{2}\delta_{ab}$.

3.2. Calculation of the trace

The trace over the γ matrices becomes

$$\mathrm{Tr}_\gamma \equiv \mathrm{Tr}\left[\not{\epsilon}_1^*(\not{p} - \not{k} + m_f)(-\not{k} + m_f)\not{\epsilon}_2^*(\not{q}_2 - \not{k} + m_f)\right] \tag{45a}$$
$$= m_f \Big(4m_f^2\, \epsilon_1^* \cdot \epsilon_2^* + \mathrm{Tr}\left[\not{\epsilon}_1^*(\not{p} - \not{k})(-\not{k})\not{\epsilon}_2^*\right] + \mathrm{Tr}\left[\not{\epsilon}_1^*(\not{p} - \not{k})\not{\epsilon}_2^*(\not{q}_2 - \not{k})\right]$$
$$+\mathrm{Tr}\left[\not{\epsilon}_1^*(-\not{k})\not{\epsilon}_2^*(\not{q}_2 - \not{k})\right]\Big) \tag{45b}$$
$$= 4m_f\Bigg(\epsilon_1^* \cdot \epsilon_2^* \left(m_f^2 - \frac{m_H^2}{2} - k^2 - 2\, k \cdot q_2\right) + 4\, \epsilon_1^* \cdot k\ \epsilon_2^* \cdot k$$
$$- 4\, \epsilon_1^* \cdot q_2\ \epsilon_2^* \cdot k\ +\ \epsilon_1^* \cdot q_2\ \epsilon_2^* \cdot q_1 \Bigg)\,, \tag{45c}$$

using the usual techniques to calculate traces of γ matrices as well as $\epsilon_i^* \cdot q_i = 0$ and $q_i^2 = 0$. Plugging in the pieces, we get

$$\tilde{\mathcal{M}}_A = \frac{m_f}{v} g_s^2 \frac{1}{2} \delta_{ab} \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\mathrm{Tr}_\gamma}{\Big((p-k)^2 - m_f^2\Big)\Big(k^2 - m_f^2\Big)\Big((q_2 - k)^2 - m_f^2\Big)} \tag{46a}$$
$$= \underbrace{\frac{2m_f^2 g_s^2 \delta_{ab}}{v}}_{\equiv c} \int \frac{\mathrm{d}^4k}{(2\pi)^4} \Bigg\{ \epsilon_1^* \cdot \epsilon_2^* \Bigg(\frac{-1}{\Big((p-k)^2 - m_f^2\Big)\Big((q_2 - k)^2 - m_f^2\Big)} - \frac{m_H^2}{2} \frac{1}{()()()}$$
$$- 2q_{2\mu} \frac{k^\mu}{()()()}\Bigg) + \epsilon_{1\mu}^* \epsilon_{2\nu}^* \left(\frac{4k^\mu k^\nu}{()()()} - 4q_2^\mu \frac{k^\nu}{()()()} + q_2^\mu q_1^\nu \frac{1}{()()()}\right)\Bigg\} \tag{46b}$$

where we cancelled $m_f^2 - k^2$ in the first term and indicated the full denominator of Eq. (46a) with three parentheses. From mere power counting, we would guess that this integral is logarithmically divergent (the integrant goes like $1/k$). Therefore, we have to use dimensional regularization, i.e. calculating the integral in a dimension D and taking the limit $D \to 4$ at the end of the calculations [Bin04]:

$$\int \frac{\mathrm{d}^4k}{(2\pi)^4} \to \mu^{4-D} \int \frac{\mathrm{d}^Dk}{(2\pi)^D}\,, \tag{47}$$

where μ has mass dimension and fixes the dimension of the integral for all D. The first step is to perform a wick rotation of the k_0 integration to get rid of the minkowskian metric: $q^2 \to -q_E^2$. Continuing with this procedure the logarithmic UV divergencies appear as poles [Ohl12] which can be subtracted using counterterms. However, we will see that the divergencies cancel in our case and we can continue without going into detail of the whole renormalization procedure.

3.3. Tensor integrals

A general one loop can be parametrized as [Den93]

$$T^N_{\mu_1 \ldots \mu_P}(p_1, \ldots, p_{N-1}, m_0, \ldots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2} \int \mathrm{d}^D q \frac{q_{\mu_1} \cdots q_{\mu_P}}{D_0 D_1 \cdots D_{N-1}}\,,$$
$$D_0 = q^2 - m_0^2 + \mathrm{i}\varepsilon, \qquad D_i = (q + p_i)^2 - m_i^2 + \mathrm{i}\varepsilon, \qquad i = 1, \ldots, N-1. \tag{48}$$

The factor $\mathrm{i}\pi^2$ is only convention as well as to use $T^1 \equiv A, T^2 \equiv B, \ldots$. Note that only the scalar integrals $A_0, B_0, C_0, \ldots$ have to be calculated since all higher tensors can be reduced to scalar integrals using tensor reduction. For demonstration, we expand the tensor integrals in all possible Lorentz covariant factors which can occur, i.e. the metric and the impulses:

$$B_\mu = p_{1\mu} B_1, \tag{49a}$$
$$B_{\mu\nu} = g_{\mu\nu} B_{00} + p_{1\mu} p_{1\nu} B_{11}, \tag{49b}$$
$$C_\mu = p_{1\mu} C_1 + p_{2\mu} C_2, \tag{49c}$$
$$C_{\mu\nu} = g_{\mu\nu} C_{00} + p_{1\mu} p_{1\nu} C_{11} + p_{2\mu} p_{2\nu} C_{22} + (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu}) C_{12}\,. \tag{49d}$$

Eq. (49d) has to be symmetric under $p_1 \leftrightarrow p_2$ since Eq. (48) is symmetric. Then we can contract, e.g. $p_{1\mu} B^\mu$ with the rhs of Eq. (49a), which gives a number $p_1^2 B_1$, and with Eq. (48). There we can expand the scalar product $q \cdot p_1$ as

$$q \cdot p_1 = \frac{1}{2} \left(\left[(q + p_1)^2 - m_1^2 \right] - \left[q^2 - m_0^2 \right] - \left[p_1^2 - m_1^2 + m_0^2 \right] \right) \tag{50}$$

The first term cancels D_1, the second D_0 and the last term is independent of the integration variable q. This is why the coefficient B_1 can be reduced to B_0. In the general case, this method leads to a system of linear equations. This procedure is fully worked out and we give the necessary coefficients for our case in Appendix A. In fact, we do not even need all of these coefficients. Let us first write Eq. (46b) in terms of these standard integrals while shifting the integral in the first term $k \to k + p$, which is possible due to translation invariance in D dimensions [Bin04],

$$\tilde{\mathcal{M}}_A = c \frac{\mathrm{i}\pi^2}{(2\pi)^4} \epsilon^*_{1\mu} \epsilon^*_{2\nu} \Bigg[-g^{\mu\nu} B_0(q_1, m_f, m_f) + \left(q_2^\mu q_1^\nu - \frac{m_H^2}{2} g^{\mu\nu} \right) C_0(p, q_2, m_f, m_f, m_f)$$
$$- \left(2 g^{\mu\nu} q^{2\sigma} + 4 q_2^\mu g^{\nu\sigma} \right) C_\sigma(p, q_2, m_f, m_f, m_f) + 4 C_{\mu\nu}(p, q_2, m_f, m_f, m_f) \Bigg]\,. \tag{51}$$

Since there is always the same type of fermion running around, we omit the m_f dependence from now on. Due to transversality of the polarization vectors and the massless case of gluons, we can immediately reduce $C_\sigma(p, q_2) \to C_1(p, q_2) q_{1\sigma}$. For the same reason, we see that $C_{22}(p, q_2) q_{2\mu} q_{2\nu}$

and $C_{12}(p, q_2)p_\mu q_{2\nu}$ drop out. Note that in the second notation of $C_{\dots}$, introduced in Eq. (70), we have $C_{\dots}(p, q_2) = C_{\dots}(m_H^2, 0, 0)$ because $m_0^2 = (p - q_2)^2 = 0$. The necessary coefficients of Appendix A simplify in our case vastly to

$$C_1(m_H^2, 0, 0) = \frac{1}{m_H^2}\Big(B_0(m_H^2) - B_0(0)\Big) \,, \tag{52a}$$

$$C_{00}(m_H^2, 0, 0) = \frac{1}{4}\Big(2m_f^2 C_0(m_H^2, 0, 0) + B_0(m_H^2) + 1\Big) \,, \tag{52b}$$

$$C_{11}(m_H^2, 0, 0) = -\frac{1}{2m_H^2}\Big(B_0(m_H^2) - B_0(0)\Big) = -\frac{1}{2}C_1 \,, \tag{52c}$$

$$\begin{aligned} C_{12}(m_H^2, 0, 0) &= \frac{1}{2m_H^2}\Big(-2m_f^2 C_0(m_H^2, 0, 0) + B_0(0) - B_0(m_H) - 1\Big) \,, \\ &= \frac{1}{2m_H^2}\Big(-2m_f^2 C_0(m_H^2, 0, 0) - 1\Big) - \frac{1}{2}C_1 \,. \end{aligned} \tag{52d}$$

3.4. Scalar integrals

Now as we reduced all tensor integrals to scalar integrals, we should evaluate them. Using the Feynman trick

$$\frac{1}{xy} = \int_0^1 \frac{\mathrm{d}\xi}{((1-\xi)x + \xi y)^2} . \tag{53}$$

B_0 can be calculated as [Ohl12; Den93]

$$B_0(p_1, m_0, m_1) = \Delta - \int_0^1 \mathrm{d}\xi \ln \frac{\xi^2 p_1^2 - \xi(p_1^2 - m_1^2 + m_0^2) + m_0^2 - \mathrm{i}\varepsilon)}{\mu^2} \,, \tag{54}$$

where $\Delta = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$ is indeed divergent for $\epsilon \to 0 \Leftrightarrow D \to 4$. Note that there is a nonobvious symmetry $B_0(p_1, m_0, m_1) = B_0(p_1, m_1, m_0)$. In our case, this leads to (we omit the $\mathrm{i}\varepsilon$ where it plays no role):

$$B_0(m_h^2) = \Delta - \int \mathrm{d}\xi \ln \frac{\xi^2 m_H^2 - \xi m_H^2 + m_f^2}{\mu^2} \tag{55a}$$

$$= B_0(0) + 2 - 2x \operatorname{atanh} \frac{1}{x} \quad \text{with} \quad x = \sqrt{1 - 4\frac{m_f^2}{m_H^2}} \,. \tag{55b}$$

Using this, Eq. (52a) simplifies to

$$C_1(m_H^2, 0, 0) = \frac{2}{m_H^2}\left(1 - x \operatorname{atanh} \frac{1}{x}\right) . \tag{56}$$

There is also an $B_0(q_1)$ in Eq. (51) which is just $B_0(0)$ as seen from Eq. (55a). The Feynman trick used above can be generalized and for C_0 one obtains [Bin04]:

$$C_0(p_1, p_2, m_0, m_1, m_2) = -\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \Big[x^2 p_1^2 + y^2 p_2^2 + xy\, 2\, p_1 \cdot p_2\ - x(p_1^2 - m_1^2 + m_0^2) \\ - y(p_2^2 - m_2^2 + m_0^2) + m_0^2 - \mathrm{i}\varepsilon\Big]^{-1} . \quad (57)$$

Plugging in our loop, we get

$$\begin{aligned} C_0(p, q_2, m_f, m_f, m_f) &= -\int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \Big[x^2 m_H^2 + xym_H^2 - xm_H^2 + m_f^2 - \mathrm{i}\varepsilon\Big]^{-1} \\ &= -m_H^{-2} \int_0^1 \mathrm{d}x\, \frac{1}{x} \ln \frac{m_f^2/m_H^2 - \mathrm{i}\varepsilon}{m_f^2/m_H^2 + (-x + x^2 - \mathrm{i}\varepsilon)} \\ &= -\frac{1}{m_H^2}\left(\mathrm{Li}_2\Big[-\frac{2}{x'-1}\Big] + \mathrm{Li}_2\Big[\frac{2}{x'+1}\Big]\right) , \end{aligned} \quad (58)$$

with $x' = \sqrt{1 - 4\frac{m_f^2}{m_H^2} + \mathrm{i}\varepsilon}$ and by using the dilogarithm or Spence function $\mathrm{Li}_2(x)$ which is defined as

$$\mathrm{Li}_2(x) = -\int_0^1 \frac{\mathrm{d}t}{t} \log(1 - xt) . \quad (59)$$

It is important to keep the $+\mathrm{i}\varepsilon$ in x' because the dilogarithm has a branchcut from 1 to ∞. C_0 and C_1 can now be regarded as fully calculated functions of m_H and m_f.

3.5. Evaluation of the matrix element

We have now all the needed modules for Eq. (51) which reads

$$\tilde{\mathcal{M}}_A = \underbrace{c\frac{\mathrm{i}\pi^2}{(2\pi)^4}}_{\equiv\tilde{c}}\Big[-B_0(0)\, \epsilon_1^* \cdot \epsilon_2^* + \Big(\epsilon_1^* \cdot q_2\ \epsilon_2^* \cdot q_1\ - \frac{m_H^2}{2}\, \epsilon_1^* \cdot \epsilon_2^*\Big) C_0 - \Big(2\, \epsilon_1^* \cdot \epsilon_2^* \frac{m_H^2}{2} + (4 + 2 + 2) \\ \times\ \epsilon_1^* \cdot q_2\ \epsilon_2^* \cdot q_1\Big) C_1 + \epsilon_1^* \cdot \epsilon_2^* \Big(2m_f^2 C_0 + B_0(m_H^2) + 1\Big) + \frac{2}{m_H^2}\, \epsilon_1^* \cdot q_2\ \epsilon_2^* \cdot q_1 \Big(-2m_f^2 C_0 - 1\Big)\Big] . \quad (60)$$

The $+2 + 2$ come from C_{11} and C_{12}. Condensing the above expression:

$$\tilde{\mathcal{M}}_A = \tilde{c}\Big[\ \epsilon_1^* \cdot \epsilon_2^* \Big(2m_f^2 C_0 - \frac{m_H^2}{2} C_0 + 1\Big) + \epsilon_1^* \cdot q_2\ \epsilon_2^* \cdot q_1 \Big(C_0 - 8C_1 - 4\frac{m_f^2}{m_H^2} C_0 - \frac{2}{m_H^2}\Big)\Big]. \quad (61)$$

Let us make C_0 and C_1 dimensionless by pulling out m_H:

$$\tilde{C}_0 = m_H^2 C_0, \qquad\qquad \tilde{C}_1 = m_H^2 C_1 \; . \tag{62}$$

Hence, we obtain

$$\tilde{\mathcal{M}}_A = \tilde{c}\left[\epsilon_1^* \cdot \epsilon_2^* \left(1 - \frac{x^2}{2}\tilde{C}_0\right) + \epsilon_1^* \cdot q_2 \; \epsilon_2^* \cdot q_1 \, \frac{1}{m_H^2}\left(x^2 \tilde{C}_0 - 8\tilde{C}_1 - 2\right)\right] , \tag{63}$$

where we used x of Eq. (55b) again. We can conclude now that $\tilde{\mathcal{M}}_B$ is indeed equal to $\tilde{\mathcal{M}}_A$ because Eq. (63) is symmetric under $\epsilon_1^* \leftrightarrow \epsilon_2^*$, $q_1 \leftrightarrow q_2$ and $\tilde{\mathcal{M}}_B$ is given by Eq. (43). Therefore adding diagram B leads only to a factor of 2. Note that so far $\tilde{\mathcal{M}}_A$ is completely general and we could sum over all six quarks if we desire to do so. However, we know that the heaviest quark, the top quark, will play the dominant role since the coupling is proportional to the mass. Calculations have shown that even the second heaviest quark, i.e. the bottom quark with $m_b/m_t \approx 0.024$, has only negligible effects, cf. Fig. 2.21 of Ref. [Djo08]. Furthermore we will see in the result explicitely how surpressed other contributions are and thus we continue with the top quark only.

To avoid the necessity to add the ghost diagrams, we will use a proper polarization sum, i.e. not $-g^{\mu\nu}$ which only works in abelian theories like QED due to the Ward identity. In the massless case $q^2 = 0$ for $n_\mu \epsilon^\mu = 0$, where n_μ is an external four-vector, it reads [BDJ01]

$$\mathcal{P}_{\mu\nu} \equiv \sum_{\lambda=1}^{2} \epsilon_\mu^*(q,\lambda)\epsilon_\nu(q,\lambda) = -g_{\mu\nu} - \frac{q_\mu q_\nu}{(\,q \cdot n\,)^2} n^2 + \frac{n_\mu q_\nu + n_\nu q_\mu}{q \cdot n} \quad \text{with} \quad q \cdot n \neq 0 \, . \tag{64}$$

As seen by explicit calculation, this polarization sum fullfills $\mathcal{P}_{\mu\nu} n^\nu = 0$. We notice further that $\mathcal{P}_{\mu\nu}$ contains all possible lorentzcovariant objects. A clever choice in our case is to use $n_\mu = q_{2\mu}$ for $\mathcal{P}^1$ and vice versa. Then we get $n \cdot q \;=\; q_1 \cdot q_2 \;=\, {}^{m_H^2}\!/2$ and $\epsilon_1^* \cdot q_2 \; \epsilon_2^* \cdot q_1 \;= 0$ as well as

$$\mathcal{P}_{1\mu\nu} = \mathcal{P}_{2\mu\nu} = -g_{\mu\nu} + \frac{2}{m_H^2}\left(q_{1\mu} q_{2\nu} + q_{1\nu} q_{2\mu}\right) \, . \tag{65}$$

Thus, we obtain

$$\begin{aligned} |\mathcal{M}_A + \mathcal{M}_B|^2 &= \sum_{\lambda_1 \lambda_2, ab} 4 \left|\tilde{\mathcal{M}}_A\right|^2 \\ &= \sum_{ab} 4 \left|\tilde{c}\left(1 - \frac{x^2}{2}\tilde{C}_0\right)\right|^2 \underbrace{\left(4 - \frac{4}{m_H^2} 2\, q_1 \cdot q_2 + \frac{4}{m_H^4} 2 (\,q_1 \cdot q_2\,)^2\right)}_{=2} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{ab} \Big(\frac{2m_t^2 g_s^2 \delta_{ab}}{v} \Big)^2 \frac{1}{2^5 \pi^4} \left| 1 - \frac{x^2}{2} \tilde{C}_0 \right|^2 \\
&= \sum_{b} \frac{m_t^4 g_S^4}{m_W^2 \mathrm{s}_W^2} \frac{\alpha}{8\pi^3} \left| 1 - \frac{x^2}{2} \tilde{C}_0 \right|^2 \quad \text{with} \quad g_S^2 = 4\pi\alpha_S \\
&= \frac{\alpha_S^2 \alpha}{m_W^2 \mathrm{s}_W^2} \frac{16}{\pi} m_t^4 \left| 1 - \frac{x^2}{2} \tilde{C}_0 \right|^2 , \qquad (66)
\end{aligned}
$$

where we used that the number of gluons is equal to the dimension of the adjoint representation, i.e. $3^2 - 1 = 8$ in $SU(3)$.

3.6. Decay width

Now we plug this in Eq. (27d) and take the factor $\frac{1}{2}$ for identical particles in the final state into account:

$$
\begin{aligned}
\Gamma &= \frac{|\mathcal{M}_A + \mathcal{M}_B|^2}{8\pi m_H^2} \frac{m_H}{4} \\
&= \frac{\alpha_S^2 \alpha}{8 m_W^2 \mathrm{s}_W^2} \frac{m_H^3}{9\pi^2} \underbrace{\left| 6 \Big(\frac{m_t}{m_H} \Big)^2 \Big(1 - \frac{x^2}{2} \tilde{C}_0 \Big) \right|^2}_{\equiv F(\frac{m_t}{m_H})} . \qquad (67)
\end{aligned}
$$

The behaviour of the formfactor $F(y)$ which we have defined here is shown in Fig. 4. Eq. (67) is exactly the result indicated in Ref. [PS95] and shown in Ref. [Djo08]. Keep in mind that the formfactor is multiplied with m_H^3, so the full decay width is indeed growing for increasing m_H like the other decay widths as shown in Fig. 6. We see now that the formfactor is the only factor dependent on the mass of the quark in the loop. Since the formfactor goes to zero if $m_H \gg m_f$ contributions of the five lighter quarks can be neglected. A possible $m_H = \mathcal{O}(m_b)$ can be discarded since this would violate vacuum stability. This means, by calculating the one loop renormalization group equation Eq. (16) for the quartic coupling λ of the Higgs potential, one sees that $\lambda(Q^2)$ may become negative if the Higgs mass is too small. This would lead to a potential which is not bounded by below and the vacuum or better the vev would not be stable anymore. At a scale $\sim 1\,\text{TeV}$ this constraint gives $m_H \gtrsim 70\,\text{GeV}$ [Djo08]. This theoretical argument is supported by the experimental lower bound for the Higgs mass which has been found at the Large Electron-Positron Collider (LEP). The collision data at energies between 189 GeV and 209 GeV of the four detectors give $m_H \geq 114.1\,\text{GeV}$ at 95 % confidence level [LEP01].

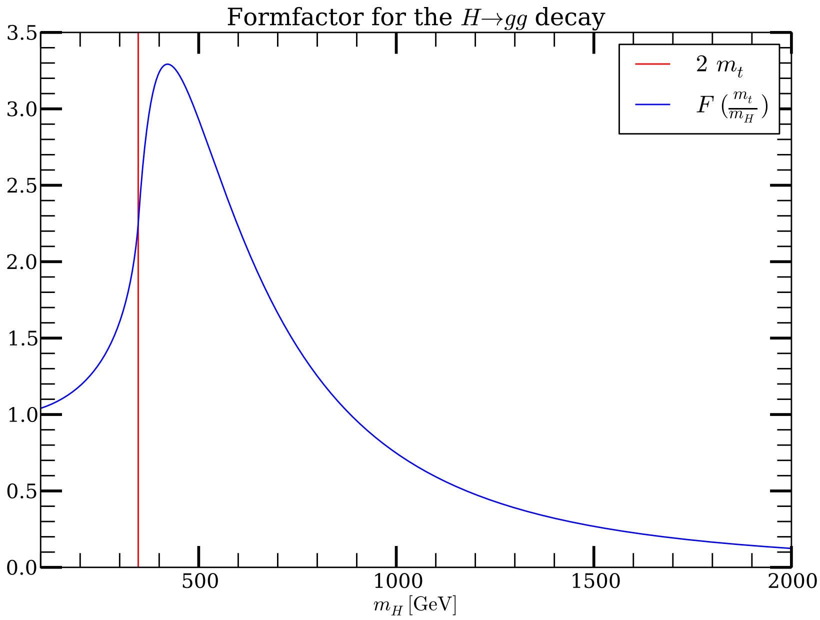


Figure 4: The calculated formfactor of Eq. (67) using $m_t = 173.5\,\text{GeV}$ [al12]. It peaks around $2m_t$, reaches 1 for $m_H \to 0$ and vanishes in the chiral limit $m_H \gg m_t$. This asymptotic behaviour emphasizes the dominant contribution of the top quark to the gluon decay.

4. Discussion

The achieved results are summarized in Table 1. Here, we used $v = \left(\sqrt{2}G_F\right)^{-1/2}$. The quark masses in $H \to \bar{f}f$ should be understood as running masses as explained in Section 2.2. We have calculated the leading order processes of the decay of the Higgs boson to all symmetric pairs of particles of the SM, fermions and gauge bosons, except $\gamma\gamma$. $H \to \gamma\gamma$ as well as $H \to \gamma Z$ have to occur at one loop order and are very similar to $H \to gg$. The calculation of these channels can be performed analogous to the digluon channel presented here, but as an additional complication, not only fermion but also W boson and lepton loops contribute. The calculation of this channel is beyond the scope of this project and is shown e.g. in Ref. [Djo08]. The numerical evaluation of the analytical expressions has been done with the most recent values from July 2012 of the Particle Data Group [al12]. Fig. 6 summarizes all obtained decay widths. The bump in $H \to gg$ at $m_H \approx 2m_t$ originates from the peak of Fig. 4. Note that the small decay widths into $e\bar{e}$, $\mu\bar{\mu}$, $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ result in small branching ratios $< 10^{-3}$. They are not shown since this is the order of the error we are making by neglecting loop corrections and $\gamma\gamma$ decays. The dominating decay channels are described in Fig. 7. We see here that even when the top decay is kinematically possible, i.e. the heaviest particle of the SM, it does not change the dominant position of the weak gauge bosons WW and ZZ. However, for experiments the dominant decay channels are not obligatorily the most important ones. E.g. the most significant evidence for the Higgs boson has been found by CMS in the $\gamma\gamma$ and

Process	Decay width
$H \to \bar{f}f$	$\Gamma = N_f \frac{m_f^2 m_H \alpha}{8 m_W^2 s_W^2} x^3$
$H \to W^+W^-, Z^0Z^0$	$\Gamma = N_V \frac{\sqrt{2} G_F m_H^3}{16\pi} x_V \left(x_V^2 + 12 \frac{m_V^4}{m_H^4} \right)$
$H \to gg$	$\Gamma = \frac{\alpha_S^2 \alpha}{8 m_W^2 s_W^2} \frac{m_H^3}{9\pi^2} F(\frac{m_t}{m_H})$

Table 1: The calculated decay widths as functions of the mass of the Higgs boson and the other parameters of the SM. Similar to x, we define $x_V = \left(1 - 4\frac{m_V^2}{m_H^2}\right)^{1/2}$. The factors were defined as $N_f = 3$ (1) for quarks (leptons) and $N_V = 1$ ($\frac{1}{2}$) for W (Z) bosons. The form factor $F(y)$ is defined in Eq. (67).

ZZ channels [Cha12], despite their low branching ratios < 0.01 as seen in Fig. 8, because they provide a high ratio of signal to background. If we compare our obtained branching ratios with the literature, we see that for $m_H > 2m_Z$ we have pretty much reproduced the same picture. Also the qualitative behaviour below the $2m_W$ threshold is correct. The obvious difference is the sharp jump at $2m_W$ in comparison with the smooth results of Ref. [Djo08]. These sharp kinematical tresholds of the ZZ and WW decays smoothen if one considers off-shell contribution of three or four body decays, shown in Fig. 5. This leads to a slowly decreasing decay rate under the threshold, cf. Fig. 2.10 in Ref. [Djo08].

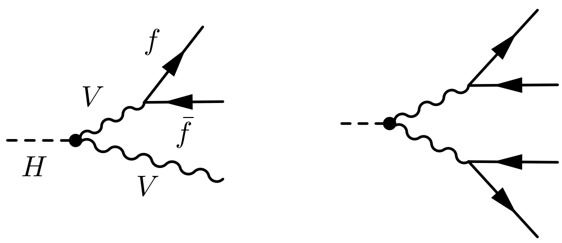


Figure 5: The decays into one virtual and one real gauge bosons in three body processes as well as four body processes with only virtual gauge bosons lead to smooth kinematic edges around $2m_W$ rather than the sharp crossover which follows from energy conservation.

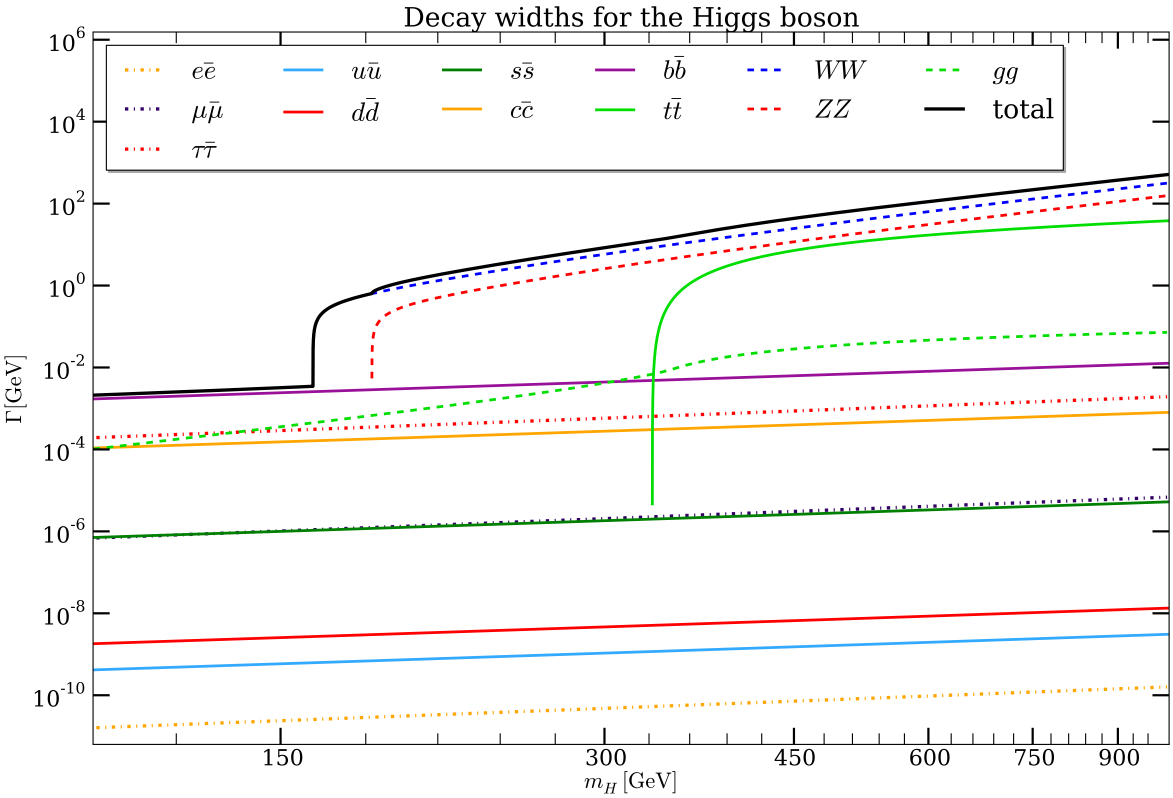


Figure 6: The decay widths into all pairs of particles of the SM expect $\gamma\gamma$ at leading-order calculated with the formulae of Table 1. We draw leptons, quarks and gauge bosons with dash-dotted, full and dashed lines, respectively. Decay widths which do not go through the whole graph come faster from zero than the numeric plot resolution (0.01 GeV). The sum of all decay widths is represented by the black line.

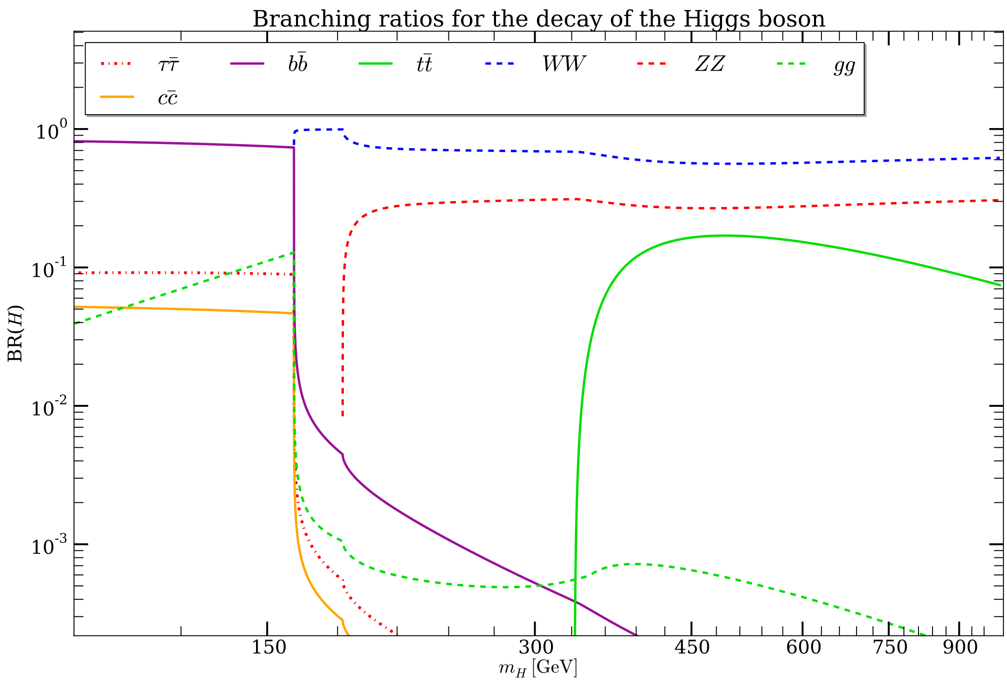


Figure 7: The branching ratios $\Gamma(H \to X)/\Gamma_{\text{ges}}$ calculated from our results in Sections 2 and 3. The linestyles and colors are the same as in Fig. 6. Before the $2m_W$ threshold the decay is dominated by $b\bar{b}$ and to a lesser extent $\tau\bar{\tau}$ and gg. After the threshold the situation changes completely and WW and ZZ become the main decay products.

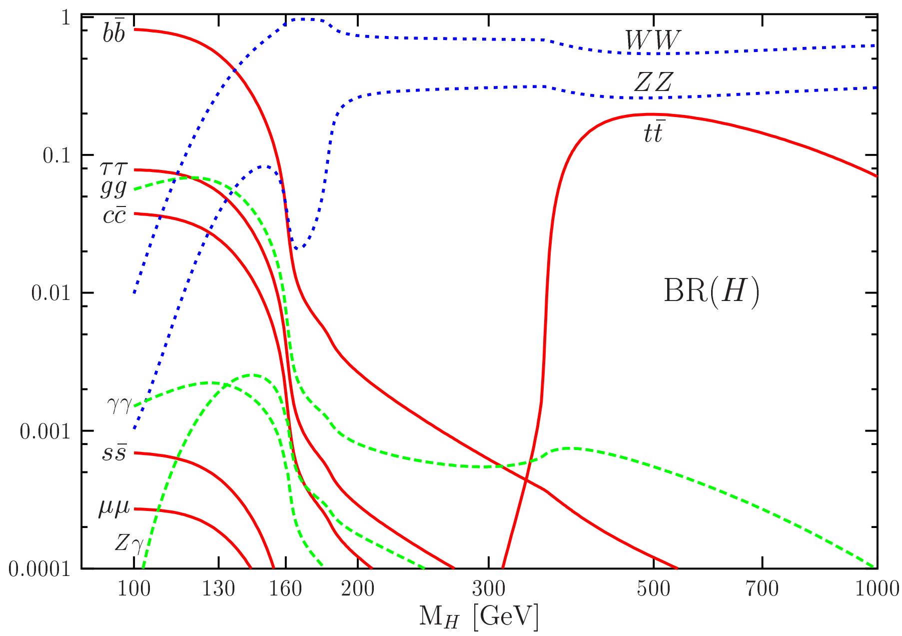


Figure 8: The branching ratio including next to-leading-order (NLO) and partially NNLO corrections as well as three and four body calculations. The smooth threshold of WW, ZZ change the look of all branching ratios below $2m_W$ and come from the decays into off-shell particles. Taken from Ref. [Djo08].

A. Tensor coefficients

The following coefficients have been taken from Ref. [Den93] and are based on the work of Passarino and Veltman. The coefficient for the two-point function B_μ is

$$B_1(p^2, m_0, m_1) = \frac{m_1^2 - m_0^2}{2p^2}\Big(B_0(p^2, m_0, m_1) - B_0(0, m_0, m_1)\Big) - \frac{1}{2}B_0(p^2, m_0, m_1). \tag{68}$$

The three-point function is defined as

$$C_{...} = C_{...}(p_1, p_2, M_0, M_1, M_2) = C_{...}(m_1^2, m_0^2, m_2^2, M_0, M_1, M_2) \tag{69a}$$

$$= \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq \frac{\cdots}{[q^2 - M_0^2][(q+p_1)^2 - M_1^2][(q+p_2)^2 - M_2^2]} \tag{69b}$$

and can be called in two different ways, where

$$p_1^2 = m_1^2, \quad p_2^2 = m_2^2, \quad p_1p_2 = -\frac{1}{2}(m_0^2 - m_1^2 - m_2^2). \tag{70}$$

The coefficients for C_μ are

$$C_1 = -\frac{4}{\kappa^2}\Big[m_2^2 R^{3,1} + \frac{1}{2}(m_0^2 - m_1^2 - m_2^2)R^{3,2}\Big], \tag{71a}$$

$$C_2 = -\frac{4}{\kappa^2}\Big[\frac{1}{2}(m_0^2 - m_1^2 - m_2^2)R^{3,1} + m_1^2 R^{3,2}\Big], \tag{71b}$$

$$\kappa = \kappa(m_0^2, m_1^2, m_2^2), \tag{71c}$$

$$R^{3,1} = \frac{1}{2}\Big[B_0(m_2^2, M_0, M_2) - (m_1^2 - M_1^2 + M_0^2)C_0 - B_0(m_0^2, M_2, M_1)\Big], \tag{71d}$$

$$R^{3,2} = \frac{1}{2}\Big[B_0(m_1^2, M_0, M_1) - (m_2^2 - M_2^2 + M_0^2)C_0 - B_0(m_0^2, M_2, M_1)\Big]. \tag{71e}$$

Here the Källén function $\kappa(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ has been used. Finally we give the the tensor coefficients of $C_{\mu\nu}$

$$C_{00} = \frac{1}{4}\Big[B_0(m_0^2, M_2, M_1) + (M_0^2 - M_1^2 + m_1^2)C_1 + (M_0^2 - M_2^2 + m_2^2)C_2 + 1 + 2M_0^2 C_0\Big], \tag{72a}$$

$$C_{11} = -\frac{4}{\kappa^2}\Big[m_2^2(R_1^{3,1} - C_{00}) + \frac{1}{2}(m_0^2 - m_1^2 - m_2^2)R_1^{3,2}\Big], \tag{72b}$$

$$C_{21} = -\frac{4}{\kappa^2}\Big[\frac{1}{2}(m_0^2 - m_1^2 - m_2^2)(R_1^{3,1} - C_{00}) + m_1^2 R_1^{3,2}\Big] = \tag{72c}$$

$$C_{12} = -\frac{4}{\kappa^2}\Big[m_2^2 R_2^{3,1} + \frac{1}{2}(m_0^2 - m_1^2 - m_2^2)(R_2^{3,2} - C_{00})\Big], \tag{72d}$$

$$C_{22} = -\frac{4}{\kappa^2}\Big[\frac{1}{2}(m_0^2 - m_1^2 - m_2^2)R_2^{3,1} + m_1^2(R_2^{3,2} - C_{00})\Big], \tag{72e}$$

with

$$R^{3,00} = M_0^2 C_0 + B_0(m_0^2, M_2, M_1), \tag{72f}$$

$$R_1^{3,1} = \frac{1}{2}\Big[\qquad\qquad - (m_1^2 - M_1^2 + M_0^2)C_1 - B_1(m_0^2, M_2, M_1)\Big], \tag{72g}$$

$$R_2^{3,1} = \frac{1}{2}\Big[B_1(m_2^2, M_0, M_2) - (m_1^2 - M_1^2 + M_0^2)C_2 + (B_0 + B_1)(m_0^2, M_2, M_1)\Big], \tag{72h}$$

$$R_1^{3,2} = \frac{1}{2}\Big[B_1(m_1^2, M_0, M_1) - (m_2^2 - M_2^2 + M_0^2)C_1 - B_1(m_0^2, M_2, M_1)\Big], \tag{72i}$$

$$R_2^{3,2} = \frac{1}{2}\Big[\qquad\qquad - (m_2^2 - M_2^2 + M_0^2)C_2 + (B_0 + B_1)(m_0^2, M_2, M_1)\Big]. \tag{72j}$$

The identity $C_{12} = C_{21}$, which has to be fullfilled based on symmetry arguments, is not anymore obvious above. Eq. (72) provides two ways to calculate the same coefficient which can be used as consistency check.

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