Option Hedging with Binomial Tree and Black-Scholes pricing model

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TO: Zhixin Wu, Professor of Mathematics
FROM: Bijay Ranabhat, Hengyi Zhou, Bingyun Chen
RE: Option Hedging of a Portfolio Based on Historical and Real-Time Data

Introduction

Delta hedging is one of the ways to minimize the risk of a portfolio. The rationale, like other hedging methods, is that one can create a portfolio consist of different securities. These the payoff of these securities are suppose to be negatively correlated so that the winnings can offset the losses, hopefully the total value of the portfolio does not change. In other words, the purpose of such conduct is to make the portfolio riskless. In out case, we assume we are buying stock of MSFT. and shorting its call options. (All the data is generated from Yahoo Finance). The payoff of shorted call options and longed stocks are negatively correlated. Therefore, theoretically, we can buy/short certain numbers of stocks/ options to make the portfolio riskless.

In delta hedging, as the name indicates, consists the parameter delta which is also known as the hedge ratio. Delta is the ratio of the change in the option price to the change in the stock price in a designated period of time.For example, if a call option has a delta of 0.1, that means for every dollar increases in the stock, there is a $0.1 increase in the value of the call option.

According to Random Walk theory, the occurrence of an event that is based on a series of random movements cannot be predict, such as the market movements, or movements of a specific stock. However, we can divide the movements of stock as up and down. Thus, we are able to use binomial trees to display the movements, and then estimate related parameters.

However, Binomial model is considered simple and unrealistic when comes to practice. In reality, if we are to use binomial tree for option pricing, the life of the option is typically divide in to 30 or more times steps. On the other hand, as the number of steps increases , we can get a relatively more precise result. Also, as increases the steps , the prices converge to Black-Scholes-Merton model. However, Black-Scholes-Merton model can be only used to European option pricing.

In our excel, we conducted two hedges. One of them is based on historical data, and the other one is based on real-time market information. The purpose of this project is to compare these two hedges and draw possible conclusions and findings.

Findings

The project highlighted the significance of Delta Hedging to preserve the value of an asset-derivative portfolio. Comparing the option prices calculated from both of the models with the actual prices, it seems that the price predictions of Black-Scholes model is more accurate than the Binomial model. However, Black-Scholes model can only be used to calculate European options unlike Binomial model which can be used to calculate both European and American options. On increasing the number of steps in a Binomial model, the options price predictions eventually converges to that obtained from the Black-Scholes model.

Although, hedging with historical data and hedging with real-time data have similar results for this project, we believe that hedging with real-time data is better as hedging with historical data can not preserve the portfolio on the long run from the ever changing trends of the stock market. This can be seen by the difference is the values of implied and historical volatility.

Discussions

Methods:

As mentioned in the introduction, we used Black-Scholes model and Binomial model to perform Delta hedging. Delta hedging is done by buying (or selling) delta shares of the underlying asset to offset any short (or long) positions in options. The value of delta is calculated as the change in option prices divided by the change in price of the underlying asset.

Black-Scholes model is used to theoretically calculate the option prices. It is based on the assumptions that percentage changes in the stock price in a short period of time are normally distributed. It uses various inputs such as time value, volatility, stock price, strike price, dividend rate, risk-free interest rate. It is based on the returns of the stock and the volatility of the stock price. Based on this model, we calculated the first day and last day values for European options.

Volatility of the stock is the measure of uncertainty about the returns provided by the stock. Volatility can be calculated from historical data of the underlying asset or real-time data from the exchange. To calculate the historical volatility, we observed the stock price for the past 50 days and estimated the daily volatility. As all of the inputs in this project are annualized values, we annualized the daily volatility by multiplying the daily volatility by the square root of 252. As we calculated the volatility, we set the time range for our hedging, with options maturing on June 8. Based on this we calculated the number of trading days till expiry for both the first and the last days. This number was divided by 252 to obtain annualized time value as an input in the Black-Scholes model. All other inputs in the Black-Scholes model were obtained from Yahoo Finance and the U.S. Department of Treasury.

Apart from historical volatility, implied volatility obtained from the market based on the option prices are also used in Black-Scholes model. Based on the option price obtained, we used Excel solver as shown in “Implied Volatility” sheet to iterate through the values of volatility that matches the option price.

To generate a Binomial Tree Pricing Model, there are certain parameters to be obtained, including strike price of the option, volatility, up movement percentage(U) and down movement percentage(D). Up movement percentage refers to by what percentage the stock price will increase. Down movement percentage follows the same rationale. They can be calculated based on volatility. As mentioned, we are using the historical volatility at this point. The U can be calculated as **e^($\sigma\sqrt{\Delta\square}$)** (equation 1), where σstands for the volatility and $\Delta\square$ stands for the designated period. The D can be calculated as **e^($-\square\sqrt{\square\square}$)**, which is interpreted as the decreasing factor. Then, we can construct a binomial tree.

By the method stated above, we are able to calculate deltas, option payoffs and stock prices at every node, and thus calculate the premium of the option. As mentioned, such model is flawed if the number of steps is small. If the number of steps is big enough, the model becomes relatively accurate as the stock prices converge towards a continuous distributions. In addition, Binomial tree pricing can be used to estimate prices of American options as well, while Black-Scholes-Merton model fails to do so. Due to the fact that we are able to calculate the payoffs at every node.

As mentioned, we are hedging based on delta. We started our hedge on 27th of April and ended it on 10th of May. We are able to get our delta from the binomial tree, which is the hedge ratio. The rationale of hedging, again, is by longing stock and shorting options. The hedge ratio is the benchmark of the number of stocks and options. Specifically, we obtained a hedge ratio of 0.492. That leads us to buy 492 share of stock while short 5 call options.

Black-Scholes Pricing Engines							
	First Day			27-Apr	Option Premiums	Calculated	Market
Inputs		Outputs			First Day		
Intial Stock Price (S0)	95.82	d1	0.04116264		Call Option Premium (c)	4.21352941	3
Dividend Rate (q)	1.72%	d2	-0.0659746		Put Option Premium (p)	4.30382195	1.65
Risk Free Rate (r)	2.18%	N(d1)	0.51641688				
Volatility (σ)	31.051%	N(d2)	0.47369903				
Time Period (T) From First Day	0.119047619						
Lowest OTM Strike Price (K)	96						
	Last Day			10-May	Last Day		
Inputs		Outputs			Call Option Premium (c)	3.70175347	1.84
Intial Stock Price (S0)	97.91	d1	0.03884514		Put Option Premium (p)	3.70701034	2.22
Dividend Rate (q)	1.72%	d2	-0.0507923				
Risk Free Rate (r)	2.18%	N(d1)	0.51549307				
Volatility (σ)	31.05%	N(d2)	0.47974552				
Time Period (T) From Last Day	0.083333333						
Lowest OTM Strike Price (K)	98						

Analysis

Based on the historical data, the Black-Scholes model calculated the call option value to be a bit different from the that given by the market. This is understandable as the volatility used by the market is certainly more accurate than what we have. Our volatility coefficient was calculated to be 31.05% which is much higher than the implied volatility of 17.87%.

Comparing the results for option prices from Black-Scholes model and the Binomial model, we do not see much difference. However, the option price for Binomial model at 4.33 was seen to be higher than that of the Black-Scholes model at 4.21.

Also, comparing the market prices for call options from the first day and the last day with the call option prices obtained from the Black-Scholes model, we found stark differences among them. The model calculated prices were far higher(4.21 and 3.70) for first and last day were higher than the market prices(3, 1.84).

The Binomial tree model we used generated some numbers that was going to be used in hedging. We constructed a five-step binomial tree. The U and D are calculated to be approximately 1.0490 and 0.9532 by the equation 1. That is, for every node, the stock price can either goes up by 4.9% or goes down by 4.68%. The volatility is generated from previous Black-Scholes model. The premium is calculated to be around $4.33. Most importantly the delta, hedge ratio, is calculated to be 0.492.

As the premium in the market being \$3.00, a premium of \$4.33 is higher. Therefore, it indicates that the premium generated from historical data might not be accurate in the sense that the premium does not reflect the true value of the option. That is, potential arbitrage opportunity can be attained.

During the course of hedging, we recorded the implied volatility from Yahoo Finance. Also, using the data from the market, we calculated the implied volatility through Solver in Excel. The value of implied volatility at 22.82% was calculated to be higher than the market recorded implied volatility. One of the reasons for this difference could be the the vast amount of information available to the market compared to the limited set of data we had.

Inputs	
Delta for Binomial Model	0.492
Hedging Period	10 Days
Initial Portfolio Value	1,000,000
First Day of Trading	
Stock Price	95.82
Call Option Price	3
Last Day of Trading	
Stock Price	97.91
Call Option Price	1.84

				95.82	4.5228
				0	91.337
	100.52	0.98132			0
	4.5228				110.63
				105.456	14.632
				9.45643	100.52
			100.5228	0.98132	4.5228
			4.522808		100.52
				95.82	4.5228
				0	91.337
		95.82	0.49238		0
		0			100.52
				95.82	4.5228
				0	91.337
			91.33721	0	0
			0		91.337
				87.0641	0
				0	82.991
95.82	0.4924				0
0					110.63
				105.456	14.632

Static Hedging Portfolio						
	Purchase Amount	Intial Price	Value	Final Price	Final Value	Final Portfolio Value
MSFT Call Options	-5	3	-1500	1.84	-920	1001614.55
MSFT Stock	495	95.82	47430.9	97.91	48465.45	
Totals			45930.9		47545.45	

To perform static hedging, we set up a static portfolio based on the deltas(historical and real-time) obtained from the Binomial model. No changes were made to the hedge.With an initial portfolio value of \$1,000,000, the final portfolio values for the historical delta and real time delta came out to be \$1,001,614.5 and 1001587.38. The differences were calculated to be 1614.5 and 1587.38. This shows that real time volatility hedging preserves the portfolio value better than historical volatility hedging.

Limitations:

Even though we conducted the analysis meticulously, we believe that the models used in this project do have some limitations. Lack of enough real-time data can be seen as a limitations while predicting prices through the models.

Black-Scholes model and Binomial models are based on various assumptions which have contradictions in real world scenario. Black-Scholes model assumes that there is no early exercise which is only the case for European options thus making the model unfit for American options. It also assumes that the there are no dividend payouts which consequently affects the valuation.

The Binomial model is also based on assumptions that are not possible in the real world. It assumes that there are only two prices for the asset in the next period which is not the true. It also assumes that there are no dividend payments, thus ignoring the change in valuation of the asset. For Binomial model, the larger the number of steps, the accurate its pricing, eventually converging to the pricing prediction of the Black-Scholes model. However, the model is flexible for a large enough value for the number of steps.

Conclusions

This project is an example of delta hedging with binomial pricing model and Black-Scholes model. It seem to us while Black-Scholes model appears to provides better accuracy, it fails to do one job that binomial model, on ther hand, can do. That is, American option pricing. Due to the fact that every nodes can be taken into account in a binomial tree. In addition, as the steps of binomial tree increases, it will converge to the Black-schole model.

Recognizing the difference between the implied volatility and the historical volatility, we come to a conclusion that volatility generated from historical data is less accurate, compared to volatility from real-time market data.

References

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