Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2015-2016

Take Home Exam 4

Due date: 24 December 2015, 23:55

Question 1

Consider A is a relation defined on R (real numbers) where $A = \{(a, b) : |a - b| < 4, a, b \in R\}$. Prove/disprove each of the following:

a. A is reflexive

b. A is symmetric

c. A is transitive

Question 2

Given the set $S = \{x - y\sqrt{5} : x, y$ are rational numbers and $x - y\sqrt{5} \neq 0\}$. Assume the relation T is defined on the set S by a T b if a/b is a rational number.

a. Prove that T is an equivalence relation.

b. Find the distinct equivalence classes of T. Show all the steps clearly.

Question 3

Use Warshall's algorithm to find the transitive closure of the following relation.
R=$\{(1, 2), (2, 4), (4, 1), (4, 3)\}$
Show all the computation steps clearly.

Question 4

Answer the questions for the poset $(\{4, 8, 12, 18, 24, 36, 54, 72, 96, 120, 144\}, |)$

a. Find the maximal elements.

b. Find the minimal elements.

c. Is there a greatest element?

d. Is there a least element?

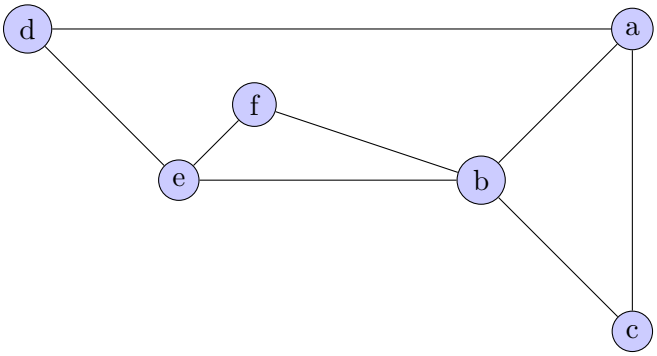
e. Find all upper bounds of $\{4, 18\}$.

f. Find the least upper bound of $\{4, 18\}$, if it exists.

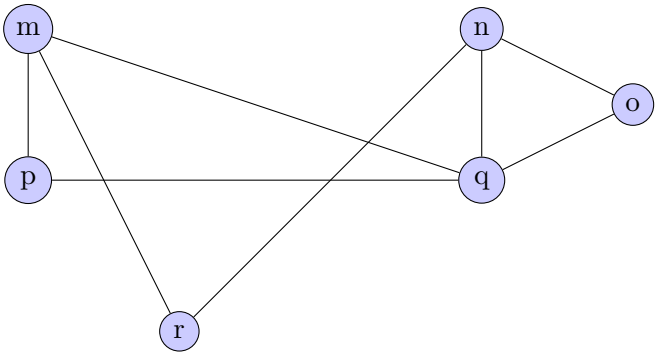
g. Find all lower bounds of $\{120, 144\}$.

h. Find the greatest lower bound of $\{120, 144\}$, if it exists.

Question 5



Graph G



Graph H

Given two graphs G and H, determine whether G and H are isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

Question 6

Let G is an undirected simple bipartite graph with bipartition (V_1, V_2) and suppose that all the vertices in G has exactly the same degree $n > 0$. Prove the following:

a) $|V_1| = |V_2|$

b) A perfect matching must exist in G

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.

2. Show your work by explaining your solution. Writing only the final result will not get you full points.

3. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.

4. **Late Submission: Not allowed**

5. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

6. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.

7. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "the4.tex", when you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the4.tex
```