#Input Side Economics: Single input single output production function worked out problem and comparative Statics complete solution in Maple. Author: Bijesh Mishra. #

restart;

Digits := 3;

$$Digits := 3 \tag{1}$$

$x := x$; $c1 := 2512$; $c2 := 180$; $c3 := -1.5$; $c4 := 0$; $p := p$; $r := r$; $b := 0$;

$x := boat$; $c1 := 2512$; $c2 := 180$; $c3 := -1.5$; $c4 := 0$; $p := 2$; $r := 2000$; $b := 0$;

$$x := boat$$

$$c1 := 2512$$

$$c2 := 180$$

$$c3 := -1.5$$

$$c4 := 0$$

$$p := 2$$

$$r := 2000$$

$$b := 0 \tag{2}$$

 # Quadratic Production function :

$quad := c1 \cdot x + c2 \cdot x^2 + c3 \cdot x^3 + c4$;

$$quad := -1.5\ boat^3 + 180\ boat^2 + 2512\ boat \tag{3}$$

 # Cobb Douglas Production Function:

$Z := z$; $m := m$; $n := n$;

 # change values here to change the Cobb Douglas function equation given by eq. 3. and change eq. 6 to "cobb" to run optimization using cobb douglas production function with two input and onw output.

$$Z := z$$

$$m := m$$

$$n := n \tag{4}$$

 # Change production function

$cobb := Z \cdot x^m$; *#Cobb Douglas Production Function.*

$$cobb := z\ boat^m \tag{5}$$

$y := quad$; **#cobb or quad.**

$$y := -1.5\ boat^3 + 180\ boat^2 + 2512\ boat \tag{6}$$

$APP := simplify\left(\frac{y}{x}\right)$; $AVP := p \cdot APP$;

#APP = Average physical product and AVP = Averge Value product.

$$APP := -1.5\ boat^2 + 180.\ boat + 2510.$$

$$AVP := -3.0\ boat^2 + 360.\ boat + 5020. \tag{7}$$

$MPP := diff(y, x)$; $MVP := MPP \cdot p$; $SOC\_y := diff(MPP, x)$; $TOC\_y := diff(SOC\_y, x)$;

#MPP = Mraginal Physical productivity (f1) **and** *MVP = Marginal Value Productivity), Second* **and** *third order conditions of production function.*

$$MPP := -4.5\ boat^2 + 360\ boat + 2512$$

$$MVP := -9.0\ boat^2 + 720\ boat + 5024$$

$$SOC\_y := -9.0\ boat + 360$$

$$TOC\_y := -9.0 \tag{8}$$

$Elasticity := \frac{MPP}{APP};$

$$Elasticity := \frac{-4.5\ boat^2 + 360\ boat + 2512}{-1.5\ boat^2 + 180.\ boat + 2510.} \tag{9}$$

$MFC := MVP;$ *#Marginal Value Productivity (MVP) = Marginal Factor Cost (MFC).*

$$MFC := -9.0\ boat^2 + 720\ boat + 5024 \tag{10}$$

$VC := r \cdot x;\ FC := b;\ Cost := (VC + FC);$ *#VC = Variable cost, FC = Fixed Cost, Cost = Total Cost.*

$$VC := 2000\ boat$$

$$FC := 0$$

$$Cost := 2000\ boat \tag{11}$$

$TVP := p \cdot y;$

$$TVP := -3.0\ boat^3 + 360\ boat^2 + 5024\ boat \tag{12}$$

$profit := TVP - Cost;$

$$profit := -3.0\ boat^3 + 360\ boat^2 + 3024\ boat \tag{13}$$

$FOC\_profit := diff(profit, x);$

$$FOC\_profit := -9.0\ boat^2 + 720\ boat + 3024 \tag{14}$$

$xstar := solve(FOC\_profit = 0, x);$ ***#XStar:***

 # The positive value in the result is the demand needed.

#For unconstrained profit maximization, three conditions are must: 1) FOC = 0, SOC < 0 and TOC < 0. So, Check SOC and TOC to make sure this is demand at maximum profit.

$$xstar := -4., 84. \tag{15}$$

$SOC\_profit := diff(FOC\_profit, x);$ ***#SOC > 0 for maximum.***

$$SOC\_profit := -18.0\ boat + 720 \tag{16}$$

$SOC\_profit\_Check := eval(SOC\_profit, x = xstar[2]);$

 # If this value is negative, xstar could be profit maximizing demand. Check TOC for final confirmation.

$$SOC\_profit\_Check := -790. \tag{17}$$

$TOC\_profit := diff(SOC\_profit, x);$ ***# Third order condition < 0 for maximum.***

$$TOC\_profit := -18.0 \tag{18}$$

$TOC\_profit\_Check := eval(TOC\_profit, x = xstar);$

 # If this is also negative, all three conditions for profit maximization holds which confirms the demand is maximum demand.

$$TOC\_profit\_Check := -18.0 \tag{19}$$

$CostStar := r \cdot xstar[2] + b;$ ***#Total cost to run number of boats that maximize profit.***

$$CostStar := 168000. \tag{20}$$

$ystar := eval(y, x = xstar[2]);$ ***#Total fish caught using number of boats that maximize profits.***

$$ystar := 591000. \tag{21}$$

$ProfitStar := p \cdot ystar - CostStar;$

#Total profit from fishing using number of boat that maximize profit.

$$ProfitStar := 1.01\ 10^6 \tag{22}$$

$ElasticityStar := \frac{eval(MPP, x = xstar[2])}{eval(APP, x = xstar[2])};$

$$ElasticityStar := 0.130 \quad \textbf{(23)}$$

$Stage\_I\_II := solve(Elasticity = 1, x)$; **# *Select positive value.***

$$Stage\_I\_II := 60.0, -0.0111 \quad \textbf{(24)}$$

$Stage\_II\_III := solve(Elasticity\ = 0,\ x)$; **# *Select positive value.***

$$Stage\_II\_III := 86.5, -6.46 \quad \textbf{(25)}$$

#***Economic Region of Production:***

 # If unlimited capital, and single variable input = production will be in second stage of production only.

 # Perfect competition in factor and product market, negative factor price, positive product price, unconstrained capital = Stage III.

 # Perfect competition in product and factor market, non-negative prices, unconstrained capital = Stage II. Stage I, if MVP > r and TVP > Cost.

#Comparative Statics for Factor Demand:

 # Find x where AVP is Maximum to determine max AVP and max r. Take FOC of AVP, equate to 0, and solve for x to get x_not. Replace x_not in AVP to get Max AVP which is Max r.

$AVP\_f1 := simplify(diff(AVP, x))$; *#FOC of AVP.*

$$AVP\_f1 := -6.0\ boat + 360. \quad \textbf{(26)}$$

$Max\_Avp\_x := solve(AVP\_f1 = 0, x)$; *#* x^o **in** *lecture note.*

$$Max\_Avp\_x := 60. \quad \textbf{(27)}$$

$Max\_AVP :=\ eval(AVP, x = Max\_Avp\_x)$; *#Maximum AVP = Max r.*

 # Demand function (XStar) is valid for r <= Max_AVP and 0 Otherwise.

#This value gives what is the maximum input price a firm should pay to be in business. Do not pay more than this amount to make profit from business.

$$Max\_AVP := 15800. \quad \textbf{(28)}$$

 # HW3_Q2_a: #Use positive value.

$MaximumBoat\_2a := (xstar);$

$$MaximumBoat\_2a := -4., 84. \qquad (29)$$

$Totfish\_Caught := eval(y,\ x = MaximumBoat\_2a[2]);$

$$Totfish\_Caught := 591000. \qquad (30)$$

$Profit\_2a := eval(profit,\ [y = ystar,\ x = MaximumBoat\_2a[2]]);$

 # This also confirst TOC i.e. positive profit which could be 0 as well. (TVP > Cost).

$$Profit\_2a := 1.01\ 10^{6} \qquad (31)$$

#HW3_Q2_b: #use positive value.

 # New boat will be added when there is profit. So, number of boat reach to maximum when profit is zero under perfect comptition when everyone maximize their profit. So, the individual profit function is a maximum profit function for individual and total profit is zero. So, equate proft function of individual $(boat)$ **and** *solve* **for** *x1* **or** *b*.

$$Profit\_Individual := simplify\left(\frac{profit}{x}\right);$$

$$Profit\_Individual := -3.0\ boat^2 + 360.\ boat + 3020. \tag{32}$$

$Total\_Boats\_Supported := solve(Profit\_Individual = 0, x\ );$ ***#Positive value is the right answer.***

$$Total\_Boats\_Supported := -7.87, 128. \tag{33}$$

#HW3_Q2_c:

 # Cooperative is formed and share profit equally, then total profit divides among people and the total profit also changes with total number of boats. The profit function for individual boat is equal to the average profit function. #The number of boats also can be calculated by differentiating APP wrt x, equate result to zero and solve for x.

$f\_coop := simplify\left(\frac{y}{x}\right);$ ***#production function of individual boat.***

$$f\_coop := -1.5\, boat^2 + 180.\, boat + 2510. \quad \textbf{(34)}$$

$profit\_coop := simplify\left(\frac{profit}{x}\right);$

$$profit\_coop := -3.0\, boat^2 + 360.\, boat + 3020. \quad \textbf{(35)}$$

$f1\_coop\_avg\_profit := diff(profit\_coop, x);$

$$f1\_coop\_avg\_profit := -6.0\, boat + 360. \quad \textbf{(36)}$$

$boats\_coop := solve(f1\_coop\_avg\_profit = 0, x);$ ***#This is the right answer.***

$$boats\_coop := 60. \quad \textbf{(37)}$$

HW3_Q2_d:
 # The maximum profit is where Elasticity is 1 at 60 unit of x. At this point MPP and APP are equal.
Stage_I_II_Boundary ≔ *Stage_I_II* ; # ***Positive value is the right answer.***

$$Stage\_I\_II\_Boundary := 60.0, -0.0111 \quad (38)$$

Stage_II_III_Boundary ≔ *Stage_II_III* # ***Positive value is the right answer.***

$$Stage\_II\_III\_Boundary := 86.5, -6.46 \quad (39)$$