Decomposition of Aggregate Energy and Gas Emission Intensities for Industry: A Refined Divisia Index Method

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Several methods for decomposing energy consumption or energy-induced gas emissions in industry have been proposed by various analysts. Two commonly encountered problems in the application of these methods are the existence of a residual after decomposition and the handling of the value zero in the data set. To overcome these two problems, we modify the often used Divisia index decomposition method by replacing the arithmetic mean weight function by a logarithmic one. This refined Divisia index method can be shown to give perfect decomposition with no residual. It also gives converging decomposition results when the zero values in the data set are replaced by a sufficiently small number. The properties of the method are highlighted using the data of the Korean industry.

INTRODUCTION

Since the early 1980s, many studies concerning the decomposition of changes in industrial energy consumption have been reported. A recent survey by Ang (1995) listed more than 50 studies and many different decomposition methods. Decomposition has generally been carried out on changes in energy consumption or in aggregate energy intensity over time. Aggregate energy intensity, given by total industrial energy consumption divided by total industrial production, is often taken as an energy performance indicator in industrial energy demand analysis.

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The main purpose of decomposition studies is to estimate the relative contributions arising from changes in product mix and in sectoral energy intensity to changes in the energy indicator being decomposed. The results obtained are useful in energy policy analysis and energy demand forecasting. For instance, changes in sectoral energy intensity can be influenced through various energy policy measures while the composition of industrial activity is effectively beyond the control of energy policy makers. An understanding of their relative contributions is useful in energy policy formulation and implementation. More recently, with the growing concern about global warming and air pollution, a number of studies applying the same methodology to study energy-induced emissions of CO_2 and other gases have been reported.

Two common problems associated with the application of decomposition methods are the existence of a residual in the results obtained and the treatment of the value zero in the data set. As mentioned in Ang and Lee (1994), a large residual will defeat the purpose of decomposition since this means a large part of the observed change in the energy indicator being decomposed is left unexplained. They proposed using the magnitude of the residual as a criterion for judging the desirability of a decomposition method. Decomposition carried out based on highly disaggregated data, which involves a large number of industrial sectors, is generally preferred to that based on aggregated data, since using more disaggregated data allows the impact of changes in product mix to be captured more fully. However, with a large number of sectors, not all the fuel types are used in all the sectors. This leads to zero values for some fuel types in the data set, which in turn lead to computational problems in some decomposition methods. This issue was first pointed out in Liu et al. (1992b) in a study involving the effect of interfuel substitution.

By far the most often used decomposition methods in industrial energy and energy-induced gas emission studies are the so-called Laspeyres index method and the Divisia index method, respectively based on the Laspeyres and Divisia indices in economics and statistics. In the Laspeyres index method, each main effect is isolated by measuring a change in energy consumption or aggregate energy intensity associated with a change in the corresponding variable, say from year 0 to year *T*, while holding all the other variables constant at their respective values in year 0. In addition to the main effects, this method also gives one or more interaction effects, depending on the number of the main effects studied. These interaction effects do not mean much in economic terms, and because of them the sum of the estimated main effects is different from the observed change in the aggregate energy intensity. Studies using the Laspeyres index method include Park (1992) and Hsu and Hsu (1993). In the Divisia index method, the estimated effects are formulated in terms of the weighted average of logarithmic changes of the relevant variables. The application of this method to industrial energy decomposition, originally proposed in Boyd et al. (1987 and 1988), always leads to a residual due to approximation of the theoretical and continuous Divisia indices. In addition computational problems arise when the values of the variables given in the logarithmic form are zero, a problem often encountered when the method is used to study interfuel substitution or energy-induced emissions gas emissions. Liu et al. (1992a) and Ang (1994) proposed a general framework for decomposition method formulation and introduced the adaptive weighting Divisia index methods. Although these methods generally lead to small residuals, the difficulty in handling zero values in the data set remains unresolved.

In this paper, we refine the Divisia index method mentioned above through using a logarithmic weight function. This refined Divisia index method leaves no residual and can effectively handle the value zero in the data set. The computations involved, however, are only marginally more complex than the Laspeyres and conventional Divisia index methods, and are much simpler than the adaptive weighting Divisia index methods. Our study will be based on the decomposition of the aggregate energy intensity index. The proposed procedure, however, can be easily extended and applied to the decomposition of changes in industrial energy consumption. Decomposition results obtained using the data of the Korean industry are presented to highlight the properties of the refined method.

THE LASPEYRES AND DIVISIA INDEX DECOMPOSITION APPROACHES

Define the following variables which, in empirical studies, are normally measured on an annual basis. Energy consumption is measured in an energy unit and industrial production in a monetary unit, and industry is disaggregated into a specific number of sectors which defines the level of sector disaggregation:

E	=	Total industrial energy consumption
E_i	=	Energy consumption in industrial sector i
Y	=	Total industrial production
Y_i	=	Production of sector i
y_i	=	Production share of sector i ($= Y_i / Y$)
I	=	Aggregate energy intensity ($= E / Y$)
I_i	=	Energy intensity of sector i ($= E_i / Y_i$)

For simplicity, we use lower case letters to represent share variables. The aggregate energy intensity may be expressed in the following form:

$$\begin{aligned} I &= \Sigma_i \, E_i \, / \, Y \\ &= \Sigma_i \, (Y_i \, / \, Y)\,(E_i \, / \, Y_i) \\ &= \Sigma_i \, y_i \, I_i \end{aligned} \tag{1}$$

Let D_{tot} be the ratio of the aggregate energy intensity of year T to that of year 0

$$D_{tot} = I_T \, / \, I_0 \tag{2}$$

Decomposition is often carried out on this ratio which we shall call the aggregate energy intensity index. Year 0 and year T can be consecutive years or the beginning and ending years of a time period respectively.

The Laspeyres Index Approach

Taking year 0 as the base year and expanding I_T, it can be shown that

$$I_T = \Sigma_i \, y_{i,T} \, I_{i,0} + \Sigma_i \, y_{i,0} \, I_{i,T} + \Sigma_i \, (y_{i,T} - y_{i,0})\,(I_{i,T} - I_{i,0}) - \Sigma_i \, y_{i,0} \, I_{i,0} \tag{3}$$

After dividing by I_0, Equation (3) may be expressed in the form

$$D_{tot} = D_{str} + D_{int} + D_{itr} - 1 \tag{4}$$

where

$$D_{str} = \Sigma_i \, y_{i,T} \, I_{i,0} \, / \, I_0 \tag{5}$$

$$D_{int} = \Sigma_i \, y_{i,0} \, I_{i,T} \, / \, I_0 \tag{6}$$

$$D_{itr} = \Sigma_i \, (y_{i,T} - y_{i,0})\,(I_{i,T} - I_{i,0}) \, / \, I_0 \tag{7}$$

For simplicity, we shall use D_{str} and D_{int} generally to represent the estimated structural and intensity effects given by all the decomposition methods considered. The term D_{itr} gives the interaction effect of changes in industrial structure and sectoral energy intensity. The relationship between D_{tot} and the decomposition effects are given in the additive form which differs from the Divisia index approach described below.

The Divisia Index Approach

Following the original definition of Divisia indices, we assume that all the variables in Equation (1) are continuous and given as functions of time t. Applying the theorem of instantaneous growth rate to Equation (1) leads to

$$\mathrm{d}\ln(I)/\mathrm{d}t = \Sigma_i\, w_i\, [\mathrm{d}\ln(y_i)/\mathrm{d}t + \mathrm{d}\ln(I_i)/\mathrm{d}t] \tag{8}$$

where $w_i = E_i \,/\, E$ is also a function of t. Integrating Equation (8) with respect to t over the time interval 0 to T and rearranging the terms yields

$$\ln(I_T/I_0) = \int_0^T \sum_i w_i\, [\mathrm{d}\ln(y_i)/\mathrm{d}t]\mathrm{d}t + \int_0^T \sum_i w_i\, [\mathrm{d}\ln(I_i)/\mathrm{d}t]\mathrm{d}t \tag{9}$$

Taking the exponential, Equation (9) may be expressed in the multiplicative form

$$D_{tot} = D_{str}\, D_{int} \tag{10}$$

where

$$D_{str} = \exp\Big[\int_0^T \sum_i w_i\, [\mathrm{d}\ln(y_i)/\mathrm{d}t]\, \mathrm{d}t \Big] \tag{11}$$

$$D_{int} = \exp\Big[\int_0^T \sum_i w_i\, [\mathrm{d}\ln(I_i)/\mathrm{d}t]\, \mathrm{d}t \Big] \tag{12}$$

In empirical studies, the discrete versions of Equations (11) and (12) are needed because the data available are discrete. A common practice is to use the arithmetic mean weight scheme which results in

$$D_{str} = \exp\,[\, \Sigma_i\, (w_{i,T} + w_{i,0})/2\, \ln\, (y_{i,T}\,/y_{i,0})\,] \tag{13}$$

$$D_{int} = \exp\,[\, \Sigma_i\, (w_{i,T} + w_{i,0})/2\, \ln\, (I_{i,T}\,/I_{i,0})\,] \tag{14}$$

The Divisia index decomposition method presented above has been adopted in many studies (see the survey given in Ang, 1995). It will be called the conventional Divisia index method. A problem associated with its application is the existence of a residual in the results obtained since after approximation of the product of D_{str} and D_{int} in Equation (10) is no longer exactly equal to D_{tot}. We therefore rewrite Equation (10) as

$$D_{tot} = D_{str} D_{int} D_{rsd} \tag{15}$$

where the term D_{rsd} signifies a residual and the relationship, for consistency, is given in the multiplicative form. The term D_{rsd} is computed by dividing D_{tot} by the product of D_{str} and D_{int}, and the decomposition is perfect if it is exactly equal to 1.

THE LOGARITHMIC MEAN WEIGHT SCHEME

In his search for an ideal log-change index formula, Sato (1976) proposed the following weight function which gives the logarithmic mean of x and y:

$$L(x,y) = (y-x) \;/\; \ln (y/x) \tag{16}$$

It was defined that $L(x,x) = x$, which is the limit of $L(x,y)$ as $y \rightarrow x$. According to Törnqvist et al. (1985), this logarithmic function was originally proposed in Törnqvist (1935). Törnqvist et al. (1985) specified that x and y must be positive numbers and $L(x,y)$ has the range of $(xy)^{1/2} < L(x,y) < (x+y)/2$ when x is not equal to y. It can be seen that the weight function is symmetric, i.e., $L(x,y) = L(y,x)$. Sato (1976) pointed out that this weight function has other desirable properties that weight functions are expected to have.

Instead of approximating Equations (11) and (12) using the arithmetic mean weight scheme, we propose the use of the logarithmic mean weight scheme. Replacing x and y in Equation (16) by $w_{i,0}$ and $w_{i,T}$ respectively yields

$$L(w_{i,0}, w_{i,T}) = (w_{i,T} - w_{i,0}) \;/\; \ln (w_{i,T} / w_{i,0}) \tag{17}$$

However, the sum of this weight function, when taken over all sectors, is not unity. This sum is always slightly less than unity, which can be seen from a property of the weight function mentioned above, i.e., $(xy)^{1/2} < L(x,y) < (x+y)/2$. To fulfill the basic property of weight functions, that the sum is unity, Equation (17) may be normalized, as suggested in Sato (1976). The normalized weight function can be written as

$$w_i^* = L(w_{i,0}, w_{i,T}) \;/\; \Sigma_k L(w_{k,0}, w_{k,T}) \tag{18}$$

where the summation in the denominator on the right-hand side is taken over all sectors. Normalization is also common in other statistical applications when problems similar to the above arise. For instance, in the decomposition of a time series with seasonal variation, the estimated seasonal factors are normalized so that the sum for all the seasons in a year is unity. Hence, the formulae for the refined Divisia index method for aggregate energy intensity decomposition are

$$D_{str} = \exp [\Sigma_i w_i^* \ln (y_{i,T} / y_{i,0})] \tag{19}$$

$$D_{int} = \exp [\Sigma_i w_i^* \ln (I_{i,T} / I_{i,0})] \tag{20}$$

Application of Equations (19) and (20) leaves no residual in Equation (15), i.e., $D_{rsd} = 1$, and the proof of perfect decomposition is given in Appendix 1.

We have decomposed the change in the aggregate energy intensity of the Korean industry from 1981 to 1993 using the refined Divisia, conventional Divisia and Laspeyres index methods. The aggregate energy intensity of Korean industry decreased by about 28 percent during the period; the aggregate energy intensity index in 1993 is 0.721 (1981 = 1). The data set used contains nine industrial sectors, and some details about the data and the sources are given in Appendix 2. The decomposition results obtained are summarized in Table 1. The results for the refined Divisia index method show perfect decomposition, i.e., $D_{rsd} = 1$. The estimated effects given by the conventional Divisia index method are very close to those of the refined method but D_{rsd} is not exactly equal to 1, and the Laspeyres index method gives a negative interaction effect.

Table 1. Results of Decomposition of Aggregate Energy Intensity Index (1993/1981) for Korean Industry by Method.

Method	D_{tot}	D_{str}	D_{int}	D_{rsd}	Source Equations
Refined Divisia index	0.72119	0.90798	0.79429	1.00000	(15), (19), (20)
Conventional Divisia index	0.72119	0.90695	0.79455	1.00079	(15), (13), (14)
Laspeyres index	0.72119	0.92522	0.80849	-0.01252[a]	(4) - (7)

[a]The estimated interaction effect D_{itr}.

DECOMPOSITION OF ENERGY-INDUCED GAS EMISSIONS

Extensions of the decomposition methodology described above have been made to study energy-induced emissions of CO_2 and other gases. Related studies include Torvanger (1991), Lin and Chang (1996), and Ang and Pandiyan (1997) and the conventional Divisia index method was used in all these studies. Decomposition of energy-induced emissions requires energy consumption data to be given for individual fuel types and electricity from which total CO_2

emissions are calculated and then decomposed. With total energy consumption broken down into consumption by fuel type, zero values often exist in the data set, especially when fairly disaggregated data are involved. As an illustration, we describe the methodology for decomposing the energy-induced aggregate CO_2 intensity index for industry in this section and discuss how the refined Divisia index method may be applied to overcome the problem of zero values in the data set in the next section.

The following additional variables are defined:

C_{ij} = Total CO_2 emissions arising from consumption of fuel j in sector i

C_i = Total CO_2 emissions arising from energy consumption in industrial sector i ($= \Sigma_j C_{ij}$)

C = Total CO_2 emissions arising from industrial energy consumption ($= \Sigma_i C_i$)

e_{ij} = Consumption share of fuel j in sector i ($= E_{ij} / E_i$)

U_{ij} = CO_2 emission coefficient of fuel j in sector i, given by emissions per unit of energy use

Z = The aggregate CO_2 intensity (C / Y)

The aggregate CO_2 intensity may be written as

$$\begin{aligned} Z &= \Sigma_{ij}\, U_{ij}\, E_{ij} / Y \\ &= \Sigma_{ij}\, (U_{ij})\, (Y_i / Y)\, (E_{ij} / E_i)\, (E_i / Y_i) \\ &= \Sigma_{ij}\, U_{ij}\, y_i\, e_{ij}\, I_i \qquad (21) \end{aligned}$$

We define

$$D_{tot} = Z_T / Z_0 \qquad (22)$$

which we shall refer to as the aggregate CO_2 intensity index. The formulae for the conventional Divisia index method which are equivalent to those presented earlier may be derived in the same manner as before. The resulting formulae are as follows:

$$D_{tot} = D_{emc}\, D_{str}\, D_{fsh}\, D_{int}\, D_{rsd} \tag{23}$$

where

$$D_{emc} = \exp [\ \Sigma_{ij}\ (w_{ij,T} + w_{ij,0})/2\ \ln\ (U_{ij,T}\ /U_{ij,0})] \tag{24}$$

$$D_{str} = \exp [\ \Sigma_{ij}\ (w_{ij,T} + w_{ij,0})/2\ \ln\ (y_{i,T}\ /y_{i,0})] \tag{25}$$

$$D_{fsh} = \exp [\ \Sigma_{ij}\ (w_{ij,T} + w_{ij,0})/2\ \ln\ (e_{ij,T}\ /e_{ij,0})] \tag{26}$$

$$D_{int} = \exp [\ \Sigma_{ij}\ (w_{ij,T} + w_{ij,0})/2\ \ln\ (I_{i,T}\ /I_{i,0})] \tag{27}$$

and $w_{ij} = C_{ij}/C$, the CO_2 emission of fuel j in sector i as a share of total emissions in industry. The meanings of the terms D_{str} and D_{int} are similar to those in energy decomposition. The term D_{emc} gives the weighted effect of changes associated with CO_2 emission coefficients and is commonly referred to as the emission coefficient effect. The term D_{fsh} gives the weighted effect of changes in sectoral fuel shares and is called the fuel share effect. Torvanger (1991) gives a detailed description of this method and the meanings of the terms.

We define

$$L(w_{ij,0},\ w_{ij,T}) = (w_{ij,T} - w_{ij,0})\ /\ \ln\ (w_{ij,T}\ /w_{ij,0}) \tag{28}$$

$$w_{ij}{*} = L(w_{ij,0},\ w_{ij,T})\ /\ \Sigma_{uv}\ L(w_{uv,0},\ w_{uv,T}) \tag{29}$$

where the summation in the denominator on the right-hand side of Equation (29) is taken over all fuel types and sectors. The formulae in the refined Divisia index decomposition method are

$$D_{emc} = \exp [\ \Sigma_{ij}\ w_{ij}^{*}\ \ln\ (U_{ij,T}\ /U_{ij,0})] \tag{30}$$

$$D_{str} = \exp [\ \Sigma_{ij}\ w_{ij}^{*}\ \ln\ (y_{i,T}\ /y_{i,0})] \tag{31}$$

$$D_{fsh} = \exp [\ \Sigma_{ij}\ w_{ij}^{*}\ \ln\ (e_{ij,T}\ /e_{ij,0})] \tag{32}$$

$$D_{int} = \exp [\ \Sigma_{ij}\ w_{ij}^{*}\ \ln\ (I_{i,T}\ /I_{i,0})] \tag{33}$$

and $D_{rsd} = 1$ because of perfect decomposition.

DECOMPOSITION OF THE AGGREGATE CO_2 INTENSITY INDEX FOR KOREAN INDUSTRY

Consider the case where fuel j had not been used in sector i in year 0 and it was introduced in the sector in year T. We then have $e_{ij,0} = 0$ such that D_{fsh} in Equations (26) and (32) are indeterminate. Similarly, the same problem arises when a fuel ceased to be used in year T but it had been used in year 0, i.e., $e_{ij,T} = 0$. In both cases, the corresponding $w_{i,j}$ is also equal to zero. These cases can arise even when a limited number of sectors is considered, particularly for fuel types such as coal and natural gas.

To overcome the above-mentioned problem, Ang and Pandiyan (1997) proposed that, say in Equation (26), those terms in the summation with $e_{ij,0}$ or $e_{ij,T}$ equal to zero be excluded from the calculation of D_{fsh}. This is a simplification and the disadvantage is that the fuels concerned are ignored in the estimation of the fuel share effect. Liu et al. (1992b) suggested the use of a small value, δ, to replace the value zero in the data set. If the decomposition results obtained converge as δ approaches zero, this approach is theoretically more appealing than that of simply ignoring those terms with a zero value. We shall adopt this approach and compare the decomposition results given by the conventional and the refined Divisia index methods using the data of the Korean industry.

We consider CO_2 emissions arising from the following four energy types: coal, oil, gas, and electricity. Details about the data and the computations of CO_2 emissions are given in Appendix 2. Several zero values (i.e. for $E_{i,j}$ and hence $e_{i,j}$) appear in the data set because coal and gas were not used in a number of industrial sectors in the Korean industry in 1981 or 1993. These cases occur even though industry is broken down into nine sectors only. The CO_2 intensity index in 1993 is 0.648 (1981 = 1). The decomposition results obtained are summarized in Table 2. In the computations, all the zero values in the data were replaced by δ the value of which was varied in step from 10^{-8} to 10^{-20}. The refined Divisia index method performs well; the results obtained not only converge as δ approaches zero but also show perfect decomposition in all cases. In contrast, the conventional Divisia index method gives a D_{fsh} which is highly sensitive to the value of δ assumed. In addition, the results do not converge and the residuals are far from 1 in all cases.

Referring to the source equations indicated in Table 2, it can be seen that the replacement of zero values in $E_{i,j}$ (hence in e_{ij}) with different values of δ have a direct effect on the values of the logarithmic terms in Equations (26) and (32) but this is not the case for the logarithmic terms in the other equations. However, different values for δ also have a very minor effect on the weights w_{ij} and w_{ij}^*, and hence the calculated values of all indices. In Table 2, this minor effect is captured in the estimated indices given by the refined Divisia index method which change slightly as δ varies. Due to a different weighting scheme, this effect is relatively smaller and is not reflected in the results presented for the conventional Divisia index method.

Table 2. Results of Decomposition of Aggregate CO_2 Intensity Index (1993/1981) for Korean Industry

Method	δ	D_{tot}	D_{emc}	D_{str}	D_{fsh}	D_{int}	D_{rsd}
Refined Divisia	10^{-8}	0.64839	0.90493	0.91472	1.01446	0.77214	1.00000
	10^{-12}	0.64839	0.90492	0.91470	1.01447	0.77216	1.00000
	10^{-16}	0.64839	0.90492	0.91469	1.01448	0.77216	1.00000
	10^{-20}	0.64839	0.90491	0.91468	1.01449	0.77217	1.00000
Source equation		(22)	(30)	(31)	(32)	(33)	(23)
Conventional Divisia	10^{-8}	0.64839	0.90529	0.91337	1.07379	0.77281	0.94496
	10^{-12}	0.64839	0.90529	0.91337	1.10265	0.77281	0.92023
	10^{-16}	0.64839	0.90529	0.91337	1.13228	0.77281	0.89615
	10^{-20}	0.64839	0.90529	0.91337	1.16271	0.77281	0.87269
Source equation		(22)	(24)	(25)	(26)	(27)	(23)

SOME APPLICATION ISSUES

Related to the residual, past studies have shown that the conventional Divisia index method performs fairly well in general, especially when decomposition is carried out using aggregated data such that changes in product mix and sectoral intensity are not drastic. However, in situations where these changes are drastic, which may occur when decomposition is carried out using highly disaggregated data, its performance can be shown to deteriorate. Changes can also be drastic if decomposition is carried out of the aggregate intensity for individual fuels. For instance, in a study that focuses on coal consumption only, the sectoral coal intensity (i.e., I_i) for some industrial sectors can be very different between year 0 and year T because of fuel substitution. In such situations the refined Divisia index method is preferred to the conventional method.

As to studies involving fuel share variables, such as those related to interfuel substitution or energy-induced gas emissions, the refined Divisia index method is clearly preferred to the conventional Divisia index method due to the high likelihood of having zero values in the data set. This problem can be overcome by aggregating the specific sectors with related sectors so that the application of the conventional method poses no problem. However, this practice is not recommended since aggregation results in fewer sectors and hence a loss in useful information. In using the refined method, a check for zero values in the data set should be carried out before decomposition. We recommend that these zero values be replaced by a number which is at least two orders of magnitude smaller than the smallest sectoral fuel consumption terms (i.e., E_{ij}) found in the data set to ensure that meaningful decomposition results are obtained.

CONCLUSION

Since the early 1980s, many different decomposition methods have been proposed by various analysts in industrial energy demand analysis. In more recent years, one of the most widely used methods has been the conventional Divisia index method. We have, in this study, proposed a refinement to the conventional Divisia index method through replacing the arithmetic weight function by a normalized logarithmic weight function. The improvements made are that the refined method leaves no residual and can effectively handle the value zero in the date set. With these improvements, the refined method can be generally used to decompose energy, environmental or any other similar indicators irrespective of the level of sector disaggregation and the values in the data set. The extra computational burden, as compared to the conventional Divisia index method, is very marginal and its application is simple and straightforward. We have also presented decomposition results obtained using data from Korean industry to highlight the properties of the refined method.

APPENDIX 1: Proof of Perfect Decomposition Using the Refined Divisia Method

We decompose the aggregate energy intensity to show the property of perfect decomposition for the refined Divisia decomposition method. The proof can be extended to the case of aggregate CO_2 intensity decomposition. From Equations (1) and (2),

$$\begin{aligned} D_{tot} &= \Sigma_i\, y_{i,T}\, I_{i,T} \,/\, \Sigma_i\, y_{i,0}\, I_{i,0} \\ &= \exp\,[\ln\,(\Sigma_i\, y_{i,T}\, I_{i,T} \,/\, \Sigma_i\, y_{i,0}\, I_{i,0})] \end{aligned} \qquad (A.1)$$

The product $D_{str}\, D_{int}$ on the right-hand side of Equation (10) is given by the product of Equations (19) and (20) and can be written as

$$D_{str}\, D_{int} = \exp\,\{\, \Sigma_i\, w_i^*\, \ln\,[(y_{i,T}\, I_{i,T})/(y_{i,0}\, I_{i,0})]\} \qquad (A.2)$$

After taking the logarithm of Equations (A.1) and (A.2), we wish to show that the difference, Δ, given by

$$\Delta = \ln\,(\Sigma_i\, y_{i,T}\, I_{i,T} \,/\, \Sigma_i\, y_{i,0}\, I_{i,0}) - \Sigma_i\, w_i^*\, \ln[(y_{i,T}\, I_{i,T})/(y_{i,0}\, I_{i,0})] \qquad (A.3)$$

is equal to zero. Since $\Sigma_k w_k^* = 1$, $I_{i,0}\, y_{i,0} = E_{i,0} / Y_0$ and $I_{i,T}\, y_{i,T} = E_{i,T} / Y_T$, we can rewrite Equation (A.3) and show $\Delta = 0$ as follows:

$$\begin{aligned} \Delta &= \Sigma_i\, w_i^*\, [\, \ln\,\{(\Sigma_k\, E_{k,T} / Y_T)/(\Sigma_k\, E_{k,0} / Y_0)\} - \ln\{(E_{i,T} / Y_T)/(E_{i,0} / Y_0)\}] \\ &= \Sigma_i\, w_i^*\, [\, \ln\,(E_{i,0} / \Sigma_k\, E_{k,0}) - \ln(E_{i,T} / \Sigma_k\, E_{k,T})] \\ &= \Sigma_i\, w_i^*\, \ln\,(w_{i,0} / w_{i,T}) \\ &= \Sigma_i\, (w_{i,0} - w_{i,T}) \,/\, \Sigma_k\, L(w_{k,0}, w_{k,T}) \\ &= \Sigma_i\, (E_{i,0} / E_0 - E_{i,T} / E_T) \,/\, \Sigma_k\, L(w_{k,0}, w_{k,T}) \\ &= 0 \end{aligned}$$

APPENDIX 2: Energy and Production Data for Korean Industry

Korean energy and industrial production data for 1981 and 1993 were collected respectively from Korea, Ministry of Resources and Energy (1994) and National Statistical Office (1994). Industry comprises the following nine sectors: food, textiles, wood, paper, chemical, non-metal, basic metal, machinery, and other industry. Industrial production data represent gross output at constant prices of 1985. More details about the energy consumption and production data are given in Choi et al. (1995).

In the decomposition of the aggregate CO_2 intensity, the CO_2 emissions were estimated by multiplying consumption of coal, oil, gaseous fuels and electricity in Korean industry by their respective CO_2 emission coefficients given in tonnes of carbon per unit of energy used. The sum of the emissions over all the energy types in a year divided by the industrial production in that year gives the aggregate CO_2 intensity. The CO_2 emission coefficient for electricity was determined as follows. Total CO_2 emissions, at the national level, were calculated by first multiplying consumption of individual fossil fuels in electricity generation by their respective CO_2 emission coefficients. The sum of emissions over all these fuels was divided by the total electricity used by all final consumers to give the emission coefficient for electricity. This weighted coefficient for electricity, which is the national average, is assumed to be applicable to the industrial sector. As the fuel mix in electricity generation varies from one year to another, the CO_2 emission coefficient for electricity changes over time. More details about the estimation and the assumptions made are given in Ang and Pandiyan (1997). A similar approach was also adopted in Torvanger (1991).

REFERENCES

Ang, B. W. (1994). "Decomposition of Industrial Energy Consumption: The Energy Intensity Approach." *Energy Economics* 16(3): 163-174.

Ang, B. W. and S. Y. Lee (1994). "Decomposition of Industrial Energy Consumption: Some Methodological and Application Issues." *Energy Economics* 16(2): 83-92.

Ang, B. W. (1995). "Decomposition Methodology in Industrial Energy Demand Analysis." *Energy* 20(11): 1081-1095.

Ang, B. W. and G. Pandiyan (1997). "Decomposition of Energy-induced CO_2 Emissions in Manufacturing." *Energy Economics* (in press).

Boyd, G., J. F. McDonald, M. Ross, and D. A. Hanson (1987). "Separating the Changing Composition of US Manufacturing Production from Energy Efficiency Improvements: A Divisia Index Approach." *The Energy Journal* 8(2): 77-96.

Boyd, G.A., D.A. Hanson and T. Sterner (1988), "Decomposition of Changes in Energy Intensity - A Comparison of the Divisia Index and Other Methods." *Energy Economics* 10(4): 309-312.

Choi, K. H., B. W. Ang, and K. K. Ro (1995). "Decomposition of the Energy-Intensity Index with Application for the Korean Manufacturing Industry." *Energy* 20(9): 835-842.

Hsu, G. J. Y. and A. C. Hsu (1993), "Energy Intensity in Taiwan's Industrial Sectors: Divisia Index vs Laspeyres Index.", *Proceedings of the 16th Annual International Conference on the International Association for Energy Economics*, July 27-29, Bali Indonesia, pp. 9.23 - 9.38.

Korea, Ministry of Resources and Energy and Korea Energy Economics Institute (1994). *Yearbook of Energy Statistics*, Seoul.

Korea, National Statistical Office (1994). *Report on Mining and Manufacturing Survey*, Seoul.

Lin, S. J. and T. C. Chang (1996). "Decomposition of SO_2, NO_x and CO_2 Emissions from Energy Use of Major Economic Sectors in Taiwan." *The Energy Journal* 17(1): 1-17.

Liu, X. Q., B. W. Ang, H. L. Ong (1992a). "The Application of the Divisia Index to the Decomposition of Changes in Industrial Energy Consumption." *The Energy Journal* 13(4): 161-177.

Liu, X. Q., B. W. Ang, and H. L. Ong (1992b). "Interfuel Substitution and Decomposition of Changes in Industrial Energy Consumption." *Energy* 17(7): 689-696.

Park, S. H. (1992). "Decomposition of Industrial Energy Consumption - An Alternative Method." *Energy Economics* 14(4): 265-270.

Sato, K. (1976). "The Ideal Log-change Index Number." *The Review of Economics and Statistics* 58: 223-228.

Törnqvist, L. (1935). "A Memorandum Concerning the Calculation of Bank of Finland Consumption Price Index." (In Swedish), unpublished memo, Bank of Finland, Helsinki.

Törnqvist, L., P. Vartia, and Y. Vartia (1985). "How Should Relative Changes be Measured?" *The America Statistician* 39(1): 43-46.

Torvanger, A. (1991). "Manufacturing Sector Carbon Dioxide Emissions in Nine OECD Countries, 1973-87." *Energy Economics* 13(3): 168-186.