 # Q7:

restart;

 # r1 := r1; r2 := r2; p := p; b := 0; Co := Co; yo := yo;

 # Cobb Douglas Production Function:

$Z := 2; m := \frac{1}{8}; n := \frac{1}{4};$

$$Z := 2 \quad \textbf{(1)}$$

$$m := \frac{1}{8} \quad \textbf{(1)}$$

$$n := \frac{1}{4} \quad \textbf{(1)}$$

$cobb := Z \cdot x1^m x2^n;$

$$cobb := 2\, x1^{1/8}\, x2^{1/4} \quad \textbf{(2)}$$

$y := cobb;$ ***#Change production function (quad or cobb).***

$$y := 2\, x1^{1/8}\, x2^{1/4} \quad \textbf{(3)}$$

$VC := r1 \cdot x1 + r2 \cdot x2; FC := b; TotalCost := VC + FC;$

 # Cost Functions: Variable Cost (VC), Fixed Cost (FC), Total Cost (TC)

$$VC := r1\, x1 + r2\, x2$$

$$FC := b$$

$$TotalCost := r1\, x1 + r2\, x2 + b \quad \textbf{(4)}$$

$TVP := p \cdot y; Profit\_Function := TVP - TotalCost;$ *# TVP = Total Value Product, Profit_fun = Profit.*

$$TVP := 2\, p\, x1^{1/8}\, x2^{1/4}$$

$$Profit\_Function := 2\, p\, x1^{1/8}\, x2^{1/4} - r1\, x1 - r2\, x2 - b \quad \textbf{(5)}$$

$Cost := TotalCost;$

$$Cost := r1\, x1 + r2\, x2 + b \quad \textbf{(6)}$$

$LC := Cost + \lambda \cdot (yo - y);$ *# Lagrangean function. λ is lagrangean multiplier.*

$$LC := r1\, x1 + r2\, x2 + b + \lambda \left(yo - 2\, x1^{1/8}\, x2^{1/4}\right) \quad \textbf{(7)}$$

$LCf1 := diff(LC, x1); LCf11 := diff(LCf1, x1);$ *#FOC and SOC of lagrangean function wrt x1*

$$LCf1 := r1 - \frac{\lambda\, x2^{1/4}}{4\, x1^{7/8}}$$

$$LCf11 := \frac{7\, \lambda\, x2^{1/4}}{32\, x1^{15/8}} \quad \textbf{(8)}$$

$LCf2 := diff(LC, x2); LCf22 := diff(LCf2, x2);$ *#FOC and SOC of lagrangean function wrt x2*

$$LCf2 := r2 - \frac{\lambda\, x1^{1/8}}{2\, x2^{3/4}}$$

$$LCf22 := \frac{3\, \lambda\, x1^{1/8}}{8\, x2^{7/4}} \quad \textbf{(9)}$$

$LCf12 := diff(LCf1, x2); LCf21 := diff(LCf2, x1);$

 # Cross differentiation of LCf1 and LCf2 wrt x2 and x1 respectively. Gives interdependence of

factors.

$$LCf12 := -\frac{\lambda}{16\, x1^{7/8}\, x2^{3/4}}$$

$$LCf21 := -\frac{\lambda}{16\, x1^{7/8}\, x2^{3/4}} \qquad \textbf{(10)}$$

$LCF\lambda := diff(LC, \lambda);$ *#FOC of lagrangean function wrt λ.*

$$LCF\lambda := yo - 2\, x1^{1/8}\, x2^{1/4} \qquad \textbf{(11)}$$

$LCf1\lambda := solve(LCf1, \lambda);$ **# λ from LCf1.**

$$LCf1\lambda := \frac{4\, r1\, x1^{7/8}}{x2^{1/4}} \qquad \textbf{(12)}$$

$LCf2\lambda := solve(LCf2, \lambda);$ **# λ from LCf2.**

$$LCf2\lambda := \frac{2\, r2\, x2^{3/4}}{x1^{1/8}} \qquad \textbf{(13)}$$

$EP\_C\_x1 := solve(LCf1\lambda = LCf2\lambda, x1);$ **# Expansion path X1**

$$EP\_C\_x1 := \frac{r2\, x2}{2\, r1} \qquad \textbf{(14)}$$

$EP\_C\_x2 := solve(LCf1\lambda = LCf2\lambda, x2);$ **# Expansion Path X2**

$$EP\_C\_x2 := \frac{2\, r1\, x1}{r2} \qquad \textbf{(15)}$$

 # 7 a.:

$x2s\_cost := (solve((eval(LCF\lambda, x1 = EP\_C\_x1)), x2));$

#X2Star: *Cost Minimizing Input x2 Demand Function, [Constrained Input Deman Function x2.]*

$$= \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}$$

$$x2s\_cost := \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\frac{\sqrt{3}}{2} + \frac{I}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \qquad \textbf{(16)}$$

$$\frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}},$$

$$\frac{yo^4\, 2^{5/6}\, r1^4}{8\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(-\frac{\sqrt{3}}{2} + \frac{I}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}},$$

$$\frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(-\frac{\sqrt{3}}{2} - \frac{I}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}},$$

$$\frac{yo^4\, 2^{5/6}\, r1^4}{8\left(-\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}},$$

$$\frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\frac{1}{2}-\frac{I\sqrt{3}}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}, \frac{yo^4\, 2^{5/6}\, r1^4}{8\left(\frac{\sqrt{3}}{2}-\frac{I}{2}\right)^4 \left(\sqrt{2}\, r2\, yo^4\, r1^{11}\right)^{1/3}}$$

x1s_cost := simplify(eval(EP_C_x1, x2 = x2s_cost));
 # X1Star: *Cost Minimizing Input x1 Demand Function. [Constrained Input Deman Function x1.]*

$$x1s\_cost := \frac{r2\, yo^4\, 2^{2/3}\, r1^3}{16\left(r2\, yo^4\, r1^{11}\right)^{1/3}} \tag{17}$$

 # 7 b:
CostStar := simplify(eval(TotalCost, [x1 = x1s_cost, x2 = x2s_cost])); ***#CostStar:*** *Minimum Cost* **for** *the production of given level of output. Indirect Conditional Cost Function.*

$$CostStar := \frac{3\, 2^{2/3}\, r1^4\, r2\, yo^4 + 16\left(r2\, yo^4\, r1^{11}\right)^{1/3} b}{16\left(r2\, yo^4\, r1^{11}\right)^{1/3}} \tag{18}$$

 # 7 c: Shephard's Lemma:
ConstantOutput_Input_Demand_x1 := simplify(diff(CostStar, r1));
 # This should be equal to x1s_cost. Constant-Output input Demand Function x1.

$$ConstantOutput\_Input\_Demand\_x1 := \frac{r2\, yo^4\, 2^{2/3}\, r1^3}{16\left(r2\, yo^4\, r1^{11}\right)^{1/3}} \tag{19}$$

ConstantOutput_Input_Demand_x2 := simplify(diff(CostStar, r2));
 # This should be equal to x1s_cost. Constant-Output input Demand Function x2.

$$ConstantOutput\_Input\_Demand\_x2 := \frac{r1^4\, yo^4\, 2^{2/3}}{8\left(r2\, yo^4\, r1^{11}\right)^{1/3}} \tag{20}$$