Dynamics of Lens Movement

Presentation towards a technical interview with Flanders Make

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Problem

Goal: Realize an actuation system for the following application:

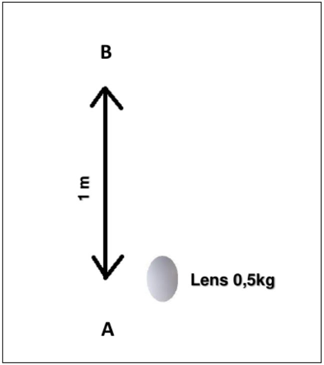

Figure: 1

Realize an actuation system for the following application.

A lens needs to be moved up and down between positions A and B (Fig. 1). The (vertical) distance between A and B is 1m. The lens needs to be standing still in point A for 0.2 seconds, has 0.2 seconds to move up to point B, should be standing still in point B for 0.2 seconds and has 0.2 seconds to move down again to point A. The trajectory followed for the up and down motion is free, only the stationary position in A and B for 0.2 seconds is critical. This motion has to be continuously repeated.

System Dynamics

- x_1 position above point A and x_2 is the velocity of the lens when at height x_1.

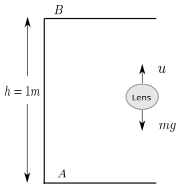

Figure: Free body diagram

- The dynamics of the lens is

$$m\ddot{x}_1 = -mg + u, \tag{1}$$

where u is the control input, and g is the acceleration due to gravity.

State-Space Representation

- The system dynamics in state space form is given as

$$\dot{x}(t) = Ax(t) + Bu(t) + Wg \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \qquad W = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \tag{3}$$

- Controllability: Yes , i.e.,

$$\mathrm{rank}(B|AB) = 2 \tag{4}$$

Control Objective

- Let the point A be the initial condition such that $x_0 = (0, 0)^\top$ and the reference point B as $x_f = (1, 0)^\top$.
- **Goal:** Design a minimum energy control gain to drive the lens from point A to point B.
- **Approach:** Use L_2-control (*minimum norm control*) to drive the lens from point A to point B.
- The minimum norm control can be understood as the finding a control that take minimum energy to drive the system from initial point to the final point.

Minimum Norm Control

- Consider the system dynamics

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{5}$$

- Let x_0 be the initial condition and x_f be the final state.
- The goal is to drive the system from state $x(t_0) = x_0$ to the state x_f in time T. i.e. $x(T) = x_f$ while minimizing the performance index of the form,

$$J = \frac{1}{2}\int_{t_0}^{T} \|u(t)\|^2 \, \mathrm{dt}\,. \tag{6}$$

- The minimum energy control for the desired control objective is[1]

$$u_{AB}^*(t) = (e^{A(T-t)}B)^* \left(\int_{t_0}^{T} (e^{A(T-s)}B)(e^{A(T-s)}B)^* \, \mathrm{ds} \right)^{-1} (x(T) - e^{A(T-t_0)}x_0). \tag{7}$$

[1] Chapter 8. minimum norm control. In Leigh, J., editor, Functional Analysis and Linear Control Theory, volume 156 of Mathematics in Science and Engineering, pages 79–102. Elsevier.

Minimum Energy Control

- For the dynamics of the lens movement:

$$\dot{x}(t) = Ax(t) + Bu(t) + Wg, \tag{8}$$

the minimal energy control to drive the state from $x_0 = x(0)$ to $x_f = x(T)$ while minimizing the performance index,

$$J = \frac{1}{2}\int_0^T \|u(t)\|^2 \,\mathrm{dt}\,. \tag{9}$$

is

$$\begin{aligned} u^*_{AB}(t) = (e^{A(T-t)}B)^* \left(\int_{t_0}^{T} (e^{A(T-s)}B)(e^{A(T-s)}B)^* \,\mathrm{ds} \right)^{-1} \times \\ \left(x(T) - e^{A(T-t_0)}x_0 - \int_{t_0}^{T} e^{A(T-s)}Wg \,\mathrm{ds} \right). \end{aligned} \tag{10}$$

Switching Mechanism

- Let $\sigma_k, k \in \mathbb{N}$ denote the switching instance for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. The switching patter in each switching instance is defined as follows,

$$u^{\sigma_k}(t) = \begin{cases} mg & \forall t \in [t_k, 0.2), \\ u^*_{AB}(t) & \forall t \in [t_k + 0.2, 0.4), \\ mg & \forall t \in [t_k + 0.4, 0.6), \\ u^*_{BA}(t) & \forall t \in [t_k + 0.6, 0.8). \end{cases} \tag{11}$$

Simulation Results

Position vs Time

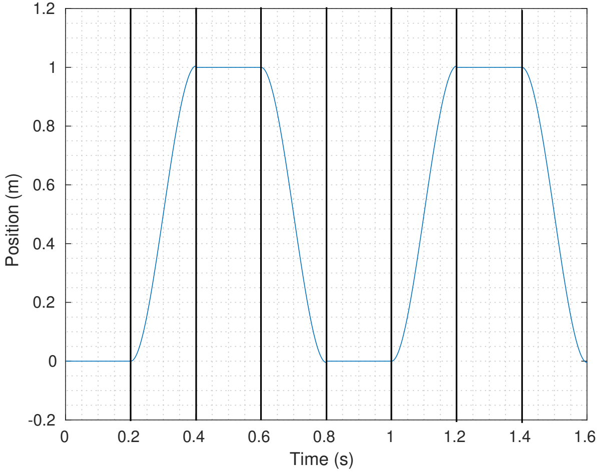

Figure: Plot showing the change of position with time.

Velocity vs Time and Control vs Time

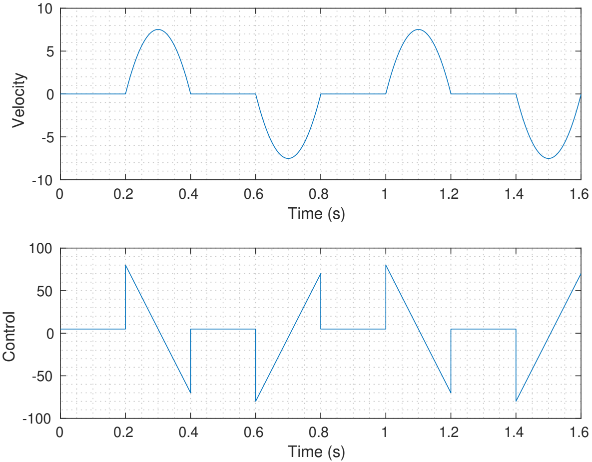

Figure: Control and Velocity vs Time.

Velocity vs Time and Control vs Time

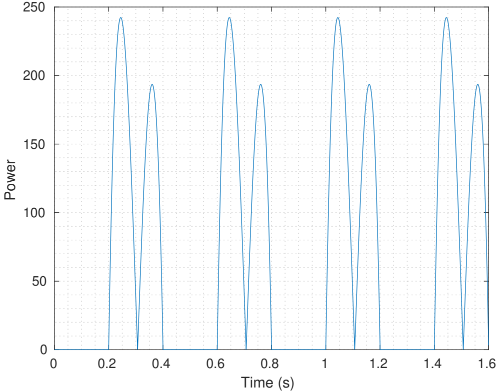

Figure: Power required at each instant. Power available form the electric source is $P = V * I = 3680W$.

Simulation for $T = 1$

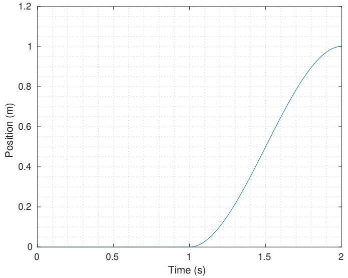

Figure: Plot showing the change of position with time. ($T = 1$ sec).

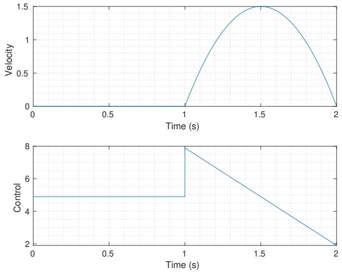

Figure: Control and Velocity vs Time ($T = 1$ sec).

Conclusion

- A minimum energy control approach to drive a system between two points is proposed
- The proposed model is validated by the simulation results.

Thank You!