

Student Information

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Answer 1

$$\begin{aligned}& \overline{(A \setminus B)} \cap \overline{(B \setminus A)} \\& \{x \mid \neg(x \in (A \setminus B)) \cap \neg(x \in (B \setminus A))\} \\& \{x \mid \neg(x \in (A \cap \overline{B})) \cap \neg(x \in (B \cap \overline{A}))\} \\& \{x \mid \neg(x \in A \wedge x \notin B) \wedge \neg(x \in B \wedge x \notin A)\} \\& \{x \mid (x \notin A \vee x \in B) \wedge (x \notin B \vee x \in A)\} \\& \{x \mid ((x \notin A) \wedge (x \notin B \vee x \in A)) \vee ((x \in B) \wedge (x \notin B \vee x \in A))\} \\& \{x \mid (((x \notin A) \wedge (x \notin B)) \vee ((x \notin A) \wedge (x \in A))) \vee (((x \in B) \wedge (x \notin B)) \vee ((x \in B) \wedge (x \in A)))\} \\& \{x \mid ((x \notin A) \wedge (x \notin B)) \vee ((x \in B) \wedge (x \in A))\} \\& \{x \mid \neg((x \in A) \vee (x \in B)) \vee ((x \in B) \wedge (x \in A))\} \\& \{x \mid x \in ((A \cap B) \cup (\overline{A \cup B}))\} \\& (A \cap B) \cup \overline{(A \cup B)}\end{aligned}$$

Answer 2

a)

$$\begin{aligned}& \{a, b \mid (a \in A \cap B) \wedge (b \in C \cap D) \rightarrow (a \in A \wedge b \in C) \cap (a \in B \wedge b \in D)\} \\& \{a, b \mid (a \in A \wedge a \in B) \wedge (b \in C \wedge b \in D) \rightarrow (a \in A \wedge b \in C) \wedge (a \in B \wedge b \in D)\} \\& \{a, b \mid (a \in A) \wedge (b \in C) \wedge (a \in B) \wedge (b \in D) \rightarrow (a \in A) \wedge (b \in C) \wedge (a \in B) \wedge (b \in D)\}\end{aligned}$$

b)

Suppose that g is not onto but gof is onto

There is an element $z_1 \in Z$ such that $g(y) \neq z_1, \forall y, y \in Y$

Suppose $x_1 \in X, y_1 \in Y$ and $f(x_1) = y_1$

$g(y_1) \neq z_1$ then $g(f(x_1)) \neq z_1$ then $\text{gof}(x_1) \neq z_1$

$\forall x, x \in X, \text{gof}(x) \neq z_1$ then gof is not onto

It is a contradiction

So, if gof is onto, then g must be onto

Answer 3

$$\begin{aligned}& \{x, y \mid f(x) = y, x \in f^{-1}(A \cup B), y \in (A \cup B)\} \\& (y \in (A \cup B)) \leftrightarrow (f(x) \in A \vee f(x) \in B) \\& (f(x) \in A \vee f(x) \in B) \leftrightarrow (x \in f^{-1}(A) \vee x \in f^{-1}(B))\end{aligned}$$

$$(x \in f^{-1}(A) \vee x \in f^{-1}(B)) = f^{-1}(A) \cup f^{-1}(B)$$

Answer 4

a)

n is odd positive, then $n=2m+1$ $m \in \mathbb{Z}$

$$n^2-1 = (n-1)(n+1) = (2m)(2m+2) = 4(m)(m+1)$$

One of (m) and (m+1) is even, then $m(m+1)$ is dividible by 2

$$4m(m+1) = (n^2-1) \text{ is dividible by } 8$$

b)

k is an integer, $a=2nk+b$

$$a^2=4n^2k^2 + b^2 + 4nkb$$

$$a^2=4n(k^2+k) + b^2$$

$$a^2 \equiv b^2 \pmod{4n}$$

Answer 5

Answer 6

$$(x.y).(x.z) = \text{lcm}(x.y,x.z).\text{gcd}(x.y,x.z)$$

$$\text{lcm}(x.y,x.z) = x^2.y.z / \text{gcd}(x.y,x.z) = x^2.y.z / x.\text{gcd}(y,z) = x.y.z / \text{gcd}(y,z)$$

$$x.y.z = x.\text{gcd}(y,z).\text{lcm}(y,z)$$

$$x.\text{gcd}(y,z).\text{lcm}(y,z) / \text{gcd}(y,z) = x.\text{lcm}(y,z)$$