PROJECT PROPOSAL FOR CS 359

(PARALLEL COMPUTING LAB)

IMPLEMENTATION OF

BIDIRECTIONAL LU FACTORIZATION

Under Guidance of:

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OBJECTIVE:

In this project we will implement the parallel solution for Bidirectional LU Factorization, which is used to solve system of linear equations.

INTRODUCTION:

A system of linear algebraic equations has form $Ax = b$, where A is given $m\ x\ m$ matrix, b is given m-vector, and x is unknown solution m-vector to be computed. If a unique solution is known to exist, and the coefficient matrix is full, a direct method for solving general linear system is by computing LU factorization.

BIDIRECTIONAL LU FACTORIZATION:

The basic idea is to use left-multiplication of $A \in \mathbb{C}^{mxm}$ by (elementary) lower triangular matrices, $L_1, L_2, \ldots, L_{m-1}$ to convert A to upper triangular form, i.e.,

$$L_{m-1}\, L_{m-2} \ldots L_2 L_1 A = U \quad \Leftrightarrow \quad \tilde{L}A = U$$

Note that the product of lower triangular matrices is a lower triangular matrix, and the inverse of a lower triangular matrix is also lower triangular. Therefore,

$$\tilde{L}A = U \quad \Leftrightarrow \quad A = LU$$

where $L = \tilde{L}^{-1}$ is unit lower triangular and U is upper triangular.

This approach can be viewed as triangular triangularization.

Linear system $Ax = b$ then becomes $LUx = b$.

$$LUx = b, where\ Ux = y$$

Now by performing two steps:

1. Solve lower triangular system $Ly = b$ by forward-substitution to obtain vector y.
2. Solve upper triangular system $Ux = y$ by back-substitution to obtain solution x to original system.

The value of x is obtained.

Moreover, consider the problem $AX = B$ (i.e., many different right-hand sides that are associated with the same system matrix). In this case we need to compute the factorization $A = LU$ only once, and then

$$AX = B \quad \Leftrightarrow \quad LUX = B$$

and we proceed as before:

1. Solve LY = B by many forward substitutions (in parallel).
2. Solve UX = Y by many back substitutions (in parallel).

In order to appreciate the usefulness of this approach note that the operations count for the matrix factorization is $O(m^3)$, while that for forward and back substitution is $O(m^2)$.

SUMMARY:

The aim for this project is parallelize the LU Factorization, forward and backward substitutions.