#Output Side Economics: Complete Worked out file for Profit Maximization, Cost Minimization and Output Maximization (Expenditure Demand Function) using Quadratic **and *Cobb Douglas Production Function*** **in** ***Perfectly Competitive Market. Author: Bijesh Mishra.*** **#\` \`*WORK ON PROGRESS***

restart;
Digits := (3); *#Limit upto three digits after decimal.*

$$Digits := 3 \tag{1}$$

 # r1 := 8; r2 := 10; p := 10; b := 0; Co := 936; yo := 385; # labor, capital, output price, fixed cost, investment, output demanded.

 # Cobb Douglas Production Function:

$Z := 4; m := \frac{1}{2}; n := \frac{1}{4}; Y := Y;$

 # change values here to change the Cobb Douglas function equation given by eq. 3. and change eq. 6 to "cobb" to run optimization using cobb douglas production function with two input and onw output. Y is the LHS of the production function.

$cobb := Z \cdot x1^m x2^n$; *#Cobb Douglas Production Function.*

$$cobb := Z\, x1^m\, x2^n \tag{2}$$

 # Quadratic Production function (two input one output):

$b1 := 6; b2 := 9; c1 := -0.2; c2 := -0.3; d1 := 0.4; a := 0; Y := Y;$

$$b1 := 6$$

$$b2 := 9$$

$$c1 := -0.2$$

$$c2 := -0.3$$

$$d1 := 0.4$$

$$a := 0$$

$$Y := Y \tag{3}$$

 # change values here to change the quadratic equation and set up y to "quad" to run optimization using quadratic production function with two input and one output. Same procedure works for Cobb Douglas production function which can be done by simply inserting Cobb douglas production function instead of quadratic function. Y is the LHS of the production function.

$quad := b1 \cdot x1 + b2 \cdot x2 + c1 \cdot x1^2 + c2 \cdot x2^2 + d1 \cdot x1 \cdot x2 + a;$

$$quad := -0.2\, x1^2 + 0.4\, x1\, x2 - 0.3\, x2^2 + 6\, x1 + 9\, x2 \tag{4}$$

 # Change function to quadratic (quad) or Cobb Douglas (cobb) production function with one output and two inputs. This process works for both types of function.

$y := cobb;$

$$y := Z\,x1^{m}\,x2^{n} \tag{5}$$

$APP1 := simplify\left(\frac{y}{x1}\right); APP2 := simplify\left(\frac{y}{x2}\right);\ AVP1 := simplify(APP1 \cdot p)\ ; AVP2 := simplify(APP2 \cdot \mathbf{p});$

 # APP1 & APP2 are average physical productivity. AVP1 & AVP2 Average value productivity which is obtained by multiplying price of output with respective APPs.

$$APP1 := Z\,x1^{m-1}\,x2^{n}$$
$$APP2 := Z\,x1^{m}\,x2^{n-1}$$
$$AVP1 := Z\,x1^{m-1}\,x2^{n}\,p$$
$$AVP2 := Z\,x1^{m}\,x2^{n-1}\,p \tag{6}$$

$f1 := \frac{\partial}{\partial x1}(y);\ \ f2 := \frac{\partial}{\partial x2}(y);\ MVP1 := p \cdot f1; MVP2 := p \cdot f2;\ MFC1 := MVP1; MFC2 := MVP2;$

#MPP1, MPP2, MVP1, MVP2, MVP1, MVP2

$$f1 := \frac{Z\,x1^{m}\,m\,x2^{n}}{x1}$$
$$f2 := \frac{Z\,x1^{m}\,x2^{n}\,n}{x2}$$
$$MVP1 := \frac{p\,Z\,x1^{m}\,m\,x2^{n}}{x1}$$
$$MVP2 := \frac{p\,Z\,x1^{m}\,x2^{n}\,n}{x2}$$
$$MFC1 := \frac{p\,Z\,x1^{m}\,m\,x2^{n}}{x1}$$
$$MFC2 := \frac{p\,Z\,x1^{m}\,x2^{n}\,n}{x2} \tag{7}$$

$f11 := \frac{\partial}{\partial x1}(f1);\ f22 := \frac{\partial}{\partial x2}(f2);\ f12 := \frac{\partial^2}{\partial x1\,\partial x2}(y);\ f21 := \frac{\partial^2}{\partial x2\,\partial x1}(y);$ **#SOC of f1, #SOC of f2.**

 # f12 & f21 are Factor Inerdependence (competitive, independent or complementary).

$$f11 := \frac{Z\,x1^{m}\,m^{2}\,x2^{n}}{x1^{2}} - \frac{Z\,x1^{m}\,m\,x2^{n}}{x1^{2}}$$
$$f22 := \frac{Z\,x1^{m}\,x2^{n}\,n^{2}}{x2^{2}} - \frac{Z\,x1^{m}\,x2^{n}\,n}{x2^{2}}$$
$$f12 := \frac{Z\,x1^{m}\,m\,x2^{n}\,n}{x1\,x2}$$
$$f21 := \frac{Z\,x1^{m}\,m\,x2^{n}\,n}{x1\,x2} \tag{8}$$

$MRTS := simplify\left(\left(\frac{f1}{f2}\right)\right)$; #*Marginal Rate of Technical Subsitution* $\cdot \left(MRTS_{21}\right)$

$$MRTS := \frac{m\,x2}{x1\,n} \tag{9}$$

$\frac{r1}{r2} = \frac{f1}{f2}$ ***# Marginal Revenue = Marginal Rate of Technical Subsitution (MR = MRTS$_{21}$)***

$$\frac{r1}{r2} = \frac{m\,x2}{x1\,n} \tag{10}$$

$SOC := simplify(\,f2\cdot f2\cdot f11 - 2\cdot f1\cdot f2\cdot f12 + f1\cdot f1\cdot f22)$; #***Second order condition.***

$$SOC := -x1^{3\,m-2}\,x2^{3\,n-2}\,Z^3\,m\,n\,(m+n) \tag{11}$$

$Curvature := simplify\left(\left(\frac{1}{f2^3}\right)\cdot SOC\right)$; ***# Curvature.***

 # This curvature is derived from production function. Gives change in slope of the isoquant and used to determine convexity of the isoqunt.

 # Also, if you know the equation of isoquant, then curvature is the second derivative of the equation for isoquant.

$$Curvature := -\frac{x2\,m\,(m+n)}{x1^2\,n^2} \tag{12}$$

 # Elasticity of Factor Subsitution (σ):

$Elasticity\_of\_Factor\_Subsitution := \frac{f1\cdot f2\cdot(\,f2\cdot x2 + f1\cdot x1)}{(x1\cdot x2)\cdot(2\cdot f1\cdot f2\cdot f12 - f1\cdot f1\cdot f22 - f2\cdot f2\cdot f11)}$;

$$Elasticity\_of\_Factor\_Subsitution := \left(Z^2\,(x1^m)^2\,m\,(x2^n)^2\,n\,(Z\,x1^m\,m\,x2^n + Z\,x1^m\,x2^n\,n)\right) \Bigg/ \left(x1^2\,x2^2\left(\frac{2\,Z^3\,(x1^m)^3\,m^2\,(x2^n)^3\,n^2}{x1^2\,x2^2} - \frac{Z^2\,(x1^m)^2\,m^2\,(x2^n)^2\left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right)}{x1^2} - \frac{Z^2\,(x1^m)^2\,(x2^n)^2\,n^2\left(\frac{Z\,x1^m\,m^2\,x2^n}{x1^2} - \frac{Z\,x1^m\,m\,x2^n}{x1^2}\right)}{x2^2}\right)\right) \tag{13}$$

 # ElasticityOfFS $:= eval(E\_FS, [x1 = 2, x2 = 3])$;

 # Enter value of x1 and x2 as desired to calculate elasticity of factor subsitution.

 # Elasticity of Production or Functional Coefficient (ε): Also gives return to scale (constant (= 1) , increasing (> 1) or decreasing (< 1)).

$E\_x1 := \frac{f1}{APP1}$; ***# Elasticity of production for factor x1. MPP1 divided by APP1.***

$$E\_x1 := \frac{x1^m\,m}{x1\,x1^{m-1}} \tag{14}$$

$E\_x2 := \frac{f2}{APP2}$; **# Elasticity of production for factor x2. (MPP2 divided by APP2).**

$$E\_x2 := \frac{x2^n\, n}{x2\, x2^{n-1}} \tag{15}$$

$Functional\_Coefficient := simplify(E\_x1 + E\_x2)$;

#Function coefficient or elasticity of production for production function.

$$Functional\_Coefficient := m + n \tag{16}$$

$Fun\_Coeff\_Alt := simplify\left(\frac{(f1 \cdot x1 + f2 \cdot x2)}{y}\right)$;

#Fun_Coeff_Alt and Fun_Coeff are same just calculated in different ways.

$$Fun\_Coeff\_Alt := m + n \tag{17}$$

 # Homogenous function:

 # if one common value in term of t^r *is obtained* for *entire equation, the funciton is homogenous of degree* "r" and *has constant proportion of* return to *scale of r . The r value is also equal* to *functional ceofficient* (**ε**). *The degree of homogenity of MPPs* **and** *APPs of homogenous production functions are* "r-1".

$Homogen := simplify(eval(y, [x1 = t \cdot x1, x2 = t \cdot x2]))$;

$$Homogen := Z\,(t\, x1)^m\,(t\, x2)^n \tag{18}$$

 # Ridgelines x1 and x2:

$Ridgeline\_x1 := solve(f1 = 0, x1)$; ***# MPP1 = 0 and Stage III begins for input x2.***

$$Ridgeline\_x1 := (\ ) \tag{19}$$

$Ridgeline\_x2 := solve(f2 = 0, x2)$; ***# MPP2 = 0 and stage III begins for input x1.***

$$Ridgeline\_x2 := (\ ) \tag{20}$$

 # General Cost and Profit Function:

$VC := r1 \cdot x1 + r2 \cdot x2$; $FC := b$; $Cost\_fun := VC + FC$;

#VC = Variable Cost, FC = Fixed Cost, Cost_fun = Total Cost.

$$VC := r1\, x1 + r2\, x2 \tag{21}$$

$$FC := b \tag{21}$$

$$Cost\_fun := r1\, x1 + r2\, x2 + b \tag{21}$$

$TVP := p \cdot y$; $Profit\_fun := TVP - Cost\_fun$; *# TVP = Total Value Product, Profit_fun = Profit.*

$$TVP := p\, Z\, x1^m\, x2^n$$

$$Profit\_fun := p\, Z\, x1^m\, x2^n - r1\, x1 - r2\, x2 - b \tag{22}$$

 # Cost Minimization subject to Output Constraints (Min C st (yo - y)): Least Cost Combination of Two Factors of Production. Necessary for output side economics.

$Cost := Cost\_fun;$

$$Cost := r1\,x1 + r2\,x2 + b \tag{23}$$

$LC := Cost + \lambda \cdot (yo - y);$ *# λ is lagrangean multiplier.*

$$LC := r1\,x1 + r2\,x2 + b + \lambda\left(yo - Z\,x1^m\,x2^n\right) \tag{24}$$

$LCf1 := diff(LC, x1); LCf11 := diff(LCf1, x1);$ *#FOC and SOC of lagrangean function wrt x1*

$$LCf1 := r1 - \frac{\lambda\,Z\,x1^m\,m\,x2^n}{x1}$$

$$LCf11 := -\frac{\lambda\,Z\,x1^m\,m^2\,x2^n}{x1^2} + \frac{\lambda\,Z\,x1^m\,m\,x2^n}{x1^2} \tag{25}$$

$LCf2 := diff(LC, x2); LCf22 := diff(LCf2, x2);$ *#FOC and SOC of lagraangean function wrt x2*

$$LCf2 := r2 - \frac{\lambda\,Z\,x1^m\,x2^n\,n}{x2}$$

$$LCf22 := -\frac{\lambda\,Z\,x1^m\,x2^n\,n^2}{x2^2} + \frac{\lambda\,Z\,x1^m\,x2^n\,n}{x2^2} \tag{26}$$

$LCf12 := diff(LCf1, x2); LCf21 := diff(LCf2, x1);$
 # Cross differentiation of LCf1 and LCf2 wrt x2 and x1 respectively. Gives interdependence of factors.

$$LCf12 := -\frac{\lambda\,Z\,x1^m\,m\,x2^n\,n}{x1\,x2}$$

$$LCf21 := -\frac{\lambda\,Z\,x1^m\,m\,x2^n\,n}{x1\,x2} \tag{27}$$

$LCF\lambda := diff(LC, \lambda);$ *#FOC of lagraangean function wrt λ.*

$$LCF\lambda := yo - Z\,x1^m\,x2^n \tag{28}$$

$LCf1\lambda := solve(LCf1, \lambda);$ ***# λ from LCf1.***

$$LCf1\lambda := \frac{r1\,x1}{Z\,x1^m\,m\,x2^n} \tag{29}$$

$LCf2\lambda := solve(LCf2, \lambda);$ ***# λ from LCf2.***

$$LCf2\lambda := \frac{r2\,x2}{Z\,x1^m\,x2^n\,n} \tag{30}$$

$\frac{r1}{f1} = \frac{r2}{f2};$

$$\frac{r1\,x1}{Z\,x1^m\,m\,x2^n} = \frac{r2\,x2}{Z\,x1^m\,x2^n\,n} \tag{31}$$

$\frac{r1}{LCf1} = \frac{r2}{LCf2};$

$$\frac{r1}{r1 - \frac{\lambda\,Z\,x1^m\,m\,x2^n}{x1}} = \frac{r2}{r2 - \frac{\lambda\,Z\,x1^m\,x2^n\,n}{x2}} \tag{32}$$

$EP\_C\_x1 := solve(LCf1\lambda = LCf2\lambda, x1)$; **# Expansion path X1 is the function of r1, r2, x2.**

$$EP\_C\_x1 := \frac{r2\, x2\, m}{n\, r1} \tag{33}$$

$EP\_C\_x2 := solve(LCf1\lambda = LCf2\lambda, x2)$; **# Expansion Path X2 is the function of r1, r2, x1**

$$EP\_C\_x2 := \frac{r1\, x1\, n}{m\, r2} \tag{34}$$

$EP\_C\_x1\_LHS := x1$; $EP\_C\_x2\_LHS := x2$;

$$EP\_C\_x1\_LHS := x1$$

$$EP\_C\_x2\_LHS := x2 \tag{35}$$

$EP\_C\_x1\_Inverse\_X2 := solve(EP\_C\_x1 = EP\_C\_x1\_LHS,\ x2)$;

 # Expansion path x1 is equted to x1 and **solved for x2**. *Inverse of expansion path x1* and ***equivalent* to *Expansion path x2*** *as function of x1, r1, r2,.*

$$EP\_C\_x1\_Inverse\_X2 := \frac{r1\, x1\, n}{m\, r2} \tag{36}$$

$EP\_C\_x2\_Inverse\_X1 := solve(EP\_C\_x2 = EP\_C\_x2\_LHS,\ x1)$;

 # Expansion path x2 is equated to x2 and **solved for x1**. *Inverse of expansion path x2* and ***equivalent* to *Expansion path x1*** *as function of x2, r1, r2.*

$$EP\_C\_x2\_Inverse\_X1 := \frac{r2\, x2\, m}{n\, r1} \tag{37}$$

$Y\_In\_Terms\_of\_x1 := (eval(y,\ x2 = EP\_C\_x2))$;

 # x2 in y is replaced by expansion path x2 to convert ***y as function of x1, r1, r2.***

$$Y\_In\_Terms\_of\_x1 := Z\, x1^{m} \left(\frac{r1\, x1\, n}{m\, r2}\right)^{n} \tag{38}$$

$Y\_In\_Terms\_of\_x2 := (eval(y, x1 = EP\_C\_x1))$;

 # Expansion Path x1 is replaced in y to convert ***y as function of x2, r1, r2.***

$$Y\_In\_Terms\_of\_x2 := Z \left(\frac{r2\, x2\, m}{n\, r1}\right)^{m} x2^{n} \tag{39}$$

$Y\_In\_Terms\_of\_x1\_Inverse := (eval(y,\ x2 = EP\_C\_x1\_Inverse\_X2))$;

 # Expansion Path x2 is replaced in y to convert ***y as function of x1, r1, r2.***

$$Y\_In\_Terms\_of\_x1\_Inverse := Z\, x1^{m} \left(\frac{r1\, x1\, n}{m\, r2}\right)^{n} \tag{40}$$

$Y\_In\_Terms\_of\_x2\_Inverse := (eval(y,\ x1 = EP\_C\_x2\_Inverse\_X1))$;

 # Expansion Path x1 is replaced in y to convert ***y as function of x2, r1, r2.***

$$Y\_In\_Terms\_of\_x2\_Inverse := Z \left(\frac{r2\, x2\, m}{n\, r1}\right)^{m} x2^{n} \tag{41}$$

$x2s\_cost := (solve((eval(LCF\lambda, x1 = EP\_C\_x1)),\ x2))$;

 # X2Star: Cost Minimizing input x2 demand function. #Use small value

$$x2s\_cost := \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m + n}} \tag{42}$$

$x1s\_cost := eval(EP\_C\_x1, x2 = x2s\_cost)$;

 # X1Star: Cost Minimizing input x1 demand function. #use small value.

$$x1s\_cost := \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{n\, r1} \tag{43}$$

LCf1λStar := *eval*(*LCf1λ*, [*x1* = *x1s_cost*, *x2* = *x2s_cost*]);
#Lagrangean multiplier λ1 Star #Use positive value

$$LCf1\lambda Star := \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}}{n\, Z \left(\frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{n\, r1}\right)^{m} \left(\mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\right)^{n}} \tag{44}$$

LCf2λStar := *eval*(*LCf2λ*, [*x1* = *x1s_cost*, *x2* = *x2s_cost*]);
#Lagrangean multiplier λ2 Star #Use positive value.

$$LCf2\lambda Star := \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}}{n\, Z \left(\frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{n\, r1}\right)^{m} \left(\mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\right)^{n}} \tag{45}$$

VC_Star := *simplify*(*eval*(*VC*, [*x1* = *x1s_cost*, *x2* = *x2s_cost*])); *#Variable Cost Star.*
 # Under perfect condition, VC is homogenous of degree $\frac{1}{\varepsilon}$ **in** *output* **and** ε **in** *factor levels*
. ε *is the functional coefficient* **and** *gives* **return to** *scale of production function.*
 # For Cobb Douglas production function, ε = *m* + *n.*

$$VC\_Star := \frac{r2\, \mathrm{e}^{\frac{-\ln\left(\frac{r2\, m}{n\, r1}\right) m - \ln\left(\frac{Z}{yo}\right)}{m+n}}\, (m+n)}{n} \tag{46}$$

Total_Cost_Star := *eval*(*Cost*, [*x1* = *x1s_cost*, *x2* = *x2s_cost*]); *#* ***Total Cost*** *for cost minimization.*

$$Total\_Cost\_Star := \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{n} + r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}} + b \tag{47}$$

Average_Cost_Star := $\frac{Total\_Cost\_Star}{yo}$; *#Aaverage Total Cost (ATC).*

$$Average\_Cost\_Star := \frac{\frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{n} + r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}} + b}{yo} \tag{48}$$

Average_variable_Cost_Star := $\frac{VC\_Star}{yo}$; *#Average Variable Cost.*

$$Average\_variable\_Cost\_Star := \frac{r2\, \mathrm{e}^{\frac{-\ln\left(\frac{r2\, m}{n\, r1}\right) m - \ln\left(\frac{Z}{yo}\right)}{m+n}} (m+n)}{n\, yo} \tag{49}$$

Marginal_Cost_Star := *diff* (*Total_Cost_Star*, *yo*); #*Marginal Cost.*

$$Marginal\_Cost\_Star := \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{yo\, (m+n)\, n} + \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}}{yo\, (m+n)} \tag{50}$$

$$Variable\_Cost\_Flexibity := \frac{Marginal\_Cost\_Star}{Average\_variable\_Cost\_Star};$$

#This is equivalent to lagrangean multiplier coefficient.

$$Variable\_Cost\_Flexibity := \frac{\left(\frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{yo\, (m+n)\, n} + \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}}{yo\, (m+n)}\right) n\, yo}{r2\, \mathrm{e}^{\frac{-\ln\left(\frac{r2\, m}{n\, r1}\right) m - \ln\left(\frac{Z}{yo}\right)}{m+n}} (m+n)} \tag{51}$$

Inflection_point := *diff* (*Marginal_Cost_Star*, *yo*); #*Inflection point of VC and TC.*

$$Inflection\_point := -\frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{yo^2\, (m+n)\, n} + \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}\, m}{yo^2\, (m+n)^2\, n} - \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}}{yo^2\, (m+n)} + \frac{r2\, \mathrm{e}^{-\frac{\ln\left(\frac{r2\, m}{n\, r1}\right) m + \ln\left(\frac{Z}{yo}\right)}{m+n}}}{yo^2\, (m+n)^2} \tag{52}$$

 # Total Revenue:
$TR := p \cdot y;$

$$TR := p\, Z\, x1^m\, x2^n \tag{53}$$

 # Average Revenue: is the price in perfect competition.

$$AR := \frac{TR}{y};$$

$$AR := p \tag{54}$$

 # Marginal Revenue: In perfect condition, marginal revenue is price.

$MR := \frac{MFC}{MPP}$; *# Marginal factor cost divided by Marginal physical productivity.*

$$MR := \frac{MFC}{MPP} \tag{55}$$

 # Profit Maximization: See in note (Topics 12, p 3 & 4) . Do FOC, SOC and TOC. For TOC, Total Revenue (TR) - Variable Cost (VC) > 0.
 # Supply Function: See in note. (Topics 12, p 3 & 4)
 # Homogenity of product sypply is degree zero in price.
 # Comparative Static:

$$dx1Star\_dr1 := \frac{f22}{p\cdot(f11\cdot f22 - f12^2)}$$; #Should be < 0.Equivalent to diff(dx1star, r1)

$$dx1Star\_dr1 := \left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right) \Bigg/ \left(p\left(\left(\frac{Z\,x1^m\,m^2\,x2^n}{x1^2} - \frac{Z\,x1^m\,m\,x2^n}{x1^2}\right)\left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right) - \frac{Z^2\,(x1^m)^2\,m^2\,(x2^n)^2\,n^2}{x1^2\,x2^2}\right)\right) \tag{56}$$

$$dx1Star\_dr2 := -\frac{f12}{p\cdot(f11\cdot f22 - f12^2)};$$

Sign depends. Equivalent to diff(dx1star, r2); Also equal to diff(dx2Star, r1) #Cross Effect.

$$dx1Star\_dr2 := -(Z\,x1^m\,m\,x2^n\,n) \Bigg/ \left(x1\,x2\,p\left(\left(\frac{Z\,x1^m\,m^2\,x2^n}{x1^2} - \frac{Z\,x1^m\,m\,x2^n}{x1^2}\right)\left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right) - \frac{Z^2\,(x1^m)^2\,m^2\,(x2^n)^2\,n^2}{x1^2\,x2^2}\right)\right) \tag{57}$$

$$dx1Star\_dp := \frac{(f2\cdot f12 - f1\cdot f22)}{p\cdot(f11\cdot f22 - f12^2)};$$

#sign depends.Equivalent to diff(dx1star, p); Also equal to -diff(ystar, r1). Notice negative sign.

$$dx1Star\_dp := \left(-\frac{Z\,x1^m\,m\,x2^n\left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right)}{x1} + \frac{Z^2\,(x1^m)^2\,m\,(x2^n)^2\,n^2}{x1\,x2^2}\right) \Bigg/ \left(p\left(\left(\frac{Z\,x1^m\,m^2\,x2^n}{x1^2} - \frac{Z\,x1^m\,m\,x2^n}{x1^2}\right)\left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right) - \frac{Z^2\,(x1^m)^2\,m^2\,(x2^n)^2\,n^2}{x1^2\,x2^2}\right)\right) \tag{58}$$

$$dystar\_dr2 := \frac{(f2\cdot f11 - f1\cdot f12)}{p\cdot(f11\cdot f22 - f12^2)};$$

#equivalent to diff(dystar, r2). Also equal to -diff(x2star, p). Notice negative sign.

$$dystar\_dr2 := \left(-\frac{Z^2\,(x1^m)^2\,m^2\,(x2^n)^2\,n}{x1^2\,x2} + \frac{\left(\frac{Z\,x1^m\,m^2\,x2^n}{x1^2} - \frac{Z\,x1^m\,m\,x2^n}{x1^2}\right)Z\,x1^m\,x2^n\,n}{x2}\right) \Bigg/ \left(p\left(\left(\frac{Z\,x1^m\,m^2\,x2^n}{x1^2} - \frac{Z\,x1^m\,m\,x2^n}{x1^2}\right)\left(\frac{Z\,x1^m\,x2^n\,n^2}{x2^2} - \frac{Z\,x1^m\,x2^n\,n}{x2^2}\right) - \frac{Z^2\,(x1^m)^2\,m^2\,(x2^n)^2\,n^2}{x1^2\,x2^2}\right)\right) \tag{59}$$

$$dystar\_dp := \frac{(2\cdot f1\cdot f2\cdot f12 - f1\cdot f1\cdot f22 - f2\cdot f2\cdot f22)}{p\cdot(f11\cdot f22 - f12^2)}$$; # Equivalent to diff(ystar, p).

$$dystar\_dp := \left(\frac{2 Z^3 \left(x1^m\right)^3 m^2 \left(x2^n\right)^3 n^2}{x1^2 x2^2} - \frac{Z^2 \left(x1^m\right)^2 m^2 \left(x2^n\right)^2 \left(\frac{Z x1^m x2^n n^2}{x2^2} - \frac{Z x1^m x2^n n}{x2^2} \right)}{x1^2} - \frac{Z^2 \left(x1^m\right)^2 \left(x2^n\right)^2 n^2 \left(\frac{Z x1^m x2^n n^2}{x2^2} - \frac{Z x1^m x2^n n}{x2^2} \right)}{x2^2} \right) \Bigg/ \left(p \left(\left(\frac{Z x1^m m^2 x2^n}{x1^2} - \frac{Z x1^m m x2^n}{x1^2} \right) \left(\frac{Z x1^m x2^n n^2}{x2^2} - \frac{Z x1^m x2^n n}{x2^2} \right) - \frac{Z^2 \left(x1^m\right)^2 m^2 \left(x2^n\right)^2 n^2}{x1^2 x2^2} \right) \right) \tag{60}$$

$$dx2Star\_dr2 := \frac{f11}{p \cdot \left(f11 \cdot f22 - f12^2 \right)};$$

$$dx2Star\_dr2 := \left(\frac{Z x1^m m^2 x2^n}{x1^2} - \frac{Z x1^m m x2^n}{x1^2} \right) \Bigg/ \left(p \left(\left(\frac{Z x1^m m^2 x2^n}{x1^2} - \frac{Z x1^m m x2^n}{x1^2} \right) \left(\frac{Z x1^m x2^n n^2}{x2^2} - \frac{Z x1^m x2^n n}{x2^2} \right) - \frac{Z^2 \left(x1^m\right)^2 m^2 \left(x2^n\right)^2 n^2}{x1^2 x2^2} \right) \right) \tag{61}$$