Homework exercise sheet 7

submission date: 30-01-2015

Exercise 1: Supernova Ia Light Curve

About 25 days after the maximum light, an average SN of type Ia shows the spectrum of a blackbody with an effective temperature of $6000 \pm 1000K$. The observed absorption lines of the shell's photosphere are found to be shifted by an additional 3.166% after correcting for the motion of the host galaxy and the expansion of the universe.

a) Calculate the luminosity and absolute bolometric magnitude of the SN shell 25 days after maximum light, which is typically reached 17 ± 3 days after the event.
Hint: Derive the velocity of the shell using the doppler shift. From the velocity you can derive the radius after 25 days.

b) Assuming that the brightness of an average type Ia SN declines at a rate of $0.065\,\mathrm{mag\,day}^{-1}$, calculate the absolute bolometric magnitude at maximum light.

Note: For both (a) and b)) exercises, try to give values for the lower and upper limit based on the temperature uncertainty.

Exercise 2: Neutron star

For a neutron star of $\mathrm{M} = 1.4\mathrm{M}_\odot$ and $\mathrm{R} = 10^4 m$, adopt the linearly decreasing density model $\rho(\mathrm{r}) = \rho_\mathrm{c}(1 - \frac{\mathrm{r}}{\mathrm{R}})$. Let the star consist solely of free neutrons which, in degeneracy, behave as electrons. Assume the star is supported solely by completely degenerate, nonrelativistic neutron pressure. Find the following:

a) the approximate central pressure,

b) the neutron number density n_n and mass density ρ_c at the center.

c) Take the degeneracy pressure and the pressure obtained from hydrostatic equilibrium. If you equate both, what would be the radius of the neutron star?

d) Find an approximate upper mass limit by requiring the Schwarzschild radius to equal the neutron star radius obtained in part (c), and the maximum rotation frequency that such a neutron star could reach.

Hint: the equation of state of a non relativistic degenerate electron gas is

$$\mathrm{P_e} = \frac{1}{20}\left(\frac{3}{\pi}\right)^{2/3}\frac{\mathrm{h}^2}{\mathrm{m_e}}\mathrm{n_e}^{5/3}.$$

Exercise 3: Polytropes

In general the system of basic stellar structure differential equations cannot be solved analytically. However, there exists a family of approximate solutions to the stellar structure that can be handled more easily. These are known as polytropes and are hypothetical stellar models in which the pressure is proportional to some power of the density:

$$P = K\rho^{\gamma} = K\rho^{(n+1)/n}$$

where K and $\gamma > 0$ are constants. The above equation is called polytropic equation of state and n is called polytropic index.
Such models can approximate the stellar structure in particular situations (for example for an adiabatic gas the pressure can be written as a function of density alone).

a) In which cases, under appropriate simplifying assumptions would a polytropic equation of state with index $n = 1.5$ provide a meaningful model for a stellar structure?

b) Show that polytropes with index $n = 3$ ("Eddington standard model") can be used to represent stars in radiative equilibrium.
Hint: Consider the contribution to total pressure due to both gas pressure and radiation pressure and use the following parametrization: $P_g = \beta P$ with $0 \leq \beta \leq 1$.