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Answer 1

a) It is reflective.
For (a,b), in this relation, for every 'a' there is a 'b' such that a=b, because |a-b|=0<4 when a=b.

b) It is symmetric.
If |a-b|<4, then |b-a|<4. It means that if (a,b) exists, then (b,a) also exits.

c) It is not transitive.
Assume that the relation contains (a,b),(b,c) and (a,c) such that (a-b)=3, |3|<4 and (b-c)=2, |2|<4. So (a-c)=5, but $|5| \nless 4$. Then (a,c) does not exist in the relation. And it is not transitive.

Answer 2

a) In order to be an equivalence relation, T must be reflexive, symmetric and transitive.
It is reflexive.
Given a∈S, we can clearly see that a/a=1 and it is rational, so aTa.

It is symmetric.
Assume a,b ∈S are given and aTb. a/b is a non-zero raitonal number by definition, then it's inverse $\frac{1}{a/b}$=b/a is also a rational number.

It is transitive.
Assume a,b,c are given with aTb, bTc. By definition of T, a/b and b/c are rational numbers. Then, product of them (a/b)·(b/c)= a/c is also rational, aTc.

b) We can reach the distinct equivalence classes by providing a representative from each of them.
Consider the subset A= {k-$\sqrt{5}$: k is rational} ∪ 1 of S.
Elements of A represent all distinct equivalence classes of T.
a=(x-y$\sqrt{5}$) ∈S. If y=0, then a=x, so a/1=x is also rational and aT1.
If y≠0 then k=x/y. So we can see that $\frac{a}{k-\sqrt{5}} = \frac{x-y\sqrt{5}}{(x/y)-\sqrt{5}}$ = y, so aT(k-$\sqrt{5}$).
Think that (k-$\sqrt{5}$) and (l-$\sqrt{5}$) are different elements of A.
Let B $= \frac{k-\sqrt{5}}{l-\sqrt{5}} = (\frac{k-\sqrt{5}}{l-\sqrt{5}})\cdot(\frac{l+\sqrt{5}}{l+\sqrt{5}}) = \frac{k\cdot l+\sqrt{5}(k-l)-5}{l^2-5}$

We can conclude this as $\sqrt{5} = \dfrac{B \cdot (l^2 - 5) - k \cdot l + 5}{k - l}$ and it is rational which we know is not the case.
Therefore, k-$\sqrt{5}$ and l-$\sqrt{5}$ are not equivalent. Also we can see that for every element k-$\sqrt{5}$ of A, k-$\sqrt{5}/1 =$ k-$\sqrt{5}$ is also irrational and so k-$\sqrt{5}$ is not equivalent to 1.
Hence, every two distinct elements of A are not equivalent and represent different classes.

Answer 3

Here is the adjaceny matrix for the listing elements of (1,2,3,4). $R_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

For R_1 There is a new path from 1 to 4 going through (1, 2, 4)
There is a new path from 1 to 3 going through (1, 2, 4, 3)
There is a new path from 1 to 1 going through (1, 2, 4, 1)

$$R_1 = \begin{bmatrix} \underline{1} & 1 & \underline{1} & \underline{1} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

For R_2 There is a new path from 2 to 1 going through (2, 4, 1)
There is a new path from 2 to 3 going through (2, 4, 3)
There is a new path from 2 to 2 going through (2, 4, 1, 2)

$$R_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \underline{1} & \underline{1} & \underline{1} & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

For R_3 There is no new path from 3. Same.

$$R_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

For R_4 There is a new path from 4 to 2 going through (4, 1, 2)
There is a new path from 4 to 4 going through (4, 1, 2, 4)

$$R_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & \underline{1} & 1 & \underline{1} \end{bmatrix}$$

R_4 is the matrix of the transitive closure.

Answer 4

a) [54,96,120,144]
b) [4,18]
c) No.
d) No.
e) [36,72,144]
f) 36
g) [4,8,16,24]
h) 24

Answer 5

Both have 6 nodes and 8 edges.
Degrees of G is : [4,3,3,2,2,2,2] in order of (b,a,e,c,d,f)
Degrees of H is : [4,3,3,2,2,2,2] in order of (q,m,n,o,p,r)
So their degrees are same and they have the same matrices.
That means they are isomorphic.

Answer 6

a) If they have the same degree for each node, the numbers of edges are equal.
So, the number of edges are equal from V_1 to V_2 and from V_2 to V_1.
Then, $| V_1 | = | V_2 |$