FOURIER PTYCHOGRAPHIC RECONSTRUCTION USING WEIGHTED REPLACEMENT IN THE FOURIER DOMAIN

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ABSTRACT

Fourier ptychographic microscopy (FPM) is an attractive method to extend the resolution beyond the conventional limit defined by a microscope optics, sharing properties with ptychographic, synthetic aperture imaging and phase retrieval. The algorithm uses a sequence of low-resolution (LR) images acquired under angularly varying illumination to reconstruct a high-resolution (HR) image. However, traditional FPM may trap to the sub-optimal solution, since brute-force replacement in the Fourier domain is applied. To address this problem, we propose here a weighted replacement for Fourier ptychographic microscopy (WFPM). We employ the weighted average of spectrums corresponding to different illumination angle to replace the overlapped regions in the Fourier domain. A series of experimental results demonstrate that the reconstructed image using the proposed WFPM shows a better quality and a faster convergence compared with the results obtained by FPM.

Index Terms— Fourier ptychographic, phase retrieval

1. INTRODUCTION

In microscopy, the imaging quality of an optical system is fundamentally limited by the space-bandwidth product (SBP), forcing the users to make a compromise in achievable image resolution and field of view (FOV). To overcome the challenge between the resolution and FOV, Zheng et. al [1] proposed a computational imaging, called Fourier ptychographic microscopy (FPM). FPM requires a number of low-resolution images. Based on the relation between Fourier domain and the angle of illumination, the images describing different ranges of the sample spectrum are captured under angularly varying illumination, and then the algorithm stitches these different spectrum regions together in Fourier domain to produce a high-resolution (HR) image. Recently, FPM has attracted considerable research interests. These studies can be roughly classified into two groups.

In the first group, the researchers make efforts to improve the quality of the FPM reconstructions or to correct for the spatially varying aberrations of microscopy systems. [1] provided a simple procedure to correct the aberration introduced by a defocus distance. [2] reported a pupil characterization method and efficiently recovered 2D aberration maps by parameter fitting. Using this characterization in an image deconvolution process, the resolution performance revealed a promotion largely. [3] put forward an optimization framework that implements adaptive system correction for Fourier ptychographic imaging. [4] described an approach which can recover both the spectrum in the Fourier domain and the pupil function of the imaging system simultaneously, motivated by the works of [5] [6] [7].

The second type of the studies focus on reducing both acquisition time and processing time of the FPM method. In general, the conventional FPM requires hundreds of LR-images captured under angularly varying illumination, which leads to a long acquisition time. [8] proposed Wirtinger flow optimization based Fourier Ptychography incorporating a noise relaxation constraint, to reduce exposure time significantly. [9] demonstrated that by using the multiplexing methods, both acquisition time and data capture requirements reduce greatly. [10] changed the conventional monochromatic illumination to color illumination by simultaneously turning on R/G/B LEDs for data acquisition and performed color-multiplexed imaging. Considering the relationship between a high-resolution image spectrum and its corresponding low-resolution spectrum, [11] proposed the self-learning based on FPM to reduce both acquisition time and processing time. [12] made progress in this aspect using sparsely sampled Fourier Ptychographic.

During the reconstruction of FPM, there is an overlap between neighboring Fourier domain regions to ensure the convergence. Previous methods replace the overlapped Fourier domain region by region according to a certain scanning manner. However, we found that the conventional FPM would trap in a local optimum solution by such replacement manners in many cases. Furthermore the original method has an unsatisfactory performance in noise reduction and the speed of convergence. To address these problems, we propose a new replacement scheme called weighted replacement

Fourier ptychographic microscopy (WFPM), to improve the reconstruction quality and convergence speed.

Here is the outline of this paper. In Section 2, we review the conventional FPM briefly, and then introduce the theory of our method. In Section 3, we verify the effectiveness of our proposed WFPM algorithm by a series of experiments on both synthetic and real captured data. Finally, we conclude this paper with some summaries and discussions in Section 4.

2. THEORY AND METHOD

2.1. Fourier Ptychographic microscopy

In microscopy, while we get a HR image by using a high numerical aperture (NA) objective lens, the FOV will be small correspondingly, and vice versa. FPM bypasses the above problem incorporating phase retrieval [13] and synthetic aperture microscopy [14].

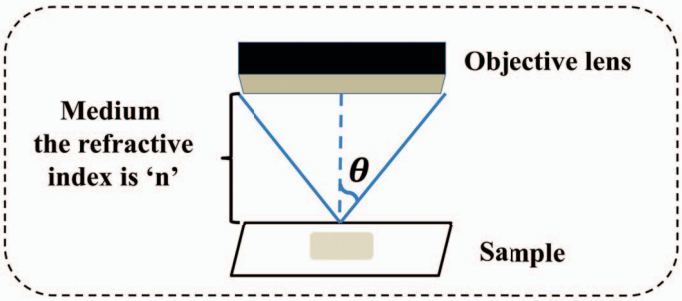

Fig. 1. The numerical aperture

Here $NA = n * sin\theta$, the parameters are shown in Fig. 1, with n representing the refractive index of the medium between the sample and the objective lens, θ signifying half of the objective lens aperture angle.

The experimental set-up consists of a microscope, a camera and a programmable LED matrix. The programmable LED matrix is placed under the sample stage as an illumination source.

During image acquisition, each LED is turned on to provide an oblique wave-vector for each captured low-resolution image. Based on the assumption that an angular variation of the illumination corresponds to a shift of the sample spectrum center in the Fourier domain. The shifting value is decided by the wave-vector, as mentioned in [1].

Here we overview the FPM as follows and the iterative recovery procedure is shown as Fig. 2.

(1) Initialize: Give an original guess of the reconstructed high resolution object function, $\sqrt{I_h}e^{i\varphi_h}$ (subscript $'h'$ stands for high-resolution).

(2) Extract: Fourier transform to the initial guess to generate a spectrum in Fourier domain, and then choose a sub-region from the spectrum to create a low-resolution image, $\sqrt{I_l}e^{i\varphi_l}$.

(3) Replace: Replace the modulus of the low-resolution image obtained by step **(2)** with the corresponding measured low-resolution image modulus to form a new low-resolution image (that is, $\sqrt{I_l}e^{i\varphi_l} \rightarrow \sqrt{I_m}e^{i\varphi_l}$, subscript $'l'$ stands for low-resolution, $'m'$ stands for measurment).

(4) Update: The Fourier transform of the new low-resolution image created by step **(3)** updates the corresponding sub-region of the spectrum.

(5) Repeat the extract-replace-update for all N measured low images.

(6) Repeat: Repeat step **(2)** — step **(5)** until a convergent solution is achieved.

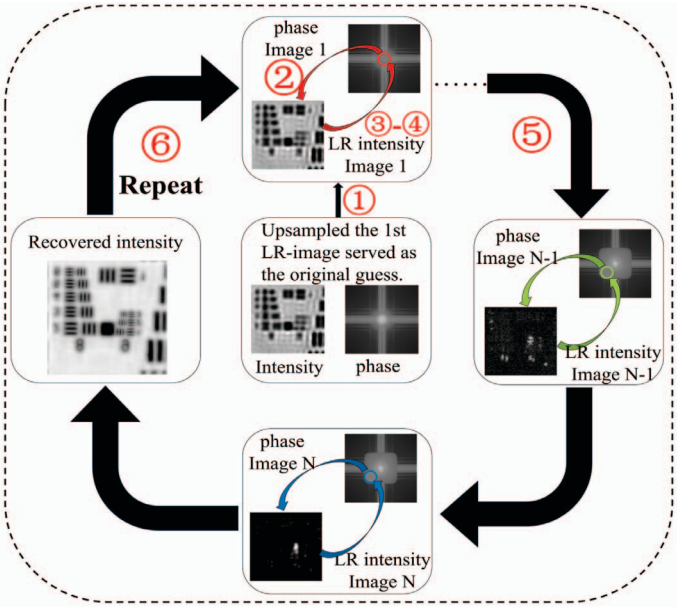

Fig. 2. Iterative recovery procedure of original FPM

2.2. Principle of WFPM

According to the workflow of the FPM, we can get the point that the Exact-Replace-Update procedure is an extremely important step, which may affect the quality of reconstruction directly. In conventional FPM, there is an overlapping between neighboring Fourier domains corresponding to spatially adjacent illuminations to ensure the convergence, which is the foundation of FPM. It is clear that the operation of update is a simple pattern in conventional FPM, that is, $'the\ latter\ updates\ the\ former'$. As is shown in Fig. 3, the black square is an overlapping region of 3 different sub-regions corresponding to 3 adjacent illumination LEDs (the red, blue and green circular domains). The black square is finally updated by the green square in the 3^{rd} circle using conventional FPM based on the raster scanning manners. However, we find that the update is not a good scheme. For the reason that the overlapping area is related to 3 different sub-regions, and the black square should be decided by the

elements of red, blue and green squares jointly. Therefore, we propose a weighted replacement (here, the replacement stands for the update (stpe **4**)), whose work flows are presented as follows.

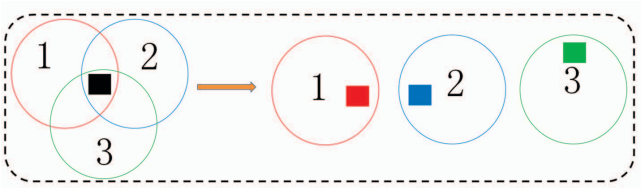

Fig. 3. The schematic of replacement

We assume that the Fourier transform of the initial guess is $\mathbf{F}_h$, namely, the guess for the spectrum of sample. $\mathbf{F}_{subi}$ represents the selective sub-region from the spectrum (subscript $'sub'$ stands for sub-region and 'i' stands for the i^{th} LED). As is shown in the Fig. 3, the different circles represent different sub-regions. $\mathcal{F}^{-1}(\mathbf{F}_{subi}) = \sqrt{\mathbf{I}_{li}}e^{j\varphi_{li}}$ (subscript 'l' stands for low-resolution) to generate a new low-resolution image corresponds to **step (2)**. It is clear that in one iteration the Extract-Replace-Update would be implemented for N times with order. Therefore, N sub-regions can be achieved.

Assuming that $\mathbf{F}$ is an overlapping region which is the result of M sub-regions. In Fig. 3, F is the black square region, and the 3 sub-regions correspond to different colored circular domains. Then, we put forward the following optimization equation:

$$\{\mathbf{F}\} = argmin \sum_{i=1}^{M} \sum_{u,v} \left\{ \hat{\mathbf{F}}_{subi}(u,v) - \mathbf{F}(u,v) \right\}^2, \quad (1)$$

Where (u,v) is the coordinate in the spatial frequency domain (Fourier domain). $\hat{\mathbf{F}}_{subi}$ is the region corresponding to F in the sub-region $\mathbf{F}_{subi}$. As shown in Fig.3, the $\hat{\mathbf{F}}_{subi}$ $(i = 1, 2, 3)$ represent the red, blue and green square respectively. The gradient of the optimization function can be obtained as:

$$\frac{d\{\mathbf{F}\}}{d\mathbf{F}(u,v)} = 2M\mathbf{F}(u,v) - 2\sum_{i=1}^{M} \hat{\mathbf{F}}_{subi}(u,v). \quad (2)$$

Then setting the derivative with respect to F to zero, we can obtain the optimal F to satisfy the **Eq. (1)** :

$$\frac{d\{\mathbf{F}\}}{d\mathbf{F}(u,v)} = 0$$
$$\Rightarrow \mathbf{F}(u,v) = \frac{\sum_{i=1}^{M} \hat{\mathbf{F}}_{subi}(u,v)}{M}. \quad (3)$$

Through the above analysis, we can conclude that the overlapping region $\mathbf{F}$ should be updated as an average of regions corresponding to $\mathbf{F}$ in each individual sub-region, namely, the black square in Fig.3 is updated by the weighted summation of red, blue and green square. And the optimum weight vector is $W_0 = (\underbrace{\frac{1}{M}, \frac{1}{M}, \frac{1}{M} \cdots \frac{1}{M}, \frac{1}{M}}_{M})$. Accordingly, the conventional FPM can be seen as a weight vector $W = (\underbrace{0, 0, 0, \ldots 0, 1}_{M})$. Furthermore, the weighted replacement would reduce the pernicious influence of some 'bad' results in iteration, so WFPM algorithm has a faster convergence rate compared to the FPM algorithm theoretically.

3. RESULTS

In this section, we demonstrate the effectiveness of the WFPM algorithm from the experimental results.

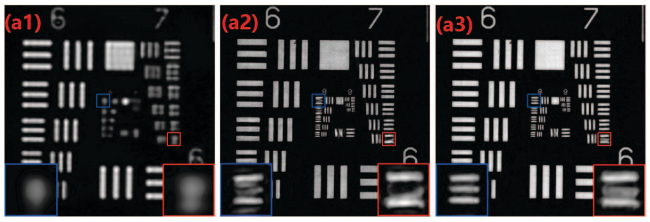

Fig. 4. Experimental reconstruction of USAF target. (a1) The raw data. (a2) The renconstruction image with FPM. (a3) The reconstruction image with WFPM.

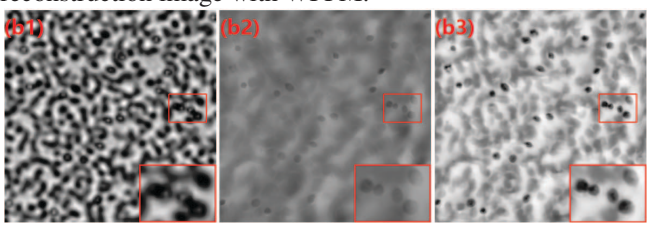

Fig. 5. Reconstruction of Blood Smear data set. (b1) The raw data. (b2) Reconstructed image using FPM. (b3) Reconstructed image using WFPM.

We utilize two types of samples to validate our approach : the USAF sample and Blood Smear sample [1]. The USAF sample is captured with a $2\times$, 0.08 NA (0.1NA for Blood Smear) objective lens. A programmable $15 * 15$ LED matrix is placed at $84.6mm$ ($90.88mm$ for Blood Smear) beneath the sample as a variable illumination source for USAF. And 225 low-resolution images are required to reconstruct the high-resolution image. Pixel size at the object plane is $2.72\mu m$ ($1.845\mu m$ for Blood Smear) in USAF experiment. For two experiments, the central wavelength of the LED is $635nm$.

As is shown in Fig. 4(a3), reconstructed lines and the space between them in group 8 element 1 and element 2 are more easier to identify than Fig. 4(a2). In addition, the noise is highly degraded by our method. The contour of the blood

cells cannot be recognized clearly and it is hard to distinguish white blood cells from red blood cells in Fig. 5(b2). A high quality image can be achieved using the WFPM algorithm, as shown in Fig. 5(b3). Results show a better performance of our WFPM method than that of the conventional FPM method. This is mainly due to the fact that we replace the sub-regions of spectrum with the weighted replacement rather than the simple $'the\ latter\ updates\ the\ former'$ in conventional FPM.

Next we compare the convergence of both algorithms by calculating the value of S in each iteration:

$$S_{itr} = \frac{\sum_r |I_{itr}(r) - I_{itr-1}(r)|}{\alpha \times M \times N}, \tag{4}$$

where $r = (x, y)$ denotes the lateral coordinates at the sample plane. $'itr'$ represents the number of iteration. I_{itr} is the reconstructed intensity image after itr^{th} iteration. The parameter α is a constant to ensure S in an appropriate range. Here, we set $\alpha = 0.5$, and $M \times N$ is the size of the reconstructed image. The results are shown in Fig. 6.

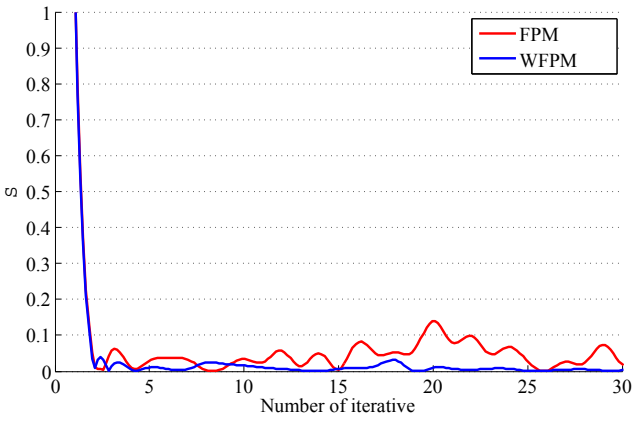

Fig. 6. Convergence comparison between the traditional FPM algorithm and the WFPM algorithm using the results of Blood Smear

From Fig. 6, WFPM algorithm has a significantly faster convergence rate compared to the conventional FPM. Besides, there is a high fluctuation in FPM compared to the proposed WFPM. This is consistent with our theoretical analysis in section 2.2.

4. CONCLSION

In this paper, we put forward an optimization algorithm that performs weighted replacement for Fourier Ptychographic microscopy called WFPM. Using the WFPM algorithm, we can get a successful reconstruction after fewer iterations than conventional FPM. Moreover, implementation of WFPM algorithm provided us with improvement of reconstruction. These are largely attributed to the fact that the WFPM makes an optimization for the update of overlapping regions. We have demonstrated the validity of this optimization approach by reconstructing a standard USAF target compared to the original FPM method. Without loss of generality, we have applied our scheme to a biological dataset and received the desired results.

Finally, we would like to point out that the optimum idea could be applied to the initial guess, for the reason that we have found that the initial guess would affect the quality and speed of reconstructed procedure greatly. And we believe that the optimum guess will be a future research direction.

5. ACKNOWLEDGEMENTS

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