Introduction to Cosmology

HW1

to be handed in until Friday noon, Oct. 30, 2014

Problem 1: Fun with Units

In cosmology many different branches of physics have applications and as a by-product, one often has to switches between units.

(a) Most common for theorists are natural, or Planckian units: based on dimensional analysis, construct a length l_{Pl}, mass m_{Pl}, time t_{Pl} and temperature T_{Pl} out of Newtons constant G, the speed of light c, Planck's constant $\hbar$ and Bolzmann's constant k_B.

(b) Since theorists are efficient, they usually set $\hbar \equiv c \equiv k_B \equiv 1$ and quote their final results in multiples of the Planck mass m_{pl}. To make comparisons with observations, one has to reverts to SI units at some point though. For example, if you are given an energy density of

$$\rho = m_{Pl}^4 \,, \tag{1}$$

what is its value in SI units?

(c) Astronomers are used to measure objects in the solar system and beyond; as a consequence, they like to use familiar astronomical objects to set their units, e.g. the mass of our sun $M_\odot$ as a unit for mass. What is the value of $M_\odot$ in Planckian units?

(d) A common distance unit is the light year, i.e. the distance light travels within one year. Roughly, over how many light year do you have to average the energy density in our universe today to arrive at a homogeneous Universe?

Problem 2: Hubbles Law (Mukhanov P.1.1)

Consider three arbitrary observers in space, A, B, C, and let $\vec{r}_{BA}$ denote the vector from A to B. In order for a general expansion law, $\vec{v} = \vec{f}(\vec{r}, t)$ with $\vec{v}$ the "velocity"[1], to be the same for all observers, the function $\vec{f}$ must be linear

$$\vec{f}(\vec{r}_{CA} - \vec{r}_{BA}, t) = \vec{f}(\vec{r}_{CA}, t) - \vec{f}(\vec{r}_{BA}, t) \,. \tag{2}$$

Based on isotropy, i.e. symmetry under rotations, show that the only solution to this equation is given by Hubbles law, that is

$$\vec{v}_{BA} = H(t)\vec{r}_{BA} \,, \tag{3}$$

[1] The use of a velocity in Hubbles law is only a good approximation for low redshifts and precludes, to some extend, the correct explanation of Hubbles observation via an expansion of space-time in general relativity (see subsequent lectures).

where $\vec{v}_{BA}$ is the "velocity" of B with respect to A.

Problem 3: Taken from "Physical Foundations of Cosmology" by Mukhanov

In chapter 1, do problems 1.2, 1.5, 1.6, 1.11 and 1.14.