

**AGEC5213: ECONOMETRIC METHODS**  
**Spring 2019**

**PROBLEM SET NO. 5- due on April 29, 2018**

**Part I. Autocorrelation test, GLS and MLE (10 points)**

Consider a wheat acreage model  $\ln(A_t) = \beta_1 + \beta_2 \ln(P_t) + e_t$ , where  $A_t$  was area of wheat (in thousands of hectares) sown in Oklahoma in year  $t$  and  $P_t$  was the price of wheat. This specification was, in fact, a simplification of a slightly more general specification; the price of wheat was measured relative to the price of Barley. Specifically, let

$W_t$  = price of wheat in year  $t$  (dollar/ton), and

$B_t$  = price of jute in year  $t$  (dollar/ton).

The price that was utilized was  $P_t = W_t/B_t$ . Since barley is the crop which competes with wheat for the use of the land, it is the price of wheat *relative* to the price of barley that affects farmers' decisions about how much wheat to plant. Data on  $A_t$ ,  $W_t$  and  $B_t$  appear in the file, HW5-DATA1.xls

- (a) Consider the model  
$$\ln(A_t) = \beta_1 + \beta_2 \ln(W_t) + \beta_3 \ln(B_t) + e_t$$
  
Give interpretations for  $\beta_2$  and  $\beta_3$ . Show that this model is equivalent to the original model if  $\beta_2 = -\beta_3$ .
- (b) Estimate the model in part (a) using least squares and test for the presence of autocorrelated errors using the DW test at the 5% significance level.
- (c) Re-estimate the model assuming the existence of AR(1) errors (GLS). Are there any noticeable changes in the results?
- (d) Using the estimation results from (b) and (c), test  $H_0: \beta_2 = -\beta_3$  against  $H_1: \beta_2 \neq -\beta_3$  at the 5% significance level. Do hypothesis test results differ between (b) and (c)?
- (e) Predict  $\ln(A)$  for the next two years assuming that  $W_{T+1} = W_{T+2} = 500$  and  $B_{T+1} = B_{T+2} = 500$ .

**Part II. Two-Stage Least Square Estimation (5 points)**

Y K L P R W

Consider the following production functions as:

$$\ln(Y_t) = \beta_1 + \beta_2 \ln(K_t) + \beta_3 \ln(L_t) + e_{1t} \quad (1)$$

$$\ln(K_t) = \alpha_1 + \alpha_2 \ln(P_t) + \alpha_3 \ln(Y_t) + \alpha_4 \ln(R_t) + e_{2t} \quad (2)$$

$$\ln(L_t) = \lambda_1 + \lambda_2 \ln(P_t) + \lambda_3 \ln(Y_t) + \lambda_4 \ln(W_t) + e_{3t} \quad (3),$$

where  $Y_t$ ,  $K_t$  and  $L_t$  are output, capital input, and labour input, and  $P_t$ ,  $R_t$  and  $W_t$  are corresponding prices. Given that  $Y_t$ ,  $K_t$  and  $L_t$  are simultaneously determined by equations (1), (2) and (3), and  $P_t$ ,  $R_t$  and  $W_t$  are determined from outside, it seems appropriate to treat the three equations as a simultaneous system where  $Y_t$ ,  $K_t$  and  $L_t$  are the endogenous variables and  $P_t$ ,  $R_t$  and  $W_t$  are exogenous variables. Data for this example (23 observations) are in the file HW5-DATA2

Y = output  
K = Capital input  
L = labor input

P = Price output  
R = Price of capital input  
W = price of labor input.

- ☒ (a) Is the production function (1) identified?
- ☒ (b) Find OLS estimates of the production function, and comment the problem of OLS.
- ☒ (c) Find two stage least squares estimates of  $\beta_2$  and  $\beta_3$ . Comment on the results.

### Part III. Instrumental Variable/Two-Stage Least Square Estimation (5 points)

Consider a production function for local beer production as:

$$Y_t = \beta_1 + \beta_2 mgt_t + \beta_3 cap_t + \beta_4 lab_t + e_{1t},$$

where  $y_t$  is an index of beer output for  $t$ -th brewery, taking into account both quantity and quality,  $mgt_t$  is a variable reflecting the efficiency of management,  $cap_t$  and  $lab_t$  are indices of capital input and labor inputs. Because Kelly cannot get data on management efficiency,  $mgt_t$ , she collects data on the number of years of experience on the brewery business ( $xpert_t$ ) of each brewery manager and uses that variable in place of  $mgt_t$ . Data for this example (75 observations) are in the file *HW5-DATA3*

- ☒ (a) Estimate the equation using OLS.
- ☒ (b) Estimate the instrumental variable/2SLS with instruments age, cap, lab, trend, and trend2, and compare the results with OLS results.
- ☒ (c) Run Hausman test to check on the validity of the instrumental variables.

W R P J K Y

W = price of labor input  
R = price of capital input  
P = price of output

J = labor input  
K = capital input  
Y = output