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Computer Organization an Architecture

Logic GATES and Boolean algebra

ABOUT ME : MURALIKRISHNA BUKKASAMUDRAM

- MTech with 20 years of Experience in Teaching GATE and Engineering colleges
- IIT NPTEL Course topper in Theory of computation with 96 %
- IGIP Certified (Certification on International Engineering educator)
- GATE Qualified
- Trained more than 50 Thousand students across the country
- Area of Expertise : TOC,OS,COA,CN,DLD

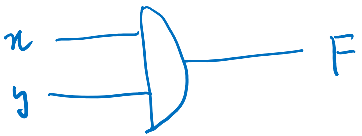
Logic GATES and Boolean algebra

GATE :– Manipulation of binary information is done by the Logic Circuit is called a GATE

x , y

	x	y
$x'y'$	0	0
$x'y$	0	1
xy'	1	0
xy	1	1

AND



$$F = xy$$

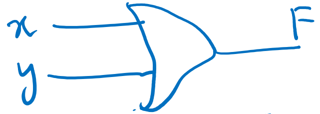
Truth Table

x	y	F
0	0	0
0	1	0
1	0	0
1 ✓	1 ✓	1 ✓

OR

$$F = x'y + xy' + xy$$
$$= x'y + x(y'+y)$$
$$= x'y + x$$
$$= (x'+x)(y+x)$$
$$= y+x$$
$$= x+y$$

OR



$$\boxed{F = x+y}$$

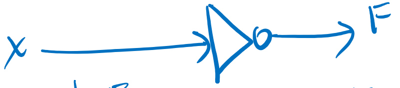
x	y	F
0	0	0
0	1	1 ✓
1	0	1 ✓
1	1	1 ✓

Logic GATES and Boolean algebra

NAND

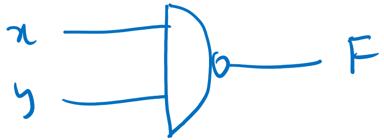
Not AND

NOT (Inverter)



x	F
0	1
1	0

$F = x'$



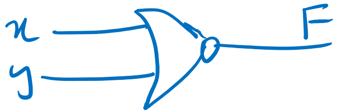
$F = (xy)'$

Truth Table

x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

$$F = x'y' + x'y + xy'$$
$$= x'(y' + y) + xy'$$
$$= x' + xy'$$
$$= (x' + x)(x' + y')$$
$$= x' + y'$$
$$= (xy)'$$

NOR [Not OR]



$F = (x+y)'$

x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

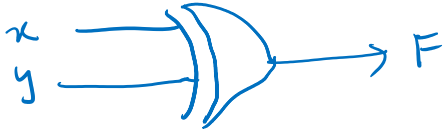
$F = x'y'$

$F = (x+y)'$

Logic GATES and Boolean algebra

EX-OR GATE

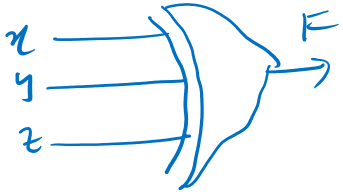
x	y	F
0	0	0
0	1	1 ✓
1	0	1 ✓
1	1	0



$F = x'y + xy'$

$F = x \oplus y$

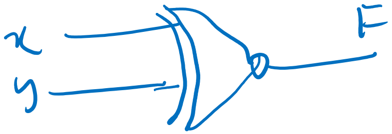
x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



$F = x \oplus y \oplus z$

odd function.

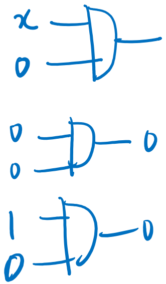
EX-NOR Gate



$F = x \odot y$

x	y	F
0	0	1 ✓
0	1	0
1	0	0
1	1	1 ✓

$F = x'y' + xy = x \odot y$



Boolean Algebra

(1) $x \cdot 0 = 0$

(2) $x \cdot 1 = x$

(3) $x + 1 = 1$

(4) $x + 0 = x$

(5) $x + y = y + x$

(6) $xy = yx$

(7) $(x')' = x$

(8) $(x+y)' = x'y'$

(9) $(xy)' = x' + y'$

(10) $x + yz = (x+y)(x+z)$

(11) $x(yz) = (xy)z$

(12) $x + (y+z) = (x+y) + z$

(13) $x \cdot x = x$

(14) $x + x = x$

Boolean functions

By using n-boolean Variables, we can have "2^{2^n}" boolean functions.

$x \quad y$ $\qquad 2^{2^2} = 16$

x, y, z

$2^{2^3} = 2^8 = 256$

Logic GATES and Boolean algebra

x	y	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

$F_0 = 0$

$F_1 = xy$

$F_2 = xy'$

$F_3 = xy' + xy$
$= x(y' + y)$

$F_3 = x$

$F_4 = x'y$

$F_5 = x'y + xy$
$= (x' + x)y$

$F_5 = y$

$F_6 = x'y + xy'$

$F_6 = x \oplus y$

$F_7 = x'y + xy' + xy$
$= x'y + x(y' + y)$
$= x'y + x = (x' + x)(y + x) = y + x$

$F_8 = x'y'$

$F_8 = (x + y)'$

$F_9 = x'y' + xy$

$F_9 = x \odot y$

$F_{10} = x'y' + xy'$
$= (x' + x)y'$

$F_{10} = y'$

$F_{11} = x'y' + xy' + xy$
$= x'y' + x(y' + y)$
$= (x' + x)(y' + x)$
$= (y' + x)$

$F_{12} = x'y' + x'y$
$= x'(y' + y)$
$= x'$

$$F_{13} = x'y' + x'y + xy$$
$$= x'y' + (x' + x)y$$
$$= x'y' + y$$
$$= (x' + y)(y' + y)$$
$$F_{13} = (x' + y)$$

$$F_{14} = x'y' + x'y + xy'$$
$$= x'(y' + y) + xy'$$
$$= x' + xy'$$
$$= (x' + x)(x' + y')$$
$$= (x' + y')$$
$$= (xy)'$$

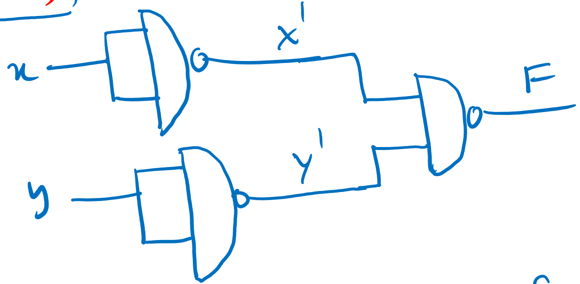
$$F_{15} = 1$$

Note :— NAND and NOR functions are called Universal building blocks.

Logic GATES and Boolean algebra

OR

$$\boxed{F = x + y}$$



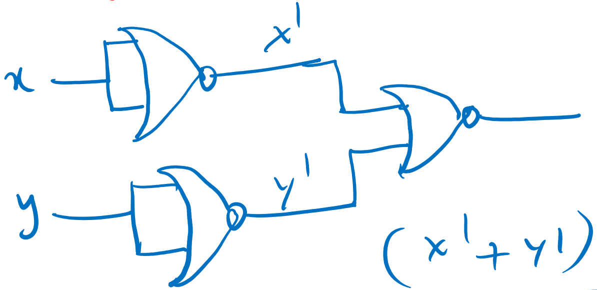
$$F = (x' \cdot y')' = (x')' + (y')'$$
$$= x + y$$

→ OR is realized using NAND Gates

AND

$$F = xy$$

[AND is realized with NOR Gates]



$$(x' + y')' = (x')' \cdot (y')'$$
$$\boxed{F = x \cdot y}$$

Logic GATES and Boolean algebra

Dual Function

$0 \to 1$

$1 \to 0$

$\underline{x}$

$\underline{y}$

$F = x + y$

$F^D = xy$

OR – AND

AND – OR.

$\underline{x+0}$

$\underline{x \cdot 1}$

$\boxed{x'y + xy'}$

$(x'+y)(x+y')$

$\underline{x'x} + x'y' + yx + \underline{yy'}$

$\underline{x'y' + xy}$

n

2^{2^n} ✓

Self Dual

F $\quad$ F^D

if $F = F^D$ then F is said to be self dual.

$F = x$ ✓

$F^D = x$

x, y

$2^{2^{2-1}} = 4$

$\left(2^{2^{n-1}}\right)$ is the total no. of functions which are self dual using n-variables

$x \quad y \quad x' \quad y'$

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Logic GATES and Boolean algebra

$$(((P+Q)' + (Q+R)')' + ((P+R)' + (Q+R)')')'$$

$$(P+Q)' + (Q+R)' \cdot (P+R)' + (Q+R)'$$

$$(P'Q' + Q'R')(P'R' + Q'R')$$

$$P'Q'R' + P'Q'R' + P'Q'R' + Q'R'$$

$$P'Q'R' + Q'R'$$

$$(P'+1)(Q'R')$$

$$Q'R' = \overline{(Q+R)}$$

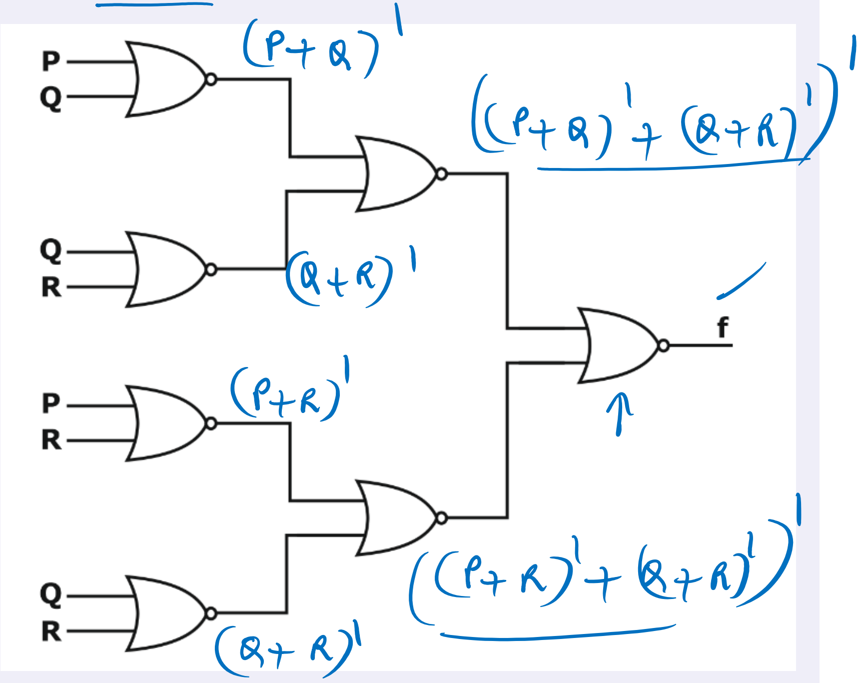
1. What is the Boolean expression for the output **f** of the combinational logic circuit of NOR gates given below ?

A. $\overline{Q + R}$

B. $\overline{P + Q}$

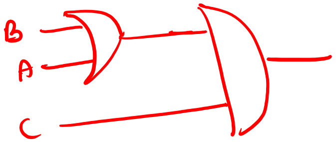
C. $\overline{P + R}$

D. $\overline{P + Q + R}$

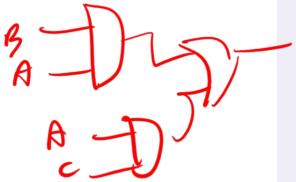


Logic GATES and Boolean algebra

$(B+A)\,C$



$BA + AC$



2. Simplify the following expressions using Boolean Algebra

(a) $A + AB = A(1+B) = A\cdot 1 = A$

(b) $AB + AB' = A(B+B') = A\cdot 1 = A$

(c) $A'BC + AC$ $\quad (A'B + A)C = (A'+A)(B+A)\cdot C = (B+A)C$

(d) $\boxed{A'B + ABC' + ABC} = A'B + AB(C'+C) = A'B + AB$
$= (A'+A)B$
$= B$

(e) $AB + A(CD + CD')$

(f) $(BC' + A'D)(AD' + CD')$

$AB + A[C(D+D')]$

$AB + AC$

$A[B+C]$

$BC'AD' + BC'CD' + A'DAD' + A'DCD'$

$BC'AD'$

Logic GATES and Boolean algebra

3. Consider Boolean function below

F = xy′z + x′y′z + xyz

Simplify the above expression using Boolean Algebra?

$xy'z + x'y'z + xyz$

$(x + x')y'z + xyz$

$y'z + xyz$

$(y' + xy)z = (y' + x)(y' + y)z$

$(y' + x)z$

4.The function AB′C + A′BC + ABC′ + A′B′C + AB′C′ is equivalent to

A. AC′ + AB + A′C

B. AB′ + AC′ + A′C

C. A′B + AC′ + AB′

D. A′B + AC + AB′

$A[BC' + B'C']$

$AC'[B + B'] = AC'$

$(A'B + A'B')C$

$A'[B + B']C = A'C$

$AC' + A'C + AB'C$

$AC' + (A' + AB')C$

$AC' + (A' + A)(A' + B')C$

$\boxed{AC' + A'C + B'C}$

5. Let f (A, B) = A′+ B. Simplified expression for function f (f (X + Y, Y), Z) is

A. X′ + Z

B. XYZ

C. XY′ + Z

D. None

$f(A,B) = A'+B$

$f(f(x+y, y), z)$

$f(x'+y, z)$

$(x'+y)'+z$

$xy'+z$

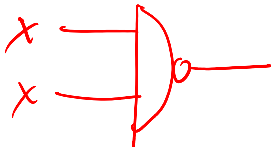
$f(x+y, y) = x'+y$

$(x+y)'+y$

$x'y'+y$

$(x'+y)(y'+y)$

Logic GATES and Boolean algebra



$(x \cdot x)' = (x)'$

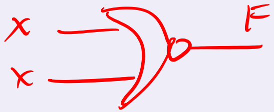
6. Which of the following expression is not equivalent to $\overline{X}$

A. X NAND X

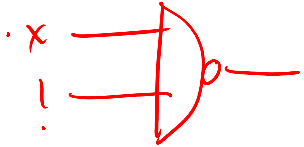
B. X NOR X

C. X NAND 1

D. X NOR 1 $= 0$



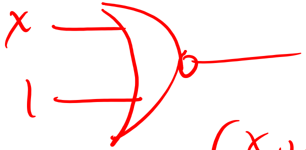
$(x + x)' = x'$



$(x \cdot 1)' = x'$

x'

$(x \cdot 1)' = x'$



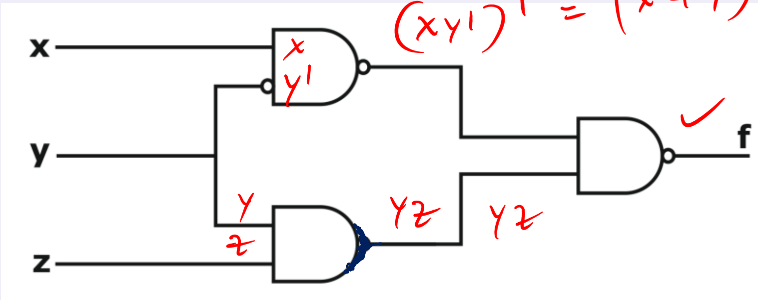
$(x+1)'$

$(1)' = 0$

Logic GATES and Boolean algebra

7. Consider the following circuit

$(xy')' = (x'+y)$



A. f is Independent of x

B. f is Independent of y

C. f is Independent of z

D. None of These

$((x'+y)\cdot yz)'$

$(x'+y)' + (yz)'$

$xy' + y' + z'$

$((xy')'\cdot(yz))'$

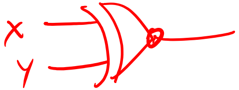
$((xy')')' + (yz)'$

$xy' + yz'$

$(x+1)y' + z'$

$y' + z'$

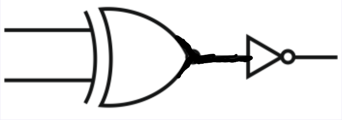
Logic GATES and Boolean algebra



$xy + x'y'$

$x \odot y$

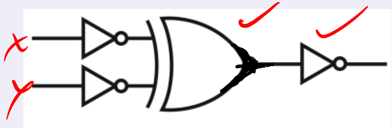
8. Which one of the following circuit is NOT equivalent to a 2-input XNOR gate ?

A. 

$(x'y + xy')' = (x'y)' \cdot (xy')'$

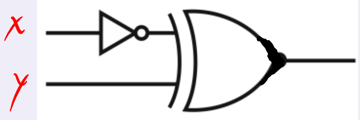
$(x+y')(x'+y)$

$xy + x'y'$

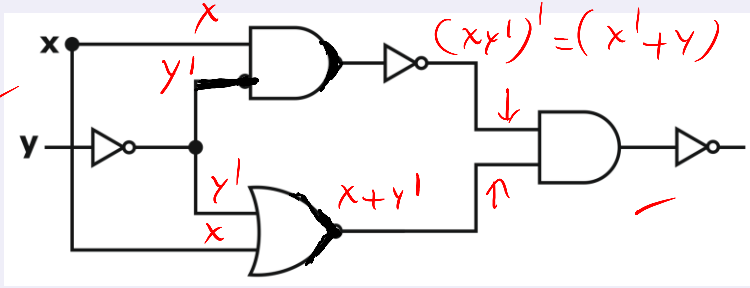
B. 

$(x')'y' + x'(y')'$

$xy' + x'y$

C. 

$(x')'y + x'y' = xy + x'y'$

D. 

$(xy')' = (x'+y)$

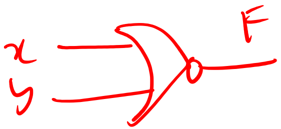
$((x'+y)(x+y'))'$

$(x'+y)' + (x+y')'$

$xy' + x'y$

Logic GATES and Boolean algebra

$(AB+C)$



$F = (x+y)'$

9. What is the minimum number of GATES required to implement the Boolean function (AB+C) if we have to use two input NOR Gates only ?

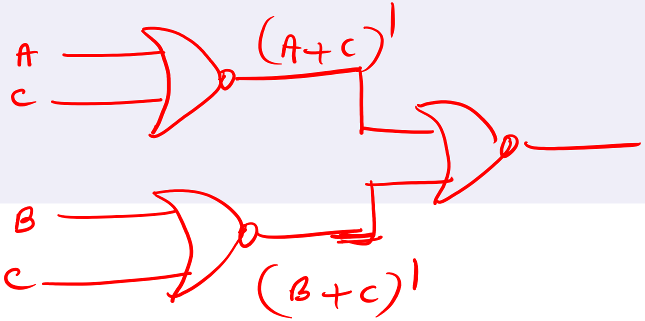
A. 2

B. 3 ✓

C. 4

D. 5

$(AB+C) = (A+C)(B+C)$



$((A+C)' + (B+C)')'$

$= ((A+C)')' \cdot ((B+C)')' = (A+C)(B+C)$

$AB+C$ ✓