

Book Problems Chp 2: 1(a-c), 2, 5, 7, 8, 9

Remark: Students enrolled in STAT 4463 need not work the challenge problems on homework assignments labeled “Grad Students only”.

Remark 2: Don’t forget to write in complete sentences and always justify your answers.

Setup: Generate data as below and consider estimating  $f$  as one of the following functions

$$f_1(X) = \beta_0 + \beta_1 X$$

$$f_2(X) = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3$$

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```
set.seed(1)
x<- seq(from = -2, to = 2, by=.1)
y<- 100 + 2*x - x^2 + rnorm(41)
```

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Code

1. What is  $f(x)$ ,  $\hat{f}_1(x)$  and  $\hat{f}_2(x)$  for this data? Use `lm()` to get estimates.
2. Construct a plot with the true  $f(x)$  and estimates of  $\hat{f}_1(x)$  and  $\hat{f}_2(x)$  superimposed. Each line should be a different color and type.
3. Compute  $f(0)$  and  $\hat{f}_1(0)$  and identify them on your plot. Is the difference between these values reducible or irreducible error? Explain why in a sentence or two.
4. Compute  $f(0)$  and  $\hat{f}_2(0)$  and identify them on your plot. Is the difference between these values reducible or irreducible error? Explain why in a sentence or two.
5. Get  $y_{21}$  and  $f(0)$  and identify them on your plot. Is the difference between these values reducible or irreducible error? Explain why in a sentence or two.
6. Suppose you have test data  $(x, y) = (-1, 94), (0, 100), (1, 100)$ . Get the test MSE for both functions based on this test data.
7. (Grad Student only) Regenerate data above but change `set.seed(1)` to `set.seed(2)`. Compare  $\hat{f}_1(0)$  and  $\hat{f}_2(0)$  for this data to your previous answers for  $\hat{f}_1(0)$  and  $\hat{f}_2(0)$ . Is this difference attributable to the bias or variance of your predictions?
8. (Grad Students only) Provide an estimate for the bias of  $\hat{f}_2(0)$  using your two  $\hat{f}_2(0)$  answers and  $f(0)$ .

9. Will  $\hat{f}_1$  or  $\hat{f}_2$  generally have smaller variance?
10. Will  $\hat{f}_1$  or  $\hat{f}_2$  generally have the smallest (amount of) bias?
11. Explain, in a sentence or two, whether you should choose  $f_1$  or  $f_2$  to minimize the mean squared test error when  $n$  is “large”. Your answer must include terms “Bias” and “Variance”.