Homework exercise sheet 2

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Exercise 1: Saha Equation

a) With fixed Temperature T, what does the Saha equation predict about the degree of ionization when the density of the gas (i.e. density of electrons) increases? Justify your answer by physical arguments.

b) In a gas/plasma that consists only of atomic hydrogen in equilibrium state (binding energy $\epsilon_H = 13.6\text{eV}$), we assume that the number of nucleons is conserved, i.e. $n_{i+1} + n_i = n$, with n the total number density. The degree of ionization $n_{i+1}/n = n_\mathrm{e}/n$ in this case can be expressed by a simple formula:

$$\frac{(n_{i+1}/n)^2}{1 - n_{i+1}/n} = \frac{4 \times 10^{-9}}{\rho} T^{3/2} e^{-\frac{1.6\times10^5}{T}} \qquad \text{[in cgs]}.$$

In the lecture, 6 components of the ISM are listed. Calculate the ionization degrees of those 6 components of the ISM under upper assumptions.

For what phase(s) of the ISM are these assumptions not justified and why?

Exercise 2: Planck's law

We consider a gas consisting of atoms with only two energy states $E_0 < E_1$, the number density of the atoms in the ground state (energy level ~ 0) is called n_0. Following Boltzmann, at a temperature T the number density in the excited state n_1 is described as follows:

$$\frac{n_0}{n_1} = e^{\frac{E_1 - E_0}{kT}}.$$

The transition between the two energy levels can emit and absorb photons with energy $h\nu = E_1 - E_0$. We can differentiate between the following three processes:

- Absorption:
$$\frac{dn_{0-1}}{dt} = \beta u(\nu) n_0 d\nu,$$
- Spontaneous emission:
$$\frac{dn_{1-0}}{dt} = \alpha n_1 d\nu,$$
- Stimulated emission:
$$\frac{dn_{1-0}}{dt} = \beta u(\nu) n_1 d\nu,$$

where α and β are constants for this transition and $u(\nu)$ is the spectral radiance of the photons in the gas.

In thermal equilibrium, absorption and emission have to be in balance. Show, that from this assumption Planck's law for the spectral radiance $u(\nu)$ can be derived. To eliminate the constants α and β, use either the Rayleigh-Jeans law or Wien's displacement law.

Exercise 3: Dark nebulae

If a light ray of intensity I_0 enters a cloud of dust, it undergoes the following attenuation:

$$\triangle m = -2.5 log(\frac{I}{I_0})$$
$$I = I_0 \cdot e^{-\kappa \cdot D}$$

Here, the attenuation coefficient κ is given by:

$$\kappa \approx 3n\sigma$$

where σ is the cross-section area of a dust particle, n is the number density and D the linear diameter of the cloud.

We know this dark nebula as "Coalsack Nebula". The distance to it is 170 pc, the apparent diameter is 3°. The size of the dust particles is on average 1μm and the extiction $\triangle m$ by the cloud is 1.8^m.

a) Calculate the particle density n. In comparison, what is the typical particle density in an ultra high vacuum?

b) Determine the mass density of the dark nebula, assuming the density of a single particle to be 1000 kg m^{-3}.

c) Calculate the total mass of the cloud in units of solar mass.