Home Exercise sheet 5

submission date: 01-16-2014

Exercise 1: Hydrogen burning

The energy generated per unit time and unit mass of matter for the proton-proton reaction is:

$$E_{pp} = 2.5 \times 10^6 \rho X^2 (\frac{10^6}{T})^{2/3} exp[-33.8(\frac{10^6}{T})^{1/3}]\, erg\, g^{-1} s^{-1} \tag{1}$$

and the one for the CN cycle is:

$$E_{CN} = 8 \times 10^{27} \rho X X_{CN} (\frac{10^6}{T})^{2/3} f\, exp[-152.3(\frac{10^6}{T})^{1/3}]\, erg\, g^{-1} s^{-1} \tag{2}$$

where f is an electron screening factor, $f \sim 1$.
Consider the above expressions for solving the following problems:

a) At present the concentration of hydrogen, by mass, at the center of the Sun is about 70% throughout a central region out to roughly 0.122 $R_\odot$. The density is about 155 g cm−3 in the center. Take the mean temperature throughout the region to be 1.1 $\sim 10^7$ K and calculate the luminosity and surface temperature for this solar model.

b) A red-giant star whose radius is 100 $R_\odot$ is in an evolutionary state where the inner hydrogen has all been exhausted but helium burning has not yet set in. The main energy source is hydrogen burning that takes place in a shell surrounding the inert helium core. Let hydrogen burning take place at a radial distance ranging from 1.8 to 2 $\times$ 10^9 cm. The mean density in this layer is about 50 g cm^{-3}. The temperature is 5 $\times$ 10^7 K. Calculate the above parameters. Take $X_{CN} = 10^{-3}X$, $X \sim 0.5$.

Exercise 2: Quantum tunnelling

a) The coulomb barrier between two nucleons can be expressed by:

$$V = \frac{Z_A\ Z_B\ e^2}{4\pi\varepsilon_0\ r}$$

Calculate the barrier for two protons (with $Z_P = 1$ and $r \simeq 1.2 \cdot 10^{-15}$m).

b) Calculate the typical thermal energy of a proton in the core of the sun ($T = 15.6 \cdot 10^6$ K). Compare it to the result from a).

c) Consider a nucleon with energy E approaching the coulomb barrier. Far away from the barrier the wave function is a sine function. Closer the the nucleus the kinetic energy E-V becomes negative and the wave function follows the time independent

Schrödinger equation. In one dimension with the radial distance r it can be written as:

$$\left[-\frac{\hbar^2}{2m_r}\frac{\partial^2}{\partial r^2}+V\right]\ \psi_s(r)=E\psi_s(r)$$

For this exercise the wave function is given by

$$\psi_s(r)=exp(\chi r)$$

with χ

$$\chi^2=\frac{2m_r}{\hbar^2}(V-E)$$

The wave function drops exponentially during tunnelling. Behind the barrier the wave function of the particle follows again a sine, but now with a lower amplitude. Show that the wave function $\psi_s(r) = exp(\chi r)$ fulfils the Schrödinger equation above.

d) The tunnelling probability can be written as follows:

$$P_{Pen}\approx\frac{|\psi_s(r_1)|^2}{|\psi_s(r_2)|^2}=\{exp[-\chi(r_2-r_1)]\}^2$$

Where r_1 is the inner boundary of the barrier and r_2 the outer boundary - ($r_2 \gg r_1$). We now assume a rectangular barrier potential. How does the tunnelling probability changes with the height of the barrier? And how does it changes with the width of the barrier?

Exercide 3: A simple model of stellar evolution

Consider a hypothetical evolution of a star with the initial mass M_0. The core of the star grows due to the nuclear burning and each of these processes emits a certain energy Q per mass element. Additionally the star loses mass due to its stellar wind. The stellar wind can be assumed to be proportional to the (constant) luminosity L, respectively

$$\dot{M}_{wind}=-\beta L$$

where β is constant.

a) Find an expression for the mass of the core as a function of time, with the condition that M_c(t=0)=0.

b) Find an expression for the mass of the sphere as a function of time, with the condition that M_{sphere}(t=0)=M_0.

c) What is the mass of the core when the whole sphere has vanished?

d) Calculate an upper limit for M_0 for the star to become a white dwarf.

What you need:

$$Q = 5 \cdot 10^{14} J/kg$$

and

$$\beta = 10^{-14} kg/J$$

Do not think to complicated, NO formulas from the lectures are needed.

Exercise 4: Energy Output by Nucleosynthesis

a) The energy outputs of most fusion chains is in the range of MeV. What is a typical corresponding photon wavelenght?

b) The frequency (ν) distribution of a black body as a function of temperature is given by the Stefan-Boltzmann equation

$$I(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

where $I(\nu)$ is the intensity. At what wavelength has a Star with surface temperature of T= 8000K its intensity maximum and what color is it? How does this match with upper result?

Hints: Expand the Stefan-Boltzmann equation into a Taylor series to the third order for determining the maximum. Do not insert the values for the constants and the temperature until the last step.