Project 1
Due 11th March 2018
MATH336

DATE: March 4th, 2018
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RE: Construction and Comparison of Optimal Portfolios based on historical data.

Background:
As per the instructions, we have analyzed the weekly return from the beginning of 2016 to the end of 2017, of the five securities assigned. The five securities assigned were two large cap stocks(IBM, JPM), one small cap stock(AGX), one corporate bond(LQD) and one treasury bond(TLT). We have calculated and evaluated the minimum risk/maximum return portfolios and compared the data from the past two years to the data from the past 30 years by analyzing the efficient frontier formed by the securities.

Findings:
After we ran the data analysis, we found that there are three minimal risk portfolios with standard deviation 0.72%. The portfolio with the minimum risk is a combination of 5% weight on IBM, 14.94% weight on JPM, 5% weight on AGX, 50% weight on LQD and 25.06%% weight on TLT. Among the three portfolios, this one has the highest expected return but also the minimum risk so we chose it as the minimal risk portfolio. The portfolio with the highest expected return 2.71% is a combination with 5% weight on IBM, 50% weight on JPM, 35% weight on AGX, 5% weight on LQD and 5% weight on TLT. This portfolio has the highest risk of 0.54%.
However, when we do the analysis without constraint. We found that the portfolio with 98.23% weight on JPM and 1.77% weight on AGX has the highest expected return, 0.65%, and the highest risk of 3%. The portfolio with the lowest risk is a combination of 11.94% weight on JPM and 88.06% on LQD. It has a standard deviation of 0.55% and an expected return of 0.18%.

We also analyzed the optimal portfolio with 4 indices: S&P 500 index, Russell 2000 index, U.S. Corporate Investment Grade and U.S. Treasury. The highest expected return portfolio on recent 5 years has 40% weight on S&P 500 index, 50% weight on Russell 2000 index, 5% weight on CIG (U.S. Corporate Investment Grade), and 5% weight on U.S. Treasury. It is expected to have 1.58% of the portfolio in return and has 3.42% standard deviation. The highest expected return portfolio on recent 10 years has 46.45% weight on S&P 500 index, 50% weight on Russell 2000 index, 5% weight on CIG, and 5% weight on U.S. Treasury. It is expected to have 1.66% of the portfolio in return and has 3.31% standard deviation. The highest expected return portfolio on recent 20 years has 5% weight on S&P 500 index, 5% weight on Russell 2000 index, 50% weight on CIG, and 40% weight on U.S. Treasury. It is expected to have 0.64% of the portfolio in return and has 1.19% standard deviation. The highest expected return portfolio on recent 30 years has 40% weight on S&P 500 index, 50% weight on Russell 2000 index, 5% weight on CIG, and 5% weight on U.S. Treasury. It is expected to have 0.98% of the portfolio in return and has 4.03% standard deviation.

Overall, the analysis gave us important investment perceptions. For investors looking for high returns from the market, they should be wary of the risks that come with it since higher returns mean higher risk as per our analysis. Also, investing in stocks more than bonds is a reasonable option for investors who are seeking high returns. For more conservative investors, investing in bonds is a safer bet compared to investing in stocks. However, these conservative investors should accept that low risk investments yield low returns. In addition to that, we found out that for higher return on the long term, investors should invest in small cap stocks as indicated by the weights of Russell 2000 index in the optimal portfolios for the long term data and for a higher return on the short term, investors should invest in large cap stocks as shown by the weights of JPM stock in the 2 year data set.

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Discussion:

1. **Method:**

Our analysis begins by computing the expected return, standard deviation and variance on each of the securities based on the data obtained from finance.yahoo.com for the five securities mentioned in the background section. We use Microsoft Excel to work with the data for this project.

Stocks	Expected Return	Risk
IBM	0.002486032	0.027798038
JPM	0.006475641	0.030294524
AGX	0.005497342	0.052836474
LQD	0.001210218	0.006458328
TLT	0.000911535	0.016230646

Table:1

We calculate the covariance and correlation matrix for each of the security pairs to find out the statistical measurement of the risks for a mix of the assets we are working with and the correlation between expected returns and expected volatility of different investments respectively.

Furthermore, we note the highest and lowest returns and standard deviations(risks) from the securities such that we can use these highest and lowest data values as extremes for return and risk while constructing the efficient frontier for the assets. To obtain the information for constructing the efficient portfolio, we set up a weight matrix for all the assets involved, with the sum of the weights adding up to 100%. We start gathering weights from the point of minimum return among the assets, setting up solver such that the standard deviation is minimum for a particular set of weights. Based on the 30 plus different portfolio sets, we construct an efficient frontier.

Also, as per the instruction, we find the efficient frontier with constraints that the weight for each asset in a given portfolio should be between 5% and 50% inclusive. We do this by integrating the constraints in solver.

In addition to that, based on the given long-term return series for the 4 indices(S&P 500, Russell 2000, U.S. Corporate Investment Grade and, U.S. Treasury) we calculate the portfolios with maximum return such that the weight for each asset in a given portfolio is be between 5% and 50% inclusive for a time period of most recent 5,10, 20 and 30 years.

2. **Main Analysis:**

The expected return and standard deviation for each of the securities are given in Table 1. From that data, we calculated the maximum and minimum values for both of these variables.

Max Return	JPM	0.65%
Min Return	TLT	0.09%
Max Risk	AGX	5.28%
Min Risk	LQD	0.65%

Table: 2

We find out that the JPM stock gave the maximum return of 0.65% and TLT stock gave the minimum return of 0.09% among all the assets. Based on this, our efficient frontier is constructed such that the return for the every portfolio is in between 0.65% and 0.09%. This means that for a given portfolio, the weights of the stocks have to be assigned such that, the risk is minimum for each of the expected return values.
We calculated the expected return and risk for the entire assets to 1.65% and 6.91% respectively.

COVARIANCE	IBM	JPM	AGX	LQD	TLT
IBM	0.000772731	0.000459223	0.000302476	7.72E-06	-0.000102975
JPM	0.000459223	0.000917758	0.0004311	-6.97E-05	-0.000315152
AGX	0.000302476	0.0004311	0.002791693	1.22E-05	-0.000143047
LQD	7.72E-06	-6.97E-05	1.22E-05	4.17E-05	7.78E-05
TLT	-0.000102975	-0.000315152	-0.000143047	7.78E-05	0.000263434

Table: 3

We also calculated the covariance matrix and the correlation matrix for the assets. The covariance matrix is shown in Table 3. In construction of the portfolios, it is important to attempt to reduce overall risk by including assets that have a negative covariance with each other. By including assets that show a negative covariance, the risk of a portfolio is minimized. From this it is inferred that, if a portfolio consists of TLT stock, it should also consist the IBM, JPM and AGX stocks in some capacity to minimize the risk on the portfolio. Also, if the portfolio consists of LQD stock, it should also incorporate JPM stock to minimize the risk based on the covariance matrix.

Correlation	IBM	JPM	AGX	LQD	TLT
IBM	1	0.550606862	0.207940445	0.043411186	-0.230450195
JPM	0.550606862	1	0.27194176	-0.35955668	-0.647166217
AGX	0.207940445	0.27194176	1	0.03608928	-0.168424128
LQD	0.043411186	-0.35955668	0.03608928	1	0.74959217
TLT	-0.230450195	-0.647166217	-0.168424128	0.74959217	1

Table: 4

Looking at the correlation matrix as shown in Table 4, we have to pick the asset combinations for the portfolios which have near zero correlation coefficient values to limit the risk, based on the Modern Portfolio Theory(MPT).

The next step is to find the data points for the efficient frontiers. The first frontier does not have any constraints for weights on the individual assets and is depicted in Figure 1.

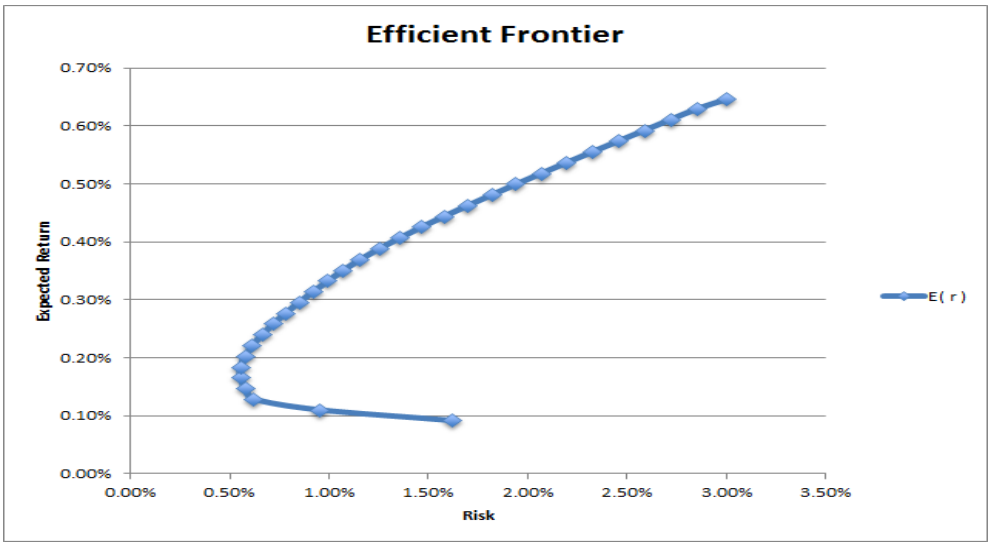

Figure 1: Unconstrained Efficient Frontier

The frontier in Figure 1 bends backwards when the risk is at 0.55%, which denote the minimum variance portfolio. In this case, we have two such portfolios with minimum variance of 0.55% and return 0.17% and 0.18%. It can be inferred that, investors should not consider investing in portfolios below the minimum variance portfolio in the efficient frontier since for all the portfolios below the minimum variance portfolio, there are portfolios above the minimum variance portfolio which carry the same risks but offer higher returns.

Moving on to the second frontier which has constraints such that the minimum and maximum weights on each security/asset should be 5% and 50% respectively. Figure 2 depicts the efficient frontier with these constraints.

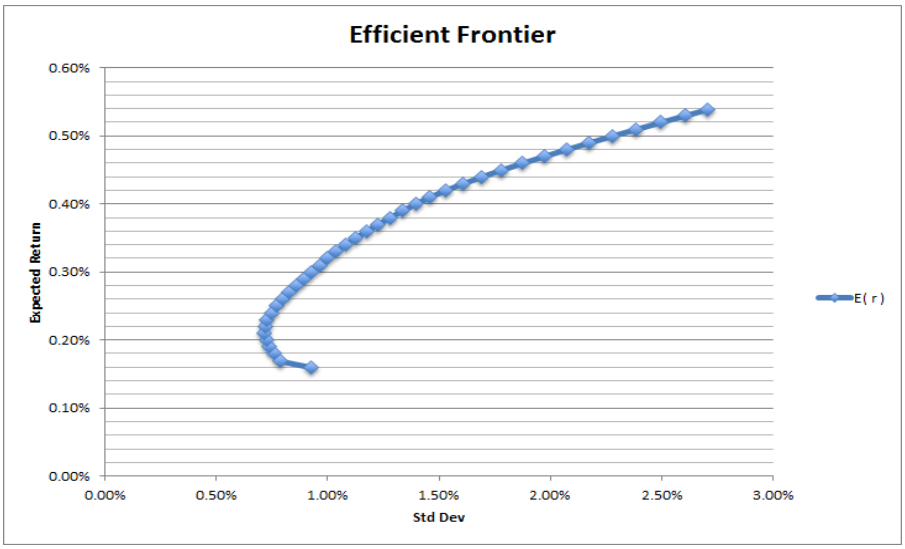

Figure 2: Efficient Frontier with Constraints

The minimum variance portfolios have a risk of 0.72% and returns of 0.20%,0.21% and 0.22%.The frontier bends backward below these minimum variance portfolios for the same reason it bent backwards for Figure 1.

On comparing the minimum variance portfolios for unconstrained and constrained frontiers, we find out that for unconstrained frontier the portfolio consists of only JPM and LQD stocks. For both of the frontiers, as we increase the return, the weights of JPM and AGX grow but that of LQD decreases. Also, the weight of IBM remains minimum for both of the frontiers regardless of the change in returns. We can infer that, a portfolio with majority of the weight on LQD has the least risk but yields a low return.

Looking at the maximum return portfolios for both of the frontiers, we see that JPM and AGX carry higher weights and IBM, LQD and TLT stocks carry the least possible weights. From this it is clear that, portfolios with only JPM and AGX yield higher returns for higher risks. This further infers that, portfolios with a mix of large cap stocks and small cap stocks give higher returns with higher risks.

Similarly, constructing optimal portfolios with maximum return of 4 indices from the most recent 5, 10, 20 and 30 years of data, we see that, with the constraints on weights applied, the portfolios mostly consist of large cap and small cap stocks for data from 5, 10 and 30 years in similar weight ratios.

Thus in general, we can state that riskier portfolios yield higher returns and vice-versa.

3. Limitations:

Even though we conducted the analysis meticulously, we believe that the project does have some limitations. One of them is the limited amount of securities we had to work with. We were given two large cap stocks, one small cap stock, a corporate bond and a treasury bond. However, the market consists of a large variety of stocks from a lot of different industries. Thus our work does not represent the trend in the entire market. Also, the small sample size for the data as we worked with weekly returns over the two year period can be identified as a limitation.

Conclusion:

Our objective for this project was to construct and compare optimal portfolios based on the given data to understand the investment trends of the financial market. We conclude that the project data complies with the high risk, high return theory of economics with stocks deemed as high risk, high return investment and bonds deemed as low risk, low return investments.

References:

1. *Investopedia*, https://www.investopedia.com/terms/e/efficientfrontier.asp. Accessed 5 Mar. 2018.

2. economistatlarge.com/portfolio-theory/efficient-frontier. Accessed 7 Mar. 2018.