Dynamics of Lens Movement

Bikash Adhikari, CRAN, University of Lorraine

April 20, 2022

Contents

1 Problem Statement: Lens

Goal: Realize an actuation system for the following application.

A lens needs to be moved up and down between positions A and B (Fig. 1). The (vertical) distance between A and B is 1m. The lens needs to be standing still in point A for 0.2 seconds, has 0.2 seconds to move up to point B, should be standing still in point B for 0.2 seconds and has 0.2 seconds to move down again to point A. The trajectory followed for the up and down motion is free, only the stationary position in A and B for 0.2 seconds is critical. This motion has to be continuously repeated.

Requirements:

- Energy efficiency is of utmost importance
- The system has to be powered via a standard net switch (230V, 16A, 50-60Hz, single faze).

2 Proposed Model

2.1 System Dynamics

The state of the lens can be described as

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{1}$$

where x_1 represents the height of the lens above the surface A and x_2 is the velocity of the lens. Also, let us assume the direction of the velocity to be positive while going from point A to point B and negative from point B to point A, a general convention to identify the direction of the movement.

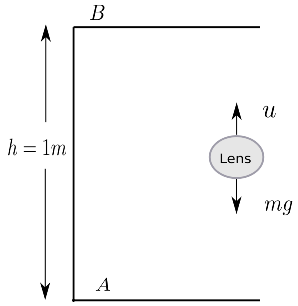


Figure 1: Free body diagram

Then the dynamics of the lens is as follows,

$$m\ddot{x}_1 = -mg + u \tag{2}$$

where u is the control input, g is acceleration due to gravity. This is equivalent to

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -g + \frac{1}{m}u \end{aligned} \tag{3}$$

Now, the system dynamics in state space form is given as

$$\dot{x} = Ax + Bu + Wg \tag{4}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \qquad W = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \tag{5}$$

Remark: The system is controllable i.e., $rank(B|AB) = 2$.

2.2 Control Design

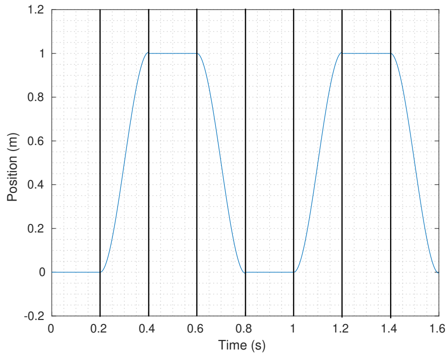
Let A be the initial point such that $x_0 = (0, 0)^\top$, and point B is the reference point such that $x_f = (1, 0)^\top$. Then the system described by the (4) is to be driven form an initial state x_0 to the desired stated x_f at time T while minimizing the performance index of the form,

$$J_{AB} = \frac{1}{2} \int_{t_0}^{T} \|u(t)\|^2 \, \mathrm{dt}\,. \tag{6}$$

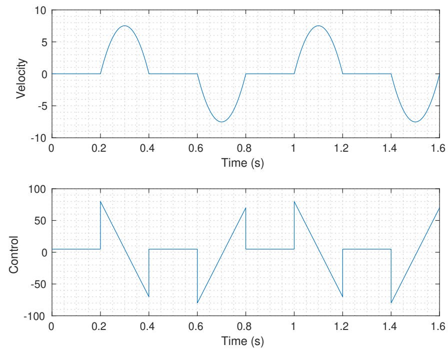
Then the minimum energy control required to drive the lens from point A to B in time T is given by the following expression[1],

$$u^*_{AB} = (e^{A(T-t)}B)^* \left(\int_{t_0}^{T} (e^{A(T-s)}B)(e^{A(T-s)}B)^* \, \mathrm{ds} \right)^{-1} \big(x(T) - e^{A(T-t_0)}x_0 - \int_{t_0}^{T} e^{A(T-s)} Wg \, \mathrm{ds} \big). \tag{7}$$

[1]Chapter 8. minimum norm control. In Leigh, J., editor, Functional Analysis and Linear Control Theory, volume 156 of Mathematics in Science and Engineering, pages 79–102. Elsevier.



(a) Plot showing the change of position with time.



(b) Control and Velocity vs Time.

Figure 2: Position, Control and Velocity vs Time

Similarly, the lens can be driven back to the point B from point B by changing the reference point. In other words, for the reverse motion, the initial point x_0 should be taken as $(1, 0)$ and the reference point x_f as $(0, 0)$.

2.3 Switching Mechanism

As we have the minimum energy control for driving the system from point A to point B, now we define the switching law to achieve the desired objective of to and fro motion of the lens between the two points. Since the movement of the lens needs to stop at point A and B for a fixed time, and it has to follow a to and fro motion, a switching mechanism is defined to achieve such objective.

Let $\sigma_k, k \in \mathbb{N}$ denote the switching instance for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. The switching patter in each switching instance is defined as follows,

$$u^{\sigma_k} = \begin{cases} mg & \forall t \in [t_k, 0.2), \\ u^*_{AB} & \forall t \in [t_k + 0.2, 0.4), \\ mg & \forall t \in [t_k + 0.4, 0.6), \\ u^*_{BA} & \forall t \in [t_k + 0.6, 0.8). \end{cases} \tag{8}$$

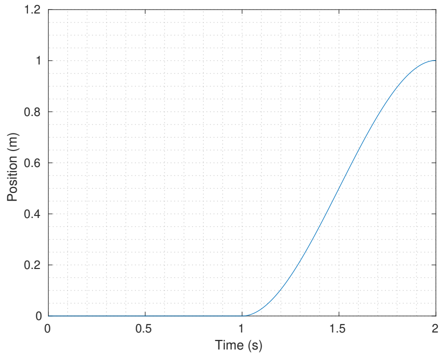
Finally, the system dynamics in the presence of the switching mechanism (18) takes the following form,

$$\dot{x}(t) = Ax + Bu^{\sigma_k} + Wg, \qquad \forall k \in \mathbb{N}. \tag{9}$$

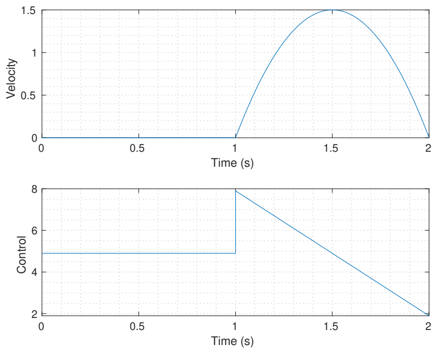
3 Simulation

In the following, the simulation results are presented. In Figure 2a, the change of the position with respect to time is plotted. We can see that the desired objective is achieved. The lens stays in point A for 0.2 sec, then moves to point B in 0.2 sec, and returns to the point A in 0.2 sec.

Then, in figure 2b, the plot shows the control and velocity versus time. It shows that the velocity is zero for the initial time when the constant control $u_1 = mg$ is applied. Then the velocity increases when the control is positive and decreases when the control is negative.



(a) Plot showing the change of position with time. ($T = 1$ sec).



(b) Control and Velocity vs Time ($T = 1$ sec).

Figure 3: Position, Control and Velocity vs Time at $T = 1$.

In figure 3a and 3b, it is shown that if the time required to move from point A to B is increased, then the control effort required also decreases. The comparison between figure 2b and 3b shows that the control effort required to move from point A to point B is high when the time allowed is 0.2 sec compared to the second case when the time of 1 sec is allowed.

Remark: The simulation results are reproducible and the MATLAB code for the simulation can be found https://github.com/bikas3121/Dynamicsoflensmovement.git.

4 Conclusion

The control input is designed for the system (4) to drive it from point A to point B. The control effort used minimum energy to drive the system from the given state to the desired state using minimum norm control. Then a switching law is defined to obtain the to and fro motion of the lens with rest in between. Thus combining both control and switching law, the desired objective of controlling the motion of the lens using the minimum energy is achieved.

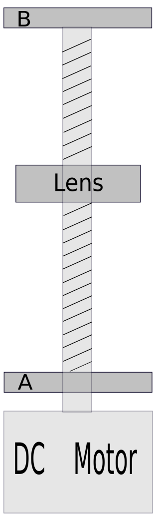
A Appendix: Dynamics of the lens actuated by a motor and a ball-screw

Let us suppose the lens is mounted on a feed-drive system that is actuated by the DC motor as shown in the figure 4a. Let us consider the system to be actuated by the DC motor with a ball-screw that change the rotational motion to the linear motion. Let $i_a(t)$ is the current, $\omega(t)$ the angular velocity of the rotor and $\theta(t)$ is the angular position.

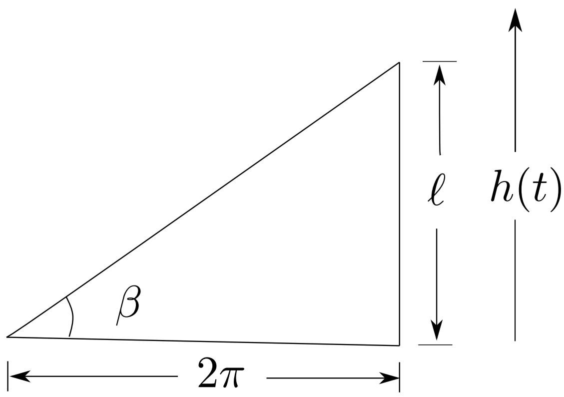
$$
\begin{aligned}
\dot{\theta}(t) &= \omega(t) \\
\dot{\omega}(t) &= \frac{K}{J_m} i_a(t) - \frac{B_m}{J_m}\omega(t) \\
\dot{i}_a(t) &= -\frac{R_a}{L_a} i_a(t) - \frac{K}{L_a}\omega(t) + \frac{1}{L_a}u(t)
\end{aligned}
\tag{10}
$$

where R_a is the armature resistance, L_a is the inductance, B_m is the frictional coefficient and K is the motor constant. The relation between the angular advance caused by the motor and the linear advance after the ball screw can be understood as below. Let β be the angle of the ball screw, l represent the step of the lead, and h represent the linear advance. Then the relation between the angular and linear motion is[2]

$$h(t) = \frac{l}{2\pi}\theta(t). \tag{11}$$



(a) Linear ball-screw feed drive actuated by the DC motor.



(b) Relation between angular and linear movement.

Figure 4

Thus, we have,

$$\theta(t) = \frac{2\pi}{l}h(t) =: Ph(t). \tag{12}$$

[2]Kim, Min-Seok & Chung, Sung-Chong. (2005). A systematic approach to design high-performance feed drive systems. International Journal of Machine Tools and Manufacture. 45. 1421-1435. 10.1016/j.ijmachtools.2005.01.032.

and in the similar manner, we also have,

$$v(t) := \dot{h}(t) = \frac{\omega(t)}{P} \tag{13}$$

Let $x_1 = h(t)$, $x_2 = v(t)$, and $x_3 = i_a(t)$, then the dynamics of the system takes the following form[3],

$$\begin{aligned} \dot{h}(t) &= v(t) \\ \dot{v}(t) &= -\frac{B_m}{J_m}v(t) + \frac{P*K}{J_m}i_a(t) \\ \dot{i}_a(t) &= -\frac{K}{P*L_a}v(t) - \frac{R_a}{L_a}i_a(t) + \frac{1}{L_a}u(t) \end{aligned} \tag{14}$$

The state space representation is given by,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{15}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B_m}{J_m} & \frac{P*K}{J_m} \\ 0 & -\frac{K}{P*L_a} & -\frac{R_a}{L_a} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

Remark: The system is controllable i.e, $rk(B|AB|A^2B) = 3$

Now the minimum energy control design follows from as in the section 2. Let us define the objective function as follows,

$$J_1 = \frac{1}{2}\int_{t_0}^{T} \|u(t)\|^2 \,\mathrm{dt}\,. \tag{16}$$

Let the point A be the initial point such that $x_0 = (0,0,0)^\top$ and the final point $B = (1,0,0)^\top$. Then the minimum energy control required to drive the lens from point A to B in time T is as follows,

$$u^*_{AB} = (e^{A(T-t)}B)^*\left(\int_{t_0}^{T}(e^{A(T-s)}B)(e^{A(T-s)}B)^*\,\mathrm{ds}\right)^{-1}\big(x(T) - e^{A(T-t)}x_0\big). \tag{17}$$

Similarly, the lens can be driven back to the point A from point B by changing the point of reference. Moreover, in physical system the reverse motion can be generated by the change of polarity in the DC motors.

A.1 Switching Mechanism:

Since the movement of the lens need to stop at point A and B for a fixed time and the also it has to follow a to and fro motion, a switching mechanism is defined to achieve such objective.

[3]E. D. Ruiz-Rojas, J. L. Vazquez-Gonzalez, R. Alejos-Palomares, A. Z. Escudero-Uribe and J. R. Mendoza-Vázquez, "Mathematical Model of a Linear Electric Actuator with Prosthesis Applications," 18th International Conference on Electronics, Communications and Computers (conielecomp 2008), 2008, pp. 182-186, doi: 10.1109/CONIELECOMP.2008.29.

Let $\sigma_k, k \in \mathbb{N}$ denote the switching instance for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. The switching patter in each switching instance is defined as follows,

$$u^{\sigma_k}(t) = \begin{cases} 0 & \forall t \in [t_k, 0.2), \\ u^*_{AB}(t) & \forall t \in [t_k + 0.2, 0.4), \\ 0 & \forall t \in [t_k + 0.4, 0.6), \\ u^*_{BA}(t) & \forall t \in [t_k + 0.6, 0.8). \end{cases} \tag{18}$$

Finally, the system dynamics (15) takes the following form in the presence of switching mechanism (18),

$$\dot{x}(t) = Ax(t) + Bu^{\sigma_k}(t). \tag{19}$$

A.2 Conclusion

For the given parameters of the DC motor and the lead screw, the minimum energy control input for the linear drive system (15) can be obtained using the expression (17). It is worth noting that the control designed in Section 2 achieves the desired objective using minimum effort, and the control input (17) for the dynamics (15) will be proportional to such minimum energy control.