Introduction to Cosmology

HW8

Friday, Dec. 18, 2015

Problem 1: Reheating

Assuming the potential of inflaton takes the form of $V(\varphi) = \frac{1}{2}m^2\varphi^2$, during the oscillating epoch ($H \ll m$), the solution of the inflaton can be written as

$$\varphi \simeq A(t)\cos(mt). \tag{1}$$

The average density of inflaton $\bar{\rho}_\varphi \sim a^{-3}$, i.e. $\bar{\rho}_\varphi a^3 = const$. Now consider the inflaton decays into other particles during oscillating around the minimum of the potential. The energy of inflaton in a fixed volume $\bar{\rho}_\varphi a^3$ will decrease. Then the continuity equation takes the form

$$\dot{\bar{\rho}}_\varphi + (3H + \Gamma)\bar{\rho}_\varphi = 0. \tag{2}$$

Here Γ is the decay rate which satisfies ($\Gamma \ll m$) and we have used the fact that the average pressure during several oscillates $\bar{p} \approx 0$.

(a) Verify that $\bar{\rho}_\varphi a^3 \sim \exp(-\Gamma\, t)$ is a solution of Eq. (2).

(b) Similar to Eq. (2), the equation of motion for the inflaton can be written as

$$\ddot{\varphi} + (3H + \Gamma)\dot{\varphi} + m^2\varphi = 0. \tag{3}$$

The decay of inflaton contributes an extra friction term. Verify that

$$\varphi \sim \frac{1}{a^{3/2}}\exp(-\frac{1}{2}\Gamma\, t)\cos(mt) \tag{4}$$

is a solution of Eq. (3).