restart;
 # Exam2 Q3 Solution:
 # a)
restart;
$y := 79 \cdot h + 75 \cdot k - 4 \cdot h^2 - 3 \cdot k^2 - 2 \cdot h \cdot k;$

$$y := -4 h^2 - 2 h k - 3 k^2 + 79 h + 75 k \quad (1)$$

#Cost and Profit Function:
$vc := w \cdot h + k \cdot r;\ fc := b;\ Cost\_fun := vc + fc;$
#VC = Variable Cost, FC = Fixed Cost, Cost_fun = Total Cost.

$$vc := w h + k r \quad (2)$$

$$fc := b \quad (2)$$

$$Cost\_fun := w h + k r + b \quad (2)$$

$TVP := p \cdot y;\ Profit\_fun := TVP - Cost\_fun;$ *# TVP = Total Value Product, Profit_fun = Profit.*

$$TVP := p \left(-4 h^2 - 2 h k - 3 k^2 + 79 h + 75 k\right)$$

$$Profit\_fun := p \left(-4 h^2 - 2 h k - 3 k^2 + 79 h + 75 k\right) - w h - k r - b \quad (3)$$

 # Profit Maximization:
$profit := Profit\_fun;$

$$profit := p \left(-4 h^2 - 2 h k - 3 k^2 + 79 h + 75 k\right) - w h - k r - b \quad (4)$$

$pf1 := diff(profit, h);\ pf11 := diff(pf1, h);$ *# pf11 < 0 for profit max.*

$$pf1 := p\,(-8 h - 2 k + 79) - w$$

$$pf11 := -8 p \quad (5)$$

$pf2 := diff(profit, k);\ pf22 := diff(pf2, k);$ *# pf22 < 0 for profit max.*

$$pf2 := p\,(-2 h - 6 k + 75) - r$$

$$pf22 := -6 p \quad (6)$$

$pf12 := diff(pf1, k);\ pf21 := diff(pf2, h);$

$$pf12 := -2 p$$

$$pf21 := -2 p \quad (7)$$

$EP\_p\_h := solve(pf1 = 0, h);$ ***# Pseudo-scale line h***

$$EP\_p\_h := -\frac{2 p k - 79 p + w}{8 p} \quad (8)$$

$EP\_p\_k := solve(pf2 = 0, k);$ ***# Pseudo-scale line w.***

$$EP\_p\_k := -\frac{2 p h - 75 p + r}{6 p} \quad (9)$$

$ks\_profit := simplify(solve((eval(pf2,\ h = EP\_p\_h)) = 0, k));$
 # kStar: Profit maximizing level of input k. Ordinary input demand function k

$$ks\_profit := \frac{221 p + w - 4 r}{22 p} \quad (10)$$

$hs\_profit := simplify(eval(EP\_p\_h, k = ks\_profit));$
 # hStar: Profit maximizing level of input h. Ordinary Input demand function h.

$$hs\_profit := \frac{162 p - 3 w + r}{22 p} \quad (11)$$

$f1 := \frac{\partial}{\partial h}(y);\quad f2 := \frac{\partial}{\partial k}(y);$

$$f1 := -8 h - 2 k + 79$$

$$f2 := -2\,h - 6\,k + 75 \qquad \textbf{(12)}$$

$f11 := \frac{\partial}{\partial h}(f1);\; f22 := \frac{\partial}{\partial k}(f2);\; f12 := \frac{\partial^2}{\partial h\,\partial k}(y);\; f21 := \frac{\partial^2}{\partial k\,\partial h}(y);$ #***SOC of f1, #SOC of f2.***

$$f11 := -8$$
$$f22 := -6$$
$$f12 := -2$$
$$f21 := -2 \qquad \textbf{(13)}$$

$w := 8;\; r := 4;\; p := 8;\; b := 0;$

$$w := 8$$
$$r := 4$$
$$p := 8$$
$$b := 0 \qquad \textbf{(14)}$$

Profit_Max_k := *ks_profit*; # = 10.

$$Profit\_Max\_k := 10 \qquad \textbf{(15)}$$

Profit_Max_h := *hs_profit*; # = 7.25

$$Profit\_Max\_h := \frac{29}{4} \qquad \textbf{(16)}$$

Profit_Max_Output := *eval*(*y*, [*h* = *hs_profit*, *k* = *ks_profit*]); # = 667.5

$$Profit\_Max\_Output := \frac{1335}{2} \qquad \textbf{(17)}$$

Profit_Max_Profit := *eval*(*profit*, [*h* = *hs_profit*, *k* = *ks_profit*]); # = 5242

$$Profit\_Max\_Profit := 5242 \qquad \textbf{(18)}$$

 # Q3.b: Quota:

$QuoProfit := p\cdot y - 7\cdot 12 - k\cdot r;$

$$QuoProfit := -32\,h^2 - 16\,h\,k - 24\,k^2 + 632\,h + 596\,k - 84 \qquad \textbf{(19)}$$

quohf1 := *diff*(*QuoProfit*, *h*);

$$quohf1 := -64\,h - 16\,k + 632 \qquad \textbf{(20)}$$

quokf1 := *diff*(*QuoProfit*, *k*);

$$quokf1 := -16\,h - 48\,k + 596 \qquad \textbf{(21)}$$

quo_k := *solve*(*quokf1* = 0, *k*);

$$quo\_k := \frac{149}{12} - \frac{h}{3} \qquad \textbf{(22)}$$

quo_h := *eval*(*quohf1*, [*k* = *quo_k*]);

$$quo\_h := -\frac{176\,h}{3} + \frac{1300}{3} \qquad \textbf{(23)}$$

h_star_quo := *solve*(*quo_h*, *h*); # = 7.38

$$h\_star\_quo := \frac{325}{44} \qquad \textbf{(24)}$$

k_star_quo := *eval*(*quo_k*, [*h* = *h_star_quo*]); # = 9.955

$$k\_star\_quo := \frac{219}{22} \qquad \textbf{(25)}$$

QuoOutLevel := *eval*(*y*, [*h* = *h_star_quo*, *k* = *k_star_quo*]); # = 667.5455

$$QuoOutLevel := \frac{7343}{11} \tag{26}$$

$QuoProfitStar := eval(QuoProfit, [k = k\_star\_quo, h = h\_star\_quo])$; # = 5216.54545

$$QuoProfitStar := \frac{57382}{11} \tag{27}$$

 # Q3.c: Tarrif

$TarProfit := p \cdot y - h \cdot 12 - k \cdot r$;

$$TarProfit := -32\, h^2 - 16\, h\, k - 24\, k^2 + 620\, h + 596\, k \tag{28}$$

$hf1 := diff(TarProfit, h)$;

$$hf1 := -64\, h - 16\, k + 620 \tag{29}$$

$kf1 := diff(TarProfit, k)$;

$$kf1 := -16\, h - 48\, k + 596 \tag{30}$$

$tar\_k := solve(kf1 = 0, k)$;

$$tar\_k := \frac{149}{12} - \frac{h}{3} \tag{31}$$

$tar\_h := eval(hf1, [k = tar\_k])$;

$$tar\_h := -\frac{176\, h}{3} + \frac{1264}{3} \tag{32}$$

$h\_star := solve(tar\_h = 0, h)$; #amount of h used. = 7.1818

$$h\_star := \frac{79}{11} \tag{33}$$

$k\_star := (eval(tar\_k, [h = h\_star]))$; #amount of k used. = 10.02272

$$k\_star := \frac{441}{44} \tag{34}$$

$TarOutputLevel := eval(y, [h = h\_star, k = k\_star])$; # = 667.42

$$TarOutputLevel := \frac{117467}{176} \tag{35}$$

$TarProfitStar := eval(TarProfit, [k = k\_star, h = h\_star])$; # = 5213.136.

$$TarProfitStar := \frac{114689}{22} \tag{36}$$

 # Q3.d:

 # Under quota, total use of h will increase and the use of k will decrease. Total output will remain same and the profit will decrease.

 # Under Tarrfi, Total use of h will decrease and the use of k will increase. Total output will remain almost constant and the profit will decrease.

 # But profit is slightly better in quota than tarrif. So, tarrif is better in this case.