Introduction to Cosmology

HW4

to be handed in on Friday noon, Nov. 20, 2014

Problem 1: Problems in "Physical Foundations of Cosmology" by Mukhanov

In chapter 2, do problems 2.2, 2.3 and 2.4 and think about the physical meanings of particle horizon, "optical" horizon, Hubble horizon and event horizon.

Problem 2: Matter-radiation equality

At early time, the universe is dominated by the radiation. But the energy density of radiation scale as a^{-4}. It decrease faster than the dust matter whose energy density scale as a^{-3}. So at some time, the energy density of the radiation and the dust matter will become equal to each other: $\rho_m = \rho_r$. It is called the matter-radiation equality. After the matter-radiation equality, the universe will be dominated by the dust matter till very late time when the cosmological constant plays an important role and leads to the late accelerating of the universe.

(a) Assume the fractions of radiation and dust matter at present are Ω_r and Ω_m, respectively. Find the expression of redshift z_{eq} at matter-radiation equality. During the radiation-dominated era and the matter-dominated era, the contribution from cosmological constant can be neglected. Find the expression of Hubble parameter H_{eq} at matter-radiation equality.

(b) Assuming a spatial flat universe and neglecting the contribution from cosmological constant, show that the time of matter-radiation equality is given by

$$t_{eq} = \frac{4(\sqrt{2}-1)}{3H_{eq}}. \tag{1}$$

Hint:

$$\int \frac{x}{\sqrt{a+b\,x}}dx = \frac{2(-2\,a+b\,x)\sqrt{a+b\,x)}}{3\,b^2} \qquad (b \neq 0). \tag{2}$$

Problem 3: Equilibrium era

As we know, the temperature of ultra-relativistic matter (e.g. radiation) falls as $1/a$, but the temperature of non-relativistic matter falls as $1/a^2$. At early time of the universe, the radiation and baryonic matters are in equilibrium. Explain why the temperature decrease approximately as $1/a$. You can explain it qualitatively. Feel free to do some quantitive calculations.

Hint: In equilibrium, the entropy and the baryon number are conserved.