Numeričke metode financijske matematike

Druga domaća zadaća

Uzeo sam funkciju $f: [0,2\pi] \to \mathbb{R}$, $f(x) = 5\sin(5x) + 5\cos(5x) + 5x$ te $n = 7 \geq 4$.

S time smo odredili i $N = 2^n = 2^7 = 128$, $x_k = \frac{2k\pi}{N}$ te $f_k = f(x_k)$ za $k = 0,1, \dots, 127$.

U drugom i trećem zadatku stavili smo da je $d = 55, d + 1 < N$ te $t_j = \frac{2j\pi}{d}$, $j = 0,1, \dots, d$

U našem slučaju:

x_k	f_k	t_j
0	5	0
0.049087385212340517	6.3104940915507415	0.11423973285781065
0.098174770424681035	7.2574638579951696	0.2284794657156213
0.14726215563702155	7.7988611791949953	0.34271919857343197
0.19634954084936207	7.9169469308575486	0.45695893143124261
0.24543692606170259	7.6193542231847173	0.5711986642890533
0.2945243112740431	6.9386308913790025	0.68543839714686394
0.34361169648638362	5.9302886599790146	0.79967813000467458
0.39269908169872414	4.6694759092246061	0.91391786286248522
0.44178646691106466	3.2464734694963817	1.028157595720296
0.49087385212340517	1.7612834146215688	1.1423973285781066
0.53996123733574564	0.31763518821292447	1.2566370614359172
0.58904862254808621	-0.98323167919507837	1.3708767942937279
0.63813600776042678	-2.0486356138608195	1.4851165271515385
0.68722339297276724	-2.8000081000695189	1.5993562600093492
0.73631077818510771	-3.1776028800419294	1.7135959928671598
0.78539816339744828	-3.1440769948782341	1.8278357257249704
0.83448554860978885	-2.686729027918525	1.9420754585827811
0.88357293382212931	-1.8182603958227093	2.0563151914405919
0.93266031903446978	-0.57601405749060319	2.1705549242984024
0.98174770424681035	0.98026372929854144	2.2847946571562132
1.0308350894591509	2.7720044488299487	2.3990343900140236
1.0799224746714913	4.7065265273619943	2.5132741228718345
1.1290098598838318	6.6825904343602121	2.6275138557296449
1.1780972450961724	8.5964667262118422	2.7417535885874558
1.227184630308513	10.348153329089662	2.8559933214452662
1.2762720155208536	11.847369412613055	2.970233054303077

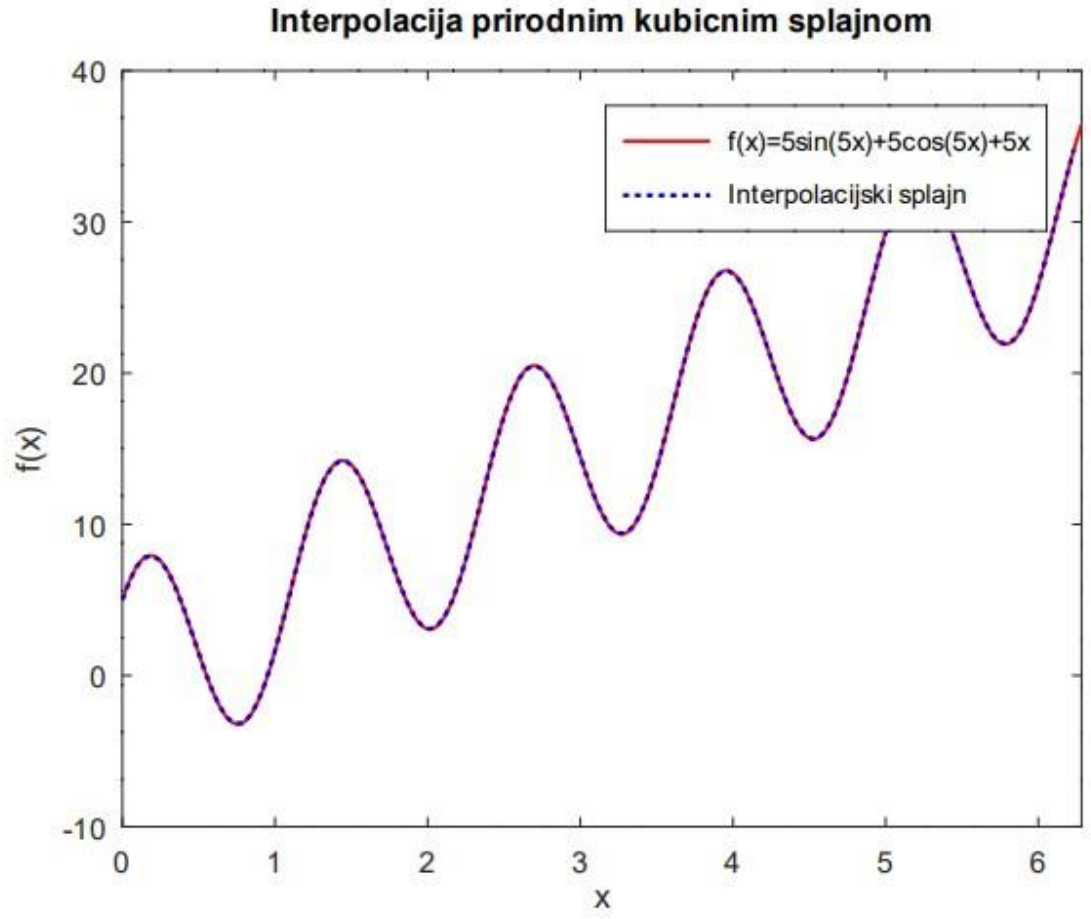
1.3253594007331939	13.018966596542171	3.0844727871608875
1.3744467859455345	13.807433156338409	3.1987125200186983
1.4235341711578751	14.180221256799264	3.3129522528765087
1.4726215563702154	14.129697787722842	3.4271919857343196
1.521708941582556	13.673601873401822	3.54143171859213
1.5707963267948966	12.853981633974485	3.6556714514499409
1.6198837120072371	11.734673926492583	3.7699111843077517
1.6689710972195777	10.397478123709677	3.8841509171655622
1.7180584824319181	8.9372532646993008	3.998390650023373
1.7671458676442586	7.4562324418065788	4.1126303828811839
1.8162332528565992	6.0578952053289976	4.2268701157389943
1.8653206380689396	4.8407652586315173	4.3411098485968047
1.9144080232812801	3.8925051943056905	4.4553495814546151
1.9634954084936207	3.2846622180862211	4.5695893143124264
2.012582793705961	3.0683797886644157	4.6838290471702368
2.0616701789183018	3.2713322069595971	4.7980687800280473
2.1107575641306422	3.896065887884582	4.9123085128858577
2.1598449493429825	4.9198467346181136	5.026548245743669
2.2089323345553233	6.2960227633878407	5.1407879786014794
2.2580197197676637	7.9568203064495764	5.2550277114592898
2.3071071049800045	9.8174061958647805	5.3692674443171002
2.3561944901923448	11.780972450961714	5.4835071771749115
2.4052818754046852	13.744538706058677	5.597746910032722
2.454369260617026	15.605124595473868	5.7119866428905324
2.5034566458293663	17.265922138535601	5.8262263757483437
2.5525440310417071	18.642098167305331	5.9404661086061541
2.6016314162540475	19.665879014038865	6.0547058414639645
2.6507188014663878	20.290612694963851	6.1689455743217749
2.6998061866787286	20.493565113259034	6.2831853071795862
2.748893571891069	20.277282683837228	
2.7979809571034093	19.669439707617766	
2.8470683423157501	18.721179643291929	
2.8961557275280905	17.504049696594464	
2.9452431127404308	16.105712460116877	
2.9943304979527716	14.624691637224155	
3.043417883165112	13.164466778213779	
3.0925052683774528	11.827270975430862	
3.1415926535897931	10.707963267948969	
3.1906800388021335	9.8883430285216338	
3.2397674240144743	9.4322471142006066	
3.2888548092268146	9.3817236451241861	
3.3379421944391554	9.7545117455850381	
3.3870295796514958	10.542978305381277	
3.4361169648638361	11.714575489310384	
3.4852043500761769	13.213791572833781	

3.5342917352885173	14.965478175711601	
3.5833791205008576	16.879354467563211	
3.6324665057131984	18.855418374561435	
3.6815538909255388	20.789940453093493	
3.7306412761378791	22.581681172624908	
3.7797286613502199	24.137958959414057	
3.8288160465625602	25.380205297746151	
3.877903431774901	26.248673929841971	
3.9269908169872414	26.706021896801683	
3.9760782021995817	26.739547781965381	
4.0251655874119221	26.361953001992973	
4.0742529726242633	25.610580515784257	
4.1233403578366037	24.545176581118529	
4.172427743048944	23.244309713710539	
4.2215151282612844	21.800661487301888	
4.2706025134736247	20.315471432427071	
4.3196898986859651	18.892468992698859	
4.3687772838983063	17.631656241944427	
4.4178646691106467	16.623314010544451	
4.466952054322987	15.942590678738732	
4.5160394395353274	15.644997971065902	
4.5651268247476677	15.763083722728451	
4.614214209960009	16.304481043928288	
4.6633015951723493	17.25145081037271	
4.7123889803846897	18.561944901923432	
4.76147636559703	20.172126461528737	
4.8105637508093704	22.000196116435063	
4.8596511360217116	23.951294827568855	
4.908738521234052	25.923189502584982	
4.9578259064463923	27.812400591185956	
5.0069132916587327	29.520404390006842	
5.056000676871073	30.959538306456064	
5.1050880620834143	32.058255134798955	
5.1541754472957546	32.765411416344165	
5.203262832508095	33.053332850172389	
5.2523502177204353	32.919473021370806	
5.3014376029327757	32.386566026760683	
5.350524988145116	31.501263850114359	
5.3996123733574573	30.331340159176019	
5.4486997585697976	28.961628121884235	
5.497787143782138	27.488935718910703	
5.5468745289944783	26.016243315937146	
5.5959619142068187	24.646531278645384	
5.6450492994191599	23.476607587707022	
5.6941366846315002	22.591305411060702	

5.7432240698438406	22.058398416450579	
5.7923114550561809	21.924538587648993	
5.8413988402685213	22.212460021477213	
5.8904862254808616	22.919616303022423	
5.9395736106932029	24.018333131365296	
5.9886609959055432	25.457467047814539	
6.0377483811178836	27.165470846635397	
6.0868357663302239	29.054681935236395	
6.1359231515425643	31.026576610252494	
6.1850105367549055	32.977675321386315	
6.2340979219672459	34.805744976292637	

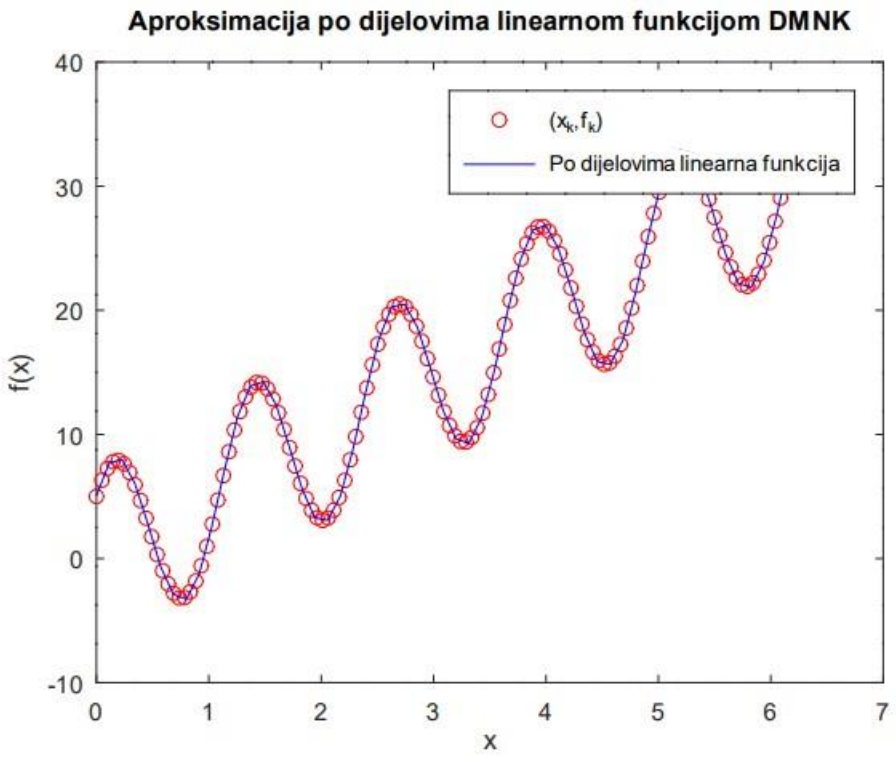
Ovi podaci mogu se pronaći i u ulazni_podaci.

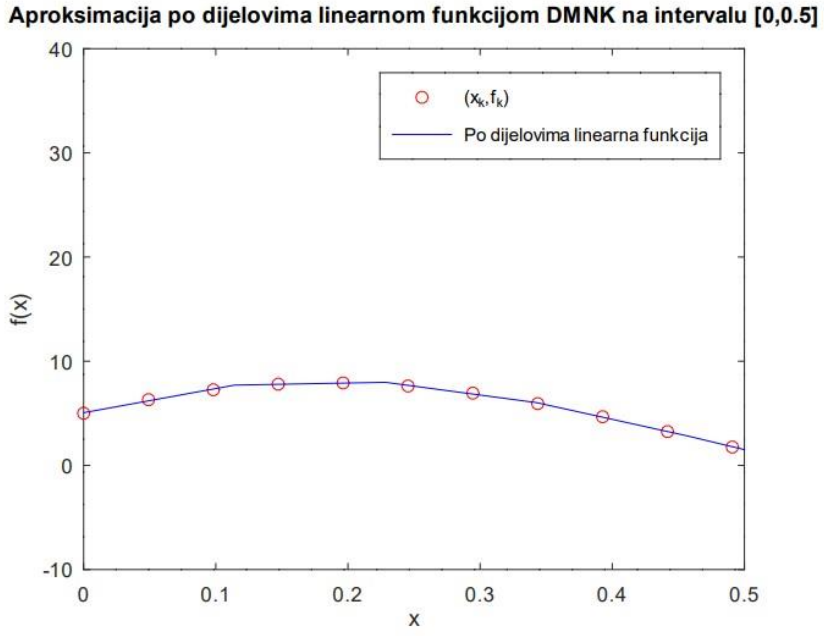
1.zadatak (DZ1)



Maksimalna greška u čvorovima (x_k, f_k) iznosi 0. Spremljena je u datoteku greska1.mat pod nazivom greska_1. Iz maksimalne greške zaključuje se da prirodni kubični splajn dobro interpolira funkciju.

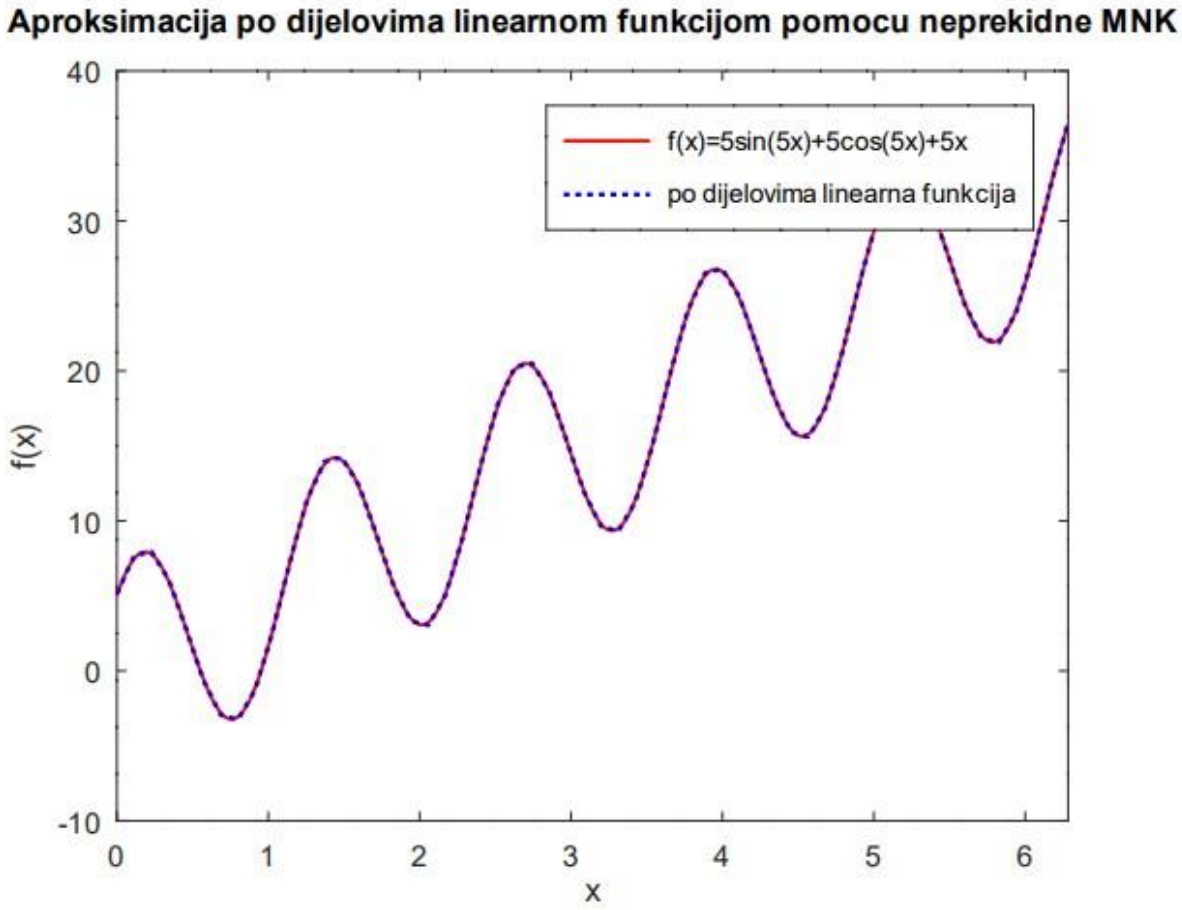
2.zadatak (DZ2)



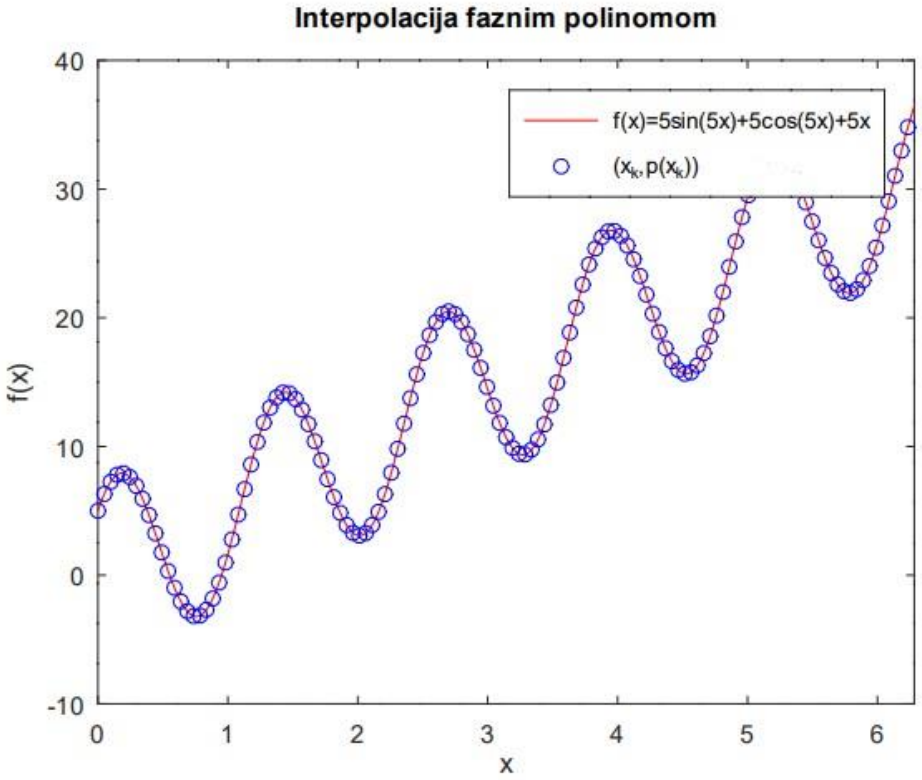


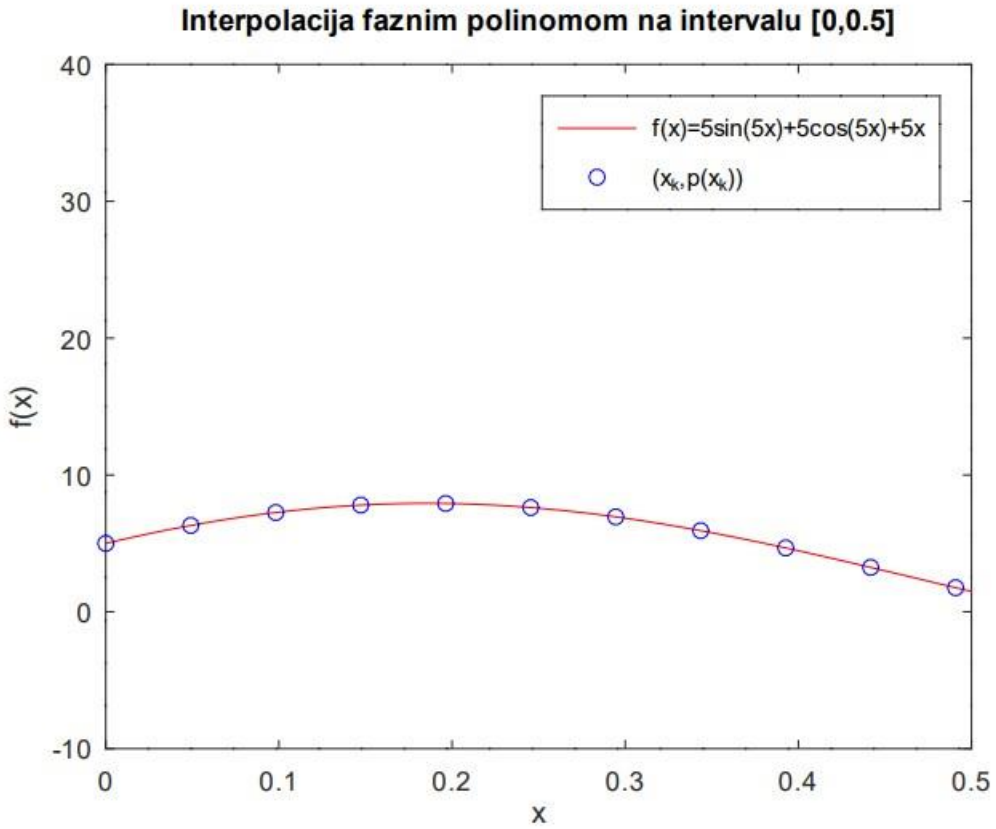
Donji prikaz sam koristio samo da bi pogledali na manjem intervalu što se događa kod aproksimacije po dijelovima linearnom funkcijom diskretnom metoda najmanjih kvadrata (DMNK u nazivima slika).

3.zadatak (DZ3)



4.zadatak (DZ4)





Maksimalna greška u čvorovima (x_k, f_k) iznosi 8.5892e-13. Spremljena je u datoteku greska4.mat pod nazivom greska_4. Vidimo da je greška vrlo mala pa zaključujemo da fazni polinom dobro interpolira funkciju.

Donji prikaz sam koristio samo da bi pogledali na manjem intervalu što se događa kod interpolacije faznim polinomom.