Stellar Structure and Evolution
—Exercises—
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Part 2

1) Assume that the equation of state of the stellar matter is described by that of an ideal gas $p = \frac{\mathcal{R}}{\mu}\rho T$ (p: pressure, ρ: density, T: temperature, μ: mean molecular weight, $\mathcal{R}$: gas constant) with the specific heat $c_V = \frac{3}{2}\frac{\mathcal{R}}{\mu}$. How big is then the thermal energy content E_T and the (gravitational) potential energy E_G of a star? Hint: Use M_r as independent variable and express E_T and E_G as an integral over M_r.

Derive from the condition of hydrostatic equilibrium (independent variable M_r) by integration over M_r the virial theorem for a star in hydrostatic equilibrium:

$$E_G = -2E_T$$

What is the relation between the total energy E_{total} and E_T? What is the sign of E_{total}? What happens, if a star loses energy by radiation (no additional energy sources)? If a specific heat would be assigned to a star, which sign would it have?

2) On the basis of order of magnitude estimates prove that a relation between pressure and density together with the requirement of hydrostatic equilibrium implies a relation between mass M and radius R of a star.

Hint: Derivatives of the form $\frac{dy}{dM_r}$ may be estimated by y/M, the density by $\rho \propto M/R^3$. Assume a power law relation between pressure and density of the form $p \propto \rho^\gamma$.

What are the mass - radius relations for $\gamma = 4/3$ and $\gamma = 5/3$? Interpret the result.

3) A stellar model: Assume that pressure and density of the stellar matter are related by an equation of state of the form $p = K\rho^2$ (p: pressure, ρ: density, K: Constant). Calculate the density as a function of radius using the requirement of hydrostatic equilibrium.

Hint: Use Poisson's equation for the gravitational potential. For the solution of the problem use $r\rho$ as dependent variable rather than ρ.

Are all solutions physically meaningful? Are there free parameters? Sketch and interpret the solution.