AGEC 5403

I. A small agribusiness plant produces items for its parent company in accordance with the following response function: $y = 6x_1 + 9x_2 - 0.2x_1^2 - 0.3x_2^2 + .4x_1x_2$
where y = units of output, x_1= units of labor, and x_2 = units of capital.

a. Input prices are r_1 per unit for labor and r_2 per unit for capital and the price of product is p and all prices are determined competitively. Derive the ordinary input demand functions, constrained input demand functions, and the expenditure demand functions for both inputs. Derive the supply function, profit function, cost function, and conditional production function.
b. Input prices are r_1 = \$8 per unit for labor and r_2 = \$10 per unit for capital. Output price is \$10. What is the profit maximizing output and input levels?
c. Input prices are r_1 = \$8 per unit for labor and r_2 = \$10 per unit for capital. The parent company has requested that the plant produce 385 units of output. Determine the least cost combination of labor and capital to produce the 385 units of output. Note: You do not have to check second order conditions. If your solution is a multi-optimum, it should be obvious which input combination is least cost.
d. If the parent company increases their request from the plant to 386 units of output, what is the approximate estimated increase in production cost if the firm continues to minimize costs? You need not resolve the problem to answer the question.
e. Assume the parent company can provide only 936 dollars for the cost of the inputs for the profit maximizing level, what is the optimum production and input levels?
f. If the parent company can provide 937 dollars for the cost of the inputs, what is the approximate expected increase in output? You need not resolve the problem to answer the question.

II. In ocean fishing (in a region of the ocean) the number of fishes caught is an increasing (first convex and then concave) function of the number of fishing boats. Assume you can represent the production function for the total number of fishes caught by the following function: $f = 2512b + 180b^2 - 1.5b^3$ where f is the number of fishes caught and b is the number of boats. Assume the price of fish is \$2 and the price (variable cost) of operating a boat is \$2,000. Assume there are no fixed costs.

a. How many fishing boats will one individual use to maximize his or her profits if that individual owned all the fishing rights in the region of the ocean and no one else could fish there?
b. How many fishermen and fisherwomen (fishing boats) will fish in the region if each boat is independently owned and everyone maximized his or her individual profit? Hint: Assume each boat catches the same number of fishes, f/b fish.
c. How many fishing boats will be used if the fishermen and fisherwomen decide to form a cooperative and share equally in the profits? Assume each fisherman or fisherwoman owns one boat. Everyone wants to maximize his or her share of the profit, so the cooperative maximizes profit per boat.
d. Explain your results relative to the stages of production.

 # Complete Solution.
 # Complete Worked out File for Profit Maximization, Cost Minimization and Output Maximization #

restart;
 # Digits := (3); *#Limit upto three digits after decimal.*

 # r1 := 8; r2 := 10; p := 10; b := 0; Co := 936; yo := 385; # labor, capital, output price, fixed cost, investment, output demanded.

#Cobb Douglas Production Function:

$Z := Z;\ m := m;\ n := n;$

 # change values here to change the Cobb Douglas function equation given by eq. 3. and change eq. 6 to "cobb" to run optimization using cobb douglas production function with two input and onw output.

$$Z := Z$$
$$m := m$$
$$n := n \tag{1}$$

$cobb := Z\cdot x1^{m} x2^{n}$; *#Cobb Douglas Production Function.*

$$cobb := Z\, x1^{m}\, x2^{n} \tag{2}$$

#Quadratic Production function (two input one output):

$b1 := 6;\ b2 := 9;\ c1 := -0.2;\ c2 := -0.3;\ d1 := 0.4;\ a := 0;$

 # change values here to change the quadratic equation given by eq. 5. and change eq. 6 to "quad" to run optimization using quadratic production function with two input and one output.

$$b1 := 6$$
$$b2 := 9$$
$$c1 := -0.2$$
$$c2 := -0.3$$
$$d1 := 0.4$$
$$a := 0 \tag{3}$$

$quad := b1\cdot x1 + b2\cdot x2 + c1\cdot x1^{2} + c2\cdot x2^{2} + d1\cdot x1\cdot x2 + a\ ;$

$$quad := -0.2\, x1^{2} + 0.4\, x1\, x2 - 0.3\, x2^{2} + 6\, x1 + 9\, x2 \tag{4}$$

$y := quad$;

#Change function to quadratic (quad) or Cobb Douglas (cobb) production function with one output and two inputs. This process works for both types of function.

$$y := -0.2\, x1^2 + 0.4\, x1\, x2 - 0.3\, x2^2 + 6\, x1 + 9\, x2 \tag{5}$$

$\# \, APP1 := \frac{y}{x1}$; $APP2 := \frac{y}{x2}$; *#Average physical productivity (only in perfect competition)*

$\# \, AVP1 := \frac{y \cdot p}{x1}$; $AVP2 := \frac{y \cdot p}{x2}$; *#Average value product (only in perfect competition)*

$f1 := \frac{\partial}{\partial x1}(y)$; $f2 := \frac{\partial}{\partial x2}(y)$; $MVP1 := p \cdot f1$; $MVP2 := p \cdot f2$; *#MPP1, MPP2, MVP1, MVP2*

$$f1 := -0.4\, x1 + 0.4\, x2 + 6$$
$$f2 := 0.4\, x1 - 0.6\, x2 + 9$$
$$MVP1 := p\, (-0.4\, x1 + 0.4\, x2 + 6)$$
$$MVP2 := p\, (0.4\, x1 - 0.6\, x2 + 9) \tag{6}$$

$f11 := \frac{\partial}{\partial x1}(f1)$; $f22 := \frac{\partial}{\partial x2}(f2)$; $f12 := \frac{\partial^2}{\partial x1\, \partial x2}(y)$; $f21 := \frac{\partial^2}{\partial x2\, \partial x1}(y)$;

#SOC of f1, #SOC of f2, #f12 & f21 are Factor Inerdependence.

$$f11 := -0.4$$
$$f22 := -0.6$$
$$f12 := 0.4$$
$$f21 := 0.4 \tag{7}$$

$mrts := simplify\left(\left(\frac{f1}{f2}\right)\right)$;

$$mrts := \frac{-0.4\, x1 + 0.4\, x2 + 6.}{0.4\, x1 - 0.6\, x2 + 9.} \tag{8}$$

$\frac{r1}{r2} = \frac{f1}{f2}$ *#MRTS = MR*

$$\frac{r1}{r2} = \frac{-0.4\, x1 + 0.4\, x2 + 6}{0.4\, x1 - 0.6\, x2 + 9} \tag{9}$$

$SOC := simplify(f2 \cdot f2 \cdot f11 - 2 \cdot f1 \cdot f2 \cdot f12 + f1 \cdot f1 \cdot f22)$; *#second order condition.*

$$SOC := -0.032\, x1^2 + (0.064\, x2 + 0.96)\, x1 - 0.048\, x2^2 + 1.44\, x2 - 97.2 \tag{10}$$

$Curvature := simplify\left(\left(\frac{1}{f2^3}\right) \cdot SOC\right)$;

$$Curvature := \frac{-0.032\, x1^2 + (0.064\, x2 + 0.96)\, x1 - 0.048\, x2^2 + 1.44\, x2 - 97.2}{(0.4\, x1 - 0.6\, x2 + 9.)^3} \tag{11}$$

 # Profit Maximization:

$profit := p \cdot y - r1 \cdot x1 - r2 \cdot x2 - b;$

$$profit := p\left(-0.2\,x1^2 + 0.4\,x1\,x2 - 0.3\,x2^2 + 6\,x1 + 9\,x2\right) - r1\,x1 - r2\,x2 - b \quad \textbf{(12)}$$

$pf1 := diff(profit, x1);$

$$pf1 := p\,(-0.4\,x1 + 0.4\,x2 + 6) - r1 \quad \textbf{(13)}$$

$pf2 := diff(profit, x2);$

$$pf2 := p\,(0.4\,x1 - 0.6\,x2 + 9) - r2 \quad \textbf{(14)}$$

$EP\_p\_x1 := solve(pf1 = 0, x1);$

$$EP\_p\_x1 := \frac{0.5000000000\,(2.\,p\,x2 + 30.\,p - 5.\,r1)}{p} \quad \textbf{(15)}$$

$EP\_p\_x2 := solve(pf2 = 0, x2);$

$$EP\_p\_x2 := \frac{0.3333333333\,(2.\,p\,x1 + 45.\,p - 5.\,r2)}{p} \quad \textbf{(16)}$$

$profit\_x2 := eval(pf2,\ x1 = EP\_p\_x1);$

$$profit\_x2 := p\left(\frac{0.2000000000\,(2.\,p\,x2 + 30.\,p - 5.\,r1)}{p} - 0.6\,x2 + 9\right) - r2 \quad \textbf{(17)}$$

$x2s\_profit := simplify(solve(profit\_x2 = 0, x2));$ ***#X2Star***

$$x2s\_profit := \frac{75.\,p - 5.\,r1 - 5.\,r2}{p} \quad \textbf{(18)}$$

$x1s\_profit := simplify(eval(EP\_p\_x1, x2 = x2s\_profit));$ ***#X1Star***

$$x1s\_profit := \frac{90.\,p - 7.5\,r1 - 5.\,r2}{p} \quad \textbf{(19)}$$

$SupplyStar := simplify(eval(y, [x1 = x1s\_profit, x2 = x2s\_profit]));$ ***#SupplyStar***

$$SupplyStar := \frac{607.5\,p^2 - 3.75\,r1^2 - 5.\,r1\,r2 - 2.5\,r2^2}{p^2} \quad \textbf{(20)}$$

$ProfitStar := simplify(p \cdot (y = SupplyStar) - r1 \cdot (x1 = x1s\_profit) - r2 \cdot (x2 = x2s\_profit) - b);$
#ProfitStar

$$ProfitStar := \left(-0.2\,x1^2 + (0.4\,x2 + 6.)\,x1 - 0.3\,x2^2 + 9.\,x2\right)p - 1.\,r1\,x1 - 1.\,r2\,x2 - 1.\,b = \frac{607.5\,p^2 + (-90.\,r1 - 75.\,r2 - 1.\,b)\,p + 3.75\,r1^2 + 5.\,r1\,r2 + 2.5\,r2^2}{p} \quad \textbf{(21)}$$

$\frac{r1}{f1} = \frac{r2}{f2};$

$$\frac{r1}{-0.4\,x1 + 0.4\,x2 + 6} = \frac{r2}{0.4\,x1 - 0.6\,x2 + 9} \quad \textbf{(22)}$$

$\frac{r1}{pf1} = \frac{r2}{pf2};$

$$\frac{r1}{p\,(-0.4\,x1 + 0.4\,x2 + 6) - r1} = \frac{r2}{p\,(0.4\,x1 - 0.6\,x2 + 9) - r2} \quad \textbf{(23)}$$

#Cost Minimization:

$Cost := r1 \cdot x1 + r2 \cdot x2 + b;$

$$Cost := r1\, x1 + r2\, x2 + b \quad \textbf{(24)}$$

$LC := Cost + \lambda \cdot (yo - y);$ *# λ is lagrangean multiplier.*

$$LC := r1\, x1 + r2\, x2 + b + \lambda \left(yo + 0.2\, x1^2 - 0.4\, x1\, x2 + 0.3\, x2^2 - 6\, x1 - 9\, x2\right) \quad \textbf{(25)}$$

$LCf1 := diff(LC, x1);$

$$LCf1 := r1 + \lambda\, (0.4\, x1 - 0.4\, x2 - 6) \quad \textbf{(26)}$$

$LCf2 := diff(LC, x2);$

$$LCf2 := r2 + \lambda\, (-0.4\, x1 + 0.6\, x2 - 9) \quad \textbf{(27)}$$

$LCF\lambda := diff(LC, \lambda);$

$$LCF\lambda := yo + 0.2\, x1^2 - 0.4\, x1\, x2 + 0.3\, x2^2 - 6\, x1 - 9\, x2 \quad \textbf{(28)}$$

$LCf1\lambda := solve(LCf1, \lambda);$

$$LCf1\lambda := -\frac{2.500000000\, r1}{x1 - 1.\, x2 - 15.} \quad \textbf{(29)}$$

$LCf2\lambda := solve(LCf2, \lambda);$

$$LCf2\lambda := \frac{5.\, r2}{2.\, x1 - 3.\, x2 + 45.} \quad \textbf{(30)}$$

$EP\_C\_x1 := solve(LCf1\lambda = LCf2\lambda, x1);$

$$EP\_C\_x1 := \frac{0.5000000000\, (3.\, r1\, x2 + 2.\, r2\, x2 - 45.\, r1 + 30.\, r2)}{r1 + r2} \quad \textbf{(31)}$$

$EP\_C\_x2 := solve(LCf1\lambda = LCf2\lambda, x2);$

$$EP\_C\_x2 := \frac{2.\, r1\, x1 + 2.\, r2\, x1 + 45.\, r1 - 30.\, r2}{3.\, r1 + 2.\, r2} \quad \textbf{(32)}$$

$cost\_x2 := simplify(eval(LCF\lambda, x1 = EP\_C\_x1));$

$$cost\_x2 := \frac{1}{(r1 + r2)^2} \left(\left(yo + 0.15\, x2^2 - 22.5\, x2 + 236.25\right) r1^2 + \left(-30.\, x2 + 2.\, yo + 0.2\, x2^2 - 90.\right) r2\, r1 + \left(yo + 0.1\, x2^2 - 15.\, x2 - 45.\right) r2^2 \right) \quad \textbf{(33)}$$

$x2s\_cost := solve(cost\_x2, x2);$ ***#X2Star***

$$x2s\_cost := \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2 + \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2 + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2}\Big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(1.\, \big(-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2 + \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2 + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\right)^{1/2}\big)\Big) \quad \textbf{(3}$$

$x1s\_cost := eval(EP\_C\_x1, x2 = x2s\_cost);$ ***#X1Star***

$$x1s\_cost := \frac{1}{r1 + r2} \Bigg(0.5000000000 \Bigg(3.\, r1 \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \big(225.\, r1^2 + 300.\, r1\, r2$$ (3

$$+ 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2} \big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \big(1.\, ($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}) \big) \Bigg)$$

$$+ 2.\, r2 \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2} \big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \big(1.\, ($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}) \big) \Bigg) - 45.\, r1 + 30.\, r2 \Bigg) \Bigg)$$

$CostStar\_Cost := r1 \cdot x1s\_cost + r2 \cdot x2s\_cost + b;$ **#CostStar**

$$CostStar\_Cost := \frac{1}{r1 + r2} \Bigg(0.5000000000\, r1 \Bigg(3.\, r1 \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \big(225.\, r1^2$$ (3

$$+ 300.\, r1\, r2 + 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2} \big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \big(1.\, ($$

$-225.\ r1^2 - 300.\ r1\ r2 - 150.\ r2^2$

$+ \left(-60.\ r1^4\ yo - 200.\ r1^3\ r2\ yo - 260.\ r1^2\ r2^2\ yo - 160.\ r1\ r2^3\ yo - 40.\ r2^4\ yo + 36450.\ r1^4 + 121500.\ r1^3\ r2\right.$

$\left.+ 157950.\ r1^2\ r2^2 + 97200.\ r1\ r2^3 + 24300.\ r2^4\right)^{1/2} \Big)\Big)\Big)$

$+ 2.\ r2 \left(\frac{1}{3.\ r1^2 + 4.\ r1\ r2 + 2.\ r2^2} \Big(225.\ r1^2 + 300.\ r1\ r2 + 150.\ r2^2 \right.$

$+ \left(-60.\ r1^4\ yo - 200.\ r1^3\ r2\ yo - 260.\ r1^2\ r2^2\ yo - 160.\ r1\ r2^3\ yo - 40.\ r2^4\ yo + 36450.\ r1^4 + 121500.\ r1^3\ r2\right.$

$\left.+ 157950.\ r1^2\ r2^2 + 97200.\ r1\ r2^3 + 24300.\ r2^4\right)^{1/2} \Big), - \frac{1}{3.\ r1^2 + 4.\ r1\ r2 + 2.\ r2^2} \Big(1. \Big($

$-225.\ r1^2 - 300.\ r1\ r2 - 150.\ r2^2$

$+ \left(-60.\ r1^4\ yo - 200.\ r1^3\ r2\ yo - 260.\ r1^2\ r2^2\ yo - 160.\ r1\ r2^3\ yo - 40.\ r2^4\ yo + 36450.\ r1^4 + 121500.\ r1^3\ r2\right.$

$\left.+ 157950.\ r1^2\ r2^2 + 97200.\ r1\ r2^3 + 24300.\ r2^4\right)^{1/2} \Big)\Big)\Big) - 45.\ r1 + 30.\ r2 \Big)\Big)$

$+ r2 \left(\frac{1}{3.\ r1^2 + 4.\ r1\ r2 + 2.\ r2^2} \Big(225.\ r1^2 + 300.\ r1\ r2 + 150.\ r2^2 \right.$

$+ \left(-60.\ r1^4\ yo - 200.\ r1^3\ r2\ yo - 260.\ r1^2\ r2^2\ yo - 160.\ r1\ r2^3\ yo - 40.\ r2^4\ yo + 36450.\ r1^4 + 121500.\ r1^3\ r2\right.$

$\left.+ 157950.\ r1^2\ r2^2 + 97200.\ r1\ r2^3 + 24300.\ r2^4\right)^{1/2} \Big), - \frac{1}{3.\ r1^2 + 4.\ r1\ r2 + 2.\ r2^2} \Big(1. \Big($

$-225.\ r1^2 - 300.\ r1\ r2 - 150.\ r2^2$

$+ \left(-60.\ r1^4\ yo - 200.\ r1^3\ r2\ yo - 260.\ r1^2\ r2^2\ yo - 160.\ r1\ r2^3\ yo - 40.\ r2^4\ yo + 36450.\ r1^4 + 121500.\ r1^3\ r2\right.$

$\left.+ 157950.\ r1^2\ r2^2 + 97200.\ r1\ r2^3 + 24300.\ r2^4\right)^{1/2} \Big)\Big)\Big) + b$

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ystar_cost := eval(y, [x1 = x1s_cost, x2 = x2s_cost]); #Ystar
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ystar_cost := **(3**

$- \frac{1}{(r1 + r2)^2} \Bigg(0.05000000000 \Bigg(3.\ r1 \left(\frac{1}{3.\ r1^2 + 4.\ r1\ r2 + 2.\ r2^2} \Big(225.\ r1^2 \right.$

$+ 300.\ r1\ r2 + 150.\ r2^2$

$+ \left(-60.\ r1^4\ yo - 200.\ r1^3\ r2\ yo - 260.\ r1^2\ r2^2\ yo - 160.\ r1\ r2^3\ yo - 40.\ r2^4\ yo + 36450.\ r1^4 + 121500.\ r1^3\ r2\right.$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\big)^{1/2}\Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ \big(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\big)^{1/2}\big)\Big)\Big)$$

$$+ 2.\, r2\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ \big(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\big)^{1/2}\Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ \big(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$\left.+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\big)^{1/2}\big)\Big)\Big) - 45.\, r1 + 30.\, r2\right)^2\Big)$$

$$+ \frac{1}{r1 + r2}\left(0.2000000000\left(3.\, r1\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(225.\, r1^2 + 300.\, r1\, r2\right.\right.\right.$$

$$+ 150.\, r2^2$$

$$+ \big(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4\big)^{1/2}\Big),\ -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ \big(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)$$

$$+ 2.\, r2 \left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$\left.+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big) - 45.\, r1 + 30.\, r2\right)$$

$$\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)\Big)$$

$$- 0.3$$

$$\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2\right.$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big), -\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)^2$$

$$+ \frac{1}{r1 + r2} \Bigg(3.000000000 \Bigg(3.\, r1 \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(225.\, r1^2 + 300.\, r1\, r2$$

$$+ 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big), - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big)$$

$$+ 2.\, r2 \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big), - \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(1.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big)\Big) - 45.\, r1 + 30.\, r2\Big)\Big)$$

$$+ \Bigg(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(9\, \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2$$

$$+ (-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2$$

$$+ 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4)^{1/2}\Big)\Big),$$

$$- \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\Big(9.\, \Big(-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2})))$

LCf1λStar = eval(LCf1λ, [x1 = x1s_cost, x2 = x2s_cost]); ***#Lagrangean multiplier λ1 Star***

$LCf1\lambda Star = -(2.500000000\,r1)$ **(3**

$\left(\frac{1}{r1 + r2} \left(0.5000000000 \left(3.\,r1 \left(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \left(225.\,r1^2 + 300.\,r1\,r2\right.\right.\right.\right.\right.$

$+ 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}),\ -\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} (1.\,($

$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2})))$

$+ 2.\,r2 \left(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \left(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2\right.\right.$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}),\ -\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} (1.\,($

$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$

$+ (-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2$

$+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4)^{1/2}))) - 45.\,r1 + 30.\,r2)) + ($

$$- \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \Big(1.\,\Big(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left.+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big)\Big),\ \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\Big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left.+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big)\Big)\Big) - 15.\Big)$$

$LCf2\lambda Star = eval(LCf2\lambda, [x1 = x1s\_cost, x2 = x2s\_cost]);$ ***#Lagrangean multiplier λ2 Star***

$$LCf2\lambda Star = (5.\,r2) \Big/ \Bigg(\frac{1}{r1 + r2} \Bigg(1.000000000 \Bigg(3.\,r1 \Bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(225.\,r1^2 \tag{3}$$

$$+ 300.\,r1\,r2 + 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left.+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big),\ - \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\Big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left.+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big)\Big)\Bigg)$$

$$+ 2.\,r2 \Bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(225.\,r1^2 + 300.\,r1\,r2 + 150.\,r2^2$$

$$+ \left(-60.\,r1^4\,yo - 200.\,r1^3\,r2\,yo - 260.\,r1^2\,r2^2\,yo - 160.\,r1\,r2^3\,yo - 40.\,r2^4\,yo + 36450.\,r1^4 + 121500.\,r1^3\,r2\right.$$

$$\left.+ 157950.\,r1^2\,r2^2 + 97200.\,r1\,r2^3 + 24300.\,r2^4\right)^{1/2}\Big),\ - \frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\Big(1.\,\Big($$

$$-225.\,r1^2 - 300.\,r1\,r2 - 150.\,r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2 \right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big) \Big) \Big) - 45.\, r1 + 30.\, r2 \Big) \Big) + \Bigg($$

$$-\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(3.\, \Big(225.\, r1^2 + 300.\, r1\, r2 + 150.\, r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2 \right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big) \Big), \frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} \Big(3.\, \Big($$

$$-225.\, r1^2 - 300.\, r1\, r2 - 150.\, r2^2$$

$$+ \left(-60.\, r1^4\, yo - 200.\, r1^3\, r2\, yo - 260.\, r1^2\, r2^2\, yo - 160.\, r1\, r2^3\, yo - 40.\, r2^4\, yo + 36450.\, r1^4 + 121500.\, r1^3\, r2 \right.$$

$$\left. + 157950.\, r1^2\, r2^2 + 97200.\, r1\, r2^3 + 24300.\, r2^4 \right)^{1/2} \Big) \Big) \Big) + 45. \Big)$$

$$\frac{r1}{f1} = \frac{r2}{f2};$$

$$\frac{r1}{-0.4\, x1 + 0.4\, x2 + 6} = \frac{r2}{0.4\, x1 - 0.6\, x2 + 9} \tag{40}$$

$$\frac{r1}{LCf1} = \frac{r2}{LCf2};$$

$$\frac{r1}{r1 + \lambda\, (0.4\, x1 - 0.4\, x2 - 6)} = \frac{r2}{r2 + \lambda\, (-0.4\, x1 + 0.6\, x2 - 9)} \tag{41}$$

 # Expenditure Minimization:

$Ly := y + \mu\cdot(Co - Cost);$

$$Ly := -0.2\,x1^2 + 0.4\,x1\,x2 - 0.3\,x2^2 + 6\,x1 + 9\,x2 + \mu\,(-r1\,x1 - r2\,x2 + Co - b) \quad \textbf{(42)}$$

$Lyf1 := diff(Ly, x1);$

$$Lyf1 := -0.4\,x1 + 0.4\,x2 + 6 - \mu\,r1 \quad \textbf{(43)}$$

$Lyf2 := diff(Ly, x2);$

$$Lyf2 := 0.4\,x1 - 0.6\,x2 + 9 - \mu\,r2 \quad \textbf{(44)}$$

$Ly\mu := diff(Ly, \mu);$

$$Ly\mu := -r1\,x1 - r2\,x2 + Co - b \quad \textbf{(45)}$$

$Lyf1\mu := solve(Lyf1 = 0, \mu);$

$$Lyf1\mu := -\frac{0.4000000000\,(x1 - 1.\,x2 - 15.)}{r1} \quad \textbf{(46)}$$

$Lyf2\mu := solve(Lyf2 = 0, \mu);$

$$Lyf2\mu := \frac{0.2000000000\,(2.\,x1 - 3.\,x2 + 45.)}{r2} \quad \textbf{(47)}$$

$EP\_Ly\_x1 := solve(Lyf1\mu = Lyf2\mu, x1);$

$$EP\_Ly\_x1 := \frac{0.5000000000\,(3.\,r1\,x2 + 2.\,r2\,x2 - 45.\,r1 + 30.\,r2)}{r1 + r2} \quad \textbf{(48)}$$

$EP\_Ly\_x2 := solve(Lyf1\mu = Lyf2\mu, x2);$

$$EP\_Ly\_x2 := \frac{2.\,r1\,x1 + 2.\,r2\,x1 + 45.\,r1 - 30.\,r2}{3.\,r1 + 2.\,r2} \quad \textbf{(49)}$$

$expd\_x2 := simplify(eval(Ly\mu, x1 = EP\_Ly\_x1));$

$$expd\_x2 := \frac{(-1.5\,x2 + 22.5)\,r1^2 + ((-2.\,x2 - 15.)\,r2 + Co - 1.\,b)\,r1 - 1.\,r2^2\,x2 + (Co - 1.\,b)\,r2}{r1 + r2} \quad \textbf{(50)}$$

$x2s\_expd := solve(expd\_x2, x2);$ **#X2Star**

$$x2s\_expd := \frac{2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} \quad \textbf{(51)}$$

$x1s\_expd := eval(EP\_Ly\_x1, x2 = x2s\_expd);$ **#X1Star**

$$x1s\_expd := \frac{1}{r1 + r2}\Bigg(0.5000000000\bigg(\frac{1}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2}\big(3.\,r1\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)\big) + \frac{2.\,r2\,(2.\,Co\,r1 + 2.\,Co\,r2 - 2.\,b\,r1 - 2.\,b\,r2 + 45.\,r1^2 - 30.\,r1\,r2)}{3.\,r1^2 + 4.\,r1\,r2 + 2.\,r2^2} - 45.\,r1 + 30.\,r2\bigg)\Bigg) \quad \textbf{(52)}$$

$ystar\_expd := eval(y, [x1 = x1s\_expd, x2 = x2s\_expd]);$ **#YStar**

$$ystar\_expd := \quad \textbf{(53)}$$

$$
-\frac{1}{(r1+r2)^2}\Bigg(0.05000000000
$$

$$
\Bigg(\frac{3.\,r1\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}
$$

$$
+\frac{2.\,r2\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}-45.\,r1+30.\,r2\Bigg)
$$

$$
{}^2\Bigg)
$$

$$
+\frac{1}{(r1+r2)\,(3.\,r1^2+4.\,r1\,r2+2.\,r2^2)}\Bigg(0.2000000000\,\Bigg(1\Big/(3.\,r1^2+4.\,r1\,r2
$$

$$
+2.\,r2^2)\,(3.\,r1\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2))
$$

$$
+\frac{2.\,r2\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}-45.\,r1+30.\,r2\Bigg)
$$

$$
(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)\Bigg)
$$

$$
-\frac{0.3\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)^2}{(3.\,r1^2+4.\,r1\,r2+2.\,r2^2)^2}
$$

$$
+\frac{1}{r1+r2}\Bigg(3.000000000\,\Bigg(\frac{1}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}\,(3.\,r1\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2
$$

$$
-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2))
$$

$$
+\frac{2.\,r2\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}-45.\,r1+30.\,r2\Bigg)
$$

$$
\Bigg)+\frac{9\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}
$$

CostStar_expd := *eval*(*Cost*, [*x1* = *x1s_expd*, *x2* = *x2s_expd*]); **#CostStar**

CostStar_expd := **(54)**

$$
\frac{1}{r1+r2}\Bigg(0.5000000000\,r1\,\Bigg(\frac{1}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}\,(3.\,r1\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2
$$

$$
-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2))
$$

$$
+\frac{2.\,r2\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}-45.\,r1+30.\,r2\Bigg)
$$

$$
\Bigg)+\frac{r2\,(2.\,\mathit{Co}\,r1+2.\,\mathit{Co}\,r2-2.\,b\,r1-2.\,b\,r2+45.\,r1^2-30.\,r1\,r2)}{3.\,r1^2+4.\,r1\,r2+2.\,r2^2}+b
$$

$LyμStar := eval(Lyμ, [x1 = x1s\_expd, x2 = x2s\_expd])$; ***#Lagrangean Multiplier Star***

$$LyμStar := -\frac{1}{r1 + r2}\left(0.5000000000\, r1\left(\frac{1}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2}\left(3.\, r1\left(2.\, Co\, r1 + 2.\, Co\, r2 - 2.\, b\, r1 - 2.\, b\, r2 + 45.\, r1^2 - 30.\, r1\, r2\right)\right) + \frac{2.\, r2\left(2.\, Co\, r1 + 2.\, Co\, r2 - 2.\, b\, r1 - 2.\, b\, r2 + 45.\, r1^2 - 30.\, r1\, r2\right)}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} - 45.\, r1 + 30.\, r2\right)\right) - \frac{r2\left(2.\, Co\, r1 + 2.\, Co\, r2 - 2.\, b\, r1 - 2.\, b\, r2 + 45.\, r1^2 - 30.\, r1\, r2\right)}{3.\, r1^2 + 4.\, r1\, r2 + 2.\, r2^2} + Co - b \quad \textbf{(55)}$$

 # *HW3_QI_b*: *Profit Maximizing input and output levels:*

$r1 := 8; r2 := 10; p := 10; b := 0; Co := 936; yo := 385;$
 # labor, capital, output price, fixed cost, investment, output demanded.

$$r1 := 8$$

$$r2 := 10$$

$$p := 10$$

$$b := 0$$

$$Co := 936$$

$$yo := 385 \tag{56}$$

$Profit\_Maximizing\_Labor := x1s\_profit;$

$$Profit\_Maximizing\_Labor := 79.00000000 \tag{57}$$

$Profit\_Maximizing\_Capital := x2s\_profit;$

$$Profit\_Maximizing\_Capital := 66.00000000 \tag{58}$$

$Profit\_Maximizing\_Output := MaxprofOut;$

$$Profit\_Maximizing\_Output := MaxprofOut \tag{59}$$

$Max\_profit := ProfitStar;$

$$Max\_profit := -2.0\,x1^2 + 10\,(0.4\,x2 + 6.)\,x1 - 3.0\,x2^2 + 80.\,x2 - 8.\,x1 = 4694.000000 \tag{60}$$

 # *HW3_QI_c*: *Least combination of labor and capital to produce 385 unit of output.*

$Lesat\_Combn\_Labor := x1s\_cost;$

$$Lesat\_Combn\_Labor := (146.6666666, 36.66666666) - 1.666666666 \tag{61}$$

$Least\_combn\_capital := x2s\_cost;$

$$Least\_combn\_capital := 120.0000000, 30.00000000 \tag{62}$$

$Least\_Cost := CostStar\_Cost;$

$$Least\_Cost := (2373.333334, 593.3333334) - 13.33333334 \tag{63}$$

$Output\_Least\_Cost := simplify(ystar\_cost);$

$$Output\_Least\_Cost := -0.0001543209877\,(5280.000000, 1320.000000)^2 + (1977.777778, 494.4444444) - 10.55555555 + 0.01111111111\,(120.0000000, 30.00000000)\,(5280.000000, 1320.000000) - 0.3\,(120.0000000, 30.00000000)^2 \tag{64}$$

 # *HW3_QI_d*: *Appriximate estimated increase in production cost due to unit increase in output:*
#LCf1λStar and LCf1λStar are equal but opposite in direction.

$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := LCf1\lambda Star;$

$$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := LCf1\lambda Star \tag{65}$$

$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := LCf2\lambda Star;$

$$Increase\_Cost\_Per\_Unit\_Increae\_In\_Production := LCf2\lambda Star \tag{66}$$

 # *HW3_Q1_e* : *Optimum production* and *input levels* :

$Labor\_Input\_for\_Optimum\_Prod := x1s\_expd;$

$$Labor\_Input\_for\_Optimum\_Prod := 57.00000000 \tag{67}$$

$Capital\_Input\_for\_Optimum\_Prod := x2s\_expd;$

$$Capital\_Input\_for\_Optimum\_Prod := 48.00000000 \tag{68}$$

$Optimum\_production := ystar\_expd;$

$$Optimum\_production := 527.4000000 \tag{69}$$

$Optimum\_Prod\_Cost := CostStar\_expd;$

$$Optimum\_Prod\_Cost := 936.0000000 \tag{70}$$

 # HW3 _Q1_f : Approximate estimated increase in output per unit cost:

$Estimated\_Output\_increase\_per\_unit\_cost := Ly\mu Star;$

$$Estimated\_Output\_increase\_per\_unit\_cost := 0. \tag{71}$$

THE END

Two variable Quadratic production function

```
> restart;with(plots):setoptions3d(axes=boxed);with(plottools):
> a1:= 6;
```

$$a1 := 6 \quad (1)$$

```
> a2:= 9;
```

$$a2 := 9 \quad (2)$$

```
> b1:=-0.2;
```

$$b1 := -0.2 \quad (3)$$

```
> b2:=-.03;
```

$$b2 := -0.03 \quad (4)$$

```
> b3:=0.4; a:=0;
```

$$b3 := 0.4$$

$$a := 0 \quad (5)$$

```
> yv:= a1*x1+a2*x2+b1*x1^2+b2*x2^2+b3*x1*x2-a;
```

$$yv := -0.2\, x1^2 + 0.4\, x1\, x2 - 0.03\, x2^2 + 6\, x1 + 9\, x2 \quad (6)$$

```
> x2s:= [solve(yv=y0, x2)];
```

$$x2s := \left[6.666666667\, x1 + 150. + 0.6666666667 \sqrt{85.\, x1^2 + 4950.\, x1 - 75.\, y0 + 50625.}\,,\right. \quad (7)$$
$$\left. 6.666666667\, x1 + 150. - 0.6666666667 \sqrt{85.\, x1^2 + 4950.\, x1 - 75.\, y0 + 50625.}\right]$$

```
> x2p :=x2s[1];
```

$$x2p := 6.666666667\, x1 + 150. + 0.6666666667 \sqrt{85.\, x1^2 + 4950.\, x1 - 75.\, y0 + 50625.} \quad (8)$$

```
> x2n:=x2s[2];
```

$$x2n := 6.666666667\, x1 + 150. - 0.6666666667 \sqrt{85.\, x1^2 + 4950.\, x1 - 75.\, y0 + 50625.} \quad (9)$$

```
> f1:=diff(yv,x1);
```

$$f1 := -0.4\, x1 + 0.4\, x2 + 6 \quad (10)$$

```
> f11:=diff(f1,x1);
```

$$f11 := -0.4 \quad (11)$$

```
> f2:=diff(yv,x2);
```

$$f2 := 0.4\, x1 - 0.06\, x2 + 9 \quad (12)$$

```
> f22:=diff(f2,x2);
```

$$f22 := -0.06 \quad (13)$$

```
> f12:=diff(f2,x1);
```

$$f12 := 0.4 \quad (14)$$

```
> subelas:=(f1*f2*(f1*x1+f2*x2))/(x1*x2*(2*f1*f2*f12-f1*f1*f22-f2*
```

```
f2*f11));
```

$$subelas := ((-0.4\,x1+0.4\,x2+6)\,(0.4\,x1-0.06\,x2+9)\,((-0.4\,x1+0.4\,x2+6)\,x1 + (0.4\,x1-0.06\,x2+9)\,x2))\Big/\left(x1\,x2\,\left(0.8\,(-0.4\,x1+0.4\,x2+6)\,(0.4\,x1-0.06\,x2 + 9) + 0.06\,(-0.4\,x1+0.4\,x2+6)^2 + 0.4\,(0.4\,x1-0.06\,x2+9)^2\right)\right) \quad (15)$$

```
> subelass:=simplify(subelas);
```

$$subelass := -\left(50.\,\left(0.4\,x1^2-0.8\,x1\,x2-6.\,x1+0.06\,x2^2-9.\,x2\right)\,(0.4\,x1-0.06\,x2 + 9.)\,(0.2\,x1-0.2\,x2-3.)\right)\Big/\left(\left(1.36\,x1^2-2.72\,x1\,x2-40.8\,x1+0.204\,x2^2-61.2\,x2 - 1944.\right)\,x1\,x2\right) \quad (16)$$

```
> curv:=-(1/f2^3)*(f2*f2*f11-2*f1*f2*f12+f1*f1*f22);
```

$$curv := -\frac{1}{(0.4\,x1-0.06\,x2+9)^3}\left(-0.4\,(0.4\,x1-0.06\,x2+9)^2-0.8\,(-0.4\,x1+0.4\,x2 + 6)\,(0.4\,x1-0.06\,x2+9)-0.06\,(-0.4\,x1+0.4\,x2+6)^2\right) \quad (17)$$

```
> curvs:=simplify(curv);
```

$$curvs := \frac{-0.0544\,x1^2+(0.1088\,x2+1.632)\,x1-0.00816\,x2^2+2.448\,x2+77.76}{(0.4\,x1-0.06\,x2+9.)^3} \quad (18)$$

```
> ridge1:= solve(f1,x2);
```

$$ridge1 := x1-15. \quad (19)$$

```
> ridge2:= solve(f2,x2);
```

$$ridge2 := 6.666666667\,x1+150. \quad (20)$$

```
> maxmin:= solve({f1,f2},{x1,x2});
```

$$maxmin := \{x1=-29.11764706,\, x2=-44.11764706\} \quad (21)$$

```
> pm2:=f11*f22-f12*f12;
```

$$pm2 := -0.136 \quad (22)$$

```
> pm1:=f11;
```

$$pm1 := -0.4 \quad (23)$$

```
> mrts:= f1/f2;
```

$$mrts := \frac{-0.4\,x1+0.4\,x2+6}{0.4\,x1-0.06\,x2+9} \quad (24)$$

```
> x2nx1:=diff(x2n,x1);
```

$$x2nx1 := 6.666666667-\frac{0.3333333334\,(170.\,x1+4950.)}{\sqrt{85.\,x1^2+4950.\,x1-75.\,y0+50625.}} \quad (25)$$

```
> x2nx1s:=simplify(x2nx1);
```

$$x2nx1s := \frac{6.666666667\,\sqrt{85.\,x1^2+4950.\,x1-75.\,y0+50625.}-56.66666668\,x1-1650.000000}{\sqrt{85.\,x1^2+4950.\,x1-75.\,y0+50625.}} \quad (26)$$

```
> x2nx1x1:=diff(x2nx1,x1);
```

$$x2nx1x1 := \frac{0.1666666667\ (170.\ x1 + 4950.)^2}{(85.\ x1^2 + 4950.\ x1 - 75.\ y0 + 50625.)^{3/2}} - \frac{56.66666668}{\sqrt{85.\ x1^2 + 4950.\ x1 - 75.\ y0 + 50625.}} \tag{27}$$

```
> x2nx1x1s:=simplify(x2nx1x1);
```

$$x2nx1x1s := \frac{1.215000\ 10^6 + 4250.000001\ y0}{(85.\ x1^2 + 4950.\ x1 - 75.\ y0 + 50625.)^{3/2}} \tag{28}$$

```
> yvp:=eval(yv,{x1=12,x2=10});
```

$$yvp := 178.20 \tag{29}$$

```
> isoqx1x1:=eval(x2nx1x1s,{x1=12,x2=10,y0=yvp});
```

$$isoqx1x1 := 0.05488354621 \tag{30}$$

```
> curvv:=eval(curvs,{x1=12,x2=10});
```

$$curvv := 0.05488354621 \tag{31}$$

```
> mrtsv:=eval(mrts,{x1=12,x2=10});
```

$$mrtsv := 0.3939393939 \tag{32}$$

```
> isoqx1:=eval(x2nx1,{x1=12,x2=10,y0=yvp});
```

$$isoqx1 := -0.393939394 \tag{33}$$

```
> subelassv:=eval(subelass,{x1=12,x2=10});
```

$$subelassv := 0.8809034910 \tag{34}$$

```
> x2pa:=eval(x2p,y0=140);
```

$$x2pa := 6.666666667\ x1 + 150. + 0.6666666667\ \sqrt{85.\ x1^2 + 4950.\ x1 + 40125.} \tag{35}$$

```
> x2pb:=eval(x2p,y0=150);
```

$$x2pb := 6.666666667\ x1 + 150. + 0.6666666667\ \sqrt{85.\ x1^2 + 4950.\ x1 + 39375.} \tag{36}$$

```
> x2pc:=eval(x2p,y0=160);
```

$$x2pc := 6.666666667\ x1 + 150. + 0.6666666667\ \sqrt{85.\ x1^2 + 4950.\ x1 + 38625.} \tag{37}$$

```
> x2na:=eval(x2n,y0=140);
```

$$x2na := 6.666666667\ x1 + 150. - 0.6666666667\ \sqrt{85.\ x1^2 + 4950.\ x1 + 40125.} \tag{38}$$

```
> x2nb:=eval(x2n,y0=150);
```

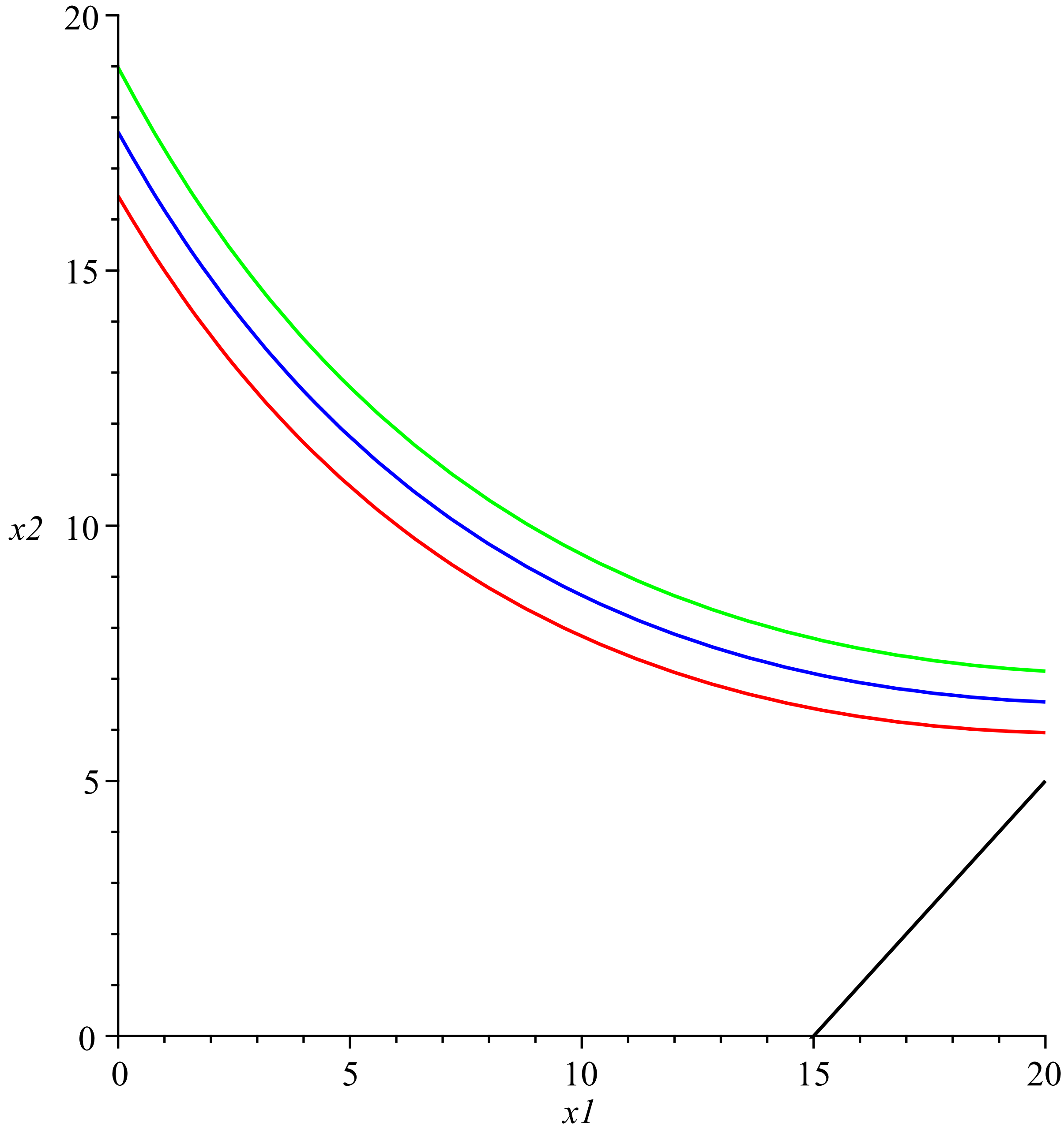
$$x2nb := 6.666666667\ x1 + 150. - 0.6666666667\ \sqrt{85.\ x1^2 + 4950.\ x1 + 39375.} \tag{39}$$

```
> x2nc:=eval(x2n,y0=160);
```

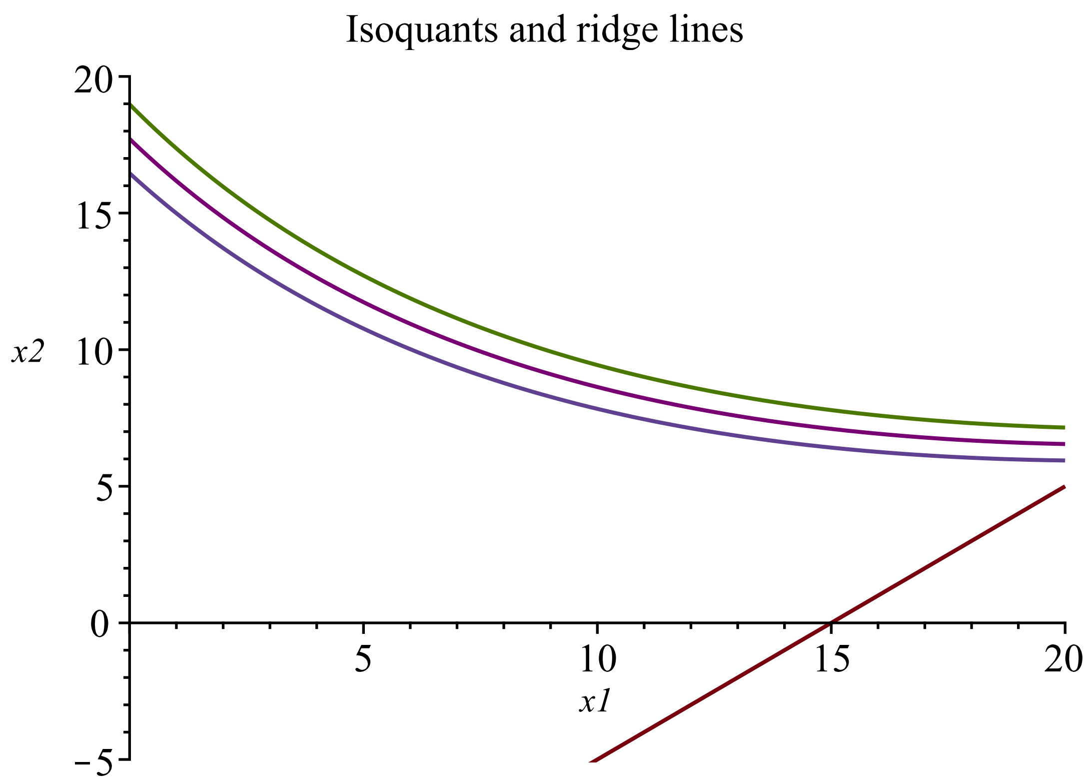
$$x2nc := 6.666666667\ x1 + 150. - 0.6666666667\ \sqrt{85.\ x1^2 + 4950.\ x1 + 38625.} \tag{40}$$

```
> p150:=implicitplot({yv=150},x1=0..20,x2=0..20,color=blue):
> p160:=implicitplot({yv=160},x1=0..20,x2=0..20,color=green):
```

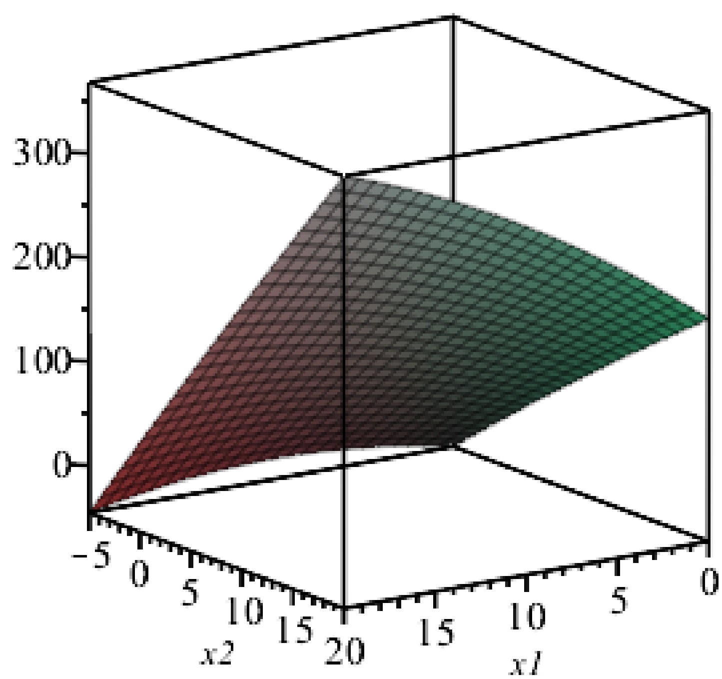
```
> ridgeplot:=plot({ridge1,ridge2},x1=0..20,x2=0..20,color=black):
> p140:=implicitplot({yv=140},x1=0..20,x2=0..20,color=red):
> plots[display](p140,p150,p160,ridgeplot);
```



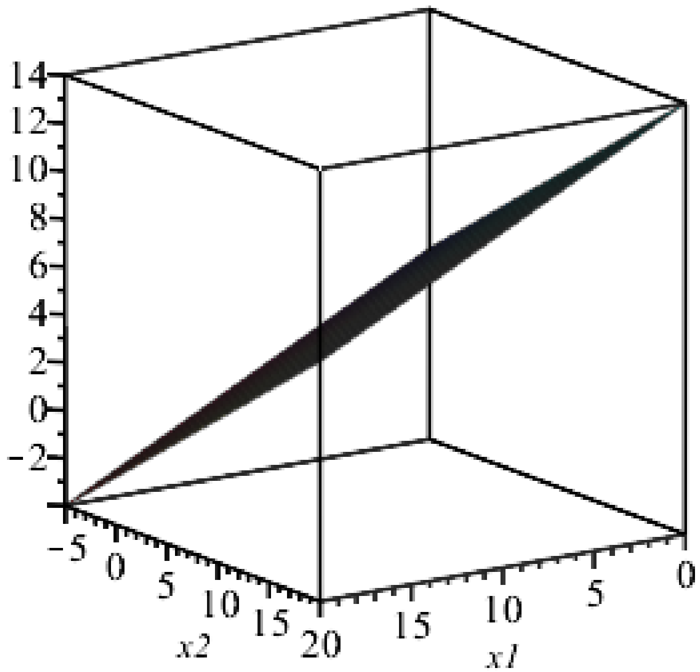

```
> plot({x2pa,x2pb,x2pc,x2na,x2nb,x2nc,ridge1,ridge2},x1=-0..20,x2=
  -5..20,title="Isoquants and ridge lines");
```



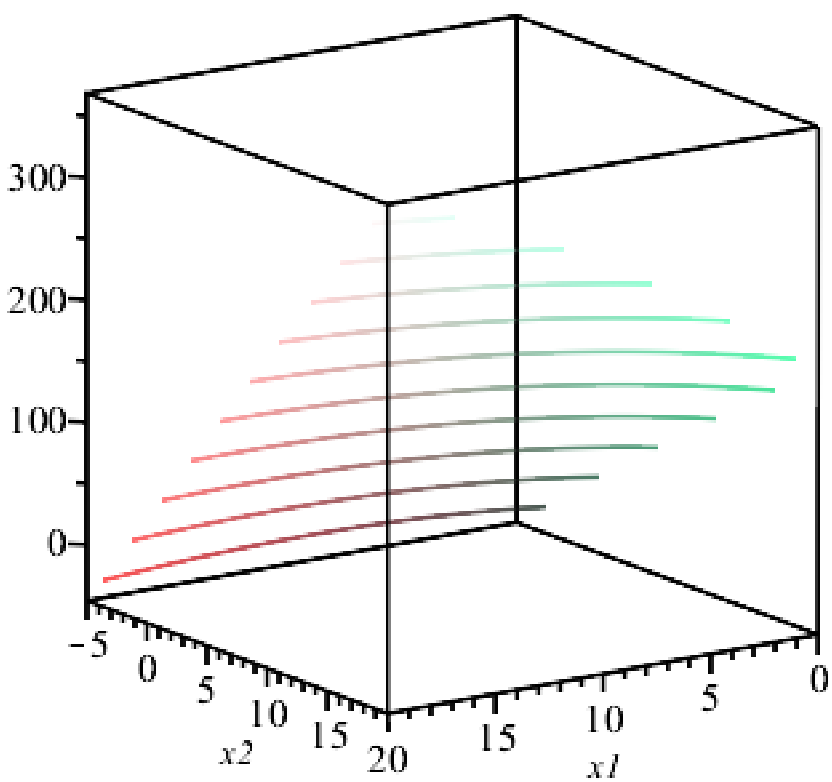

```
> plot3d(yv,x1=0..20,x2=-5..20);
```




```
> plot3d(f1,x1=0..20,x2=-5..20);
```




```
> plot3d(yv,x1=0..20,x2=-5..20,style=contour);
```



#Complete Solution

$x1 := b; a1 := 2512; a2 := 180; a3 := -1.5; p := 2; VC := 2000; FC := 0;$

$$x1 := b$$
$$a1 := 2512$$
$$a2 := 180$$
$$a3 := -1.5$$
$$p := 2$$
$$VC := 2000$$
$$FC := 0 \quad \textbf{(1)}$$

$f := a1 \cdot x1 + a2 \cdot x1^2 + a3 \cdot x1^3;$

$$f := -1.5\, b^3 + 180\, b^2 + 2512\, b \quad \textbf{(2)}$$

$profit := f \cdot p - (x1 \cdot VC + FC);$

$$profit := 3024\, b - 3.0\, b^3 + 360\, b^2 \quad \textbf{(3)}$$

$EP\_x1 := diff(profit, x1);$

$$EP\_x1 := 3024 - 9.0\, b^2 + 720\, b \quad \textbf{(4)}$$

$x1\_demand := solve(EP\_x1 = 0, b);$ *#This is just a demand but not demand at maximum profit.*

$$x1\_demand := -4., 84. \quad \textbf{(5)}$$

$APP := simplify\left(\frac{f}{x1}\right);$

$$APP := -1.5\, b^2 + 180.\, b + 2512. \quad \textbf{(6)}$$

$MPP := diff(f, x1);$

$$MPP := -4.5\, b^2 + 360\, b + 2512 \quad \textbf{(7)}$$

$AVP := p \cdot APP;$

$$AVP := -3.0\, b^2 + 360.\, b + 5024. \quad \textbf{(8)}$$

$boat := diff(AVP, x1);$

$$boat := -6.0\, b + 360. \quad \textbf{(9)}$$

#2a:

$MaximumBoat\_2a := solve(boat = 0, x1);$

$$MaximumBoat\_2a := 60. \quad \textbf{(10)}$$

#Answer: individual will use 60 boats

 # Total fish caught by 60 boats (maximum number of fish caught by 60 boats);

$Totfish := eval(f, x1 = MaximumBoat\_2a);$

$$Totfish := 474720.0 \quad \textbf{(11)}$$

$Profit\_2a := eval(profit, [f = Totfish, x1 = MaximumBoat\_2a]);$ *#under scenario in 2a..*

$$Profit\_2a := 829440.0 \quad \textbf{(12)}$$

 # 2b: *New boat will be added* **if** *there is profit. So, number of boat reach* **to** *maximum when profit is zero under perfect comption.*

$profit\_zero := Totfish \cdot p - x1 \cdot VC;$

$$profit\_zero := -2000\, b + 949440.0 \quad \textbf{(13)}$$

$maximum\_boat := solve(profit\_zero = 0, x1);$

$$maximum\_boat := 474.7200000 \quad \textbf{(14)}$$

#Ans = 474.72 boats.

*#**2c:** Cooperative is formed and share profit equally.*

$fb := simplify\left(\frac{f}{x1}\right)$; *#production function of individual boat.*

$$fb := -1.5\, b^2 + 180.\, b + 2512. \quad \textbf{(15)}$$

$profit\_fb := p \cdot fb - x1 \cdot VC - FC;$

$$profit\_fb := -1640.\, b - 3.0\, b^2 + 5024. \quad \textbf{(16)}$$

$APP\_fb := simplify\left(\frac{fb}{x1}\right);$

$$APP\_fb := \frac{-1.5\, b^2 + 180.\, b + 2512.}{b} \quad \textbf{(17)}$$

$fb\_x1\_demand := solve(\ diff(profit\_fb, x1), x1);$ *#not a maximizing demand.*

$$fb\_x1\_demand := -273.3333333 \quad \textbf{(18)}$$

$AVP\_fb := simplify(\ p \cdot APP\_fb);$

$$AVP\_fb := \frac{-3.\, b^2 + 360.\, b + 5024.}{b} \quad \textbf{(19)}$$

$boat\_fb := simplify(diff(AVP\_fb, x1));$ *# For max profit, first derivative of AVP should be 0.*

$$boat\_fb := \frac{-3.\, b^2 - 5024.}{b^2} \quad \textbf{(20)}$$

$MaxBoat\_fb := solve(boat\_fb = 0, x1);$

$$MaxBoat\_fb := -40.92269134\, \mathrm{I}, 40.92269134\, \mathrm{I} \quad \textbf{(21)}$$

 # Answer:

 # Already negative or indetermined. So, it is not profitable to operate as cooperate. Not operating any boat is most profitable.

*# **2d.** This is* **in** *the third stage of production*

$MPP\_fb := diff(fb, x1);$

$$MPP\_fb := -3.0\, b + 180. \quad \textbf{(22)}$$

$Curvature\_fb := diff(MPP\_fb, x1);$

$$Curvature\_fb := -3.0 \quad \textbf{(23)}$$

$Elasticity := \frac{MPP\_fb}{APP\_fb};$

$$Elasticity := \frac{(-3.0\, b + 180.)\, b}{-1.5\, b^2 + 180.\, b + 2512.} \quad \textbf{(24)}$$

$StatgeIII := solve(Elasticity = 0, x1);$

$$StatgeIII := 0., 60. \quad \textbf{(25)}$$

$StatgeI := solve(Elasticity = 1, x1)$

$$StatgeI := 40.92269134\, \mathrm{I}, -40.92269134\, \mathrm{I} \quad \textbf{(26)}$$