Exercise sheet 3

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Exercise 1: Gravitational potential of a uniform sphere

The relation of acceleration of a gravitational potential is given by

$$\vec{a} = -G\vec{\nabla}\phi$$

The potential of an arbitrary spherical mass distribution is

$$\phi(r) = -\int_{r_0}^{r} d\zeta a(\zeta) = G\int_{r_0}^{r} d\zeta \frac{M(\zeta)}{\zeta^2}$$

where the enclosed mass is

$$M(r) = 4\pi \int_0^r d\zeta \zeta^2 \rho(\zeta)$$

Use this to calculate the gravitational potential of

a) an infinitesimal point with mass M

b) a uniform sphere of mass M and radius r

Exercise 2: Effective temperature of a star

Most of the light emitted by a star originates from the photosphere. In this region we can assume a local thermal equilibrium and hence a black body radiation.
A black body of a temperature T emits light in the frequency range from ν to ν +dν according to

$$B_\nu = \frac{2h\nu^3}{c^2} \cdot \frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

a) Calculate the radiant flux density emitted by the sun in the upper half-space F^+. Assume that $I_\nu = B_\nu$ is isotropic. It follows, that:

$$F^+ = \sigma T^4$$

Determine σ using constants of nature.

b) Determine the luminosity of the sun and the effective temperature. (Hint: $Ł_\odot$ 3.846 $\cdot 10^{26}$ W and $R_\odot$ = 6.96$\cdot 10^5$ km.)

c) Calculate the temperature of Venus and compare your result to the measured value of 737 K. (Hint: Distance Sun-Venus = 0.723 AE and the albedo of venus A_V=0.65.)

Exercise 3: Jeans Mass

a) Explain the concept of the Jeans mass and its importance for star formation. Consider two dense cores in a cloud of hydrogen ($\mu = 1.6 \times 10^{-27}\,\mathrm{kg}$): (i) a cool core with temperature $T = 10K$ and (ii) a warm core with $T = 100\,\mathrm{K}$. Taking the number density of hydrogen to be $n = 10\,\mathrm{cm}^{-3}$ in both cases, estimate the Jeans mass and radius of both cores.

b) Assume that the pre-collapse core is in hydrostatic equilibrium and can be treated as an isothermal sphere, i.e. a sphere of gas at constant temperature T where the supporting thermal pressure is given by $P = \rho kT/\mu m_H$. Express the density and mass of the sphere as a function of radius R and the isothermal speed of sound c_s of the gas.

c) Evaluate the rate of accretion $\dot{\mathrm{M}}$, and describe its dependence with radius.

d) Considering a cloud of mass M , radius R , and angular momentum J and uniformly rotating, express the rotational kinetic energy, considering a spherical configuration.

   Assuming the gravitational binding energy of the cloud $|U| = \dfrac{3GM^2}{5R}$, calculate the R_{min} reached by the collapsing cloud if the J is conserved. Express it in solar unities. For compactness: keep the notation in terms of the specific angular momentum J/M.

   Considering a cloud extending for a parsec (R = 1pc) with a rotational frequency Ω equal to the one of the Sun ($v_{rot} = 225\,\mathrm{km\,s^{-1}}, \mathrm{r} = 8.5\,\mathrm{kpc}$), evaluate the R_{min}. Compare it with the standard radius of a star $\mathrm{R} = 10^{11}$ cm.