

CENG 223

Discrete Computational Structures

Fall '2015-2016

Take Home Exam 3

Due date: 10/12/2015

Question 1

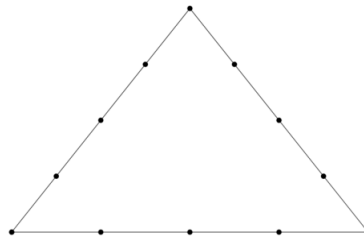
Use mathematical induction to prove that for $k \geq 1$, $(C^k - 1)$ is divisible by $(C - 1)$ where C is a constant integer.

Question 2

Use mathematical induction to prove that for $n \geq 2$,

$$\left(1 - \frac{1}{1+2}\right) \cdot \left(1 - \frac{1}{1+2+3}\right) \cdot \dots \cdot \left(1 - \frac{1}{1+2+3+\dots+n}\right) = \frac{n+2}{3n}$$

Question 3



Twelve points are marked on a triangle above. Find the number of quadrilaterals (polygons with 4 edges) that can be formed with vertices from the marked points. Note that no three vertices of the quadrilateral can be collinear. Explain your reasoning.

Question 4

Find how many solutions are there for the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 67$$

when each odd indexed variable (x_1, x_3, x_5) is a positive odd integer and each even indexed variable (x_2, x_4) is a positive even integer. **Note:** Show your work.

Question 5

Assume that a must course lasts for 2 hours while both a technical elective course and a free elective course lasts for 1 hour. Find the recurrence relation for the number of ways to arrange these three types of courses in n hours such that no three electives courses (technical/free) can occur consecutively.

Question 6

a) Find the sequence corresponding to the generating function

$$G(x) = \frac{2x^4}{2x^3 - x^2 - 2x + 1}$$

b) Find the generating function of the sequence with

$$a_n = \frac{(6^n + 1)^2}{2^n}$$

Note: You can refer to the useful generating functions table in the textbook.

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Show your work by explaining your solution. Writing only the final result will not get you full points.
3. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
4. **Late Submission: Not allowed**
5. **Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
6. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.
7. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the3.tex", when you finish your exam upload the .tex file with the same name to COW.

Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the3.tex
```