

A summary of the main properties of the expected value and variance of the estimators is presented:

	$E[c_1+c_2.u] = c_1+c_2.E[u]$	$V[c_1+c_2.u] = c_2.V[u].c_2^T$
1 - Random variable, ε		ε_n . (independent)
Expected value of ε		$E[\varepsilon] = 0$.
Variance of ε		$V[\varepsilon]_{(n,n)} = E[\varepsilon.\varepsilon^T] = I. \sigma^2$
2 - Observed response variable y $y = Y + \varepsilon$		
Expected value of y		$E[y] = Y = X.B$.
Variance of y		$V[y]_{(n,n)} = V[\varepsilon]_{(n,n)} = I. \sigma^2$
3 - Estimator of B		$\hat{B} = (X^T.X)^{-1}.X^T.y$
Expected value of \hat{B}		$E[\hat{B}] = B$
Variance of \hat{B}		$V[\hat{B}]_{(k,k)} = (X^T.X)^{-1}. \sigma^2$
4 - Estimator of Y of the model		$\hat{Y} = X. \hat{B} = L.y$
Expected value of \hat{Y}		$E[\hat{Y}] = Y$.
Variance of \hat{Y}		$V[\hat{Y}] = L. \sigma^2$
5 - Residual e		$e = y - \hat{Y} = (I-L).y$
Expected value of e		$E[e] = 0$
Variance of e		$V[e] = (I-L). \sigma^2$

6 - Sum of squares

6.1 - Residual Sum of squares = SQ residual_(1,1) = $(y - \hat{Y})^T (y - \hat{Y}) = y^T (I-L)y$

This quantity indicates the residual variation of the observed values in relation to the estimated values of the model, that is, the variation not explained by the model.

6.2 - Sum of squares of the deviation of the model = SQ model_(1,1) = $(\hat{Y} - \bar{y})^T (\hat{Y} - \bar{y}) = y^T (L-M)y$

This quantity indicates the variation of the estimated response values of the model in relation to the mean, that is, the variation explained by the model.

6.3 - Total Sum of the squares of the deviations = SQ total_(1,1) = $(y - \bar{y})^T (y - \bar{y}) = y^T (I-M)y$

This quantity indicates the total variation of the observed values in relation to the mean.

It is easy to verify the following relation:

$$SQ_{\text{total}} = SQ_{\text{model}} + SQ_{\text{residual}} \text{ or}$$

$$1 = \frac{SQ_{\text{model}}}{SQ_{\text{total}}} + \frac{SQ_{\text{residual}}}{SQ_{\text{total}}}$$

$$\text{or } 1 = R^2 + (1 - R^2)$$

where:

R^2 is the percentage of the total variation that is *explained* by the model. In matrix terms it will be:

$$R^2 = [y^T(L - M)y] \cdot [y^T(I - M)y]^{-1}$$

$1 - R^2$ is the percentage of the total variation that is not explained by the model.

The ranks of the matrices $(I-L)$, $(I-M)$ and $(L-M)$ respectively equal to $(n-k)$, $(n-1)$ and $(k-1)$, are the degrees of freedom associated with the respective sums of squares.