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Digital Logic

Simplification of Logic Functions using K-Map

ABOUT ME : MURALIKRISHNA BUKKASAMUDRAM

- MTech with 20 years of Experience in Teaching GATE and Engineering colleges
- IIT NPTEL Course topper in Theory of computation with 96 %
- IGIP Certified (Certification on International Engineering educator)
- GATE Qualified
- Trained more than 50 Thousand students across the country
- Area of Expertise : TOC,OS,COA,CN,DLD

Simplification of Logic Functions using K-Map

K-Map. [Karnagh Map]

x	y	
0	0	$x'y'$
0	1	$x'y$
1	0	xy'
1	1	xy

x, y

x, y, z

x, y, z, w

Gray code (OR) Reflected code

2-variable Map.

x \ y	0	1	
0	00	01	x'
1	10	11	x
	y'	y	

$\begin{pmatrix} 01 \\ 11 \end{pmatrix}$ ✓ $\quad \begin{pmatrix} 01 \\ 10 \end{pmatrix}$ ✗

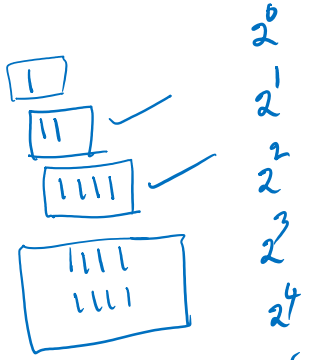
$\begin{pmatrix} 00 \\ 10 \end{pmatrix}$ ✓ $\quad \begin{pmatrix} 00 \\ 11 \end{pmatrix}$ ✗

3-Variable

x \ yz	00	01	11	10	
0	000	001	011	010	x'
1	100	101	111	110	x
	$y'z'$	$y'z$	yz	yz'	

$\begin{pmatrix} 111 \\ 110 \end{pmatrix}$

$\begin{pmatrix} 000 \\ 010 \end{pmatrix}$



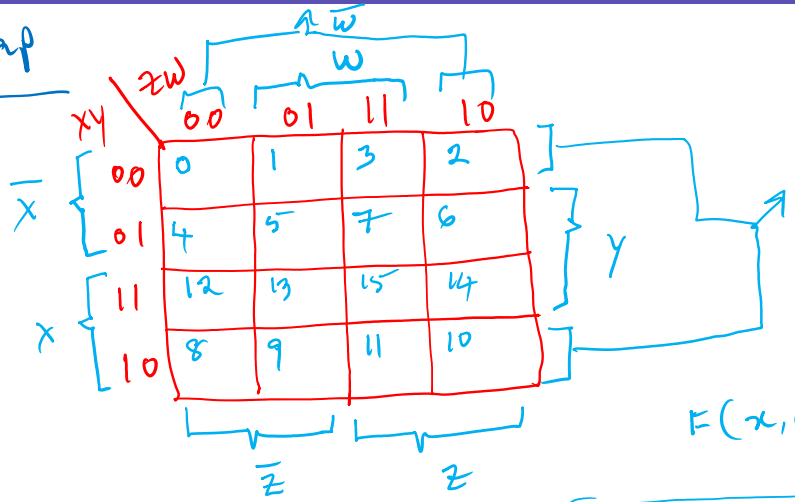
We have to plot as max. No. of adjacent 1's as possible.

Simplification of Logic Functions using K-Map

4 - Variable Map

x, y, z, w $\quad 2^4 = 16$

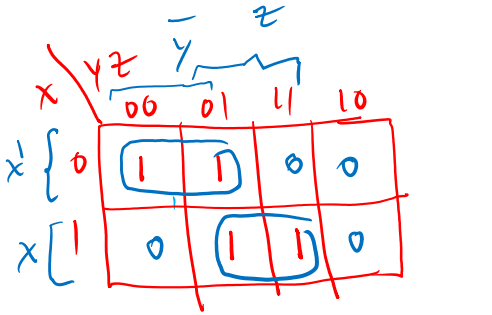
SOP [Sum of Products]
POS [Product of Sums]



→ When we plot Adjacent 1's, We get SOP.

Example

$F(x,y,z) = \Sigma(0,1,5,7)$



$F = \bar{x}\bar{y} + xz$

	x	y	z	F
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

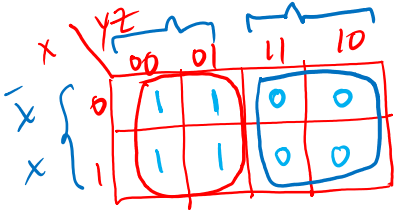
$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}z + xyz$

SOP

Simplification of Logic Functions using K-Map

→ Simplify the following boolean function.

$F(x, y, z) = \Sigma(0, 1, 4, 5)$



$\bar{x}$: $F = \bar{y}$

x : $F = \bar{y}$

$F = \bar{y}$

Product of Sums

$(x+y)(y+z)$

11 (Pair)

1111 [Quad]

1111 / 1111 } octet

SOP

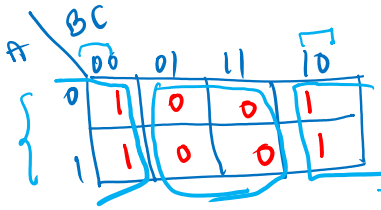
Pairing of Adjacent 1's

POS

$\bar{F} = y$

$F = \bar{y}$

→ Simplify $F(A, B, C) = \Sigma(0, 2, 4, 6)$

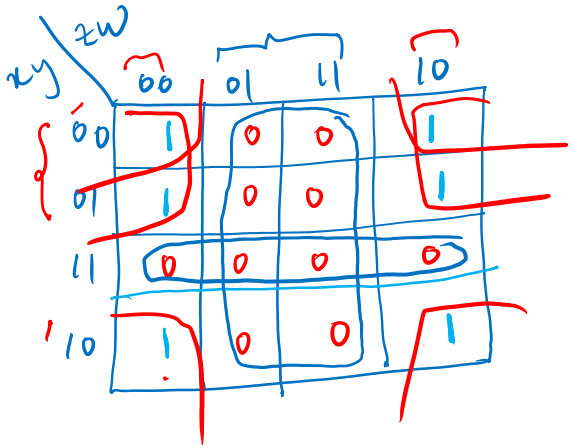


$F = \bar{C}$

$\bar{F} = C$, $F = \bar{C}$

Simplification of Logic Functions using K-Map

→ Simplification of $F(x,y,z,w) = \Sigma(0,2,4,6,8,10)$



SOP

$F = \bar{x}\bar{w} + \bar{y}\bar{w}$

POS

$\bar{F} = w + xy$

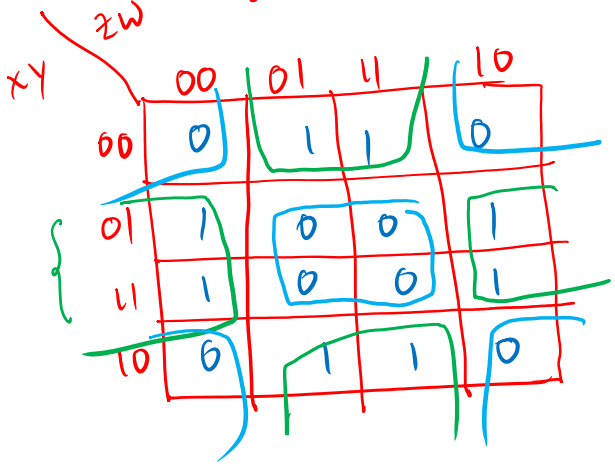
$F = (w + xy)'$

$= w' \cdot (xy)'$

$= w' \cdot (x' + y')$

Simplification of Logic Functions using K-Map

→ Simplify the following K-Map in both SOP and POS



SOP

$F = y\bar{w} + \bar{y}w$

POS

$\bar{F} = yw + \bar{y}\bar{w}$

$F = \overline{(yw + \bar{y}\bar{w})}$

$(\bar{y}w) \cdot (\bar{y}\bar{w})$

$F = (\bar{y} + \bar{w}) \cdot (y + w)$

Simplification of Logic Functions using K-Map

Don't Care Condition

Don't cares are used for further simplification of boolean functions

A Minterm that produces either 0 (or) 1 is called don't care.

Example :— $F(x,y,z) = \Sigma(0,4)$

$d(x,y,z) = \Sigma(2,6)$

Sol :—

x \ yz	00	01	11	10
0	1	0	0	X
1	1	0	0	X

SOP

$F = \bar{z}$

POS

$\bar{F} = z$

$F = \bar{z}$

Simplification of Logic Functions using K-Map

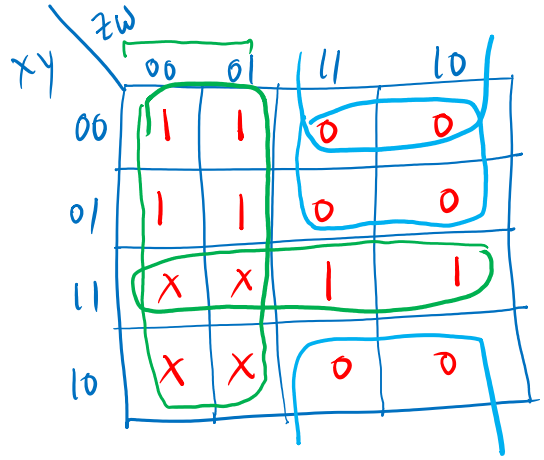
Example-2

Simplify the boolean function.

$F(x,y,z,w) = \Sigma(0,1,4,5,14,15)$

$d(x,y,z,w) = \Sigma(8,9,12,13)$.

Sol :-



SOP

$F = \bar{z} + xy$

POS

$\bar{F} = \bar{x}z + \bar{y}z$

$F = (x + \bar{z})(y + \bar{z})$

Simplification of Logic Functions using K-Map

→ Consider the following boolean function of four variables

$f(A,B,C,D) = \Sigma(2,3,6,7,8,9,10,11,12,13)$

the function is

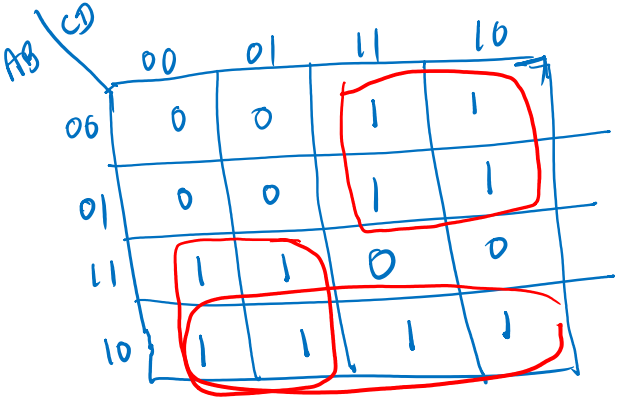
(A) Independent of one variable

(B) " two "

(C) " three "

(D) Dependent on all the variables.

Sol :-

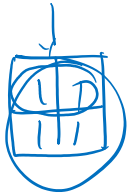


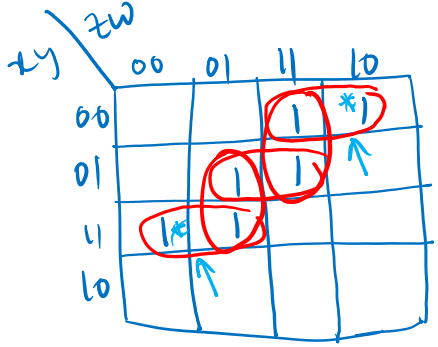
SOP

$F = \bar{A}C + A\bar{B} + A\bar{C}$

Simplification of Logic Functions using K-Map

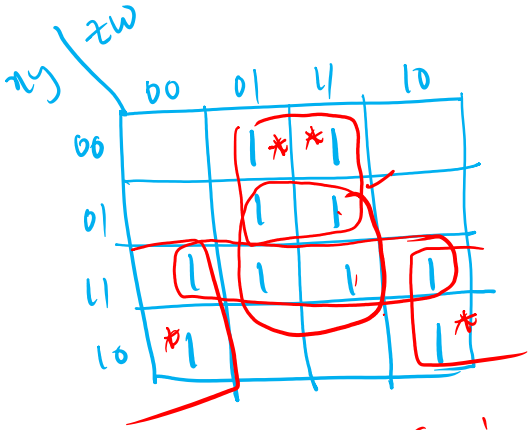
Prime implicants and Essential Prime implicants





PI's = 5

EPI's = 2



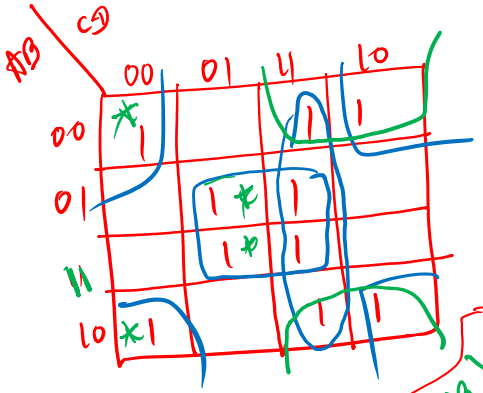
PI's = 4

EPI's = 2

EPI's = $\bar{x}w + x\bar{w}$

Simplification of Logic Functions using K-Map

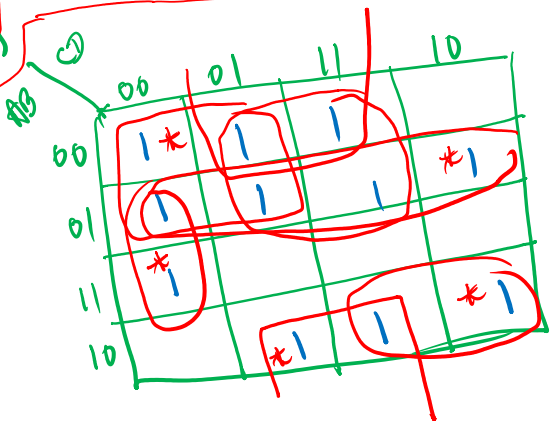
Example :- Find PI's and EPI's in the following K-Maps.



PI's = 4

EPI's = 2

$\bar{B}\bar{D}$, BD



PI's = 6

EPI's = 5