Simulation Results

To demonstrate and compare the numerical results the following budget allocation strategies are adopted:
Strategy 1: Optimal Budget Allocation, where budget is allocated to the agents depending on agent's influence power
Strategy 2: Uniform Budget Allocation, where budget is allocated uniformly to all the agents (positive to the conformists and negative to the contrarians) and
Strategy 3: Positive Budget Allocation, where budget is allocated uniformly to all the agents (even the contrarians are influenced to have the positive opinion).
The network structure we consider is the strongly connected directed graph with 10 nodes (N=10) and the number of conformists is greater than that of the contrarians ($|V|^+ > |V|^-$). The strongly connected graph is represented in Figure 1. The initial opinion are uniformly chosen between $[-1, 1]$ and represented in the Table 1.

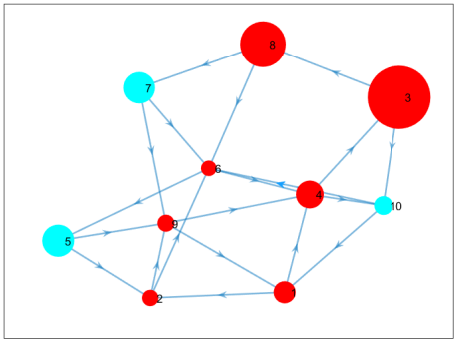

Figure 1: Strongly Connected Graph with 10 nodes, where red and cyan represents the conformists and contrarians, respectively.

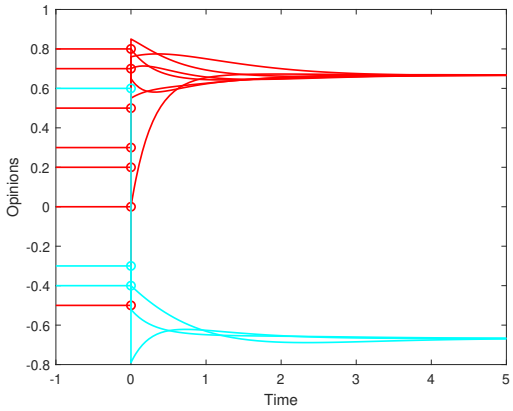

Figure 2: Dynamics of the conformists (red) and contrarians (cyan) agents.

In the Figure 1, the red and cyan nodes represents the conformist and the contrarian agents, respectively and the size of the nodes represents the agent's centrality. The centrality of all agents are also represented in Table 1. Also, in the table, the positive and negative value of c represent the conformist and contrarian agents, respectively. Considering the total available budget $B = 4$ and the maximum budget that can be allocated to each agent $\bar{u} = 0.7$ the initial cost (which is the cost before the budget allocation) is 0.8877. Using strategy 1, budget allocation is performed based on the influence power of an agent γ (represented in Table 1) while satisfying the budget constraints. The budget of [0.7, 0.7, 0.7, 0.7, 0.7, 0.5] is allocated to the agents [8, 3, 10, 9, 5, 4] and the resultant cost is 0.7331.

Agents	1	2	3	4	5	6	7	8	9	10
c	1	1	1	1	-1	1	-1	1	1	-1
Initial Opinion	0.8	0.7	0.5	0.3	-0.3	0	-0.4	0.2	-0.5	0.6
Centrality	0.075	0.054	0.224	0.095	-0.111	0.052	-0.108	0.160	0.057	-0.064
Gamma (γ)	0.128	0.112	0.101	0.085	0.078	0.067	0.065	0.052	0.016	0.015
Budget Allocation Sequence	8	3	10	9	5	4	7	6	2	1

Table 1: Data

Hence, to better understand the advantage of the designed market strategy (strategy 1), the results obtained are compared with the ones obtained from strategy 2 and 3. Using strategy 2, the total available

budget $B = 4$ is allocated uniformly among N agents and the resultant cost is 0.7728. Next, with strategy 3, the positive budget is uniformly allocated to all the agents and the cost obtained is 0.8646.

Finally, we consider the larger problem with 300 nodes, with 279 conformist and 21 contrarian agents. Of the 300 nodes, 25 are randomly chosen as popular nodes which has between 30 and 60 targets while the remaining non-popular nodes has between 1 and 29 targets. The initial opinion of the agents are uniformly distributed in $[-1, 1]$ which is generated using the formula $x_i(0) = -1 + \frac{2i}{N}$. As seen in Figure

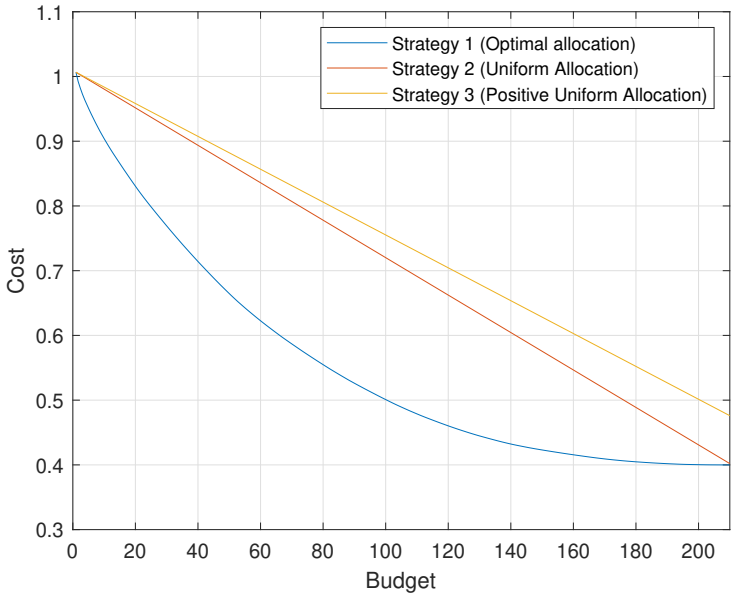

Figure 3: Cost vs the Budget.

3, the cost before budget-allocation ($B = 0$) is same for all strategies i.e., $J = 1$. It is clear from the plot that the cost function is minimum at every instance with the budget allocation using strategy 1 compared to the other two. When the total budget is allocated to every agent while satisfying the budget constraints, the minimum cost obtained with strategy 1 is ($J = 0.4$).

As for the uniform budget allocation (strategy 2) the cost obtained is always greater than the optimal cost, except for the case when the maximum possible budget is available for all agents, under budget constraints. Furthermore, if the maximum possible budget is available for all agents then it coincide with the optimal budget allocation and the cost function reach the same optimal value ($J = 0.4$). However, due to positive budget allocation the cost is significantly higher ($J = 0.48$) then the optimal case and it is because the contrarians are influenced to have the positive opinion. Hence, with the results obtained we can conclude that the strategy 1 is superior to the others.