 # AGEC 5403 Problem Set 3 Q1 Complete Solution. Bijesh Mishra
 # Complete Worked out file for Profit Maximization, Cost Minimization and Output Maximization (Expenditure Demand Function) #

restart;
 # Digits ≔ (3); *#Limit upto three digits after decimal.*
 # r1 ≔ 8; r2 ≔ 10; p ≔ 10; b ≔ 0; Co ≔ 936; yo ≔ 385; # labor, capital, output price, fixed cost, investment, output demanded.

<u>#Quadratic Production function (two input one output):</u>
$b1 := 6; b2 := 9; c1 := -0.2; c2 := -0.3; d1 := 0.4; a := 0;$
 # change values here to change the quadratic equation and set up y to "quad" to run optimization using quadratic production function with two input and one output. Same procedure works for Cobb Douglas production function which can be done by simply inserting Cobb douglas production function instead of quadratic function.

$$b1 := 6$$
$$b2 := 9$$
$$c1 := -0.2$$
$$c2 := -0.3$$
$$d1 := 0.4$$
$$a := 0 \tag{1}$$

$quad := b1 \cdot x1 + b2 \cdot x2 + c1 \cdot x1^2 + c2 \cdot x2^2 + d1 \cdot x1 \cdot x2 + a;$
$$quad := -0.2\, x1^2 + 0.4\, x1\, x2 - 0.3\, x2^2 + 6\, x1 + 9\, x2 \tag{2}$$

$y := quad;$
$$y := -0.2\, x1^2 + 0.4\, x1\, x2 - 0.3\, x2^2 + 6\, x1 + 9\, x2 \tag{3}$$

$$\# APP1 := \frac{y}{x1}; APP2 := \frac{y}{x2}; \#Average\ physical\ productivity\ (only\ in\ perfect\ competition)$$

$$\# AVP1 := \frac{y \cdot p}{x1}; AVP2 := \frac{y \cdot p}{x2}; \#Average\ value\ product\ (only\ in\ perfect\ competition)$$

$$f1 := \frac{\partial}{\partial x1}(y); \quad f2 := \frac{\partial}{\partial x2}(y); \ MVP1 := p \cdot f1; MVP2 := p \cdot f2; \ \#MPP1,\ MPP2, MVP1, MVP2$$

$$f1 := -0.4\,x1 + 0.4\,x2 + 6$$

$$f2 := 0.4\,x1 - 0.6\,x2 + 9$$

$$MVP1 := p\,(-0.4\,x1 + 0.4\,x2 + 6)$$

$$MVP2 := p\,(0.4\,x1 - 0.6\,x2 + 9) \tag{4}$$

$$f11 := \frac{\partial}{\partial x1}(f1); \ f22 := \frac{\partial}{\partial x2}(f2); \ f12 := \frac{\partial^2}{\partial x1\, \partial x2}(y); \ f21 := \frac{\partial^2}{\partial x2\, \partial x1}(y);$$

#SOC of f1, #SOC of f2, #f12 & f21 are Factor Inerdependence.

$$f11 := -0.4$$

$$f22 := -0.6$$

$$f12 := 0.4$$

$$f21 := 0.4 \tag{5}$$

$$mrts := simplify\left(\left(\frac{f1}{f2}\right)\right);$$

$$mrts := \frac{-0.4\,x1 + 0.4\,x2 + 6.}{0.4\,x1 - 0.6\,x2 + 9.} \tag{6}$$

$$\frac{r1}{r2} = \frac{f1}{f2} \ \#MRTS = MR$$

$$\frac{r1}{r2} = \frac{-0.4\,x1 + 0.4\,x2 + 6}{0.4\,x1 - 0.6\,x2 + 9} \tag{7}$$

$$SOC := simplify(f2 \cdot f2 \cdot f11 - 2 \cdot f1 \cdot f2 \cdot f12 + f1 \cdot f1 \cdot f22); \#second\ order\ condition.$$

$$SOC := -0.032\,x1^2 + (0.064\,x2 + 0.96)\,x1 - 0.048\,x2^2 + 1.44\,x2 - 97.2 \tag{8}$$

$$Curvature := simplify\left(\left(\frac{1}{f2^3}\right) \cdot SOC\right);$$

$$Curvature := \frac{-0.032\,x1^2 + (0.064\,x2 + 0.96)\,x1 - 0.048\,x2^2 + 1.44\,x2 - 97.2}{(0.4\,x1 - 0.6\,x2 + 9.)^3} \tag{9}$$

HW3_Q1_a: Derivation

 # Profit Maximization:

$profit := p \cdot y - r1 \cdot x1 - r2 \cdot x2 - b;$

$$profit := p\left(-0.2\, x1^2 + 0.4\, x1\, x2 - 0.3\, x2^2 + 6\, x1 + 9\, x2\right) - r1\, x1 - r2\, x2 - b \tag{10}$$

$pf1 := diff(profit, x1);$

$$pf1 := p\,(-0.4\, x1 + 0.4\, x2 + 6) - r1 \tag{11}$$

$pf2 := diff(profit, x2);$

$$pf2 := p\,(0.4\, x1 - 0.6\, x2 + 9) - r2 \tag{12}$$

$EP\_p\_x1 := solve(pf1 = 0, x1);$

$$EP\_p\_x1 := \frac{0.5000000000\,(2.\, p\, x2 + 30.\, p - 5.\, r1)}{p} \tag{13}$$

$EP\_p\_x2 := solve(pf2 = 0, x2);$

$$EP\_p\_x2 := \frac{0.3333333333\,(2.\, p\, x1 + 45.\, p - 5.\, r2)}{p} \tag{14}$$

$profit\_x2 := eval(pf2,\ x1 = EP\_p\_x1);$

$$profit\_x2 := p\left(\frac{0.2000000000\,(2.\, p\, x2 + 30.\, p - 5.\, r1)}{p} - 0.6\, x2 + 9\right) - r2 \tag{15}$$

$x2s\_profit := simplify(solve(profit\_x2 = 0, x2));$ ***#X2Star***

$$x2s\_profit := \frac{75.\, p - 5.\, r1 - 5.\, r2}{p} \tag{16}$$

$x1s\_profit := simplify(eval(EP\_p\_x1, x2 = x2s\_profit));$ ***#X1Star***

$$x1s\_profit := \frac{90.\, p - 7.5\, r1 - 5.\, r2}{p} \tag{17}$$

$MaxprofOut := simplify(eval(y, [x1 = x1s\_profit, x2 = x2s\_profit]));$ ***#Profit Maximizing Output Level***

$$MaxprofOut := \frac{607.5\, p^2 - 3.75\, r1^2 - 5.\, r1\, r2 - 2.5\, r2^2}{p^2} \tag{18}$$

$ProfitStar := simplify(p \cdot (y = MaxprofOut) - r1 \cdot (x1 = x1s\_profit) - r2 \cdot (x2 = x2s\_profit) - b);$
#ProfitStar

$$ProfitStar := \left(-0.2\, x1^2 + (0.4\, x2 + 6.)\, x1 - 0.3\, x2^2 + 9.\, x2\right) p - 1.\, r1\, x1 - 1.\, r2\, x2 - 1.\, b \tag{19}$$
$$= \frac{607.5\, p^2 + (-90.\, r1 - 75.\, r2 - 1.\, b)\, p + 3.75\, r1^2 + 5.\, r1\, r2 + 2.5\, r2^2}{p}$$

$\frac{r1}{f1} = \frac{r2}{f2};$

$$\frac{r1}{-0.4\, x1 + 0.4\, x2 + 6} = \frac{r2}{0.4\, x1 - 0.6\, x2 + 9} \tag{20}$$

$\frac{r1}{pf1} = \frac{r2}{pf2};$

$$\frac{r1}{p\,(-0.4\, x1 + 0.4\, x2 + 6) - r1} = \frac{r2}{p\,(0.4\, x1 - 0.6\, x2 + 9) - r2} \tag{21}$$

#Cost Minimization:

$Cost := r1 \cdot x1 + r2 \cdot x2 + b;$

$$Cost := r1\,x1 + r2\,x2 + b \tag{22}$$

$LC := Cost + \lambda \cdot (yo - y);$ # λ is lagrangean multiplier.

$$LC := r1\,x1 + r2\,x2 + b + \lambda\left(yo + 0.2\,x1^2 - 0.4\,x1\,x2 + 0.3\,x2^2 - 6\,x1 - 9\,x2\right) \tag{23}$$

$LCf1 := diff(LC, x1);$

$$LCf1 := r1 + \lambda\,(0.4\,x1 - 0.4\,x2 - 6) \tag{24}$$

$LCf2 := diff(LC, x2);$

$$LCf2 := r2 + \lambda\,(-0.4\,x1 + 0.6\,x2 - 9) \tag{25}$$

$LCF\lambda := diff(LC, \lambda);$

$$LCF\lambda := yo + 0.2\,x1^2 - 0.4\,x1\,x2 + 0.3\,x2^2 - 6\,x1 - 9\,x2 \tag{26}$$

$LCf1\lambda := solve(LCf1, \lambda);$

$$LCf1\lambda := -\frac{2.500000000\,r1}{x1 - 1.\,x2 - 15.} \tag{27}$$

$LCf2\lambda := solve(LCf2, \lambda);$

$$LCf2\lambda := \frac{5.\,r2}{2.\,x1 - 3.\,x2 + 45.} \tag{28}$$

$EP\_C\_x1 := solve(LCf1\lambda = LCf2\lambda, x1);$

$$EP\_C\_x1 := \frac{0.5000000000\,(3.\,r1\,x2 + 2.\,r2\,x2 - 45.\,r1 + 30.\,r2)}{r1 + r2} \tag{29}$$

$EP\_C\_x2 := solve(LCf1\lambda = LCf2\lambda, x2);$

$$EP\_C\_x2 := \frac{2.\,r1\,x1 + 2.\,r2\,x1 + 45.\,r1 - 30.\,r2}{3.\,r1 + 2.\,r2} \tag{30}$$

$cost\_x2 := simplify(eval(LCF\lambda, x1 = EP\_C\_x1));$

$$cost\_x2 := \frac{1}{(r1 + r2)^2}\left(\left(yo + 0.15\,x2^2 - 22.5\,x2 + 236.25\right) r1^2 + \left(-30.\,x2 + 0.2\,x2^2 + 2.\,yo - 90.\right) r2\,r1 + \left(yo + 0.1\,x2^2 - 15.\,x2 - 45.\right) r2^2\right) \tag{31}$$

$x2s\_cost := solve(cost\_x2\,,\ x2);$ **#X2Star**

$$x2s\_cost := \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\left(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2 + \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2 + 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\right), -\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\left(1.\left(-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2 + \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2 + 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\right)\right) \quad \textbf{(3}$$

$x1s\_cost := eval(EP\_C\_x1, x2 = x2s\_cost);$ **#X1Star**

$$x1s\_cost := \frac{1}{r1 + r2} \left(0.5000000000 \left(3.\, r1 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \left(225.\, r1^2 + 300.\, r1\, r2 \right.\right.\right.\right.$$ (3

$$+ 150.\, r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big)\Big)\Big)$$

$$+ 2.\, r2 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \left(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2 \right.\right.$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big)\Big)\Big) - 45.\, r1 + 30.\, r2 \Big)\Big)$$

CostStar_Cost := *r1*·*x1s_cost* + *r2*·*x2s_cost* + *b*; ***#CostStar***

$$CostStar\_Cost := \frac{1}{r1 + r2} \left(0.5000000000\, r1 \left(3.\, r1 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \left(225.\, r1^2 \right.\right.\right.\right.$$ (3

$$+ 300.\, r1\, r2 + 150.\, r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(1.\, \Big($$

$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$

$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$

$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2} \Big)\Big)\Big)$

$+ 2.\, r2 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$

$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$

$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2} \Big), - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(1.\, \Big($

$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$

$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$

$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2} \Big)\Big)\Big) - 45.\, r1 + 30.\, r2\Big)\Big)$

$+ r2 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$

$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$

$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2} \Big), - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(1.\, \Big($

$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$

$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$

$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2} \Big)\Big)\Big) + b$

```
ystar_cost := eval(y, [x1 = x1s_cost, x2 = x2s_cost]); #Ystar
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$ystar\_cost :=$ **(3**

$- \frac{1}{(r1 + r2)^2} \Bigg(0.05000000000 \Bigg(3.\, r1 \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2$

$+ 300.\, r1\, r2 + 150.\, r2^2$

$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2\right.$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\Big), -\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\Big($

$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\Big)\Big)\Big)$

$+ 2.\,r2\left(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2\right.$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\Big), -\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\Big($

$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$\left.+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\Big)\Big)\Big) - 45.\,r1 + 30.\,r2\right)^2\Big)$

$+ \frac{1}{r1 + r2}\left(0.2000000000\left(3.\,r1\left(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(225.\,r1^2 + 300.\,r1\,r2\right.\right.\right.$

$+ 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\Big), -\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\Big($

$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)$$

$$+ 2.\, r2 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big),\ - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big) - 45.\, r1 + 30.\, r2\Big)$$

$$\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big),\ - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)\Big)$$

$$- 0.3$$

$$\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big),\ - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)^2$$

$$+ \frac{1}{r1 + r2}\Bigg(3.000000000\,\Bigg(3.\, r1\,\Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(225.\, r1^2 + 300.\, r1\, r2$$

$$+ 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)$$

$$+ 2.\, r2\,\Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big) - 45.\, r1 + 30.\, r2\Big)\Big)$$

$$+ \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(9\,\Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big),$$

$$-\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(9.\, \Big(-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+\left(-60.\,r1^4\,yo-200.\,r1^3\,r2\,yo-260.\,r1^2\,r2^2\,yo-160.\,r1\,r2^3\,yo-40.\,r2^4\,yo+36450.\,r1^4+121500.\,r1^3\,r2\right.$$

$$\left.+157950.\,r1^2\,r2^2+97200.\,r1\,r2^3+24300.\,r2^4\right)^{1/2}\Big)\Big)\Big)$$

LCf1λStar := *eval*(*LCf1λ*, [*x1* = *x1s_cost*, *x2* = *x2s_cost*]); ***#Lagrangean multiplier λ1 Star***

$$LCf1\lambda Star := -(2.500000000\,r1) \qquad \textbf{(3}$$

$$\left(\frac{1}{r1+r2}\left(0.5000000000\left(3.\,r1\left(\frac{1}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}\right(225.\,r1^2+300.\,r1\,r2\right.\right.\right.\right.$$

$$+150.\,r2^2$$

$$+\left(-60.\,r1^4\,yo-200.\,r1^3\,r2\,yo-260.\,r1^2\,r2^2\,yo-160.\,r1\,r2^3\,yo-40.\,r2^4\,yo+36450.\,r1^4+121500.\,r1^3\,r2\right.$$

$$\left.+157950.\,r1^2\,r2^2+97200.\,r1\,r2^3+24300.\,r2^4\right)^{1/2}\Big),\ -\frac{1}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}\Big(1.\,\Big($$

$$-225.\,r1^2-300.\,r1\,r2-150.\,r2^2$$

$$+\left(-60.\,r1^4\,yo-200.\,r1^3\,r2\,yo-260.\,r1^2\,r2^2\,yo-160.\,r1\,r2^3\,yo-40.\,r2^4\,yo+36450.\,r1^4+121500.\,r1^3\,r2\right.$$

$$\left.+157950.\,r1^2\,r2^2+97200.\,r1\,r2^3+24300.\,r2^4\right)^{1/2}\Big)\Big)\Big)$$

$$+2.\,r2\left(\frac{1}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}\right(225.\,r1^2+300.\,r1\,r2+150.\,r2^2$$

$$+\left(-60.\,r1^4\,yo-200.\,r1^3\,r2\,yo-260.\,r1^2\,r2^2\,yo-160.\,r1\,r2^3\,yo-40.\,r2^4\,yo+36450.\,r1^4+121500.\,r1^3\,r2\right.$$

$$\left.+157950.\,r1^2\,r2^2+97200.\,r1\,r2^3+24300.\,r2^4\right)^{1/2}\Big),\ -\frac{1}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}\Big(1.\,\Big($$

$$-225.\,r1^2-300.\,r1\,r2-150.\,r2^2$$

$$+\left(-60.\,r1^4\,yo-200.\,r1^3\,r2\,yo-260.\,r1^2\,r2^2\,yo-160.\,r1\,r2^3\,yo-40.\,r2^4\,yo+36450.\,r1^4+121500.\,r1^3\,r2\right.$$

$$\left.+157950.\,r1^2\,r2^2+97200.\,r1\,r2^3+24300.\,r2^4\right)^{1/2}\Big)\Big)\Big)-45.\,r1+30.\,r2\Big)\Big)+\Big($$

$$- \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \Big(1.\,\big(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2$$

$$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$$

$$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\big)\Big), \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \Big(1.\,\big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$$

$$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\big)\Big)\Big) - 15.\Big)$$

LCf2λStar := *eval*(*LCf2λ*, [*x1* = *x1s_cost*, *x2* = *x2s_cost*]); ***#Lagrangean multiplier λ2 Star***

$$LCf2\lambda Star := (5.\,r2) \qquad (3$$

$$\Bigg(\frac{1}{r1 + r2}\Bigg(1.000000000\,\Bigg(3.\,r1\,\Bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\big(225.\,r1^2 + 300.\,r1\,r2$$

$$+ 150.\,r2^2$$

$$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$$

$$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\big), - \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$$

$$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\big)\Big)\Bigg)$$

$$+ 2.\,r2\,\Bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\big(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2$$

$$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$$

$$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}\big), - \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left. + 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big)\Big)\Big) - 45.\,r1 + 30.\,r2\Big)\Big) + \Big($$

$$-\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(3.\,\Big(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left. + 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big)\Big), \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(3.\,\Big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left. + 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big)\Big)\Big) + 45.\Big)$$

$$\frac{r1}{f1} = \frac{r2}{f2};$$

$$\frac{r1}{-0.4\,x1 + 0.4\,x2 + 6} = \frac{r2}{0.4\,x1 - 0.6\,x2 + 9} \tag{38}$$

$$\frac{r1}{LCf1} = \frac{r2}{LCf2};$$

$$\frac{r1}{r1 + \lambda\,(0.4\,x1 - 0.4\,x2 - 6)} = \frac{r2}{r2 + \lambda\,(-0.4\,x1 + 0.6\,x2 - 9)} \tag{39}$$

 # Maximize output subect to budget constraints:

$Ly := y + \mu\cdot(Co - Cost);$

$$Ly := -0.2\,x1^2 + 0.4\,x1\,x2 - 0.3\,x2^2 + 6\,x1 + 9\,x2 + \mu\,(-r1\,x1 - r2\,x2 + Co - b) \quad (40)$$

$Lyf1 := diff(Ly, x1);$

$$Lyf1 := -0.4\,x1 + 0.4\,x2 + 6 - \mu\,r1 \quad (41)$$

$Lyf2 := diff(Ly, x2);$

$$Lyf2 := 0.4\,x1 - 0.6\,x2 + 9 - \mu\,r2 \quad (42)$$

$Ly\mu := diff(Ly, \mu);$

$$Ly\mu := -r1\,x1 - r2\,x2 + Co - b \quad (43)$$

$Lyf1\mu := solve(Lyf1 = 0, \mu);$

$$Lyf1\mu := -\frac{0.4000000000\,(x1 - 1.\,x2 - 15.)}{r1} \quad (44)$$

$Lyf2\mu := solve(Lyf2 = 0, \mu);$

$$Lyf2\mu := \frac{0.2000000000\,(2.\,x1 - 3.\,x2 + 45.)}{r2} \quad (45)$$

$EP\_Ly\_x1 := solve(Lyf1\mu = Lyf2\mu, x1);$

$$EP\_Ly\_x1 := \frac{0.5000000000\,(3.\,r1\,x2 + 2.\,r2\,x2 - 45.\,r1 + 30.\,r2)}{r1 + r2} \quad (46)$$

$EP\_Ly\_x2 := solve(Lyf1\mu = Lyf2\mu, x2);$

$$EP\_Ly\_x2 := \frac{2.\,r1\,x1 + 2.\,r2\,x1 + 45.\,r1 - 30.\,r2}{3.\,r1 + 2.\,r2} \quad (47)$$

$expd\_x2 := simplify(eval(Ly\mu, x1 = EP\_Ly\_x1));$

$$expd\_x2 := \frac{(-1.5\,x2 + 22.5)\,r1^2 + ((-2.\,x2 - 15.)\,r2 + Co - 1.\,b)\,r1 - 1.\,r2^2\,x2 + (Co - 1.\,b)\,r2}{r1 + r2} \quad (48)$$

$x2s\_expd := solve(expd\_x2, x2);$ ***#X2Star***

$$x2s\_expd := \frac{2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \quad (49)$$

$x1s\_expd := eval(EP\_Ly\_x1, x2 = x2s\_expd);$ ***#X1Star***

$$x1s\_expd := \frac{1}{r1 + r2}\left(0.5000000000\left(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\left(3.\,r1\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)\right) + \frac{2.\,r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} - 45.\,r1 + 30.\,r2\right)\right) \quad (50)$$

$ystar\_expd := eval(y, [x1 = x1s\_expd, x2 = x2s\_expd]);$ ***#YStar***

$$ystar\_expd := \quad (51)$$

$$
- \frac{1}{(r1+r2)^2} \Bigg(0.05000000000
$$

$$
\Bigg(\frac{3.\,r1\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}
$$

$$
+ \frac{2.\,r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} - 45.\,r1 + 30.\,r2 \Bigg)
$$

$$
{}^{2} \Bigg)
$$

$$
+ \frac{1}{(r1+r2)\,(3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2)} \Bigg(0.2000000000 \Bigg(1 \Big/ (3.\,r1^2 + 4.\,r1\,r2
$$

$$
+ 2.\,r2^2)\,(3.\,r1\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2))
$$

$$
+ \frac{2.\,r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} - 45.\,r1 + 30.\,r2 \Bigg)
$$

$$
(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2) \Bigg)
$$

$$
- \frac{0.3\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)^2}{(3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2)^2}
$$

$$
+ \frac{1}{r1+r2} \Bigg(3.000000000 \Bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} (3.\,r1\,(2.\,Co\,r1 + 2.\,Co\,r2
$$

$$
- 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2))
$$

$$
+ \frac{2.\,r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} - 45.\,r1 + 30.\,r2 \Bigg)
$$

$$
\Bigg) + \frac{9\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}
$$

CostStar_expd := *eval*(*Cost*, [*x1* = *x1s_expd*, *x2* = *x2s_expd*]); ***#CostStar***

CostStar_expd := **(52)**

$$
\frac{1}{r1+r2} \Bigg(0.5000000000\,r1 \Bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} (3.\,r1\,(2.\,Co\,r1 + 2.\,Co\,r2
$$

$$
- 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2))
$$

$$
+ \frac{2.\,r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} - 45.\,r1 + 30.\,r2 \Bigg)
$$

$$
\Bigg) + \frac{r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} + b
$$

$Ly\mu Star := eval(Ly\mu, [x1 = x1s\_expd, x2 = x2s\_expd]);$ ***#Lagrangean Multiplier Star***

$$Ly\mu Star := -\frac{1}{r1 + r2}\left(0.5000000000\, r1\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\left(3.\, r1\, (2.\, Co\, r1 + 2.\, Co\, r2 - 2.\, b\, r1 - 2.\, b\, r2 + 45.\, r1^2 - 30.\, r1\, r2)\right) + \frac{2.\, r2\,(2.\, Co\, r1 + 2.\, Co\, r2 - 2.\, b\, r1 - 2.\, b\, r2 + 45.\, r1^2 - 30.\, r1\, r2)}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} - 45.\, r1 + 30.\, r2\right)\right) - \frac{r2\,(2.\, Co\, r1 + 2.\, Co\, r2 - 2.\, b\, r1 - 2.\, b\, r2 + 45.\, r1^2 - 30.\, r1\, r2)}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} + Co - b \tag{53}$$

 # HW3_QI_b: Profit Maximizing input and output levels:

$r1 := 8; r2 := 10; p := 10; b := 0; Co := 936; yo := 385;$
 # labor, capital, output price, fixed cost, investment, output demanded.

$$r1 := 8$$
$$r2 := 10$$
$$p := 10$$
$$b := 0$$
$$Co := 936$$
$$yo := 385 \quad \textbf{(54)}$$

$Profit\_Maximizing\_Labor := x1s\_profit;$

$$Profit\_Maximizing\_Labor := 79.00000000 \quad \textbf{(55)}$$

$Profit\_Maximizing\_Capital := x2s\_profit;$

$$Profit\_Maximizing\_Capital := 66.00000000 \quad \textbf{(56)}$$

$Profit\_Maximizing\_Output := MaxprofOut;$

$$Profit\_Maximizing\_Output := 598.6000000 \quad \textbf{(57)}$$

$Max\_profit := ProfitStar;$

$$Max\_profit := -2.0\, x1^2 + 10\,(0.4\, x2 + 6.)\, x1 - 3.0\, x2^2 + 80.\, x2 - 8.\, x1 = 4694.000000 \quad \textbf{(58)}$$

 # HW3_QI_c: Least combination of labor and capital to produce 385 unit of output.

$Lesat\_Combn\_Labor := x1s\_cost;$

$$Lesat\_Combn\_Labor := (146.6666666, 36.66666666) - 1.666666666 \quad \textbf{(59)}$$

$Least\_combn\_capital := x2s\_cost;$

$$Least\_combn\_capital := 120.0000000, 30.00000000 \quad \textbf{(60)}$$

$Least\_Cost := CostStar\_Cost;$

$$Least\_Cost := (2373.333334, 593.3333334) - 13.33333334 \quad \textbf{(61)}$$

$Output\_Least\_Cost := simplify(ystar\_cost);$

$$Output\_Least\_Cost := -0.0001543209877\,(5280.000000, 1320.000000)^2 + (1977.777778, 494.4444444) - 10.55555555 + 0.01111111111\,(120.0000000, 30.00000000)\,(5280.000000, 1320.000000) - 0.3\,(120.0000000, 30.00000000)^2 \quad \textbf{(62)}$$

 # HW3_QI_d: Appriximate estimated increase in production cost due to unit increase in output:
#LCf1λStar and LCf1λStar are equal but opposite in direction.

$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := LCf1\lambda Star;$

$$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := -\frac{20.00000000}{(26.6666666, 6.66666666) - 16.66666667} \quad \textbf{(63)}$$

$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := LCf2\lambda Star;$

$$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := \frac{50.}{(-66.6666667, -16.66666667) + 41.66666667} \quad \textbf{(64)}$$

 # HW3_Q1_e : Optimum production and input levels :
$Labor\_Input\_for\_Optimum\_Prod := x1s\_expd;$

$$Labor\_Input\_for\_Optimum\_Prod := 57.00000000 \quad \textbf{(65)}$$

$Capital\_Input\_for\_Optimum\_Prod := x2s\_expd;$

$$Capital\_Input\_for\_Optimum\_Prod := 48.00000000 \tag{66}$$

$Optimum\_production := ystar\_expd;$

$$Optimum\_production := 527.4000000 \tag{67}$$

$Optimum\_Prod\_Cost := CostStar\_expd;$

$$Optimum\_Prod\_Cost := 936.0000000 \tag{68}$$

 # HW3 _Q1_f : Approximate estimated increase in output per unit cost:

$Estimated\_Output\_increase\_per\_unit\_cost := Ly\mu Star;$

$$Estimated\_Output\_increase\_per\_unit\_cost := 0. \tag{69}$$