Introduction to Cosmology

HW3

to be handed in until Friday noon, Nov. 13, 2014

Problem 1: Distance measurement

In astronomy, a standard candle is a source that has a known luminosity. By comparing this know luminosity to the observed brightness, the distance to the object can be determined using the inverse square law. Explain why the Type Ia supernovae can be used as standard candles?

Problem 2: Luminosity distance

In flat spacetime, the distance to an object has a clear definition. But in cosmology, the universe is expanding and there are several different ways to define the distance depending on what problems we are considering and what measurements we are using. For very small redshifts (the object is close to us), the effects of cosmological expansion on the determination of distance can be neglected. But for larger redshifts, these effects will lead to differences between different definitions of distance. Two commonly used definition is the luminosity distance (have a look at Problem 1) and the angular diameter distance.

(a) What are the definitions of the luminosity distance and the angular diameter distance? What is the relation between them? Are they identical for the objects which are close to us?

(b) Considering a universe consisting of dust matter, radiation, cosmological constant and spatial curvature, derive the expressions of the luminosity distance and the angular diameter distance in terms of H_0, Ω_m, Ω_r, Ω_Λ, Ω_k and z. Here we have defined

$$\Omega_m = \frac{\rho_{m0}}{\rho_{cri}}, \quad \Omega_r = \frac{\rho_{r0}}{\rho_{cri}}, \quad \Omega_\Lambda = \frac{\Lambda}{8\pi G \rho_{cri}}, \quad \Omega_k = -\frac{3k}{8\pi G a_0^2 \rho_{cri}}. \tag{1}$$

as what we did in Problem 2 of the last homework. The critical density of the universe $\rho_{cri} = 3H_0^2/(8\pi G)$. You can use some of the conclusions in the last homework.

(c) Derive the luminosity distance for a cosmological constant dominated universe ($\Omega_\Lambda = 1$, $\Omega_m = \Omega_r = \Omega_k = 0$) and a matter dominated universe ($\Omega_m = 1$, $\Omega_\Lambda = \Omega_r = \Omega_k = 0$), respectively. Are the results consistent with the Hubble law $z = H_0 * d$? Explain why it is important to look at high-redshift observations to distinguish different universe models.

Problem 3: Conformal diagram

A conformal digram is a two-dimensional diagram that captures the causal relations between different points in spacetime. To derive the conformal digram, we need to perform a particular coordinate transformation to make sure that the infinity in the original coordinate is at finite in the new coordinate and the radial light cones are at 45°. For example, look at the Minkowski space

$$ds^2 = -dt^2 + dr^2 + r^2\, d\Omega^2. \tag{2}$$

Following the deducing in Appendix H of the book "Spacetime and Geometry" by Sean Carroll, under proper coordinate transformation, we arrive at

$$ds^2 = \omega^{-2}(T,R)\left(-dT^2 + dR^2 + \sin^2\, R\, d\Omega^2\right), \tag{3}$$

where

$$0 \leqslant R < \pi, \qquad |T| + R < \pi. \tag{4}$$

In the new coordinate (T, R), the equation for the radial light cone is similar to the one in the original coordinate $T = \pm R$. But T and R are all finite.

(a) Try to reproduce Eq. (3).

(b) Derive the equations for $t =$ constant and $r =$ constant in the new coordinate, respectively. Plot the boundary of the Minkowski space and the lines with constant t and constant r in the new coordinate.

(c) Think about how we can derive the conformal digram for FRW universe.