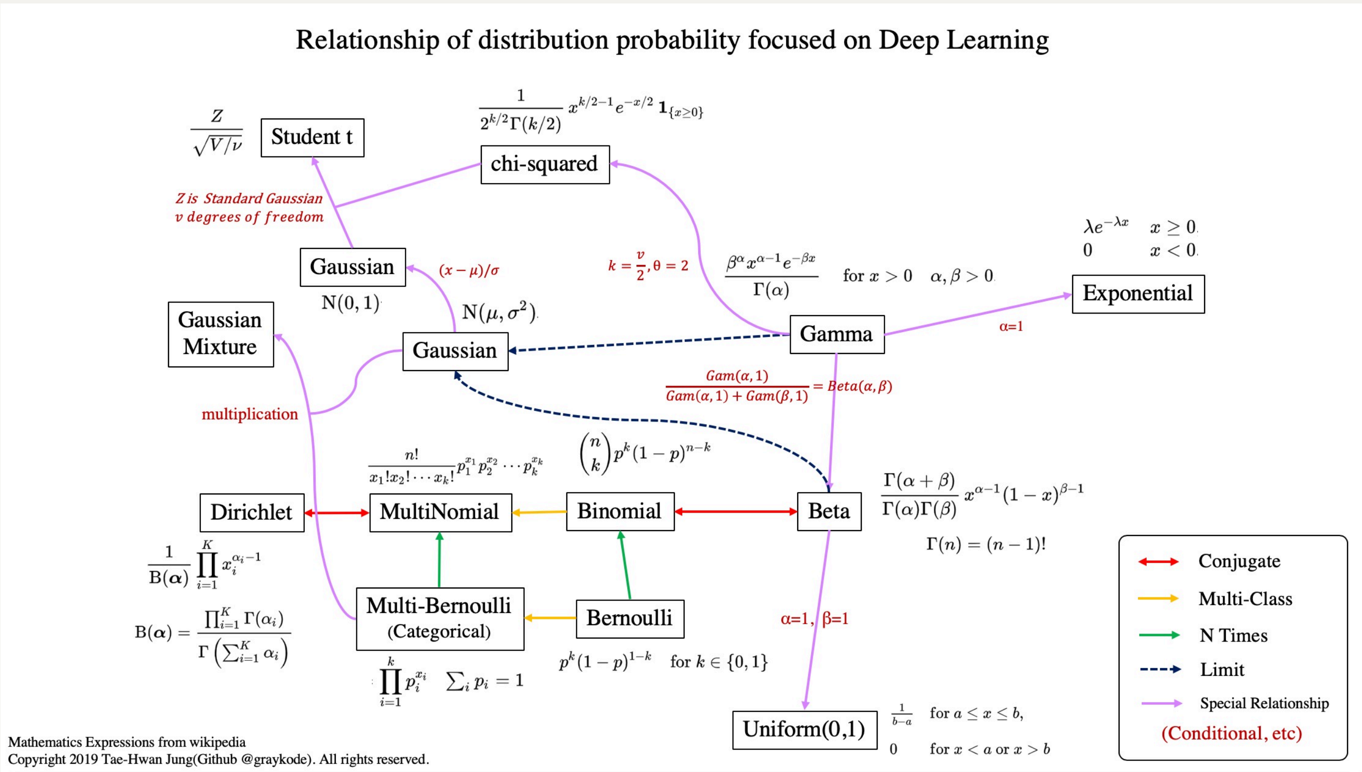
Distribution Is All U Need

[distribution-is-all-you-need] is the basic distribution probability tutorial for **most common distribution focused on Deep learning** using the python library.

For the convenience of copying code from this PDF, you can go to this repo to find the code, which was reformulated by Eren, Zhao.

This article is originally written with Jupiter notebook, which is truly awesome that I have long been looking down upon. I regret!

Overview of distribution probability



- `conjugate` means it has the relationship of **conjugate distributions**.

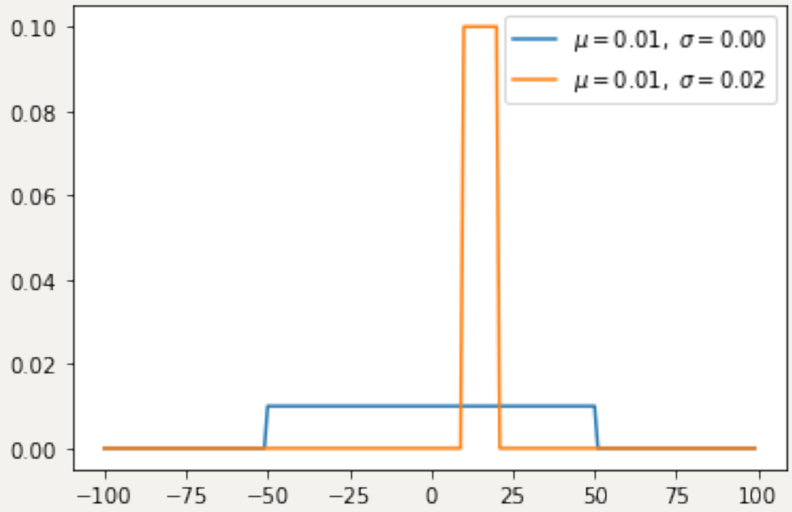
> *In Bayesian probability theory, if the posterior distributions $p(\theta \mid x)$ are in the same probability distribution family as the prior probability distribution $p(\theta)$, the prior and posterior are then called **conjugate distributions,** and the prior is called a **conjugate prior** for the likelihood function.*
> *Conjugate prior, wikipedia*

- `Multi-Class` means that Random Varivance are more than 2.
- `N Times` means that we also consider prior probability P(X).
- To learn more about probability, I recommend reading [pattern recognition and machine learning, Bishop 2006].

distribution probabilities and features

1. **Uniform distribution(continuous)**
    - Uniform distribution has same probaility value on [a, b], easy probability.

```python
import numpy as np
from matplotlib import pyplot as plt

def uniform(x, a, b):

    y = [1 / (b - a) if a <= val and val <= b
                    else 0 for val in x]

    return x, y, np.mean(y), np.std(y)

x = np.arange(-100, 100) # define range of x
for ls in [(-50, 50), (10, 20)]:
    a, b = ls[0], ls[1]
    x, y, u, s = uniform(x, a, b)
    plt.plot(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f$' % (u, s))

plt.legend()
plt.show()
```

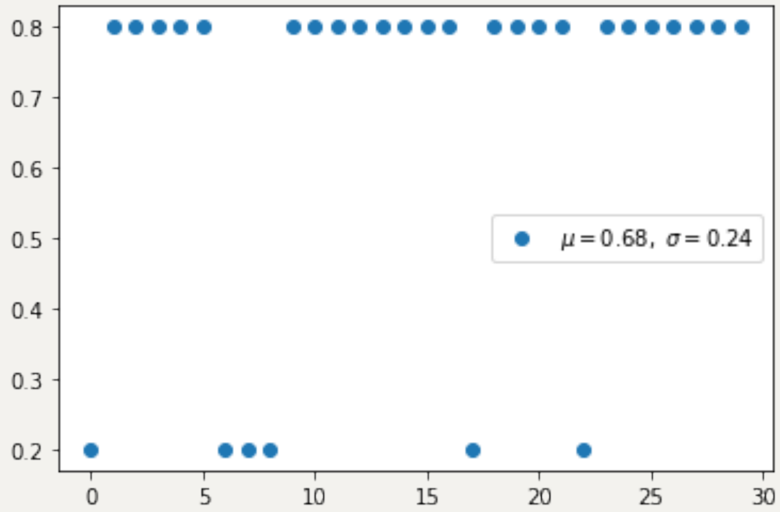


2. **Bernoulli distribution(discrete)**

- Bernoulli distribution is not considered about prior probability P(X). Therefore, if we optimize to the maximum likelihood, we will be vulnerable to overfitting.
- We use **binary cross entropy** to classify binary classification. It has same form like taking a negative log of the bernoulli distribution.

```
import random
import numpy as np
from matplotlib import pyplot as plt

n_experiment = 30
p = 0.8

def bernoulli(p, k):
    return p if k <= n_experiment * p else 1 - p

x = np.arange(n_experiment)
y = []
for _ in range(n_experiment):
    pick = bernoulli(p, k=random.randint(0, n_experiment + 1))
    y.append(pick)

u, s = np.mean(y), np.std(y)
```

```python
plt.scatter(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f$' % (u, s))
plt.legend()
plt.show()
```

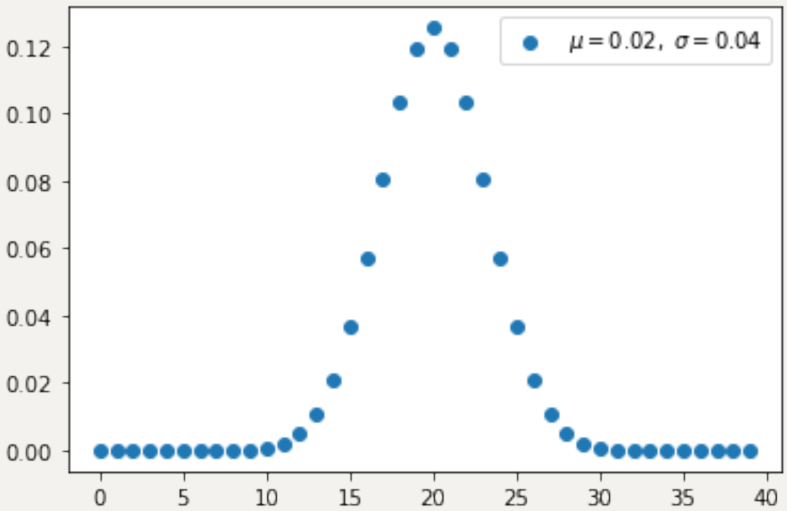


3. **Binomial distribution(discrete)**,

- Binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments.
- Binomial distribution is distribution considered prior probaility by specifying the number to be picked in advance.

```python
import numpy as np
from matplotlib import pyplot as plt

import operator as op
from functools import reduce

def const(n, r):
    r = min(r, n-r)
    numer = reduce(op.mul, range(n, n-r, -1), 1)
    denom = reduce(op.mul, range(1, r+1), 1)
    return numer / denom

def binomial(n, p):
```

```
    q = 1 - p
    y = [const(n, k) * (p ** k) * (q ** (n-k)) for k in range(n)]
    return y, np.mean(y), np.std(y)

for ls in [(0.5, 40)]:
    p, n_experiment = ls[0], ls[1]
    x = np.arange(n_experiment)
    y, u, s = binomial(n_experiment, p)
    plt.scatter(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f$' % (u, s))

plt.legend()
plt.show()
```

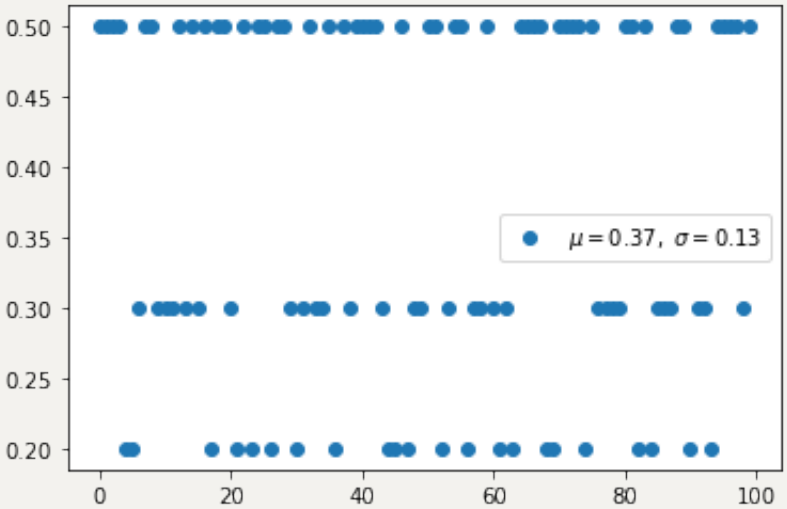


4. **Multi-Bernoulli distribution, Categorical distribution(discrete)**

- Multi-bernoulli called categorical distribution, is a probability expanded more than 2.
- **cross entopy** has same form like taking a negative log of the Multi-Bernoulli distribution.

```
import random
import numpy as np
from matplotlib import pyplot as plt

```

```
n_experiment = 100
p = [0.2, 0.3, 0.5]
x = np.arange(n_experiment)
y = []

def categorical(p, k):
    if k <= n_experiment * p[0]:
        return p[0]
    elif k <= n_experiment * (p[0] + p[1]):
        return p[1]
    else:
        return p[2]


for _ in range(n_experiment):
    pick = categorical(p, k=random.randint(0, 100))
    y.append(pick)

u, s = np.mean(y), np.std(y)
plt.scatter(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f$' % (u, s))
plt.legend()
plt.show()
```

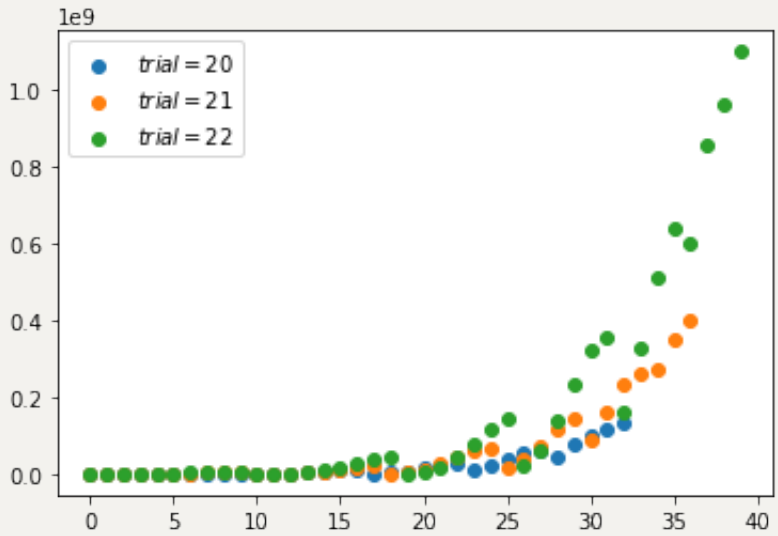


5. **Multinomial distribution(discrete)**

- The multinomial distribution has the same relationship with the categorical distribution as the relationship between Bernoull and Binomial.

```python
import numpy as np
from matplotlib import pyplot as plt

import operator as op
from functools import reduce

def factorial(n):
    return reduce(op.mul, range(1, n + 1), 1)

def const(n, a, b, c):
    """
        return n! / a! b! c!, where a+b+c == n
    """
    assert  a + b + c == n

    numer = factorial(n)
    denom = factorial(a) * factorial(b) * factorial(c)
    return numer / denom

def multinomial(n):
    """
    :param x : list, sum(x) should be `n`
    :param n : number of trial
    :param p: list, sum(p) should be `1`
    """
    # get all a,b,c where a+b+c == n, a<b<c
    ls = []
    for i in range(1, n + 1):
        for j in range(i, n + 1):
            for k in range(j, n + 1):
                if i + j + k == n:
                    ls.append([i, j, k])

    y = [const(n, l[0], l[1], l[2]) for l in ls]
    x = np.arange(len(y))
    return x, y, np.mean(y), np.std(y)
```

```
for n_experiment in [20, 21, 22]:
    x, y, u, s = multinomial(n_experiment)
    plt.scatter(x, y, label=r'$trial=%d$' % (n_experiment))

plt.legend()
plt.show()
```

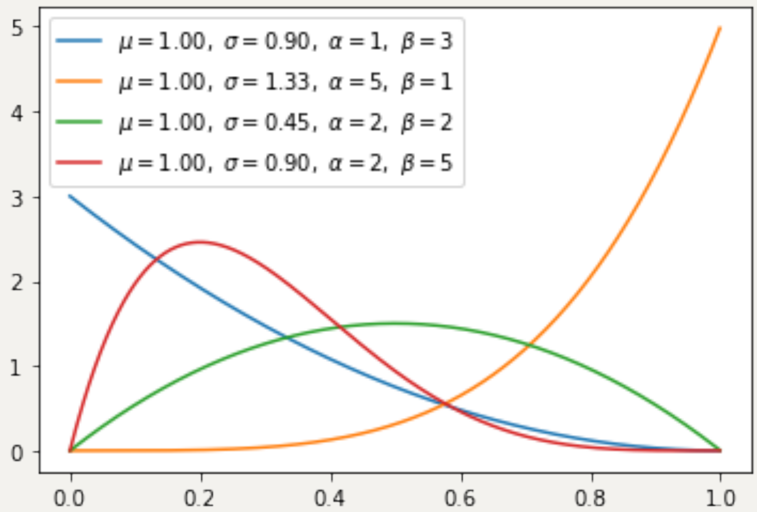


6. **Beta distribution(continuous)**

- Beta distribution is conjugate to the binomial and Bernoulli distributions.
- Using conjucation, we can get the posterior distribution more easily using the prior distribution we know.
- Uniform distiribution is same when beta distribution met special case(alpha=1, beta=1).

```
import numpy as np
from matplotlib import pyplot as plt

def gamma_function(n):
    cal = 1
    for i in range(2, n):
        cal *= i
    return cal
```

```python

def beta(x, a, b):

    gamma = gamma_function(a + b) / \
            (gamma_function(a) * gamma_function(b))
    y = gamma * (x ** (a - 1)) * ((1 - x) ** (b - 1))
    return x, y, np.mean(y), np.std(y)

for ls in [(1, 3), (5, 1), (2, 2), (2, 5)]:
    a, b = ls[0], ls[1]

    # x in [0, 1], trial is 1/0.001 = 1000
    x = np.arange(0, 1, 0.001, dtype=np.float64)
    x, y, u, s = beta(x, a=a, b=b)
    plt.plot(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f,'
                         r'\ \alpha=%d,\ \beta=%d$' % (u, s, a, b))
plt.legend()
plt.show()
```

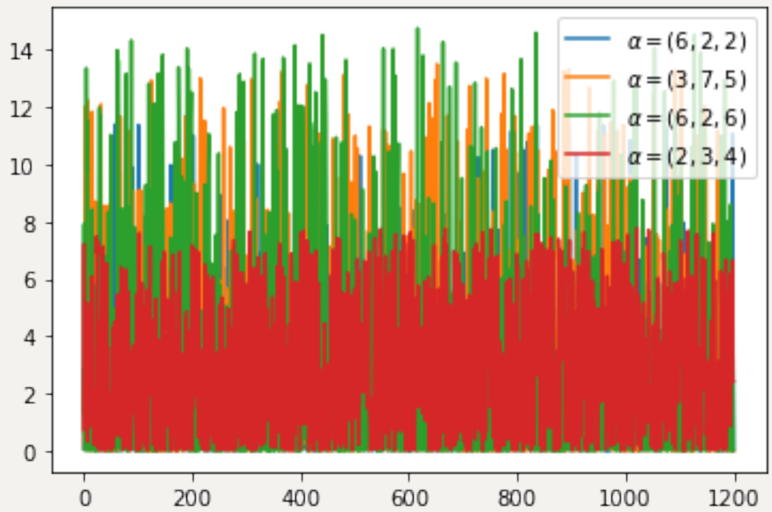


7. **Dirichlet distribution(continuous)**

- Dirichlet distribution is conjugate to the MultiNomial distributions.
- If k=2, it will be Beta distribution.

```python
from random import randint
import numpy as np
from matplotlib import pyplot as plt

def normalization(x, s):
    """
    :return: normalizated list, where sum(x) == s
    """
    return [(i * s) / sum(x) for i in x]

def sampling():
    return normalization([randint(1, 100),
            randint(1, 100), randint(1, 100)], s=1)

def gamma_function(n):
    cal = 1
    for i in range(2, n):
        cal *= i
    return cal

def beta_function(alpha):
    """
    :param alpha: list, len(alpha) is k
    :return:
    """
    numerator = 1
    for a in alpha:
        numerator *= gamma_function(a)
    denominator = gamma_function(sum(alpha))
    return numerator / denominator

def dirichlet(x, a, n):
    """
    :param x: list of [x[1,...,K], x[1,...,K], ...], shape is (n_trial, K)
    :param a: list of coefficient, a_i > 0
    :param n: number of trial
    :return:
    """
```

```python
    c = (1 / beta_function(a))
    y = [c * (xn[0] ** (a[0] - 1)) * (xn[1] ** (a[1] - 1))
         * (xn[2] ** (a[2] - 1)) for xn in x]
    x = np.arange(n)
    return x, y, np.mean(y), np.std(y)

n_experiment = 1200
for ls in [(6, 2, 2), (3, 7, 5), (6, 2, 6), (2, 3, 4)]:
    alpha = list(ls)

    # random samping [x[1,...,K], x[1,...,K], ...], shape is (n_trial, K)
    # each sum of row should be one.
    x = [sampling() for _ in range(1, n_experiment + 1)]

    x, y, u, s = dirichlet(x, alpha, n=n_experiment)
    plt.plot(x, y, label=r'$\alpha=(%d,%d,%d)$' % (ls[0], ls[1], ls[2]))

plt.legend()
plt.show()
```



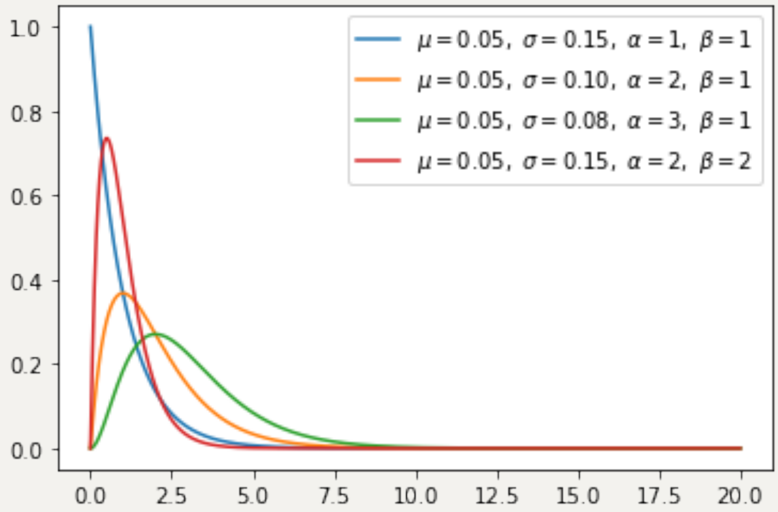
8. **Gamma distribution(continuous)**

    - Gamma distribution will be beta distribution, if `Gamma(a,1) /`

`Gamma(a,1) + Gamma(b,1)` is same with `Beta(a,b)`.

- The exponential distribution and chi-squared distribution are special cases of the gamma distribution.

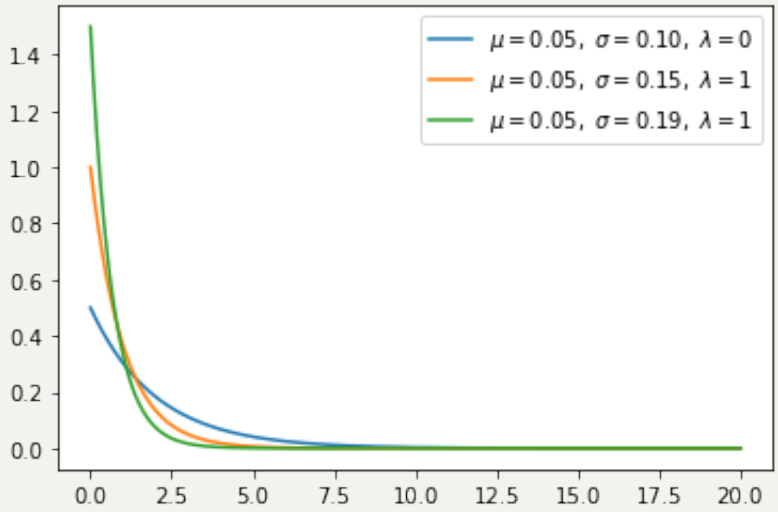
```
import numpy as np
from matplotlib import pyplot as plt

def gamma_function(n):
    cal = 1
    for i in range(2, n):
        cal *= i
    return cal

def gamma(x, a, b):
    c = (b ** a) / gamma_function(a)
    y = c * (x ** (a - 1)) * np.exp(-b * x)
    return x, y, np.mean(y), np.std(y)

for ls in [(1, 1), (2, 1), (3, 1), (2, 2)]:
    a, b = ls[0], ls[1]

    x = np.arange(0, 20, 0.01, dtype=np.float64)
    x, y, u, s = gamma(x, a=a, b=b)
    plt.plot(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f,'
                         r'\ \alpha=%d,\ \beta=%d$' % (u, s, a, b))
plt.legend()
plt.show()
```



9. **Exponential distribution(continuous)**

- Exponential distribution is special cases of the gamma distribution when alpha is 1.

```
import numpy as np
from matplotlib import pyplot as plt

def exponential(x, lamb):
    y = lamb * np.exp(-lamb * x)
    return x, y, np.mean(y), np.std(y)

for lamb in [0.5, 1, 1.5]:

    x = np.arange(0, 20, 0.01, dtype=np.float64)
    x, y, u, s = exponential(x, lamb=lamb)
    plt.plot(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f,'
                        r'\ \lambda=%d$' % (u, s, lamb))
plt.legend()
plt.show()
```

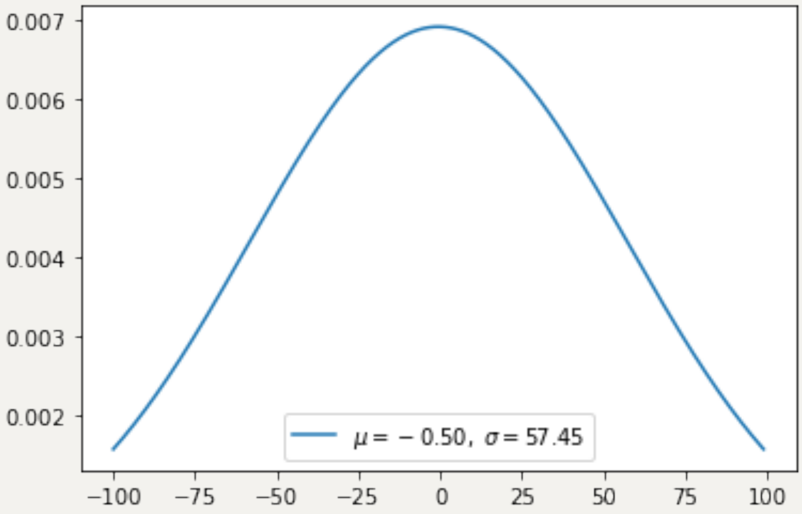


10. **Gaussian distribution(continuous)**

- Gaussian distribution is a very common continuous probability distribution
has heavier tails, meaning that it is more prone to producing values that fall far from its mean.

```
import numpy as np
from matplotlib import pyplot as plt

def gaussian(x, n):
    u = x.mean()
    s = x.std()

    # divide [x.min(), x.max()] by n
    x = np.linspace(x.min(), x.max(), n)

    a = ((x - u) ** 2) / (2 * (s ** 2))
    y = 1 / (s * np.sqrt(2 * np.pi)) * np.exp(-a)

    return x, y, x.mean(), x.std()

x = np.arange(-100, 100) # define range of x
x, y, u, s = gaussian(x, 10000)

plt.plot(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f$' % (u, s))
```

```
plt.legend()
plt.show()
```

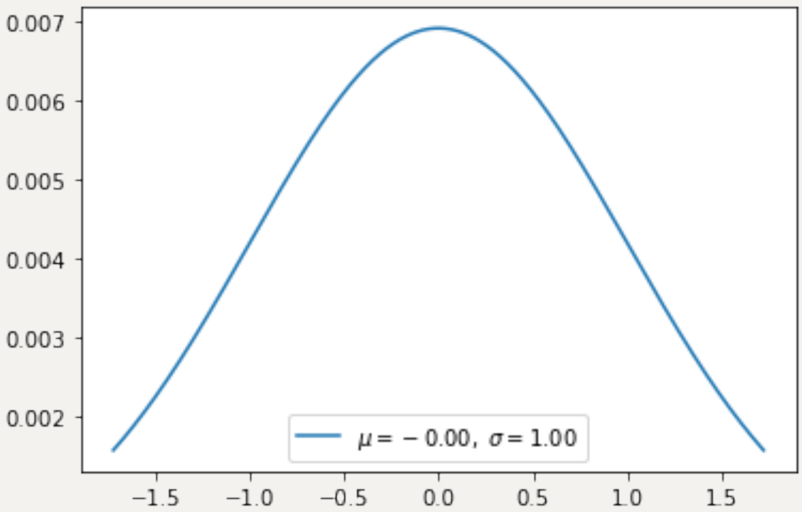


11. **Normal distribution(continuous)**

- Normal distribution is standarzed Gaussian distribution, it has 0 mean and 1 std.

```
import numpy as np
from matplotlib import pyplot as plt

def normal(x, n):
    u = x.mean()
    s = x.std()

    # normalization
    x = (x - u) / s

    # divide [x.min(), x.max()] by n
    x = np.linspace(x.min(), x.max(), n)

    a = ((x - 0) ** 2) / (2 * (1 ** 2))
    y = 1 / (s * np.sqrt(2 * np.pi)) * np.exp(-a)

    return x, y, x.mean(), x.std()
```

```

x = np.arange(-100, 100) # define range of x
x, y, u, s = normal(x, 10000)

plt.plot(x, y, label=r'$\mu=%.2f,\ \sigma=%.2f$' % (u, s))
plt.legend()
plt.show()
```

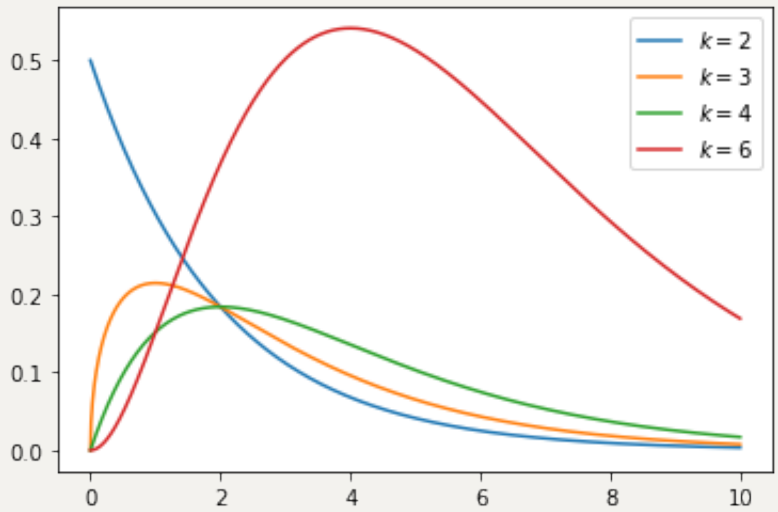


12. **Chi-squared distribution(continuous)**

- Chi-square distribution with k degrees of freedom is the distribution of a sum of the squares of k independent standard normal random variables.
- Chi-square distribution is special case of Beta distribution

```
import numpy as np
from matplotlib import pyplot as plt

def gamma_function(n):
    cal = 1
    for i in range(2, n):
        cal *= i
    return cal

def chi_squared(x, k):
```

```
    c = 1 / (2 ** (k/2)) * gamma_function(k//2)
    y = c * (x ** (k/2 - 1)) * np.exp(-x /2)

    return x, y, np.mean(y), np.std(y)

for k in [2, 3, 4, 6]:
    x = np.arange(0, 10, 0.01, dtype=np.float64)
    x, y, _, _ = chi_squared(x, k)
    plt.plot(x, y, label=r'$k=%d$' % (k))

plt.legend()
plt.show()
```

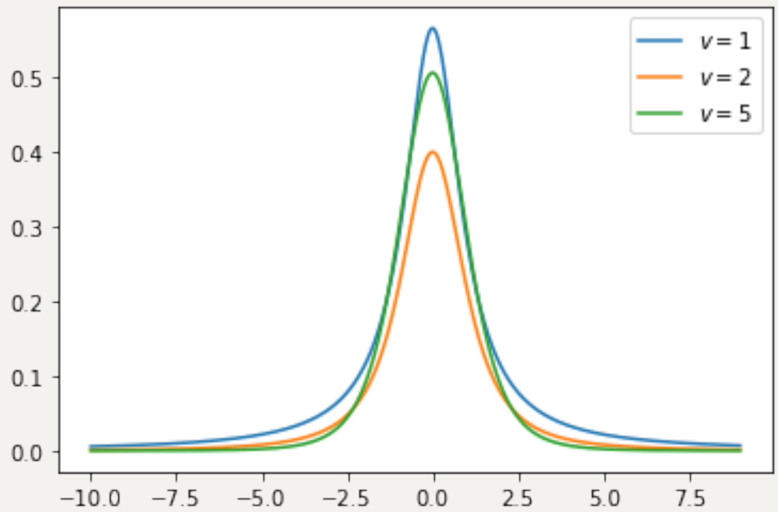


13. **Student-t distribution(continuous)**

- The t-distribution is symmetric and bell-shaped, like the normal distribution, but

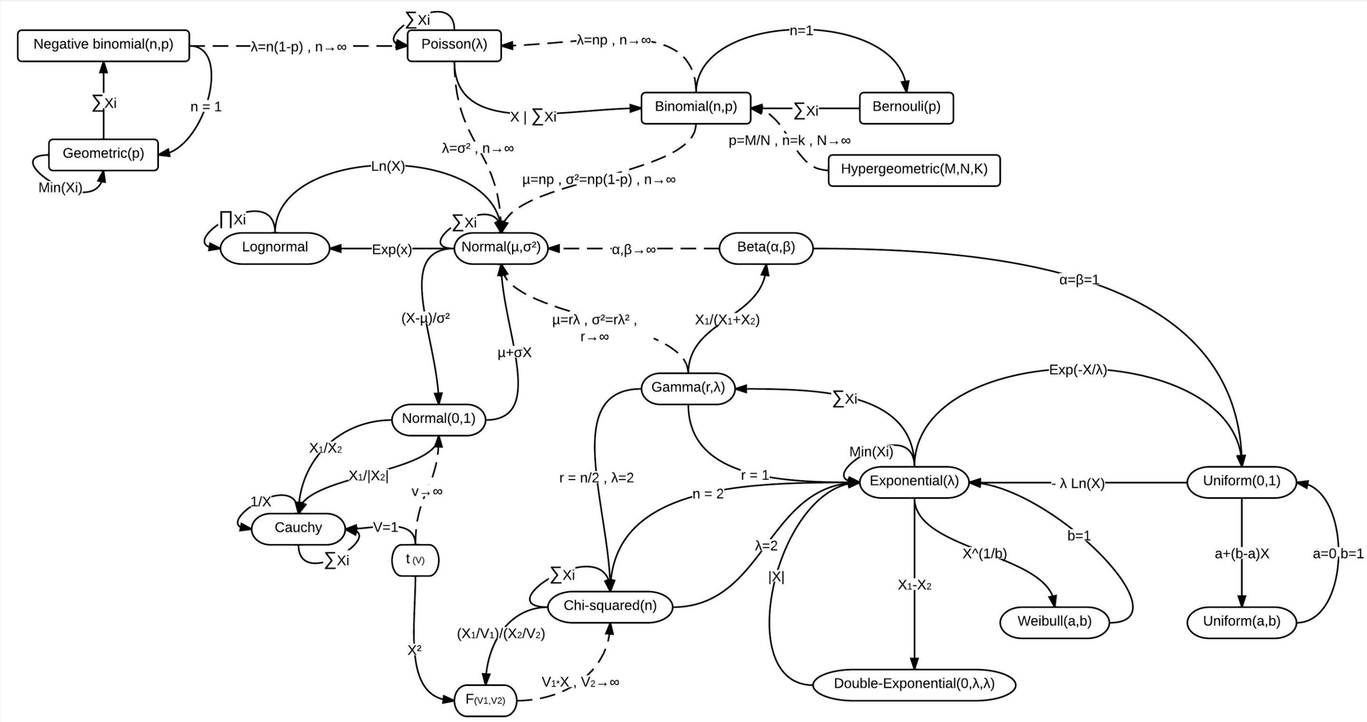
```
import numpy as np
from matplotlib import pyplot as plt

def gamma_function(n):
    cal = 1
    for i in range(2, n):
```

```
            cal *= i
        return cal

def student_t(x, freedom, n):

    # divide [x.min(), x.max()] by n
    x = np.linspace(x.min(), x.max(), n)

    c = gamma_function((freedom + 1) // 2) \
        / np.sqrt(freedom * np.pi) * gamma_function(freedom // 2)
    y = c * (1 + x**2 / freedom) ** (-((freedom + 1) / 2))

    return x, y, np.mean(y), np.std(y)

for freedom in [1, 2, 5]:

    x = np.arange(-10, 10) # define range of x
    x, y, _, _ = student_t(x, freedom=freedom, n=10000)
    plt.plot(x, y, label=r'$v=%d$' % (freedom))

plt.legend()
plt.show()
```



Author

If you would like to see the details about relationship of distribution probability, please refer to this.



- Tae Hwan Jung @graykode, Kyung Hee Univ CE(Undergraduate).
- Author Email : nlkey2022@gmail.com
- Zhao Chen Yang
- Refactored this repo in Spring 2022, during Qingming Festival, at Tsinghua University.