Stellar Structure and Evolution
—Exercises—
Wolfgang Glatzel
astromundus, October 5-16, 2015

GEORG-AUGUST-UNIVERSITÄT
GÖTTINGEN

Part 1

1) An application of the equation for radiation transport

$$\cos\Theta \frac{dI_\nu}{d\tau_\nu} = I_\nu(\tau_\nu,\Theta) - S_\nu(\tau_\nu)$$

Prove that the intensity $I_\nu(\tau_\nu = 0,\Theta)$ of the sun at the position of the observer ($\tau_\nu = 0$) is given by:

$$I_\nu(0,\Theta) = \int_0^\infty S_\nu(\tau_\nu)\exp(-\tau_\nu/cos(\Theta))d(\tau_\nu/cos(\Theta))$$

Provide an intuitive interpretation of the result.

Hint: Use (with Θ fixed) $\tau_\nu/cos(\Theta)$ as integration variable.

2) Calculate the intensity of the radiation field at the observers position of the thermal emission of an interstellar cloud. Its diameter is negligible compared to its distance. Assume LTE and constant temperature within the cloud and consider the limit of long wavelenghts. Discuss the intensity as a function of the optical depth of the cloud. Consider the limit of small and large optical depth.

How does the intensity depend on frequency ν and temperature? What is obtained for an opacity κ of the form $\kappa \propto \nu^{-2}$? Sketch the result.

Hint: Consider the radiation transport equation for the geometry given. Integrate it with respect to the optical depth (make use of integrating factors) from the observer across the cloud. Assume that the cloud is not irradiated by an external source. Is there an intuitive interpretation of the result? Then make use of the assumption of LTE and constant temperature.

3) A possible form of the radiation transport equation in the diffusion approximation is given by:

$$-\frac{1}{\rho}\frac{dp_{rad}}{dr} = \frac{\kappa L}{4\pi r^2 c}$$

p_{rad} denotes the radiation pressure, ρ the density, L the luminosity, and κ the Rosseland mean of the (specific) opacity. If an object is to be in equilibrium, the modulus of the radiative acceleration has to be smaller than that of the gravitational acceleration. What is the consequence for the luminosity of an object? Is the condition satisfied for the sun? Assume $\kappa \approx 0.34 cm^2/g$ for the opacity.