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Index decomposition analysis with multidimensional and multilevel energy data

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ABSTRACT

Index decomposition analysis (IDA) is a popular tool for analyzing changes in energy consumption over time. Traditionally, a typical IDA study uses a single dimensional energy dataset, such as industrial energy consumption by industrial sector or transportation energy consumption by transport mode. More recently, there have been a growing number of studies using more sophisticated datasets, e.g. energy consumption by geographical region and by economic sector in a single dataset. For IDA studies using energy data with multiple attributes, intermediate decomposition results can be generated using subsets of the entire dataset, and these results provide further insight into the energy system and problem studied. To ensure that these intermediate results are consistent and meaningful, the IDA method used should ideally satisfy two properties: perfect in decomposition at the subcategory level and consistency in aggregation. It is shown that the logarithmic mean Divisia index method I (LMDI-I) satisfies these two properties in both additive and multiplicative decomposition analysis. It is therefore the recommended IDA method when dealing with energy data with multiple attributes.

1. Introduction

Index decomposition analysis (IDA) has been widely used to quantitatively assess the drivers leading to changes in an aggregate energy indicator over time. This is normally done with a simple dataset which is single dimensional, such as industrial energy consumption by industrial sector, transportation energy use by transport mode, and residential energy consumption by housing type. In a single dimensional study, the effect of structure change specified is that pertains to how energy consumption is disaggregated in that particular dimension. In the above examples, this means the effect of changes in industry product mix, transport modal mix, and housing type mix, respectively. More recently, there have been a growing number of studies using multidimensional energy data. In a two-dimensional case, for example, energy consumption in a country may be presented by economic sector and by geographical region in a single dataset. In such cases, using the dataset, decomposition analysis can be conducted separately for each of the dimensions, which may lead to different effects being specified and hence different decomposition results.

Irrespective of whether the dataset is single or multidimensional, the data can be presented in more than one level of disaggregation, i.e. multilevel data. For example, when energy consumption data are presented for a few broad and meaningful industry groups and each of these groups is further disaggregated into finer industry sectors, we have two levels of sector disaggregation. This can similarly apply to the geographical dimension where energy consumption is first divided into that for several large and meaningful geographical regions of a country and each of the regions is further divided into smaller states, provinces, or counties.[1] Consider a two-dimensional dataset, say by economic sector and by geographical region, and assuming the data are presented at several different levels of disaggregation for each dimension, i.e. a dataset with multiple attributes, there are many different ways that a decomposition analysis can be conducted. The choice depends on the subset of the entire dataset that is used. These intermediate decomposition results generated can provide further insight into the energy system and problem studied. We shall refer to such IDA studies as those with multidimensional and multilevel energy data.

The majority of IDA studies in the literature are single dimensional and single disaggregation level studies.[2] More recently, there have been a growing number of studies using multidimensional and/or multilevel data. For instance, Liu et al. (2012) attempt to uncover China's greenhouse gas emission changes at both regional and sectoral dimensions. Ma (2014) examines growth in China's energy consumption by fuel type, industry sector, and geographical region, while Xu et al. (2014) study the issue by fuel type and industry sector. Voigt et al. (2014) investigate the global energy intensity changes by considering national and sectoral aspects. In the case of multilevel analysis, Petrick

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[1] There can also be a mixed multilevel structure, where energy consumption is first divided into that for several regions and the energy consumption in each of these regions is further divided by industry sector, or vice versa.

[2] Some recent examples are Ma (2010), Zha et al. (2010), Kesicki and Anandarajah (2011), Das and Paul (2014), Jimenez and Mercado (2014), Xu et al. (2012), Wang et al. (2014), and Branger and Quirion (2015). Literature surveys on IDA can be found in Ang and Zhang (2000), and Xu and Ang (2013).

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(2013) quantifies the driving forces behind Germany's CO_2 emission changes at industry subsector and plant levels. Fernández González et al. (2014) study changes of energy consumption in 27 European countries at the regional and national levels. Xu and Ang (2014a) investigate the United States' energy consumption growth at the industry sector and subsector levels.[3] Compared to single dimensional and single disaggregation level studies, multidimensional and/or multilevel studies can better reveal the drivers behind the aggregate indicator of interest from multiple perspectives and provides additional insight. It is expected more such studies will be undertaken by researchers in future.

From the methodological viewpoint, the IDA method in a multidimensional and multilevel study should ideally satisfy two specific properties on top of the several other basic ones which are considered to be desirable for an IDA method.[4] The first is perfect in decomposition at the subcategory level.[5] This property ensures the decomposition results for each subcategory to be consistent and meaningful. As such, interpretation of the drivers of change at the subcategory level can be conducted in the same way as at the aggregate level. The second property is consistency in aggregation. In a multilevel study, decomposition can be conducted using two different procedures: in a single step to give the results at the aggregate level or in two or more steps with one subcategory dealt with at a time. If the final results are independent of the procedure used, the IDA method is said to be consistent in aggregation.[6] This property is especially desirable since the choice of decomposition procedure will then be irrelevant.

As a realistic example of a multidimensional and multilevel study, consider a decomposition study that deals with changes in energy consumption in industry in China where the energy consumption and industrial activity data have two dimensions: by industrial sector and by province-level administrative division.[7] The decomposition analysis can be conducted based on two different data structure designs: (a) total energy consumption in the country is first broken down by province-level administrative division, and for each province-level administrative division, energy consumption is further broken down by industrial sector, and (b) total energy consumption in the country is first broken down by industrial sector, and for each industrial sector, energy consumption is further broken down by province-level administrative division. In each dimension, additional levels of data aggregation may be introduced, say industry groups in energy sector dimension and geographical regions in geographical dimensions. Satisfying the first property ensures perfect decomposition for each province-level administrative division in case (a) while perfect decomposition for each industry sector in case (b). Interpretation of decomposition results can therefore be conveniently conducted for each province-level administrative division or for each industry sector. Satisfying the second property implies that the same decomposition results will be obtained at the aggregate level irrespective of whether the single-step procedure or step-by-step procedure is adopted for each of the dimensions.

As another example, consider a decomposition study that investigates changes in energy consumption in the residential sector of a country where the data on energy consumption and floor space (which is taken as the activity indicator) have two dimensions: by housing type and by energy end use.[8] Assume that each dimension has two levels of data disaggregation.[9] Similarly, the decomposition analysis can be conducted based on two different data structure designs: (a) total residential energy consumption in a country is first broken down by housing type, and for each housing type, energy consumption is further broken down by end use, and (b) total energy consumption in the country is first broken down by end use, for each end use, energy consumption is further broken down by housing type. Applying an IDA method that satisfies the property of perfect decomposition at the subcategory level ensures consistent results for each housing type and each end use, while applying an IDA method that satisfies the property of consistent in aggregation ensures that the same decomposition results are obtained at the aggregate level irrespective of whether the single-step procedure or step-by-step procedure is adopted.

In this study, we shall limit our analysis to the Divisia decomposition approach since it is the most widely used in IDA.[10] In IDA, decomposition of an aggregate can be performed either additively or multiplicatively. The former involves the decomposition of an arithmetic (or difference) change of an aggregate such as total energy consumption, and the decomposed results are given in a physical unit. The latter involves the decomposition of a ratio change, and the decomposed results are expressed in indexes. Both forms are widely used by researchers and analysts and hence have to be studied for completeness. This means that, ideally, an IDA method that satisfies the two desirable properties will hold true in both additive and multiplicative decomposition analysis.

In the literature, the issue of perfect decomposition at the subcategory level for additive Divisia methods has been analyzed in Ang et al. (2009). A similar study has not been reported for multiplicative Divisia methods. The issue of consistency in aggregation has been studied for multiplicative Divisia methods in Ang and Liu (2001), and a similar study has not been reported for the additive counterpart. A formal treatment of both additive and multiplicative Divisia methods with a focus on the two properties in multidimensional and multilevel analysis is lacking. The main objective of this paper is to fill the gap. It will be further shown that among the commonly used Divisia methods, the logarithmic mean Divisia index method I (LMDI-I) is the only one that satisfies the two properties in both additive and multiplicative analysis. LMDI-I can therefore be taken as the preferred Divisia method in multidimensional and multilevel IDA studies. Our study will be useful to potential researchers and analysts as there have been a growing number of such studies being reported in the literature.

The rest of this paper is organized as follows. Section 2 presents the basics of IDA. Section 3 and Section 4 deal with the concepts of perfect decomposition at the subcategory level and consistent in aggregation in additive and multiplicative Divisia methods, respectively. A numerical example involving a multidimensional and multilevel analysis is presented in Section 5. Further discussion and conclusions are presented in Section 6.

2. Basics of IDA

Assume an aggregate indicator V which can be divided into m subcategories and where n factors contribute to its change over time, i.e. $V = \sum_{j=1}^{m} V_j = \sum_{j=1}^{m} \left(\prod_{i=1}^{n} x_{j,i} \right)$, $(i = 1, \ldots, n;\ j = 1, \ldots, m)$. The purpose of IDA is to distribute a change in V in a consistent manner to a set of pre-defined factors.[11] Additive

[3] Xu and Ang (2014a) discuss multilevel IDA and present two models, the Multilevel-Parallel (M-P) model and the Multilevel-Hierarchy (M-H) model. The multilevel analysis reported here is consistent with the M-P model.

[4] For a discussion of these basic properties, such as the factor reversal test and time-reversal test, refer to Ang and Zhang (2000) and Ang (2004).

[5] If a study involves total energy consumption that is divided by industry sector, a particular industry sector is taken as a "subcategory." Similarly, if it is divided by geographical region, a particular region is taken as a "subcategory." The term "subcategory" is used in Ang et al. (2004) and Ang et al. (2009).

[6] For the property of consistency in aggregation in index number problems, see, for example, Vartia (1976) and Diewert (1978). More on this property in the context of IDA can be found in Ang and Liu (2001).

[7] There are more than 30 province-level administrative divisions in China.

[8] Energy end use for the residential sector includes space heating, space cooling, lighting, refrigeration, etc.

[9] For an example of having two levels of data disaggregation for the end use dimension in residential energy use, see Xu and Ang (2014b).

[10] Details about the various index decomposition analysis approaches and methods can be found in Ang (2004).

[11] In IDA, the aggregate indicator can be a quantity such as total energy consumption, or an aggregate energy intensity such as energy use per value-added (for industry) or per passenger-kilometer (for passenger transportation). These two forms of aggregate indicator would entail different considerations in decomposition. In this study, as in Ang et al. (2009), our focus is on aggregate quantity indicator.

and multiplicative decomposition of the aggregate indicator can be respectively formulated as follows:

$$V^T - V^0 = \sum_{j=1}^{m} V_j^T - \sum_{j=1}^{m} V_j^0 = \sum_{j=1}^{m}\sum_{i=1}^{n} \Delta V_{j,i} + \Delta V_{rsd} \tag{1}$$

$$\frac{V^T}{V^0} = \frac{\sum_{j=1}^{m} V_j^T}{\sum_{j=1}^{m} V_j^0} = \prod_{j=1}^{m}\prod_{i=1}^{n} D_{j,i} \cdot D_{rsd} \tag{2}$$

where $\Delta V_{j,i}$ and $D_{j,i}$ are the additive and multiplicative effects associated with factor i at jth subcategory, respectively. The subscript rsd denotes a potential residual term and its existence is IDA method dependent.

To estimate the additive or multiplicative effects, we first differentiate V or $\ln V$ with respect to time t. In the additive case, we have

$$\begin{aligned}\frac{dV}{dt} &= \sum_{j=1}^{m} \frac{dV_j}{dt} = \sum_{j=1}^{m}\left(x_{j,2}\ldots x_{j,n}\frac{\partial x_{j,1}}{\partial t} + x_{j,1}x_{j,3}\ldots x_{j,n}\frac{\partial x_{j,2}}{\partial t} + \ldots + x_{j,1}\ldots x_{j,n-1}\frac{\partial x_{j,n}}{\partial t}\right)\\ &= \sum_{j=1}^{m}\left(\frac{V_j}{x_{j,1}}\frac{\partial x_{j,1}}{\partial t} + \frac{V_j}{x_{j,2}}\frac{\partial x_{j,2}}{\partial t} + \ldots + \frac{V_j}{x_{j,n}}\frac{\partial x_{j,n}}{\partial t}\right)\end{aligned} \tag{3}$$

By integrating the above differentiation, the additive difference of the aggregate indicator in time period [0, T] can be written as

$$\begin{aligned}V^T - V^0 &= \int_0^T \frac{dV}{dt}dt\\ &= \int_0^T \sum_{j=1}^{m}\left(\frac{V_j}{x_{j,1}}\frac{\partial x_{j,1}}{\partial t} + \frac{V_j}{x_{j,2}}\frac{\partial x_{j,2}}{\partial t} + \ldots + \frac{V_j}{x_{j,n}}\frac{\partial x_{j,n}}{\partial t}\right)dt\\ &= \int_0^T \sum_{j=1}^{m} V_j\left(\frac{\partial \ln x_{j,1}}{\partial t} + \ldots + \frac{\partial \ln x_{j,n}}{\partial t}\right)dt\end{aligned} \tag{4}$$

Eq. (4) can be rewritten as

$$V^T - V^0 = \sum_j \Delta V_{1,j} + \Delta V_{2,j} + \ldots + \Delta V_{n,j} \tag{5}$$

In the multiplicative case, we have

$$\begin{aligned}\frac{d\ln V}{dt} &= \frac{1}{V}\cdot\frac{dV}{dt}\\ &= \frac{1}{V}\cdot\left[\sum_j \frac{dx_{j,1}}{dt}x_{j,2},\ldots,x_{j,n} + \frac{dx_{j,2}}{dt}x_{j,1}x_{j,3},\ldots,x_{j,n} + \ldots + \frac{dx_{j,n}}{dt}x_{j,1},\ldots,x_{j,n-1}\right]\\ &= \sum_j \frac{V_j}{V}\left(\frac{d\ln x_{j,1}}{dt} + \frac{d\ln x_{j,2}}{dt} + \ldots + \frac{d\ln x_{j,n}}{dt}\right)\end{aligned} \tag{6}$$

By integrating $\frac{d\ln V}{dt}$ in the time interval [0, T], we have the following contribution of each factor to the total ratio change:

$$\begin{aligned}\int_0^T \frac{d\ln V}{dt}dt = {} & \ln\frac{V^T}{V^0}\\ &= \int_0^T \sum_j \frac{V_j}{V}\left(\frac{d\ln x_{j,1}}{dt} + \frac{d\ln x_{j,2}}{dt} + \ldots + \frac{d\ln x_{j,n}}{dt}\right)dt\end{aligned} \tag{7}$$

Eq. (7) can be rewritten as

$$\begin{aligned}\frac{V^T}{V^0} &= \exp\left(\int_0^T \sum_j \frac{V_j}{V}\left(\frac{d\ln x_{j,1}}{dt} + \frac{d\ln x_{j,2}}{dt} + \ldots + \frac{d\ln x_{j,n}}{dt}\right)dt\right)\\ &= \prod_j D_{j,1}D_{j,2},\ldots,D_{j,n}\end{aligned} \tag{8}$$

Since empirical data are available on discrete time points, the integrations in Eqs. (4) and (8) cannot be evaluated. Instead, discretization is used to estimate the decomposition effects. This is analogous to requiring a certain "integration path" be chosen. Once the "path" is defined, Eqs. (4) and (8) can be respectively calculated as

$$\begin{aligned}V^T - V^0 &\cong \sum_j w_j^A \ln\frac{x_{j,1}^T}{x_{j,1}^0} + w_j^A \ln\frac{x_{j,2}^T}{x_{j,2}^0} + \ldots + w_j^A \ln\frac{x_{j,n}^T}{x_{j,n}^0}\\ &\cong \sum_{j=1}^{m}\sum_{i=1}^{n}\Delta V_{j,i}\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{V^T}{V^0} &\cong \prod_{j=1}^{m} \exp\left(w_j^M \ln\frac{x_{j,1}^T}{x_{j,1}^0}\right)\cdot\exp\left(w_j^M \ln\frac{x_{j,2}^T}{x_{j,2}^0}\right)\cdot\ldots\cdot\exp\left(w_j^M \ln\frac{x_{j,n}^T}{x_{j,n}^0}\right)\\ &\cong \prod_{j=1}^{m}\prod_{i=1}^{n} D_{j,i}\end{aligned} \tag{10}$$

where w_j^A and w_j^M are the weights.

In the literature, different sets of weights have been proposed by researchers. This has led to several Divisia decomposition methods: Parametric Divisia method 1 (PDM1) (Ang, 1994; Ang and Lee, 1994), AMDI (Boyd et al., 1987), LMDI-I (Ang and Liu, 2001), and LMDI-II (Ang and Choi, 1997). PDM1 and AMDI are parametric, while LMDI-I and LMDI-II are non-parametric and do not rely on any specification for the weights. A summary of the decomposition formulae for AMDI, LMDI-I, and LMDI-II can be found in Ang et al. (2003) and Ang (2004). Eqs. (9) and (10) imply that the observed arithmetic and ratio changes of the aggregate indicator may not be exactly the same as the sum and product of the estimated effects, respectively. A residual term may appear in the decomposition results. In IDA, a perfect decomposition method is one that does not give a residual term. Among the above Divisia methods, LMDI-I and LMDI-II are the only ones that give perfect decomposition results at the aggregate level (Ang et al., 2003).

3. Additive Divisia methods

3.1. Perfect decomposition at the subcategory level

The perfect decomposition concept as presented in the foregoing can be extended to the subcategory level. In additive decomposition, which deals with arithmetic change, Ang et al. (2009) show that LMDI-I is the only commonly used Divisia method that gives perfect decomposition at the subcategory level. They further show that by applying the "proportionally distributed by subcategory" principle to the residual terms at the subsector level to distribute the residuals to the main effects, the final adjusted decomposition results given by other Divisia methods, such as AMDI and LMDI-II, are the same as those of LMDI-I.

3.2. Consistency in aggregation

We can extend the analysis in Ang and Liu (2001) which deals with consistency is aggregation in the multiplicative form to the additive form. Suppose the aggregate indicator, V, can be disaggregated into two subcategories, namely j and k ($j = 1, \ldots, m$; $k = 1, \ldots, p$). The change in V during time period [0, T] can be expressed as

$$V^T - V^0 = \sum_j V_j^T - V_j^0 = \sum_k V_k^T - V_k^0 = \sum_{j,k} V_{jk}^T - V_{jk}^0 \tag{11}$$

For illustration purposes, we shall use LMDI-I to show the single-step and step-by-step decomposition procedures. In the single-step procedure, the effect associated with factor i can be formulated as

$$\Delta V_i = \sum_{j,k} L\left(V_{jk}^T, V_{jk}^0\right) \ln \frac{x_{j,k,i}^T}{x_{j,k,i}^0} \tag{12}$$

This means the effects at the aggregate level are derived directly from the data at the most disaggregated level in the dataset without considering the subcategory data.

In the step-by-step procedure, we first decompose the change within subcategory j using the data for subcategory level k, and then aggregate various factors' effects to the aggregate level. In the first step, we have

$$\Delta V_{j,i} = \sum_k L\left(V_{jk}^T, V_{jk}^0\right) \ln \frac{x_{j,k,i}^T}{x_{j,k,i}^0} \tag{13}$$

In the second step, the effects at the jth subcategory level are aggregated to the aggregate level, i.e.

$$\Delta V_i = \sum_j L\left(V_j^T, V_j^0\right) \ln\left(D_{j,i}\right) \tag{14}$$

In virtue of the relationship between the multiplicative and additive effects (i.e. $D_{j,i}$ and $\Delta V_{j,i}$) when using LMDI-I, Eq. (14) can be rewritten as

$$\begin{aligned} \Delta V_i &= \sum_j L\left(V_j^T, V_j^0\right) \sum_k \frac{L\left(V_{jk}^T, V_{jk}^0\right)}{L\left(V_j^T, V_j^0\right)} \ln \frac{x_{j,k,i}^T}{x_{j,k,i}^0} \\ &= \sum_{j,k} L\left(V_{jk}^T, V_{jk}^0\right) \ln \frac{x_{j,k,i}^T}{x_{j,k,i}^0} \end{aligned} \tag{15}$$

which is the same as Eq. (12). This shows that LMDI-I is consistent in aggregation. Following the same steps, it can be shown that none of PDM1, AMDI, and LMDI-II satisfies the property of consistency in aggregation.

In multidimensional and multilevel analysis, more than one decomposition "path" can be used in the step-by-step procedure. Due to the property of consistency in aggregation when using LMDI-I, the decomposition results following different step-by-step decomposition paths are consistent. For instance, the decomposition path in the above step-by-step procedure is first at the subcategory level and then at the aggregate level, i.e. bottom-up. Alternatively, we can first decompose at the aggregate level and then at the subcategory level, i.e. top-down. Following this decomposition path, the step-by-step decomposition becomes the first step, i.e.

$$\Delta V_i = \sum_j L\left(V_j^T, V_j^0\right) \ln\left(\frac{x_{j,i}^T}{x_{j,i}^0}\right) \tag{16}$$

and the second step is

$$D_{j,i} = \exp\left(\sum_k \frac{L\left(V_{jk}^T, V_{jk}^0\right)}{L\left(V_j^T, V_j^0\right)} \ln \frac{x_{j,k,i}^T}{x_{j,k,i}^0}\right) \tag{17}$$

The item $x_{j,i}^T / x_{j,i}^0$ in Eq. (16) denotes the change in factor i at the jth subcategory, which can be further expressed by the change in the ith factor within the jth subcategory, i.e. Eq. (17). Consequently, putting Eq. (17) into Eq. (16) yields

$$\Delta V_i = \sum_{j,k} L\left(V_{jk}^T, V_{jk}^0\right) \ln \frac{x_{j,k,i}^T}{x_{j,k,i}^0} \tag{18}$$

which is identical with Eqs. (12) and (15). The above analysis can also be extended to higher levels of multidimensional and multilevel analysis. It implies that by using the additive LMDI-I, the same decomposition results can be obtained when different paths for the step-by-step decomposition are adopted. In summary, since the additive LMDI-I is consistent in aggregation, the corresponding decomposition results are both procedure (i.e. single-step or step-by-step) and path independent.

4. Multiplicative Divisia methods

4.1. Perfect decomposition at the subcategory level

The study by Ang et al. (2009) deals with perfect decomposition at the subcategory level in additive decomposition analysis. It does not include multiplicative decomposition analysis and points out that this is an area where further research may be conducted. Given below is an extension of the study to multiplicative decomposition analysis.

4.1.1. Multiplicative effect at the subcategory level

In the additive form, perfect decomposition at the subcategory level can be easily defined, which is that the decomposition of $V_j^T - V_j^0$ is perfect. The equivalence in the multiplicative form is more complex. Since each subcategory is a component of the total and the changes at all the subcategory level contribute to the change at the aggregate level, we assign a weight to each subcategory which measures the subcategory's contribution to the total change. Moreover, it is logical that the perfect decomposition property of a subcategory should also be linked to this weight and the total change at the aggregate level. We therefore propose a simple form of weight given by $a_j = \frac{V_j^T - V_j^0}{V^T - V^0}$ for subcategory j, which is the ratio of the arithmetic change of the indicator for the subcategory to that at the aggregate level. The perfect decomposition property at subcategory level j in the multiplicative form can then be formulated as

$$D_j^* = \left(\frac{V^T}{V^0}\right)^{\frac{V_j^T - V_j^0}{V^T - V^0}} \tag{19}$$

Intuitively, Eq. (19) represents the multiplicative effect of subcategory j through its contribution to the total change of the aggregate indicator. It has the following properties. First, if $V_j^T - V_j^0 = 0$, the jth subcategory does not have any contribution to the total change of the aggregate indicator and $D_j^* = 1$. If $V_j^T - V_j^0 > (<)0$, subcategory j has a positive (negative) effect on the total change in the aggregate indicator and hence $D_j^* > (<)1$. Second, if $V_j^T - V_j^0 = V^T - V^0$, the contribution of subcategory j is the same as the total change in the aggregate indicator and $D_j^* = \frac{V^T}{V^0}$. Third, if $V_1^T - V_1^0 = \lambda(V_2^T - V_2^0)$, then subcategory 1 contributes λ times that of subcategory 2 and their multiplicative effects have the relationship $D_1^* = (D_2^*)^\lambda$. Finally, if the jth subcategory can be further divided into l sectors ($k = 1, \ldots, l$), we have $D_{j,k}^* = \left(\frac{V^T}{V^0}\right)^{\frac{V_{j,k}^T - V_{j,k}^0}{V^T - V^0}}$ which satisfies the property of consistent in aggregation.

It should also be noted that when $V^T = V^0$, Eq. (19) cannot be used directly. However, given the property of the logarithmic function, i.e. L(a,b), we can rewrite Eq. (19) as

$$D_j^* = \left(\frac{V^T}{V^0}\right)^{\frac{V_j^T - V_j^0}{V^T - V^0}} = \exp\left(\frac{V_j^T - V_j^0}{L\left(V^T, V^0\right)}\right) \tag{20}$$

When $V^T = V^0$, we have the perfect decomposition property for the jth subcategory given by $D_j^* = \exp\left(\frac{V_j^T - V_j^0}{L(V^T, V^0)}\right)$ which satisfies all the aforementioned properties.[12] The weight function given by Eq. (19) is therefore reasonable and robust.

4.1.2. Distributing residual at subcategory level

For an arbitrary multiplicative Divisia method, the total effect at the subcategory level can be given by

$$D_j = \exp\left(\sum_i w_j \ln\left(\frac{x_{j,i}^T}{x_{j,i}^0}\right)\right) = \prod_i D_{j,i} \tag{21}$$

where w_j is the weights which are method dependent. The residual at subcategory j can be calculated as the ratio of the perfect decomposition effect to the effect for this subcategory given in Eq. (21):

$$D_{rsd_j} = \frac{D_j^*}{D_j} \tag{22}$$

If the residual term exists at the subcategory level, i.e. D_{rsd_j} is not equal to 1, we may distribute it among the main factors according to some principle. Ang et al. (2009) propose the principle of "proportionally distributed by subcategory" for the additive Divisia methods. Using the same concept, we introduce the principle of "distributing the log of residual proportionally by subcategory" to distribute the residual terms for the multiplicative Divisia methods. This principle can be explained through the following example. For instance, let $\alpha_{j,i}$ be the proportion for factor i at subcategory j and its value can be determined as $\alpha_{j,i} = \frac{\ln(D_{j,i})}{\ln\left(\prod_i D_{j,i}\right)}$, or alternatively, $D_{j,i} = \left[\prod_i D_{j,i}\right]^{\alpha_{j,i}}$. The term D_{rsd_j} is the total residual for the jth subcategory in the multiplicative form, and the residual distributed to factor i at jth subcategory is $(D_{rsd_j})^{\alpha_{j,i}}$. This leads to the adjusted main effect given by $D_{j,i}(D_{rsd_j})^{\alpha_{j,i}}$. With such adjustments, perfect decomposition at both the subcategory and aggregate levels is achieved. It can further be shown that that all multiplicative Divisia decomposition methods given in the form of Eq. (21) will collapse to LMDI-I if we apply such a distribution principle. The proof for a general case is given in Appendix A.[13]

4.2. Consistency in aggregation

The study by Ang and Liu (2001) shows that the multiplicative LMDI-I is consistent in aggregation. It presents several examples to show that the single-step and step-by-step decomposition procedures yield exactly the same decomposition results. The study also points out that multiplicative LMDI-II and AMDI are not consistent in aggregation. Furthermore, similar to the additive case in Section 3.2, the multiplicative LMDI-I can give consistent results independent of the decomposition path in the step-by-step procedure. Hence, the decomposition results given by the multiplicative LMDI-I are also both procedure (i.e. single-step or step-by-step) and path independent.

[12] By definition, the logarithmic function $L(V^T, V^0) = V^T$ when $V^T = V^0$.

[13] As in the additive case reported in Ang et al. (2009), the only condition for the proof is the weight w_j should depend only on subcategory and not on factors. This condition can be met for most Divisia methods in the IDA literature except the Adaptive Weighting Parametric Divisia method (AWT-PDM1) introduced in Ang (1994).

Table 1
Data for a multidimensional and multilevel analysis using IDA (arbitrary unit).

Subcategory			Year 0		Year T	
			E_0	Y_0	E_T	Y_T
Region A	Sector 1	Subsector 1	20	7	50	30
		Subsector 2	10	3	30	10
	Sector 2	Subsector 1	10	30	12	15
		Subsector 2	10	10	4	25
Region B	Sector 1	Subsector 1	20	15	30	15
		Subsector 2	20	5	10	10
	Sector 2	Subsector 1	20	15	35	50
		Subsector 2	10	15	25	25
Total			120	100	196	180

5. Numerical example

We use the data in Table 1 to illustrate the properties of Divisia methods presented above in both the additive and multiplicative forms. The total energy consumption, E, and economic output, Q, of a country are divided into two dimensions, i.e. by region and by industry sector. Further, the latter has two levels of disaggregation, namely, industry sector and subsector. This example therefore has a multidimensional and multilevel dataset. For simplicity, we use the case of two regions, two industry sectors (where Sector 1 is more energy intensive than Sector 2), and two subsectors for each industry sector.

From the data structure in Table 1, the following identity can be defined:

$$E = \sum_{ijk} E_{ijk} = \sum_{ijk} Q \frac{Q_i}{Q} \frac{Q_{ij}}{Q_i} \frac{Q_{ijk}}{Q_{ij}} \frac{E_{ijk}}{Q_{ijk}} \tag{23}$$

where i, j, and k denote region, sector and subsector, respectively. Three structure factors appear in the identity, where Q_i/Q is the regional output share in the economy, Q_{ij}/Q_i is the sector output share in ith region, and Q_{ijk}/Q_{ij} is the subsector output share in the jth sector in ith region.

Table 2
Weights for additive and multiplicative LMDI-I, LMDI-II, and AMDI in the numerical example.

		Additive
Single-step	LMDI-I	$L(E_{ijk}^T, E_{ijk}^0)$
	LMDI-II	$\frac{L(E_{ijk}^T/E^T, E_{ijk}^0/E^0)}{\sum_{ijk} L(E_{ijk}^T/E^T, E_{ijk}^0/E^0)} \cdot L(E^T, E^0)$
	AMDI	$(E_{ijk}^T + E_{ijk}^0)/2$
Step-by-step	LMDI-I	$L(E_{ijk}^T, E_{ijk}^0)$
	LMDI-II	$\frac{L(E_i^T/E^T, E_i^0/E^0)}{\sum_i L(E_i^T/E^T, E_i^0/E^0)} \frac{L(E_{ij}^T/E_i^T, E_{ij}^0/E_i^0)}{\sum_j L(E_{ij}^T/E_i^T, E_{ij}^0/E_i^0)} \frac{L(E_{ijk}^T/E_{ij}^T, E_{ijk}^0/E_{ij}^0)}{\sum_k L(E_{ijk}^T/E_{ij}^T, E_{ijk}^0/E_{ij}^0)} L(E^T, E^0)$
	AMDI	$\frac{E_i^T + E_i^0}{2} \cdot \frac{E_{ij}^T/E_i^T + E_{ij}^0/E_i^0}{2} \cdot \frac{E_{ijk}^T/E_{ij}^T + E_{ijk}^0/E_{ij}^0}{2}$
		Multiplicative
Single-step	LMDI-I	$\frac{L(E_{ijk}^T, E_{ijk}^0)}{L(E^T, E^0)}$
	LMDI-II	$\frac{L(E_{ijk}^T/E^T, E_{ijk}^0/E^0)}{\sum_{ijk} L(E_{ijk}^T/E^T, E_{ijk}^0/E^0)}$
	AMDI	$(E_{ijk}^T/E^T + E_{ijk}^0/E^0)/2$
Step-by-step	LMDI-I	$\frac{L(E_{ijk}^T, E_{ijk}^0)}{L(E^T, E^0)}$
	LMDI-II	$\frac{L(E_i^T/E^T, E_i^0/E^0)}{\sum_i L(E_i^T/E^T, E_i^0/E^0)} \frac{L(E_{ij}^T/E_i^T, E_{ij}^0/E_i^0)}{\sum_j L(E_{ij}^T/E_i^T, E_{ij}^0/E_i^0)} \frac{L(E_{ijk}^T/E_{ij}^T, E_{ijk}^0/E_{ij}^0)}{\sum_k L(E_{ijk}^T/E_{ij}^T, E_{ijk}^0/E_{ij}^0)}$
	AMDI	$\frac{E_i^T/E^T + E_i^0/E^0}{2} \cdot \frac{E_{ij}^T/E_i^T + E_{ij}^0/E_i^0}{2} \cdot \frac{E_{ijk}^T/E_{ij}^T + E_{ijk}^0/E_{ij}^0}{2}$

Table 3
Additive decomposition results for LMDI-I.

Subcategory			ΔE_{tot}	ΔE_{act}	ΔE_{str1}	ΔE_{str2}	ΔE_{str3}	ΔE_{int}	ΔE^*_{tot}
Region A	Sector 1	Subsector 1	30.0000	19.2445	−3.8563	30.0000	2.2589	−17.6471	30.0000
		Subsector 2	20.0000	10.7005	−2.1442	16.6809	−3.3191	−1.9181	20.0000
	Sector 2	Subsector 1	2.0000	6.4478	−1.2920	−5.1558	−7.6036	9.6036	2.0000
		Subsector 2	−6.0000	3.8489	−0.7713	−3.0776	6.0000	−12.0000	−6.0000
Region B	Sector 1	Subsector 1	10.0000	14.4966	2.5985	−11.5917	−5.5034	10.0000	10.0000
		Subsector 2	−10.0000	8.4800	1.5200	−6.7807	6.7807	−20.0000	−10.0000
	Sector 2	Subsector 1	15.0000	15.7551	2.8241	5.9812	7.7111	−17.2714	15.0000
		Subsector 2	15.0000	9.6223	1.7248	3.6529	−6.6376	6.6376	15.0000
Total			76.0000	88.5957	0.6036	29.7091	−0.3130	−42.5954	76.0000

Table 4
Additive decomposition results for LMDI-II using single-step procedure.

Subcategory			ΔE_{tot}	ΔE_{act}	ΔE_{str1}	ΔE_{str2}	ΔE_{str3}	ΔE_{int}	ΔE^*_{tot}	ΔE_{rsd}
Region A	Sector 1	Subsector 1	30.0000	19.4460	−3.8967	30.3141	2.2825	−17.8319	30.3141	−0.3141
		Subsector 2	20.0000	10.7346	−2.1510	16.7340	−3.3297	−1.9242	20.0637	−0.0637
	Sector 2	Subsector 1	2.0000	6.7122	−1.3450	−5.3672	−7.9154	9.9974	2.0820	−0.0820
		Subsector 2	−6.0000	4.1863	−0.8389	−3.3474	6.5260	−13.0520	−6.5260	0.5260
Region B	Sector 1	Subsector 1	10.0000	14.9542	2.6805	−11.9576	−5.6771	10.3157	10.3157	−0.3157
		Subsector 2	−10.0000	9.1441	1.6391	−7.3117	7.3117	−21.5663	−10.7831	0.7831
	Sector 2	Subsector 1	15.0000	16.1507	2.8950	6.1314	7.9047	−17.7051	15.3767	−0.3767
		Subsector 2	15.0000	9.7230	1.7428	3.6912	−6.7071	6.7071	15.1570	−0.1570
Total			76.0000	91.0512	0.7259	28.8866	0.3957	−45.0592	76.0000	0.0000

Referring to them as *str1*, *str2*, and *str3* respectively, a change in total energy consumption can be decomposed additively and multiplicatively as follows:

$$E^T - E^0 \cong \Delta E_{act} + \Delta E_{str1} + \Delta E_{str2} + \Delta E_{str3} + \Delta E_{int} \tag{24}$$

$$\frac{E^T}{E^0} \cong D_{act} \cdot D_{str1} \cdot D_{str2} \cdot D_{str3} \cdot D_{int} \tag{25}$$

where *act* denotes the activity effect and *int* the energy intensity effects in IDA. The general formulae for the Divisia decomposition approach are

$$\Delta E_{act} = \sum_{ijk} w_{ijk} \ln \frac{Q^T}{Q^0} \tag{26}$$

$$\Delta E_{str1} = \sum_{ijk} w_{ijk} \ln \frac{Q_i^T/Q^T}{Q_i^0/Q^0} \tag{27}$$

$$\Delta E_{str2} = \sum_{ijk} w_{ijk} \ln \frac{Q_{ij}^T/Q_i^T}{Q_{ij}^0/Q_i^0} \tag{28}$$

$$\Delta E_{str3} = \sum_{ijk} w_{ijk} \ln \frac{Q_{ijk}^T/Q_{ij}^T}{Q_{ijk}^0/Q_{ij}^0} \tag{29}$$

$$\Delta E_{int} = \sum_{ijk} w_{ijk} \ln \frac{E_{ijk}^T/Q_{ijk}^T}{E_{ijk}^0/Q_{ijk}^0} \tag{30}$$

Table 5
Additive decomposition results for LMDI-II using step-by-step procedure.

Subcategory			ΔE_{tot}	ΔE_{act}	ΔE_{str1}	ΔE_{str2}	ΔE_{str3}	ΔE_{int}	ΔE^*_{tot}	ΔE_{rsd}
Region A	Sector 1	Subsector 1	30.0000	19.3781	−3.8831	30.2083	2.2746	−17.7696	30.2083	−0.2083
		Subsector 2	20.0000	10.6181	−2.1277	16.5524	−3.2936	−1.9033	19.8460	0.1540
	Sector 2	Subsector 1	2.0000	7.1015	−1.4230	−5.6785	−8.3744	10.5772	2.2028	−0.2028
		Subsector 2	−6.0000	4.1541	−0.8324	−3.3217	6.4757	−12.9515	−6.4757	0.4757
Region B	Sector 1	Subsector 1	10.0000	15.2520	2.7339	−12.1958	−5.7902	10.5211	10.5211	−0.5211
		Subsector 2	−10.0000	8.9219	1.5992	−7.1341	7.1341	−21.0422	−10.5211	0.5211
	Sector 2	Subsector 1	15.0000	16.0318	2.8737	6.0862	7.8465	−17.5747	15.2634	−0.2634
		Subsector 2	15.0000	9.5936	1.7196	3.6420	−6.6178	6.6178	14.9553	0.0447
Total			76.0000	91.0512	0.6603	28.1590	−0.3451	−43.5253	76.0000	0.0000

Table 6
Additive decomposition results for AMDI using single-step procedure.

Subcategory			ΔE_{tot}	ΔE_{act}	ΔE_{str1}	ΔE_{str2}	ΔE_{str3}	ΔE_{int}	ΔE^*_{tot}	ΔE_{rsd}
Region A	Sector 1	Subsector 1	30.0000	20.5725	−4.1224	32.0702	2.4148	−18.8649	32.0702	−2.0702
		Subsector 2	20.0000	11.7557	−2.3557	18.3258	−3.6464	−2.1072	21.9722	−1.9722
	Sector 2	Subsector 1	2.0000	6.4657	−1.2956	−5.1700	−7.6246	9.6302	2.0055	−0.0055
		Subsector 2	−6.0000	4.1145	−0.8245	−3.2900	6.4140	−12.8281	−6.4140	0.4140
Region B	Sector 1	Subsector 1	10.0000	14.6947	2.6340	−11.7501	−5.5786	10.1366	10.1366	−0.1366
		Subsector 2	−10.0000	8.8168	1.5804	−7.0501	7.0501	−20.7944	−10.3972	0.3972
	Sector 2	Subsector 1	15.0000	16.1641	2.8974	6.1364	7.9113	−17.7198	15.3894	−0.3894
		Subsector 2	15.0000	10.2863	1.8438	3.9050	−7.0956	7.0956	16.0351	−1.0351
Total			76.0000	92.8703	0.3575	33.1772	−0.1552	−45.4520	80.7979	−4.7979

Table 7
Additive decomposition results for AMDI using step-by-step procedure.

Subcategory			ΔE_{tot}	ΔE_{act}	ΔE_{str1}	ΔE_{str2}	ΔE_{str3}	ΔE_{int}	ΔE_{tot}^*	ΔE_{rsd}
Region A	Sector 1	Subsector 1	30.0000	19.8600	−3.9796	30.9595	2.3311	−18.2115	30.9595	−0.9595
		Subsector 2	20.0000	10.8910	−2.1824	16.9778	−3.3782	−1.9522	20.3560	−0.3560
	Sector 2	Subsector 1	2.0000	7.5984	−1.5226	−6.0758	−8.9604	11.3173	2.3569	−0.3569
		Subsector 2	−6.0000	4.5590	−0.9136	−3.6455	7.1070	−14.2140	−7.1070	1.1070
Region B	Sector 1	Subsector 1	10.0000	15.1670	2.7187	−12.1278	−5.7579	10.4624	10.4624	−0.4624
		Subsector 2	−10.0000	9.1002	1.6312	−7.2767	7.2767	−21.4628	−10.7314	0.7314
	Sector 2	Subsector 1	15.0000	16.0592	2.8786	6.0966	7.8599	−17.6048	15.2895	−0.2895
		Subsector 2	15.0000	9.6355	1.7272	3.6580	−6.6467	6.6467	15.0206	−0.0206
Total			76.0000	92.8703	0.3575	28.5662	−0.1686	−45.0188	76.6066	−0.6066

Table 8
Multiplicative decomposition results for LMDI-I.

Subcategory			D_{tot}	D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	D_{tot}^*
Region A	Sector 1	Subsector 1	1.2137	1.1323	0.9754	1.2137	1.0147	0.8923	1.2137
		Subsector 2	1.1378	1.0715	0.9863	1.1137	0.9788	0.9877	1.1378
	Sector 2	Subsector 1	1.0130	1.0425	0.9917	0.9673	0.9521	1.0640	1.0130
		Subsector 2	0.9620	1.0252	0.9950	0.9803	1.0395	0.9255	0.9620
Region B	Sector 1	Subsector 1	1.0667	1.0981	1.0169	0.9279	0.9651	1.0667	1.0667
		Subsector 2	0.9375	1.0563	1.0099	0.9572	1.0447	0.8789	0.9375
	Sector 2	Subsector 1	1.1017	1.1071	1.0184	1.0394	1.0510	0.8945	1.1017
		Subsector 2	1.1017	1.0641	1.0112	1.0239	0.9581	1.0438	1.1017
Total			1.6333	1.7717	1.0039	1.2114	0.9980	0.7596	1.6333

$$D_{act} = \exp\left(\sum_{ijk} w^*_{ijk} \ln \frac{Q^T}{Q^0}\right) \quad (31)$$

$$D_{str1} = \exp\left(\sum_{ijk} w^*_{ijk} \ln \frac{Q_i^T/Q^T}{Q_i^0/Q^0}\right) \quad (32)$$

$$D_{str2} = \exp\left(\sum_{ijk} w^*_{ijk} \ln \frac{Q_{ij}^T/Q_i^T}{Q_{ij}^0/Q_i^0}\right) \quad (33)$$

$$D_{str3} = \exp\left(\sum_{ijk} w^*_{ijk} \ln \frac{Q_{ijk}^T/Q_{ij}^T}{Q_{ijk}^0/Q_{ij}^0}\right) \quad (34)$$

$$D_{int} = \exp\left(\sum_{ijk} w^*_{ijk} \ln \frac{E_{ijk}^T/Q_{ijk}^T}{E_{ijk}^0/Q_{ijk}^0}\right) \quad (35)$$

We shall use LMDI-I, LMDI-II, and AMDI to conduct the decomposition analysis. Results will be presented for the single-step and step-by-step decomposition procedures. The weights for the respective methods in Eqs. (26)–(35) are summarized in Table 2.

Tables 3 to 7 summarize the results for the additive decomposition analysis. For LMDI-I, as shown in Table 3, the single-step procedure and the step-by-step procedure give the same results and there is no residual at the aggregate and subcategory levels. The latter can be seen from the computed total energy consumption change (ΔE_{tot}^*) in the last column of Table 3 which tally with the respective observed change

Table 9
Multiplicative decomposition results for LMDI-II using single-step procedure.

Subcategory			D_{tot}	D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	D_{tot}^*	D_{rsd}
Region A	Sector 1	Subsector 1	1.2137	1.1338	0.9752	1.2162	1.0148	0.8913	1.2162	0.9980
		Subsector 2	1.1378	1.0718	0.9862	1.1141	0.9787	0.9877	1.1383	0.9996
	Sector 2	Subsector 1	1.0130	1.0443	0.9914	0.9659	0.9502	1.0667	1.0135	0.9995
		Subsector 2	0.9620	1.0274	0.9946	0.9786	1.0430	0.9192	0.9587	1.0034
Region B	Sector 1	Subsector 1	1.0667	1.1014	1.0175	0.9257	0.9640	1.0689	1.0689	0.9980
		Subsector 2	0.9375	1.0608	1.0106	0.9539	1.0483	0.8700	0.9328	1.0051
	Sector 2	Subsector 1	1.1017	1.1099	1.0189	1.0404	1.0524	0.8920	1.1044	0.9976
		Subsector 2	1.1017	1.0648	1.0113	1.0241	0.9576	1.0442	1.1028	0.9990
Total			1.6333	1.8000	1.0047	1.2050	1.0026	0.7476	1.6333	1.0000

Table 10
Multiplicative decomposition results for LMDI-II using step-by-step procedure.

Subcategory			D_{tot}	D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	D_{tot}^*	D_{rsd}
Region A	Sector 1	Subsector 1	1.2137	1.1333	0.9752	1.2153	1.0148	0.8916	1.2153	0.9987
		Subsector 2	1.1378	1.0709	0.9864	1.1128	0.9790	0.9878	1.1367	1.0010
	Sector 2	Subsector 1	1.0130	1.0469	0.9909	0.9640	0.9474	1.0707	1.0143	0.9987
		Subsector 2	0.9620	1.0272	0.9946	0.9788	1.0427	0.9198	0.9591	1.0031
Region B	Sector 1	Subsector 1	1.0667	1.1035	1.0178	0.9243	0.9633	1.0703	1.0703	0.9966
		Subsector 2	0.9375	1.0593	1.0104	0.9550	1.0471	0.8730	0.9343	1.0034
	Sector 2	Subsector 1	1.1017	1.1090	1.0187	1.0401	1.0520	0.8927	1.1036	0.9983
		Subsector 2	1.1017	1.0639	1.0112	1.0238	0.9582	1.0436	1.1014	1.0003
Total			1.6333	1.8000	1.0043	1.1994	0.9978	0.7550	1.6333	1.0000

Table 11
Multiplicative decomposition results for AMDI using single-step procedure.

Subcategory			D_{tot}	D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	D^*_{tot}	D_{rsd}
Region A	Sector 1	Subsector 1	1.2137	1.1320	0.9755	1.2132	1.0147	0.8926	1.2132	1.0004
		Subsector 2	1.1378	1.0719	0.9862	1.1144	0.9787	0.9876	1.1387	0.9993
	Sector 2	Subsector 1	1.0130	1.0434	0.9915	0.9666	0.9511	1.0653	1.0133	0.9997
		Subsector 2	0.9620	1.0310	0.9939	0.9759	1.0487	0.9093	0.9536	1.0088
Region B	Sector 1	Subsector 1	1.0667	1.0985	1.0170	0.9276	0.9650	1.0670	1.0670	0.9997
		Subsector 2	0.9375	1.0661	1.0115	0.9501	1.0525	0.8599	0.9273	1.0109
	Sector 2	Subsector 1	1.1017	1.1068	1.0184	1.0393	1.0509	0.8947	1.1014	1.0002
		Subsector 2	1.1017	1.0639	1.0112	1.0238	0.9581	1.0437	1.1014	1.0002
Total			1.6333	1.8000	1.0042	1.1959	1.0129	0.7317	1.6021	1.0195

Table 12
Multiplicative decomposition results for AMDI using step-by-step procedure.

Subcategory			D_{tot}	D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	D^*_{tot}	D_{rsd}
Region A	Sector 1	Subsector 1	1.2137	1.1312	0.9756	1.2119	1.0146	0.8931	1.2119	1.0015
		Subsector 2	1.1378	1.0700	0.9865	1.1112	0.9792	0.9880	1.1347	1.0027
	Sector 2	Subsector 1	1.0130	1.0483	0.9906	0.9630	0.9459	1.0728	1.0147	0.9983
		Subsector 2	0.9620	1.0287	0.9943	0.9776	1.0451	0.9155	0.9568	1.0054
Region B	Sector 1	Subsector 1	1.0667	1.1025	1.0176	0.9250	0.9636	1.0696	1.0696	0.9973
		Subsector 2	0.9375	1.0603	1.0105	0.9543	1.0479	0.8710	0.9333	1.0045
	Sector 2	Subsector 1	1.1017	1.1088	1.0187	1.0400	1.0519	0.8929	1.1033	0.9985
		Subsector 2	1.1017	1.0639	1.0112	1.0238	0.9581	1.0437	1.1014	1.0002
Total			1.6333	1.8000	1.0042	1.1915	0.9996	0.7524	1.6199	1.0083

(ΔE_{tot}) in the first column.[14] As such, the intermediate decomposition results by region, sector, and subsector are complete and can be interpreted accordingly. In contrast, the results in Tables 4 and 5 show that LMDI-II gives perfect decomposition at the aggregate level but not at the subcategory level. Since the results in the last row of the two tables are not the same, the method is not consistent in aggregation. The results in Tables 6 and 7 show that AMDI is neither perfect in decomposition (at both the aggregate and subcategory levels) nor consistent in aggregation.[15]

The corresponding results for multiplicative decomposition are shown from Tables 8 to 12 and similar conclusions can be reached. The results for LMDI-I, shown in Table 8, are those for both the single-step decomposition procedure and the step-by-step procedure, and the decomposition is perfect at the aggregate and subcategory levels.[16] As to LMDI-II and AMDI, different results are obtained when single-step and step-by-step decomposition procedures are applied, as shown in Tables 9–12, respectively. Furthermore, LMDI-II gives perfect decomposition at the aggregate level but not at the subcategory level, while in the case of AMDI, residuals exist at both the aggregate and subcategory levels.

The residuals at the subcategory level in Tables 9 and 10 for LMDI-II and Tables 11 and 12 for AMDI can be distributed according to the principle "distributing the log of residual proportionally by subcategory" introduced in Section 4.1.2 to show that the adjusted and final results are exactly the same as those of LMDI-I in Table 8. As an illustration, we distribute the residuals in a single-step decomposition procedure for LMDI-II in Table 9. Table 13 summarizes the intermediate and final results. Take the intensity factor in Subsector 1 of Sector 1 of Region A as an example. The original effect associated with this factor is 0.8913, while the total effect and residual for this subcategory are 1.2162 and 0.9980 (from Table 9), respectively. The share of the energy intensity factor at this subcategory can be calculated as $\alpha = \ln(0.8913)/\ln(1.2162)$. The part of residual to be distributed to this factor is given by $\hat{D_{rsd}} = (D_{rsd})^{\alpha} = (0.9980)^{\alpha}$ which is 1.0012. The adjusted effect is $\hat{D_{int}} = D_{int} \cdot \hat{D_{rsd}} = 0.8913 * \hat{D_{rsd}} = 0.8923$ where D_{int} is the original estimate of the intensity effect before adjustment. After distributing all the residuals, the adjusted effects as given in the second half of Table 13 are exactly same with those of LMDI-I in Table 8.

6. Discussion and conclusions

In conventional IDA studies, only the decomposition results at the aggregate level are normally computed, presented, and interpreted. For IDA studies with multidimensional and multilevel data, additional decomposition results can be derived at the subcategory level. These decomposition results provide additional information on the energy system and problems studied, such as further insight into the drivers on energy consumption at the subcategory level. To ensure that these intermediate results at the subcategory level are consistent, the IDA method used should preferably satisfy the properties of perfect in decomposition at the subcategory level and consistency in aggregation. Our study shows that among the commonly used Divisia decomposition methods, LMDI-I is the only one that satisfies these two properties. This desirable feature of LMDI-I applies to both additive and multiplication decomposition analysis.

Since the property of perfect in decomposition at the subcategory level ensures that a method is also perfect in decomposition at the aggregate level, LMDI-I therefore satisfies the property of perfect in decomposition at the aggregate level as is already well known. In contrast, another commonly used Divisia decomposition method, LMDI-II, satisfies the property of perfect in decomposition at the aggregate level but not the other two properties. Another commonly used Divisia decomposition method in the literature, AMDI, does not satisfy any of these three properties. It can therefore be concluded that in multidimensional and multilevel decomposition analysis, where there is a growing number of studies reported, LMDI-I is the preferred Divisia decomposition method and recommended for application.

Our study also extends Ang et al. (2009) which deals with additive decomposition analysis by introducing a residual distribution principle at the subcategory level for the multiplicative counterpart. It is shown that, in multiplicative decomposition analysis, by applying the principle of "distributing the log of residual proportionally by subcategory" to the residual terms at the subcategory level to LMDI-II and AMDI, the

[14] For example, in the case of Subsector 1 of Sector 1 and Region A, the computed change of total energy consumption of $\Delta E^*_{tot} = 30$ in Table 3 is the same as the observed change $\Delta E_{tot} = E_T - E_0 = 50 - 20 = 30$ for the subsector.

[15] As shown in Ang et al. (2009), the residual for AMDI and LMDI-II can be "proportionally distributed by sub-category" and the final results would be the same as those of LMDI-I.

[16] For example, in the case of Subsector 1 of Sector 1 and Region A, the computed change of total energy consumption of 1.2137 in Table 8 is the same as that computed by using Eq. (16) and the data in the first column (D_{tot}), i.e. $(196/120)^{(50-20)/(196-120)}$.

Table 13
Distribution of residuals in multiplicative LMDI-II and the adjusted effects.

Subcategory			Residual distributed to factors					Residual at subcategory
			D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	
	Sector 1	Subsector 1	0.9987	1.0003	0.9980	0.9998	1.0012	0.9980
		Subsector 2	0.9998	1.0000	0.9997	1.0001	1.0000	0.9996
	Sector 2	Subsector 1	0.9983	1.0003	1.0014	1.0020	0.9975	0.9995
		Subsector 2	0.9978	1.0004	1.0017	0.9966	1.0068	1.0034
Region B	Sector 1	Subsector 1	0.9971	0.9995	1.0024	1.0011	0.9980	0.9980
		Subsector 2	0.9957	0.9992	1.0034	0.9966	1.0102	1.0051
	Sector 2	Subsector 1	0.9974	0.9995	0.9990	0.9988	1.0028	0.9976
		Subsector 2	0.9993	0.9999	0.9998	1.0004	0.9996	0.9990
Subcategory			Adjusted effects					
			D_{act}	D_{str1}	D_{str2}	D_{str3}	D_{int}	D_{tot}^{*}
Region A	Sector 1	Subsector 1	1.1323	0.9754	1.2137	1.0147	0.8923	1.2137
		Subsector 2	1.0715	0.9863	1.1137	0.9788	0.9877	1.1378
	Sector 2	Subsector 1	1.0425	0.9917	0.9673	0.9521	1.0640	1.0130
		Subsector 2	1.0252	0.9950	0.9803	1.0395	0.9255	0.9620
Region B	Sector 1	Subsector 1	1.0981	1.0169	0.9279	0.9651	1.0667	1.0667
		Subsector 2	1.0563	1.0099	0.9572	1.0447	0.8789	0.9375
	Sector 2	Subsector 1	1.1071	1.0184	1.0394	1.0510	0.8945	1.1017
		Subsector 2	1.0641	1.0112	1.0239	0.9581	1.0438	1.1017
Total			1.7717	1.0039	1.2114	0.9980	0.7596	1.6333

decomposition results after adjustments are the same as those of LMDI-I. Through this adjustment process, a formal linkage between LMDI-II and LMDI-I, and between AMDI and LMDI-I, can be established in the multiplicative form, which complements the additive form studied in Ang et al. (2009). With this extension, the linkages among the commonly used Divisia methods in both the additive and multiplicative forms are fully established. Furthermore, with the linkage between AMDI and LMDI-I, the problem that AMDI fails where there are zero values in the data as has been known can now be resolved in both the additive and multiplicative decomposition analysis (Ang et al., 2009).

It should be noted that our study and findings are applicable to the decomposition of a quantity aggregate indicator, i.e. V in Eqs. (1) and (2) is a quantity such as total energy consumption, electricity consumption, or energy-related CO_2 emissions. This category of studies accounts for most of the reported IDA studies in recent years. However, in the IDA literature, there are also studies that deal with the decomposition of an aggregate intensity, such as the ratio of energy consumption to economic output or the ratio of CO_2 emissions to GDP. When an aggregate intensity is decomposed, our findings no longer apply. This is an area where further research may be conducted.

Appendix A. Proof of the linkages among multiplicative Divisia methods

Following Eq. (21), the effect associated with the ith factor can be formulated as:

$$
\begin{aligned}
D_{x_i} &= \prod_{j=1}^{m} \exp\left(w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}\right) \cdot \prod_{j=1}^{m} \left(\frac{\left(\frac{V^T}{V^0}\right)^{\frac{V_j^T - V_j^0}{V^T - V^0}}}{\sum_{i=1}^{n} \exp\left(w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}\right)} \right)^{\frac{\ln\left(\exp\left(w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}\right)\right)}{\sum_{i=1}^{n} \ln\left(\exp\left(w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}\right)\right)}} \\
&= \exp\left\{ \sum_{j=1}^{m} w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0} + \sum_{j=1}^{m} \left[\left(\frac{\sum_{i=1}^{n} L\left(V_j^T, V_j^0\right)}{L\left(V^T, V^0\right)} \ln \frac{x_{j,i}^T}{x_{j,i}^0} - \sum_{i=1}^{n} w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0} \right) \cdot \frac{w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}}{\sum_{i=1}^{n} w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}} \right] \right\} \\
&= \exp\left\{ \frac{\sum_{j=1}^{m} w_j L\left(V^T, V^0\right) \ln \frac{x_{j,i}^T}{x_{j,i}^0} + \sum_{j=1}^{m} \left[\left(\sum_{i=1}^{n} L\left(V^T, V^0\right) \ln \frac{x_{j,i}^T}{x_{j,i}^0} - \sum_{i=1}^{n} w_j L\left(V^T, V^0\right) \ln \frac{x_{j,i}^T}{x_{j,i}^0} \right) \cdot \frac{w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}}{\sum_{i=1}^{n} w_j \ln \frac{x_{j,i}^T}{x_{j,i}^0}} \right]}{L\left(V^T, V^0\right)} \right\}
\end{aligned}
\tag{A.1}
$$

Denote $w_j^* = w_j L(V^T, V^0)$, then the above equation becomes

$$D_{x_i} = \exp\left\{ \frac{\sum_{j=1}^{m} w_j^* \ln \frac{x_{j,i}^T}{x_{j,i}^0} + \sum_{j=1}^{m} \left[\left(\sum_{i=1}^{n} L\left(V^T, V^0\right) \ln \frac{x_{j,i}^T}{x_{j,i}^0} - \sum_{i=1}^{n} w_j^* \ln \frac{x_{j,i}^T}{x_{j,i}^0} \right) \cdot \frac{w_j^* \ln \frac{x_{j,i}^T}{x_{j,i}^0}}{\sum_{i=1}^{n} w_j^* \ln \frac{x_{j,i}^T}{x_{j,i}^0}} \right]}{L\left(V^T, V^0\right)} \right\} \tag{A.2}$$

According to Ang et al. (2009), the numerator in the above exponential function is equal to $\sum_{j=1}^{m} L(V_j^T, V_j^0) \ln \frac{x_{j,i}^T}{x_{j,i}^0}$. Thus, Eq. (A.2) will eventually become

$$D_{x_i} = \exp\left(\frac{\sum_{j=1}^{m} L\left(V_j^T, V_j^0\right) \ln \frac{x_{j,i}^T}{x_{j,i}^0}}{L\left(V^T, V^0\right)} \right) \tag{A.3}$$

Eq. (A.3) is identical to the decomposition formulae of multiplicative LMDI-I and the proof completes.

Appendix B. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.eneco.2015.06.004.

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