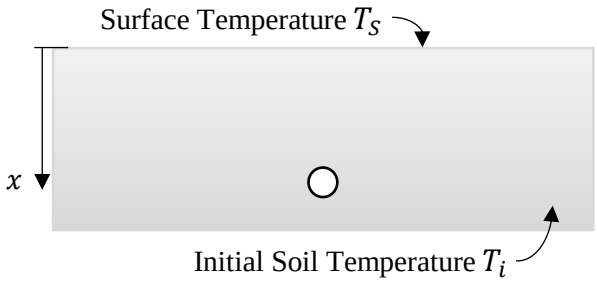
Engineering Problem #3: Soil Temperature Modeling[2]

Background

In the construction of new urban systems, one of the important considerations is the depth at which water pipes must be buried in order to ensure they do not freeze. It is possible to model this situation by considering how the temperature within the soil at a depth x relates to the surface temperature, T_S, and the initial soil temperature, T_i, over time t (in seconds). It has been found that this can be approximately represented by the equation:



$$\frac{T(x,t)-T_S}{T_i-T_S} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right) = \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-u^2}\,du$$

Here, erf, refers to a famous (and very useful) function in mathematics call the *Gauss Error Function*, or simply the *error function*. It has the integral form shown above. The parameter α represents the thermal diffusivity of the soil.

Problem Statement

Prepare a report that summarizes how the soil temperature within Toronto would change with time and depth over the course of a six-month period (December 1 – May 31). Most of the soil in Toronto can be characterized as clay, and typically the initial soil temperature for all depths at the start of December is $T_i = 14℃$. You have been asked to consider the following specific cases:

a) Demonstrate how the temperature changes with time at a 0.75 m depth over the course of the first 31 days (use increments of 1 day).
b) Show how the temperature changes with time (using 1 day increments) over the six-month period for the range of depths of 0.05 m to 5 m.
c) Determine:
    a. How deep the water lines need to be buried to ensure that the temperature does not drop below 5℃.
    b. How deep the water lines need to be buried in order for the surrounding soil temperature to change by less than 1% of the initial soil temperature (i.e., 14℃).

[2] Adapted from A. Gilat and V. Subramaniam, *Numerical Methods for Engineers and Scientists*, John Wiley & Sons, 2008, P7.27, pg. 306.