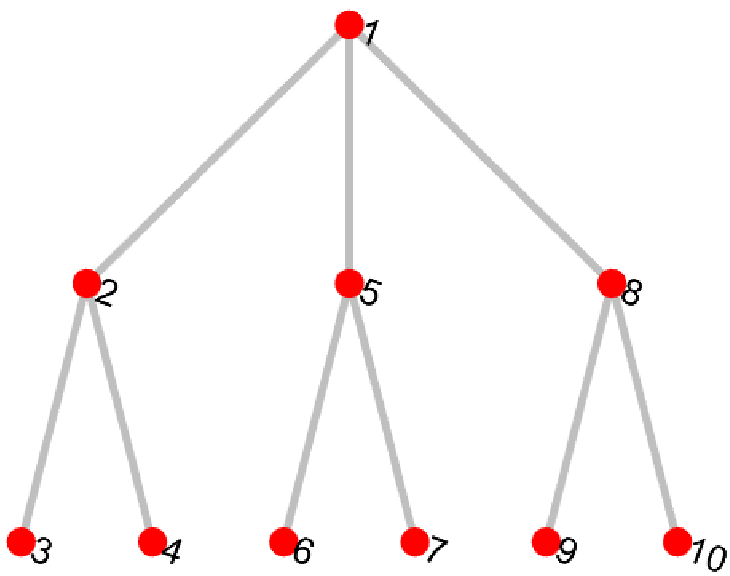
Example: Solving the emission circuit for a tree-like graph state

State preparation:

Create the adjacency matrix **Gamma** of a simple connected graph:

```
clear
filename = 'State1'; % filename
n = 10; % qubits number of target photonic state
EdgesList = [1,2;1,5;1,8; 2,3;2,4; 5,6;5,7; 8,9;8,10]; % the list for edges
Gamma = full(sparse(EdgesList,fliplr(EdgesList),ones(size(EdgesList)),n,n));
figure;
plot(graph(Gamma),'MarkerSize',10,'NodeColor','r','EdgeColor',0.5*[1,1,1], ...
    'LineWidth',3,'NodeFontSize',14);
axis off
```



Where Gamma cannot have zero row and column, which represents 'decoupled qubit'.

Create the stabilizer generators **Gen** in the following way:

```
Gen.Tableau = [eye(n),Gamma]; % Tableau matrix for generators:
Gen.SignVector = zeros(n,1); % vector that represent the signs in front of these generators: 0 for '+', 1 for '-'.
```

Where `tableau` is a way of representing Pauli matrices $\{I, X, Y, Z\}$ in $\mathbb{F}_2^2$ as $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$. In this representation, the multiplication is replaced by addition, with phases being ignored. The phases are stored in `SignVector` instead. For example, for bell state $|\Psi^+\rangle$, the generators can be $g_1 = X_1X_2, g_2 = -Z_1Z_2$, then

```
Gen.Tableau = [1,1,0,0;
               0,0,1,1];
Gen.SignVector = [0;1];
```

So we can replace **Gen** by any pure stabilizer state. Use following code to load logical $|-\rangle_L$ state of Shor code from the root directory:

```
% load('ShorCode_x1.mat','Gen');
```

Circuit solver:

Solve the circuit with **`Gen`** :

```
[Operation,InvOperation,Stat] = CircuitSolver(Gen);
```

```
        ** The protocol is SOLVED correctly! **
```

where

- **`Operation`** records the data of $\{U_{e,j}, U_{p,j}, \eta_j, \mu_j, W_j, W_0\}$ operations.
- **`InvOperation`** is the same operation set as **`Operation`** but with reversed unitaries. For instance: $P \to P^{-1} = PZ$
- **`Stat`** is other data like height function, single qubit gates number, two-qubit gates number *et al*.
- "`SOLVED correctly`" means all of the generators has reduced to $+Z_i$.

The inside mechanism of CircuitSolver is displayed in **`FlowChartMaker.mlx`** in the root directory.

Result Verification:

Verify if the output of `CircuitSolver` gives the correct state in normal time order, using stabilizer formalism.

```
[G_f_recover, G_Phi_recover] = ProtocolExecutor(InvOperation,Stat.EmittersNumber,Stat.PhotonsNumber);
Equivalence = GeneratorsEquivalence(Gen,G_Phi_recover) % check the equivalence of Tableaus and SignVectors
```

```
Equivalence = struct with fields:
       Overall: 1
       Tableau: 1
    SignVector: 1
```

where

- **`G_f_recover`** is the final outcomes stabilizer generators of "photons + emitters" system.
- **`G_Phi_recover`** is the final outcomes stabilizer generators of of photons only.

Display the circuit:

Print the TeX codes in a **`quantikz_filename.txt`** file for quantikz, which displays an easy-to-read circuit with TeX compiler.

```
Operation2quantikz(Operation,Stat.PhotonsNumber,Stat.EmittersNumber,filename);
```

```
        ** The quantikz code for latex has been saved as: quantikz_State1.txt **
```

When compiling TeX, we need to load:

`\usepackage{tikz}`

`\usetikzlibrary{quantikz}`

in the TeX compiler.

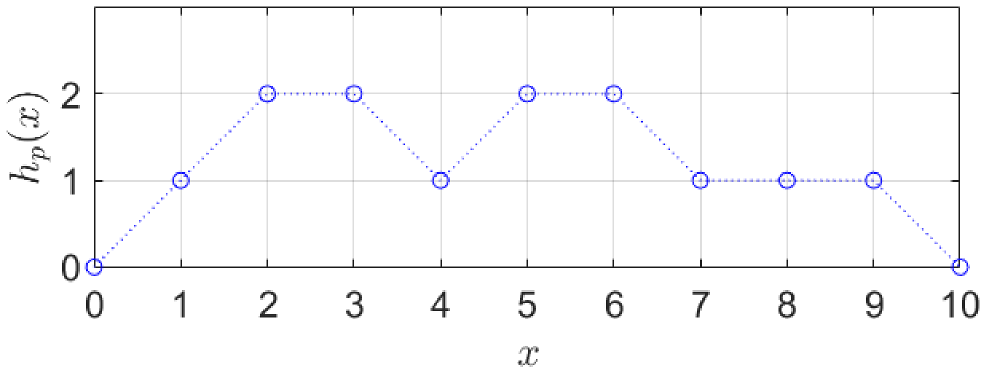
Caveat: when n > 100, the circuit display is dense, and the TeX compilation could be slow, one may want to display the circuit with raw text (less readable) by the following compromising approach, which converts and saves the circuit solution in **`ProtocolReport_filename.txt`**, by executing:

```
ProtocolPrinter(Operation,Stat,filename);
```

```
        *** The protocol has been saved as ProtocolReport_State1.txt  ***.
```

Plot the height function $h_p(x)$ for the target photonic state.

```
figure;
h = Stat.HeightFunc; % The height function of target photonic state.
plot(0:n,[0,h],':ob');
grid on
ax = gca;
xlabel('$x$','Interpreter','latex');
ylabel('$h_p(x)$','Interpreter','latex');
set(ax,'XTick',unique( round(ax.XTick)),'YTick',unique( round(ax.YTick) ), ...
    'XLim',[0,n],'YLim',[0,max(h)+1],'FontSize',14)
daspect([1,1,1]);
```



Where **h** is also accessible directly through:

```
% h = heightfunc(Gen.Tableau,'pure');
```