Homework 1

Sunday, January 18, 2026 3:31 PM

Q1:

a) $P(A=0, B=0, C=0) =$

$$1 - [0.15 + 0.05 + 0.01 + 0.14 + 0.18 + 0.27 + 0.06]$$

$$\boxed{= 0.14}$$

b)

$$P(A=0) = \sum_{c \in C} \sum_{b \in B} P(A=0, B=b, C=c)$$

$$P(A=0) = 0.15 + 0.05 + 0.01 + 0.14 = 0.35$$

$$P(A=1) = \sum_{c \in C} \sum_{b \in B} P(A=1, B=b, C=c)$$

$$P(A=1) = 0.65$$

A	P(A)
0	0.35
1	0.65

c)

$$P(A=0 \mid B=0, C=1) = \frac{P(A=0, B=0, C=1)}{P(B=0, C=1) \rightarrow ?}$$

$$P(B=0,C=1) = \sum_{a \in A} P(A=a, B=0, C=1) = 0.15 + 0.18 = 0.33$$

$$P(A=0 \mid B=0, C=1) = \frac{0.15}{0.33} = 0.455$$

$$P(A=1 \mid B=0, C=1) = \frac{P(A=1, B=0, C=1)}{0.33} = \frac{0.18}{0.33} = 0.545$$

$A \mid B=0, C=1$	$P(A \mid B=0, C=1)$
0	0.455
1	0.545

Q2:

$B \in \{0,1\}$ $M \in \{0,1\}$

$P(B=1) = 0.02$ $P(B=0) = 0.98$

$P(M=1 \mid B=1) = 0.91$

$P(M=1 \mid B=0) = 0.08$

$$P(B=1 \mid M=1) = \frac{P(M=1 \mid B=1)\, P(B=1)}{P(M=1)}$$

$P(M=1, B=1) = 0.91 \cdot 0.02 = 0.0182$

$P(M=1, B=0) = 0.08 \cdot 0.98 = 0.0784$

$P(M=1) = 0.784 + 0.0182 = 0.0966$

$$P(B=1 | M=1) = \frac{0.91 \cdot 0.02}{0.0966} = 0.188 \; \boxed{= 18.8\%}$$

Q3:

a) $A \perp E | C \qquad \mathcal{C} = \{C\}$

If A & E are d-seperated by $\mathcal{C}$, then $A \perp E | \mathcal{C}$

$A \rightarrow C \rightarrow E$: Chain, Not active

$A \rightarrow C \rightarrow B \rightarrow E$: Chain, not active

Since both acyclic paths are d-separated, we can conclude that

$A \perp E | C$ ✓

b) $A \perp D | C$?

$A \rightarrow C \leftarrow D$: Inverted fork, $C \in \mathcal{C}$, active

$A \to C \leftarrow D$: Inverted fork, $C \in \mathcal{C}$, active

Thus we cannot conclude that $A \perp D | C$

C)

$P(E=0|A=0, C=1, B=1) = 0.3$

$P(E=1|A=0, C=1, B=1) = 0.7$

From part a, $A \perp E | C$. Thus,

$P(E|A, C, B) = P(E|C, B)$ by the definition of conditional probability.

Thus,

$P(E=1|A=0, C=1, B=1) = P(E=1|A=1, C=1, B=1)$

$\boxed{= 0.7}$

d)

$P(B=1|C=0, E=0) = ?$

$P(E=0|B=1, C=0) = 0.1$

$P(E=0|B=0, C=0) = 0.8$

$P(B=1|C=0) = 0.6$

$$P(B=1|C=0, E=0) = \frac{P(B=1, C=0, E=0)}{P(C=0, E=0)}$$

$$P(B=1, C=0, E=0) = P(E=0 \mid B=1, C=0) P(B=1, C=0)$$

$$P(C=0, E=0) = P(E=0 \mid C=0) P(C=0)$$

$$P(B=1 \mid C=0, E=0) = \frac{P(E=0 \mid B=1, C=0) P(B=1 \mid C=0)}{P(E=0 \mid C=0)}$$

$$P(E=0 \mid C=0) = \frac{P(E=0, C=0)}{P(C=0)}$$

$$P(E=0, C=0) = \sum_{b \in B} P(B=b, E=0, C=0)$$

$$P(E=0 \mid C=0) = \frac{P(B=0, E=0, C=0) + P(B=1, E=0, C=0)}{P(C=0)}$$

$$P(E=0 \mid B=1, C=0) = \frac{P(B=1, E=0, C=0)}{P(B=1, C=0)}$$

$$P(E=0 \mid B=1, C=0) = \frac{P(B=1, E=0, C=0)}{P(B=1 \mid C=0) P(C=0)}$$

$$P(B=1, E=0, C=0) = P(E=0 \mid B=1, C=0) P(B=1 \mid C=0) P(C=0)$$

$$P(E=0 \mid B=0, C=0) = \frac{P(E=0, B=0, C=0)}{P(B=0, C=0)} = \frac{P(E=0, B=0, C=0)}{P(B=0 \mid C=0)\, P(C=0)}$$

$$P(E=0, B=0, C=0) = P(E=0 \mid B=0, C=0)\, P(B=0 \mid C=0)\, P(C=0)$$

$$P(E=0 \mid C=0) = P(E=0 \mid B=1, C=0)\, P(B=1 \mid C=0) + P(E=0 \mid B=0, C=0)\, P(B=0 \mid C=0)$$

$$P(E=0 \mid C=0) = 0.1 \cdot 0.6 + 0.8 \cdot 0.4 = 0.38$$

$$P(B=1 \mid C=0, E=0) = \frac{P(E=0 \mid B=1, C=0)\, P(B=1 \mid C=0)}{P(E=0 \mid C=0)}$$

$$= \frac{0.1 \cdot 0.6}{0.38} = \boxed{0.1579 = 15.79\%}$$

Q4: Code:

```julia
function f(a, bs)

    res = Vector{Vector{eltype(a)}}()
    max = Vector{eltype(a)}()

    for b in bs
        push!(res,a * b)
    end

    if length(res) == 1
        return res[1]
    end

    res_matrix =  hcat(res...)

    for row in eachrow(res_matrix)
        push!(max,maximum(row))
    end

    return max
end
```