Exam 2. Q2. Solution
 # a)
restart;
$y := 100\cdot x + 200\cdot w - 1.5\cdot x^2 - 0.5\cdot w^2 + w\cdot x;$

$$y := 100\,x + 200\,w - 1.5\,x^2 - 0.5\,w^2 + w\,x \tag{1}$$

***Cost* and *Profit Function* :**
$vc := r\cdot x + w\cdot s;\ fc := b;\ Cost\_fun := vc + fc;$
#VC = Variable Cost, FC = Fixed Cost, Cost_fun = Total Cost.

$$vc := r\,x + w\,s \tag{2}$$

$$fc := b \tag{2}$$

$$Cost\_fun := r\,x + w\,s + b \tag{2}$$

$TVP := p\cdot y;\ Profit\_fun := TVP - Cost\_fun;$ # TVP = Total Value Product, Profit_fun = Profit.

$$TVP := p\,(100\,x + 200\,w - 1.5\,x^2 - 0.5\,w^2 + w\,x)$$

$$Profit\_fun := p\,(100\,x + 200\,w - 1.5\,x^2 - 0.5\,w^2 + w\,x) - r\,x - w\,s - b \tag{3}$$

 # Profit Maximization:
$profit := Profit\_fun;$

$$profit := p\,(100\,x + 200\,w - 1.5\,x^2 - 0.5\,w^2 + w\,x) - r\,x - w\,s - b \tag{4}$$

$pf1 := diff(profit, x);\ pf11 := diff(pf1, x);$ # pf11 < 0 for profit max.

$$pf1 := p\,(100 - 3.0\,x + w) - r$$

$$pf11 := -3.0\,p \tag{5}$$

$pf2 := diff(profit, w);\ pf22 := diff(pf2, w);$ # pf22 < 0 for profit max.

$$pf2 := p\,(200 - 1.0\,w + x) - s$$

$$pf22 := -1.0\,p \tag{6}$$

$pf12 := diff(pf1, w);\ pf21 := diff(pf2, x);$

$$pf12 := p$$

$$pf21 := p \tag{7}$$

 # 2. a)
$EP\_p\_x := solve(pf1 = 0, x);$ ***# Pseudo-scale line x***

$$EP\_p\_x := \frac{0.3333333333\,(p\,w + 100.\,p - r)}{p} \tag{8}$$

$EP\_p\_w := solve(pf2 = 0, w);$ ***# Pseudo-scale line w.***

$$EP\_p\_w := \frac{p\,x + 200.\,p - s}{p} \tag{9}$$

#2. b)
$ws\_profit := simplify(solve((eval(pf2,\ x = EP\_p\_x)) = 0, w));$
 # wStar: Profit maximizing level of input w. Ordinary input demand function w

$$ws\_profit := \frac{350.\,p - 0.5\,r - 1.5\,s}{p} \tag{10}$$

$xs\_profit := simplify(eval(EP\_p\_x, w = ws\_profit));$
 # XStar: Profit maximizing level of input x. Ordinary Input demand function x.

$$xs\_profit := \frac{150.\,p - 0.5\,r - 0.5\,s}{p} \tag{11}$$

 # 2. c)

$f1 := \frac{\partial}{\partial x}(y);\quad f2 := \frac{\partial}{\partial w}(y);$

$$f1 := 100 - 3.0\,x + w$$

$$f2 := 200 - 1.0\,w + x \tag{12}$$

$f11 := \frac{\partial}{\partial x}(f1);\ f22 := \frac{\partial}{\partial w}(f2);\ f12 := \frac{\partial^2}{\partial x\,\partial w}(y);\ f21 := \frac{\partial^2}{\partial w\,\partial x}(y);$ #***SOC of f1, #SOC of f2.***

$$f11 := -3.0$$

$$f22 := -1.0$$

$$f12 := 1$$

$$f21 := 1 \tag{13}$$

$DXStar\_Ds := -\frac{f12}{p\cdot(f11\cdot f22 - f12^2)}$; ***# this is the requird result. diff(xstar, s)***

$$DXStar\_Ds := -\frac{0.5000000000}{p} \tag{14}$$

#**2 d.)**

$r := 8;\ s := 4;\ p := 2;\ b := 0;$

$$r := 8$$

$$s := 4$$

$$p := 2$$

$$b := 0 \tag{15}$$

$Profit\_Max\_w := ws\_profit;$

$$Profit\_Max\_w := 345.0000000 \tag{16}$$

$Profit\_Max\_x := xs\_profit;$

$$Profit\_Max\_x := 147.0000000 \tag{17}$$

$Profit\_Max\_Output := eval(y, [x = xs\_profit, w = ws\_profit]);$

$$Profit\_Max\_Output := 42489.00000 \tag{18}$$

 # 2.e)

$Cost := Cost\_fun;$

$$Cost := 4\,w + 8\,x \tag{19}$$

$Co := 1896;$ *#budget constraints.*

$$Co := 1896 \tag{20}$$

$Ly := y + \mu\cdot(Co - Cost);$ *#Lagrangean function.*

$$Ly := 100\,x + 200\,w - 1.5\,x^2 - 0.5\,w^2 + w\,x + \mu\,(1896 - 4\,w - 8\,x) \tag{21}$$

$Lyf1 := diff(Ly, x);\ Lyf11 := diff(Lyf1, x);$ *#FOC and SOC of Lagrangean function wrt x.*

$$Lyf1 := 100 - 3.0\,x + w - 8\,\mu$$

$$Lyf11 := -3.0 \tag{22}$$

$Lyf2 := diff(Ly, w);\ Lyf22 := diff(Lyf2, w);$ *#FOC and SOC of Lagrangean function wrt w.*

$$Lyf2 := 200 - 1.0\,w + x - 4\,\mu$$

$$Lyf22 := -1.0 \tag{23}$$

$Lyf12 := diff(Lyf1, w);\ Lyf21 := diff(Lyf2, x);$ *#Cross SOCs of Lyf1 and Lyf2 wrt w and x respectively.*

$$Lyf12 := 1$$

$$Lyf21 := 1 \tag{24}$$

$Ly\mu := diff(Ly, \mu)$; # FOC of Lagrangean function wrt μ.

$$Ly\mu := 1896 - 4\,w - 8\,x \tag{25}$$

$Lyf1\mu := solve(Lyf1 = 0, \mu)$; #Solve for μ using Lyf1μ.

$$Lyf1\mu := 12.50000000 - 0.3750000000\,x + 0.1250000000\,w \tag{26}$$

$Lyf2\mu := solve(Lyf2 = 0, \mu)$; #Solve for μ using Lyf1μ.

$$Lyf2\mu := 50. - 0.2500000000\,w + 0.2500000000\,x \tag{27}$$

$EP\_Ly\_x := solve(Lyf1\mu = Lyf2\mu, x)$;
Equate μ from above two equations and solve for x1. Expansion path x1.

$$EP\_Ly\_x := -60. + 0.6000000000\,w \tag{28}$$

$EP\_Ly\_w := solve(Lyf1\mu = Lyf2\mu, w)$;
Equate μ from above two equations and solve for x2. Expansion path x2.

$$EP\_Ly\_w := 100. + 1.666666667\,x \tag{29}$$

$ws\_expd := solve((eval(Ly\mu, x = EP\_Ly\_x)), w)$;
#X2Star: Demand function of input x2 to maximize the output subject to budget constraints. Ordinary Input Demand Function x2.

$$ws\_expd := 270. \tag{30}$$

$xs\_expd := eval(EP\_Ly\_x, w = ws\_expd)$;
#X1Star: Demand function of input x1 to maximize the output subject to budget constraints. Ordinary Input Demand Function x1.

$$xs\_expd := 102.0000000 \tag{31}$$

$ystar\_expd := eval(y, [x = xs\_expd, w = ws\_expd])$;
#YStar: Supply function. Total output produced under constrained budget.

$$ystar\_expd := 39684.00000 \tag{32}$$

$CostStar\_expd := eval(Cost, [x = xs\_expd, w = ws\_expd])$;
#CostStar: Total cost to produce given amount of output.

$$CostStar\_expd := 1896.000000 \tag{33}$$

$Profit\_Star\_Cost := ystar\_expd \cdot p - CostStar\_expd$;

$$Profit\_Star\_Cost := 77472.00000 \tag{34}$$