restart;

 # Exam2 Q1 Solution:

 # a)

$y := 1215 \cdot x^{\left(\frac{2}{5}\right)}$; # Production function.

$$y := 1215\, x^{2/5} \quad \mathbf{(1)}$$

$b := 10935$; #fixed cost.

$$b := 10935 \quad \mathbf{(2)}$$

$r := 5 \cdot x^{\frac{2}{5}}$; #labor supply function.

$$r := 5\, x^{2/5} \quad \mathbf{(3)}$$

$p := 3.50$; #price

$$p := 3.50 \quad \mathbf{(4)}$$

$profit := p \cdot y - r \cdot x - b$; # profit. r is the price of input and x is input in this case, labor.

$$profit := 4252.50\, x^{2/5} - 5\, x^{7/5} - 10935 \quad \mathbf{(5)}$$

Maximize profit:

$FOC\_profit := diff(profit, x)$;

$$FOC\_profit := \frac{1701.000000}{x^{3/5}} - 7\, x^{2/5} \quad \mathbf{(6)}$$

$xStar := solve(FOC\_profit = 0, x)$;

$$xStar := 243. \quad \mathbf{(7)}$$

$rStar := eval(r,\ x = xStar)$;

$$rStar := 45.00000000 \quad \mathbf{(8)}$$

$YStar := eval(y, x = xStar)$;

$$YStar := 10935.00000 \quad \mathbf{(9)}$$

 # b)

$Average\_Variable\_Cost := \frac{rStar}{YStar}$;

$$Average\_Variable\_Cost := 0.004115226337 \quad \mathbf{(10)}$$

$Marginal\_Cost := p$; #MC = MR

$$Marginal\_Cost := 3.50 \quad \mathbf{(11)}$$

$Average\_Total\_Cost := Average\_Variable\_Cost + \frac{b}{YStar}$;

$$Average\_Total\_Cost := 1.004115226 \quad \mathbf{(12)}$$

$ProfitStar := eval(profit, x = xStar)$;

$$ProfitStar := 16402.50000 \quad \mathbf{(13)}$$

$ProfitPerUnitY := \frac{ProfitStar}{YStar}$;

$$ProfitPerUnitY := 1.500000000 \quad \mathbf{(14)}$$

 # c)

$Demand\_p := 20 - 0.001 \cdot y$;

$$Demand\_p := 20 - 1.215\, x^{2/5} \quad \mathbf{(15)}$$

$Demand\_profit := Demand\_p \cdot y - r \cdot x - b;$

$$Demand\_profit := -5\,x^{7/5} + 1215\,x^{2/5}\left(20 - 1.215\,x^{2/5}\right) - 10935 \qquad \textbf{(16)}$$

$Profit\_Max\_Level := diff(Demand\_profit, x);$

$$Profit\_Max\_Level := -7\,x^{2/5} + \frac{486\left(20 - 1.215\,x^{2/5}\right)}{x^{3/5}} - \frac{590.4900000}{x^{1/5}} \qquad \textbf{(17)}$$

$Profit\_Max\_Level\_Input := solve(Profit\_Max\_Level = 0, x);$

$$Profit\_Max\_Level\_Input := 146.9348957 \qquad \textbf{(18)}$$

#d)

$Demand\_pStar := eval(Demand\_p,\ x = xStar);$

$$Demand\_pStar := 9.06500000 \qquad \textbf{(19)}$$

Answer: *New optimial quantity of output* **and** *input* **in** *part c* (9.065) *is greater than* **in** *part a* (3.5).