Number System

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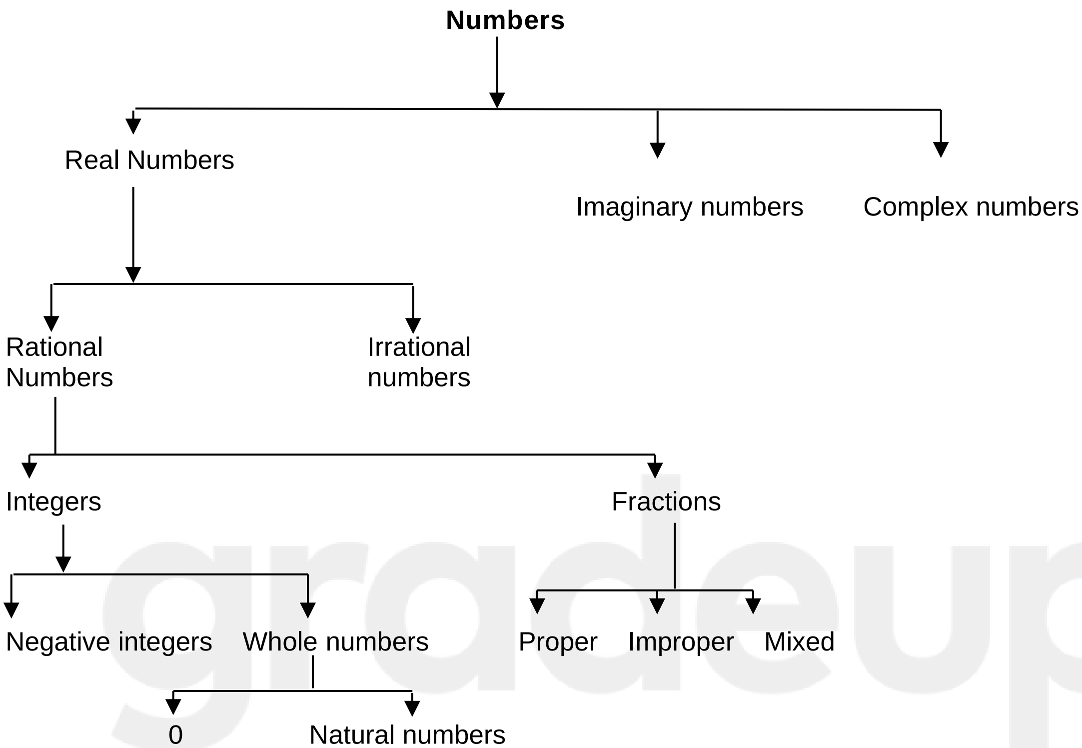
NUMBERS



- **Real numbers:**

  All the numbers, which can be represented on number line is called real number. This is made up of all the Rational and Irrational Numbers.

  Ex: 2, 1/3, √5, – 5/2

- **Imaginary numbers:** (Not required for Bank exam)

- **Complex numbers:** (Not required for Bank exam)

- **Rational numbers:** Any number that can be expressed as a ratio of the two integers is called a rational number.

  Ex: 2, 1/7, – 3/2, 9/4

- **Irrational Numbers:** Any number that cannot be expressed as the ratio of two integers is called an irrational number.

Ex: √5, –√3

- **Integers:** An integer is a whole number (not a fractional number) that can be positive, negative, or zero.

  Ex: –32, –5, 0, 1, 47, 88

- **Fractions:** A rational number, which is not integer is called fraction.

  Ex: 1/5, – 4/11, 3/8

- **Natural Numbers:** The numbers starting from 1 and including 1, 2, 3, 4, 5, and so on. are called Natural numbers.

  Ex: 3, 2, 5, 1, 7, 8

- **Prime Numbers:** All the numbers that have only two factors, 1 and the number itself, are called prime numbers.

  Hence, a prime number can only be written as the product of 1 and itself. The numbers 2, 3, 5, 7, 11…37, etc. are prime numbers.

  Ex: 2, 3, 5, 7, 11

To find whether a number N is prime or not
Find the root R (approximate) of the number N, i.e. R = √N. Divide N by every prime number less than or equal to R. If N is divisible by at least one of those prime numbers it is not a prime number. If N is not divisible by any of those prime numbers, it is a prime number.

- **Odd and Even Numbers:** All the numbers divisible by 2 are called even numbers whereas all the numbers not divisible by 2 are called odd numbers. 2, 4, 6, 8… etc. are even numbers and 1, 3, 5, 7.. etc. are odd numbers.

Remember!	$(\text{Even})^{\text{Odd}}$ = Even
Odd + Odd = Even	Even × Odd = Even
Even + Even = Even	Even × Even = Even
Odd + Even = Odd	Odd × Odd = Odd
$(\text{Odd})^{\text{Even}}$ = Odd	$(\text{Odd})^{\text{Even}} \times (\text{Even})^{\text{Odd}}$ = Even
	$(\text{Odd})^{\text{Even}} + (\text{Even})^{\text{Odd}}$ = Odd

DIVISIBILITY

- **Divisibility rule of 2, 4, 8, 16, 32, … 2^n etc**

  A number is divisible by 2, 4, 8, 16, 32, … 2^n, when the number formed by the last one, last two, last three, last four, last five... last n digits is divisible by 2, 4, 8, 16, 32, … 2^n respectively.

  Ex: 1246384 is divisible by 8 because the number formed by the last three digits of the number i.e. 384 is divisible by 8, therefore the number will be divisible by 8.

  Ex: 89764 is divisible by 4 because the number formed by the last two digits of the number i.e. 64 is divisible by 4, therefore the number will be divisible by 4.

Divisibility rule of 3 and 9

A number is divisible by 3 or 9 when the sum of the digits of the number is divisible by 3 or 9 respectively.

Ex: 313644 is divisible by 3 because the sum of the digits- 3 + 1 + 3 + 6 + 4 + 4 = 21 is divisible by 3.

Ex: 212364 is divisible by 9 because the sum of the digit- 2 + 1 + 2 + 3 + 6 + 4 = 18 is divisible by 9.

Divisibility rule of 6, 12, 14, 15, 18 etc: Whenever we have to check the divisibility of a composite number N, the number N should be divisible by all the prime factors (the highest power of every prime factor) present in N .

Divisibility by 6: the number should be divisible by both 2 and 3.

Divisibility by 12: the number should be divisible by both 3 and 4.

Divisibility by 14: the number should be divisible by both 2 and 7.

Divisibility by 15: the number should be divisible by both 3 and 5.

Divisibility by 18: the number should be divisible by both 2 and 9.

Divisibility rule of 11: If the sum of the digits at odd places and even places of a number is 0 or divisible by 11, then the number will be divisible by 11.

Ex: 2754818

Here (2 + 5 + 8 + 8) – (7 + 4 + 1) = 23 – 12 = 11 (a multiple of 11).

FACTORS

- **Factor:** If a number N (integer) is divided into integers, then these integers are factors of N or Factors are those numbers that divide the given number completely.

- **Here we will learn to calculate**
  I. Prime factorization
  II. The number of factors
  III. The sum of factors
  IV. Product of factors

- **Prime factorization:** We all know that every composite number can be written as a product of some prime numbers.
  For example, we can write 90 as $2 \times 3^2 \times 5$. This process is called prime factorization

  Let $N = X^a \times Y^b \times Z^c$ be a number, where X, Y and Z are different prime number, then
  Total number of factors of N = $(a + 1)(b + 1)(c + 1)$
  Sum of all factors of N = $\frac{(X^{a+1} - 1)}{(X - 1)} \times \frac{(Y^{b+1} - 1)}{(Y - 1)} \times \frac{(Z^{c+1} - 1)}{(Z - 1)}$
  Product of factors of N = $N^{\text{Total number of factors/2}}$
  Ex: If a number 18 is given, then find
  (i) Total number of factors
  (ii) Sum of all factors
  (iii) Product of all factors
  Sol: $18 = 2^1 \times 3^2$
  (i) Total number of factors = $(1 + 1)(2 + 1) = 2 \times 3 = 6$
  (ii) Sum of total number of factors = $\frac{(2^{1+1} - 1)}{(2 - 1)} \times \frac{(3^{2+1} - 1)}{(3 - 1)} = \frac{(4 - 1)}{1} \times \frac{(27 - 1)}{2} = 3 \times \frac{26}{2} = 39$
  (iii) Product of factors = $18^{6/2} = 18^3 = 5932.$

REMAINDER

When a number N is divided by another number D (Such that N > D), then the remainder is calculated by subtracting the maximum possible multiple of D from N.

Dividend = Quotient × Divisor + Remainder
$N = Q \times D + R$
Where: N = Number, Q = Quotient, D = Divisor and R = Remainder

Important points on the remainder

- The remainder is always less than the divisor.
- If the dividend is less than the divisor, then the remainder is dividend itself.

Ex: 8 divided by 13, then the remainder is 8 only.

- If the remainder is 0, then the divisor is called the factor of the dividend
- Two number N_1 and N_2 are divided by a number d, then remainder are R_1 and R_2, then
  When $N_1 + N_2$ is divided by a number d, then remainder = $R_1 + R_2$
  When $N_1 – N_2$ is divided by a number d, then remainder = $R_1 – R_2$
  When $N_1 \times N_1$ is divided by a number d, then remainder = $R_1 \times R_2$