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Research Paper

A comparison between single exponential smoothing (SES), double exponential smoothing (DES), holt's (brown) and adaptive response rate exponential smoothing (ARRES) techniques in forecasting Malaysia population

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Abstract

This research develops techniques which are helpful in forecasting univariate time series data. The techniques used in this study are Single Exponential Smoothing (SES), Double Exponential Smoothing (DES), Holt's (Brown) and Adaptive Response Rate Exponential Smoothing (ARRES) Techniques. For the purpose of this study, secondary data of Malaysia Population covering the period 1957 up to 2013 was obtained from the Department of Statistics Malaysia. From the result obtained, Holt's method was found to be the best method to forecast the Malaysia population since it produces the lowest Mean Square Error (MSE) value which is 38,273.3 compared to 210,480.29 for SES, 38,827.7 for DEB and 209,835.8 for ARRES techniques.

Keywords: Univariate, Forecasting, Single Exponential Smoothing, Double Exponential Smoothing, And Adaptive Response Rate Exponential *Smoothing, Holt's (Brown)*.

1. Introduction

Census 2010 revealed that the total population of Malaysia was 28.3 million, compared with 23.3 million in 2000. This gives an average annual population growth rate of 2.0 per cent for the period 2000-2010. The rate was lower compared to that of 2.6 per cent during 1991-2000 (DOS). An examination of the demographic transition showed that Malaysia experienced decline in total fertility rate from 6.0 in 1960 to 4.0 in 1980, down 3.0 in 2000 and continued to drop to 2.3 in 2010 [6]. There has been a uniformly high correlation between national income growth and falling birth rates, and between family incomes and fertility [5]. Consequently, population growth rates decreased and will eventually even turn negative. Persistent low fertility rates lead to lower population growth rates and eventually also to decreasing population sizes in most industrialized countries. There are fears that this demographic development is associated with declines in per capita GDP and possibly also increasing inequality of the wage distribution [3]. In order to estimate the Malaysia population growth until 2020, four Exponential Smoothing techniques namely Single Exponential Smoothing (SES), Double Exponential Smoothing (DES), Holt's (Brown) and Adaptive Response Rate Exponential Smoothing (ARRES) techniques will be developed and compared in order to determine which technique is the best to forecast the Malaysia population.

2. Methods

2.1. Single exponential smoothing (SES)

This forecasting method is most widely used of all forecasting techniques. It requires little computation. This method is used when data pattern is approximately horizontal (i.e., there is no neither cyclic variation nor pronounced trend in the historical data) [2]. The general equation for single exponential smoothed statistics is given as:

$$F_{t+m} = \alpha y_t + (1-\alpha)F_t$$

Where:

F_{t+m} Is the single exponential smoothed value in period $t+m$ (this is also defined as forecast value when generated out of sample) for m= 1, 2, 3, 4…

y_t Is the actual value in time period t

α Is the unknown smoothing constant to be determined with value lying between 0 and 1.

F_t Is the forecast or smoothed value for period t

2.2. Double exponential smoothing (DES)

This method is used when the data shows a trend. Exponential smoothing with a trend works much like simple smoothing except that two components must be updated each period - level and trend. The level is a smoothed estimate of the value of the data at the end of each period. The trend is a smoothed estimate of average growth at the end of each period [4]. The specific formula for simple exponential smoothing is:

$$F_{t+m} = \alpha y_t + (1-\alpha)F_t$$

$$a_t = 2S_t - S_t'$$

$$b_t = \frac{\alpha}{1-\alpha}(S_t - S_t')$$

$$S_t = \alpha y_t + (1-\alpha)S_{t-1}$$

$$S_t' = \alpha S_t + (1-\alpha)S'_{t-1}$$

Where:

S_t Be the exponentially smoothed value of y_t at time t

S'_t Be the double exponentially smoothed value of y_t at time t

α_t Computes the difference between the exponentially smoothed values

b_t Computes the adjustment factor

F_{t+m} Is the forecast for m-step-ahead period?

2.3. Holt's (brown) method

This technique not only smoothes the trend and the slope directly by using different smoothing constant but also provides more flexibility in selecting the rates at which trend and slopes are tracked [1]. The application of the Holt's method requires three equations:

$$F_{T+m} = S_T + T_T * m$$

$$T_t = \beta(S_t - S_{t-1}) + (1-\beta)T_{t-1}$$

$$S = \alpha y_t + (1-\alpha)(S_{t-1} + T_{t-1})$$

Where:

S_t Be the exponentially smoothed series

T_t Is the trend estimates

F_{t+m} Is the forecast for m-step-ahead period?

2.4. Adaptive response rate exponential smoothing (ARRES)

The development of ARRES method is an attempt to overcome the fixed constant value of α by incorporating the effect of the changing pattern of the data series into the model [1]. The ARRES technique comprises of the following basic equations:

$$F_{t+1} = \alpha_t y_t + (1-\alpha_t)F_t$$

$$\alpha_t = \left|\frac{E_t}{AE_t}\right|$$

$$E_t = \beta e_t + (1-\beta)E_{t-1}; 0 < \beta < 1$$

$$AE_t = \beta|e_t| + (1-\beta)AE_{t-1}$$

$$e_t = y_t - F_t$$

Where:

E_t	Is the smoothed average error
AE_t	Is the smoothed absolute error
α_t	Is the constant value at time t
F_{t+1}	Is the one-step-ahead forecast value

2.5. Determining the best alpha, (α) value

In this study, the best value of alpha (α) was determined by using the 'solver' in Microsoft Excel. "Best" is taken to mean that by applying a particular alpha, (α) and then a certain measurement criterion is minimized [1]. Common name given to such measurement criterion is 'error measurement'. The error measurement used in this study is the standard criterion or error measure used by most practitioners for assessing the model's fitness to a particular series of data is known as Mean Square Error (MSE). This measure is also commonly used for comparing model's forecasting performance [1]. The MSE is given as:

$$MSE = \frac{\sum_t^n e_t^2}{n}$$

Where:

$\sum_t^n$	Is the summation from period t to n?
e_t^2	Is the squared value of error at current period?
n	Is the number of period?

3. Result and discussion

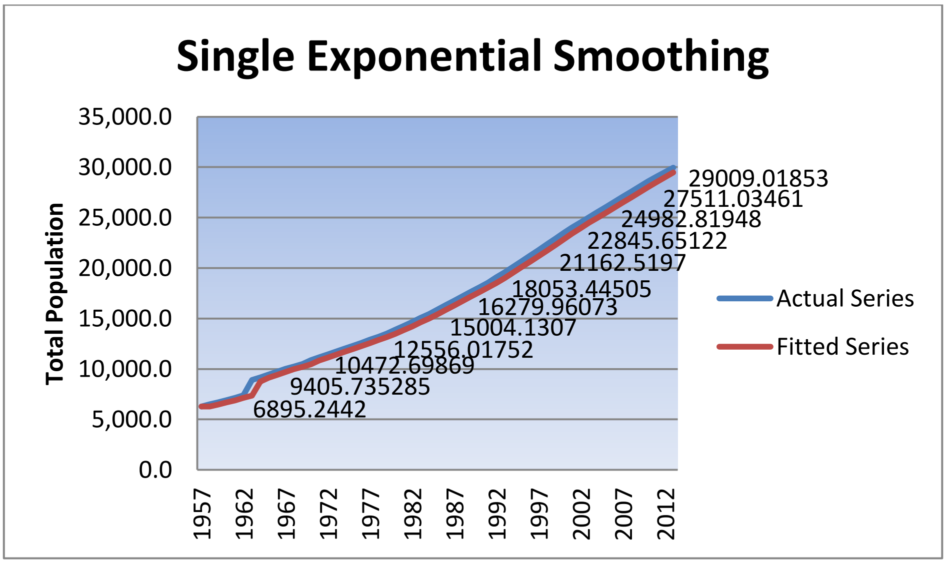

Fig. 1: Shows the Actual and SES Fitted Series of Malaysia Total Population (1957-2013). The Best Alpha Value (α) Estimated by Solver is Found to Be 1.00 and the MSE Value Obtained Is 210,480.29.

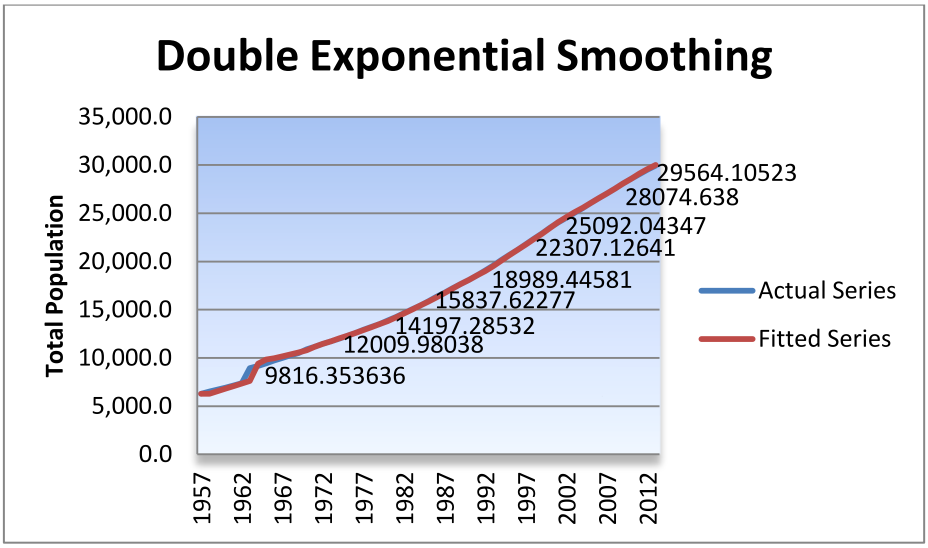

Fig. 2: Shows the Actual and DES Fitted Series of Malaysia Total Population (1957-2013). The Best Alpha Value (α) Estimated By Solver Is Found to Be 0.6 and the MSE Value Obtained Is 38,822.7.

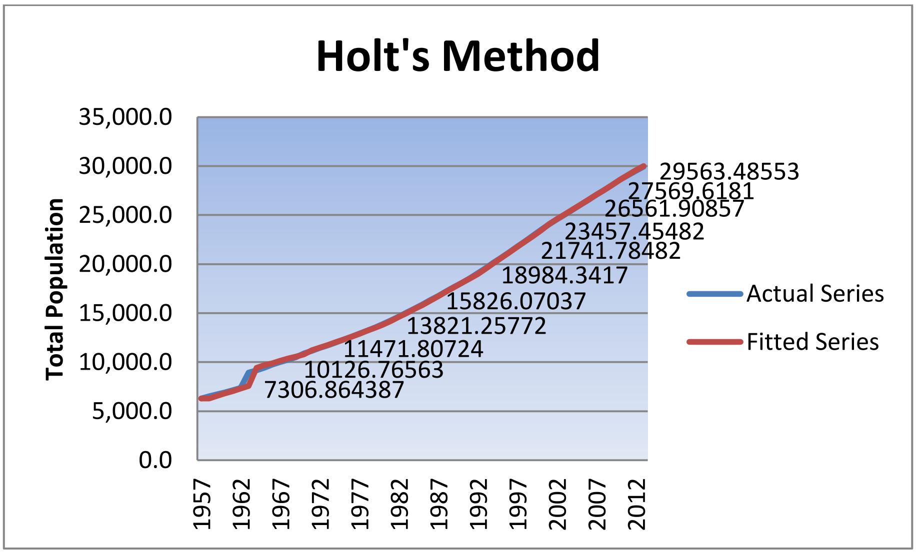

Fig. 3: Shows the Actual and Holt's Method Fitted Series of Malaysia Total Population (1957-2013). The Best Alpha (α) And Beta (β) Value Estimated by Solver are found to be 0.955 and 0.295 respectively and the MSE Value Obtained Is 38,273.3.

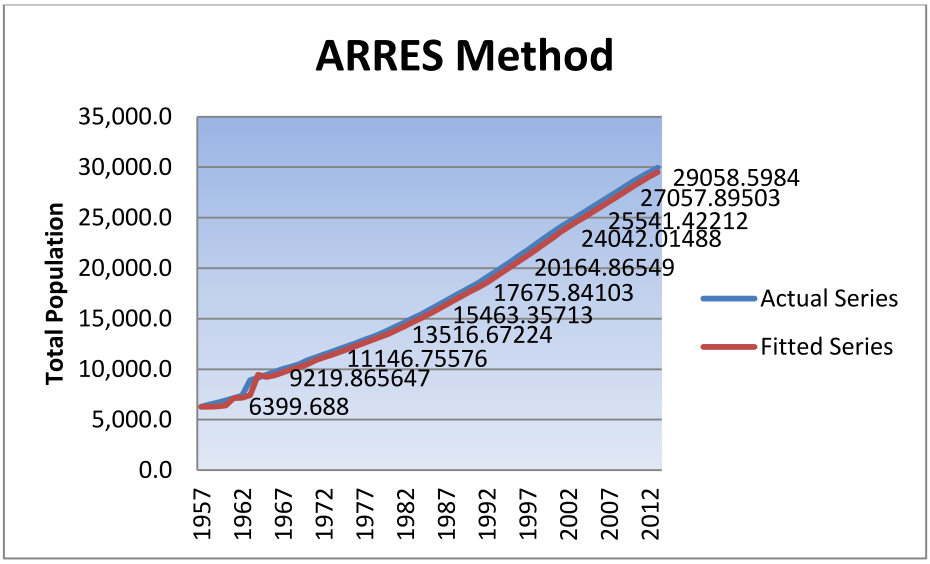

Fig. 4: Shows the Actual and ARRES Method Fitted Series of Malaysia Total Population (1957-2013). The Beta Value Estimated by Solver Is Found to be 0.0 and MSE Value Could Not is calculated. It Is Concluded that ARRES Method Is Not Applicable To This Data.

4. Conclusion

Table 1: Comparison between Methods

Method	Formula	MSE
Single Exponential Smoothing	$F_{t+m} = \alpha y_t + (1-\alpha)F_t$	210,480.29
Double Exponential Smoothing	$F_{T+m} = a_T + b_T * m$	38,822.7
Holt's (Brown)	$F_{T+m} = S_T + T_T * m$	38,273.3
Adaptive Response Rate Exponential Smoothing	$F_{t+1} = \alpha_t y_t + (1-\alpha_t)F_t$	Not Applicable

Table 1 shows the comparison between all methods used in this study. It can be concluded that Holt's (Brown) method is the most appropriate method for the Malaysia's population data since it produced the lowest MSE value which is 38,273.3 ('000). The Holt's (Brown) method is then used to forecast the Malaysia's population by 2020, and it is found that the forecast Malaysia's total population will be 33,242.43 ('000).

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