I. Estimate, Standard Error, t-statistics, and R-Squared.

Answers:

(A).

Dependent Variable: Alcohol Included Observation (N): 1519				
Variable	Coefficient (b_k)	Std. Error (SE_{bk})	T-Stat. (t)	Prob. (β_k)
C			-30.17247 (i)	
Ln(Income)		0.07984133 (ii)		
Age	-0.01448177 (iii)			
NUMKID				
R-squared	0.53139793 (iv)			
S.E. of Regression	0.07598472 (v)			

Equations used:

$t = (b_k - \beta_k)/ SE_{bk}$

S.E. Regression = √ {Sum Squared Error (SSE)/(N-k)} where, N = number of cases or observation and K is # of variables (4 in our case). (n-k) if degree of freedom.

$$t = \frac{b_k - \beta_k}{Se(bk)}$$

1. (a).

(i) $$t = \frac{0.009052 - 0.7897}{0.024050} = -30.17247$$

(ii) $$6.108620 = \frac{0.527641 - 0}{SE} \quad \text{or } SE = 0.07984133$$

(iii) $$-6.962389 = \frac{b_k - 0}{0.00208} \quad \text{or } b_k = -0.01448177$$

(v) $$S.E. \text{ of } Reg = \sqrt{\frac{SSE}{T-K}} = \sqrt{8.752896/(1519-4)} = 0.07598472281$$

(iv) $$R^2_{Adj} = 1 - \frac{SSE/(T-K)}{SST/(T-1)}$$

$$0.53047 = 1 - \frac{8.752896}{SST} \times \frac{1518}{1515} \quad ; SST = 18.67873931$$

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{8.752896}{18.67873931} = 0.5313979249$$

(B)

Interpretation of b2: with the unit percentage increase in the household income keeping other variables constant, the proportion of household income spent on the alcohol increases (+ve sign) by 0.527641 unit.

Interpretation of b3: with the unit increase in the age of person in family keeping other variables constant, the proportion of household income spent on the alcohol decreases (-ve sign) by 0.0145 unit.

Interpretation of b4: with one additional child in the household and other situation remaining unchanged, the proportion of household income spent on the alcohol decreases (-ve sign) by 0.0133 unit.

C.

Solution c, b2

reg_coeff_b2 = 0.527641

st_err_b2 = 0.00208

Computing 95% CI for b2

ME_b2 = 1.96 * st_err_b2

CI_low_b2 = reg_coeff_b2 – ME_b2

CI_high_b2 = reg_coeff_b2 + ME_b2

 # CI_b2 = [0.3711519932, 0.6841300068]

Solution c, b3

reg_coeff_b3 = -0.01448177

st_err_b3 = 0.00208

Computing 95% CI for b3

ME_b3 = 1.96 * st_err_b3

CI_low_b3 = reg_coeff_b3 - ME_b3

CI_high_b3 = reg_coeff_b3 + ME_b3

 # CI_b3 = [-0.01855857, -0.01040497]

All the t statistics values that falls inside the range of confidence interval tells that these values are not significantly different than the null value. T-statistics that falls outside this interval are significantly different than null value and thus we can reject null hypothesis at 95% confidence interval. Here both, Ln(Income) and Age are significantly different than their null values as they fall outside the these confidence interval. So, we reject null hypothesis (here absolute value of t-stat is greater than t-cric) in favor of alternative hypothesis for both b2 and b3 (ln(income) and Age). The ln(age) and the income has statistically significant effect in the proportion of household income spent on alcohol.

D.

Solution d, b4

reg_coeff_b4 = -0.013282

st_err_b4 = 0.003259

Computing 95% CI for b4

ME_b4 = 1.96 * st_err_b4

CI_low_b4 = reg_coeff_b4 - ME_b4

CI_high_b4 = reg_coeff_b4 + ME_b4

 # CI_b4 = [-0.01966964, -0.00689436]

The higher and lower critical values at 95% confidence interval is as given above. The calculated t-statistics (-4.074993) does not lies within this range of values in the probability density function and thus the null hypothesis that the household income proportion for alcohol does not depend upon the number of children in the household is rejected. The result suggests that the household income proportion for the alcohol indeed depends upon the number of children in the household.

II. Estimates, Standard Errors, and Hypothesis Test

a. What is the sample size? Answer: 25.

b.

	B	C	D	E	F	G	H
17							
18		X'X	25	0	0		
19			0	10	20		
20			0	20	50		
21		Highlight area to put inverse matrix. X'X Inverse = =MINVERSE(D18:F20). Then hit Ctrl+Shift+Enter at a time.					X'Y
22		(X'X) Inverse	0.04	0	0		100
23			0	0.5	-0.2		1
24			0	-0.2	0.1		8
25		Determinant of Matrix = mdeterm(array) and press enter. Det =MDETERM(D18:F20)					
26		Det. Mat X'X	2500				
27							
28		Multiply Matrics: =MMULT(D22:F24,H22:H24)					
29		Multiply two matrices	4	b1			
30			-1.1	b2			
31			0.6	b3			

B1, b2 and b3 are parameters.

$$(XX')^{-1} = \underset{3\times 3}{\begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.5 & -0.2 \\ 0 & -0.2 & 0.1 \end{bmatrix}} \qquad x'y = \underset{3\times 1}{\begin{bmatrix} 100 \\ 1 \\ 8 \end{bmatrix}}$$

$$b = \underset{3\times 1}{\begin{bmatrix} 4+0+0 \\ 0+0.5-1.6 \\ 0-0.2+0.8 \end{bmatrix}} = \begin{bmatrix} 4 \\ -1.1 \\ 0.6 \end{bmatrix} = \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix}$$

c.

II. (c) standard errors of b_2 & b_2

$T = 25 \quad K = 3$

$\underline{\underline{b_2}}$

$$y'y = \sum_t Y_t^2 = 450$$

$$SSE = \hat{e}'\hat{e} = y'y - b'x'y \; (SST - SSR)$$

$$= 450 - \begin{bmatrix} 4 & -1.1 & 0.6 \end{bmatrix} \begin{bmatrix} 100 \\ 1 \\ 8 \end{bmatrix} = 450 - 400 + 1.1 - 4.8 = 46.3$$

$$\hat{\sigma}^2 = \frac{\Sigma \hat{e}^2}{T-K} = \frac{SSE}{T-K} = \frac{46.3}{22} = 2.104545$$

$$Var(b) = \hat{\sigma}^2 (X'X)^{-1} = 2.104545 \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.5 & -0.2 \\ 0 & -0.2 & 0.1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0841818 & 0 & 0 \\ 0 & 1.0522725 & -0.420909 \\ 0 & -0.420909 & 0.2104545 \end{bmatrix}$$

$$se(b_2) = \sqrt{1.0522725} = 1.025803344$$

$$se(b_3) = \sqrt{0.2104545} = 0.4587532016$$

d.

II d.

$$t_{stat} = \frac{b_k - \beta_k}{se_{(bk)}}$$

$H_0 : \beta_2 = 0$
$H_A : \beta_2 \neq 0$

$$t_{stat} = \frac{-1.1 - 0}{1.0258} = -1.07$$

$$CI = b_k \pm t_{\alpha/2} \cdot SE$$

$$= -1.1 \pm 1.967 \times 1.0258 = [-3.110568, 0.910568]$$

Since t_{calc} lies within the CI, we failed to reject null hypothesis

$H_0 : \beta_3 = 1$
$H_0 : \beta_3 \neq 1$

$$t_{stat} = \frac{0.6 - 1}{0.45875} = 0.87193$$

$$CI = b_k \pm t_{\alpha/2} \cdot SE$$

$$= 0.6 \pm 1.967 \times 0.45875 = [-0.30236, 1.50236]$$

Since t_{calc} falls within the confidence interval, we failed to reject null hypothesis

e.

II e

$$R^2 = \frac{SSR}{SST} \qquad \text{Adj } R^2 = 1 - \frac{SSE/(T-K)}{SST/(T-1)}$$

$$X'y = \begin{bmatrix} \Sigma y_t \\ \Sigma x_{2t} \cdot y_t \\ \Sigma x_{3t} \cdot y_t \end{bmatrix} = \begin{bmatrix} 100 \\ 1 \\ 8 \end{bmatrix}$$

$$\sum_n y_t = 100, \quad \Sigma y_t^2 = 450 \quad \bar{y} = \frac{\Sigma y_t}{n} = \frac{100}{25} = 4$$

$$SST = \sum_n (y_t - \bar{y})^2$$
$$= \sum_n (y_t^2 - 2y_t\bar{y} + \bar{y}^2) = \sum_n (y_t^2 - 2y_t \cdot 4 + 4^2)$$
$$= \sum_n y_t^2 - 8\sum_n y_t + 16\sum_n$$
$$= 450 - 8 \cdot 100 + 16 \times 25$$
$$= 50$$

$$SSE = 46.3$$
$$SSR = SST - SSE = 50 - 46.3 = 3.7$$

$$R^2 = \frac{SSR}{SST} = \frac{3.7}{50} = 0.074$$

$$R^2_{Adj} = 1 - \frac{SSE/(T-K)}{SST/(T-1)} = 1 - \frac{(46.3)/22}{50/24} = 1 - 1.01018$$
$$= -0.010182$$

III. Elasticity, R-Squared, Adjusted R-Squared, and Hypothesis Test

a.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: lny

Number of Observations Read	94
Number of Observations Used	93
Number of Observations with Missing Values	1

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.22901	0.22901	15.94	0.0001
Error	91	1.30703	0.01436		
Corrected Total	92	1.53604			

Root MSE	0.11985	R-Square	0.1491
Dependent Mean	8.58961	Adj R-Sq	0.1397
Coeff Var	1.39524		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	7.95996	0.15817	50.32	<.0001
lnX	1	0.25108	0.06288	3.99	0.0001

a. I expect the rise in salary with the higher level of education i.e. positive relationship between these two variables. So, yes, the education has the expected sign with the salary. With the unit percentage change (say, increase) in the education level, the change in the salary (increase in this case) is by 0.25108% unit.

b. ln(y) = 7.95996 + 0.25108 Ln(x)

When x = 13, ln(y) = 7.95996 + 0.25108 ln(13) = 8.603967.

Y-hatt = EXP(8.603967) = 5453.252

c. The value of R^2 = 0.1491 (14.91%) means the ln(x) (natural log of education in years) explains 14.91% of variation in the ln(y) (natural log of salary).

d.

$$\bar{R}^2 = 1 - \frac{SSE/(T-K)}{SST/(T-1)}$$

$$SSE = 1 - \frac{SSE}{SST} \times \frac{(T-1)}{(T-K)}$$

$$= 1 - \frac{1.30703}{1.53604} \times \frac{92}{91}$$

$$= 1 - 0.860259477$$

$$= 0.13974$$

R squared explains the variation to all the independent variables in the model. So, as we add more variables into the model, we will get higher R-squared value. However, adjusted R-squared only explains the variation in the model due to significant variables in the model. So, it is not possible to increase the value of adjusted R-squared just by adding new variable into the model.

e.

The SAS System

The REG Procedure
Model: MODEL1

Test 1 Results for Dependent Variable lny				
Source	DF	Mean Square	F Value	Pr > F
Numerator	1	2.03761	141.87	<.0001
Denominator	91	0.01436		

Null Hypothesis: H_0: $\beta_2 = 1$

Alternative Hypothesis H_a: $\beta_2 \neq 1$

Choice of Probability Distribution: F Distribution

Degree of Freedom = f (1, 91)

The result shows that the elasticity of salary w.r.t. education is not one (Pr < 0.001), reject null hypothesis.

The SAS System

The REG Procedure
Model: MODEL1

Test 2 Results for Dependent Variable lny				
Source	DF	Mean Square	F Value	Pr > F
Numerator	1	36.37462	2532.53	<.0001
Denominator	91	0.01436		

Null Hypothesis: H_0: $\beta_1 = 0$

Alternative Hypothesis H_a: $\beta_1 \neq 0$

Choice of Probability Distribution: F Distribution

Degree of Freedom = f (1, 91)

We reject null hypothesis that intercept is zero ($P < 0.0001$).

The SAS System

The REG Procedure
Model: MODEL1

Test 3 Results for Dependent Variable lny				
Source	DF	Mean Square	F Value	Pr > F
Numerator	1	0.22901	15.94	0.0001
Denominator	91	0.01436		

Null Hypothesis: H_0: $\beta_2 = 0$;

Alternative Hypothesis H_a: $\beta_2 \neq 0$

Choice of Probability Distribution: F Distribution

Degree of Freedom = f (1, 91)

The result shows that the lnx is not zero ($Pr < 0.001$). We can reject null hypothesis.

f.

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: lny

Number of Observations Read	94
Number of Observations Used	90
Number of Observations with Missing Values	4

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	0.44972	0.14991	14.43	<.0001
Error	86	0.89354	0.01039		
Corrected Total	89	1.34326			

Root MSE	0.10193	R-Square	0.3348
Dependent Mean	8.59773	Adj R-Sq	0.3116
Coeff Var	1.18556		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	7.72596	0.14504	53.27	<.0001
lnX	1	0.25183	0.05350	4.71	<.0001
lne	1	0.03694	0.00990	3.73	0.0003
lnt	1	0.03354	0.01177	2.85	0.0055

Yes, b_1, b_2, R-squared and Adjusted R-squared value changed. The addition and removal of independent variables in the analysis changes the model that we are developing or using to predict the dependent variable which also changes these values.

The addition of two more variables in the model increased the value of adjusted R-squared which signifies that addition of these variables help to develop better model than before.

g.

The SAS System

The REG Procedure
Model: MODEL1

Test 1 Results for Dependent Variable lny				
Source	DF	Mean Square	F Value	Pr > F
Numerator	1	0.00050707	0.05	0.8257
Denominator	86	0.01039		

Null Hypothesis: H_0: $\beta_2 - \beta_3 = 0$;

Alternative Hypothesis H_a: $\beta_2 - \beta_3 \neq 0$

Choice of Probability Distribution: F Distribution

Degree of Freedom = f (1, 86)

We failed to reject null hypothesis based on the P value (0.8257) which is greater than 0.05 at 95% confidence interval.

SAS Code:

Import Data and Open Project:

```
PROC IMPORT OUT= WORK.bm
            DATAFILE= "C:\Users\casnrlab_agh128\Desktop\EconHW\HW1-DATA.xls"
            DBMS= EXCEL REPLACE;
     GETNAMES=YES;
     DATAROW=2;
RUN;
dbms = excel replace;
range = sheet1$
getnames = yes;
mixed = no;
scantext = yes;
scantime = yes;
run;

data bm; set bm;
lny = log (y);
lnX = log(x);
run;

# a. # e.
proc reg data = bm;
model lny = ln(x);
test lnx = 1;
test intercept = 0;
test lnx = 0;
run;
proc print;
```

```
run;

#f #g
data bm; set bm;
lny = log(y);
lnx = log(x);
lne = log(e);
lnt = log(t);

proc reg data = bm;
model lny = lnx lne lnt;
test lne-lnt = 0;

run;
proc print;
run;
```