⬤ Middle East Technical University ◈ Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2015-2016

Take Home Exam 2

Due date: 22 November 2015, 23:55

Question 1

Given non-empty sets A and B, prove that $\overline{(A \setminus B)} \cap \overline{(B \setminus A)} = (A \cap B) \cup \overline{(A \cup B)}$ using **set builder notations**.

Question 2

a. Suppose that f is a function from X cartesian product X to Y and $A, B, C, D \subseteq X$. Prove that:

$f((A \cap B) \times (C \cap D)) \subseteq f(A \times C) \cap f(B \times D)$

b. Suppose that f is a function from X to Y and g is a function from Y to Z. Prove that:

If g∘f is onto, g is onto

Question 3

Suppose that f is a function from X to Y. Given $A \subseteq Y$, $B \subseteq Y$, and $f^{-1}(A)$ is the inverse image of the set A, prove that:

$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$

Question 4

a. If n is an odd positive integer, prove that $8 \mid (n^2 - 1)$

b. Let n be positive number if $a \equiv b(\text{mod } 2n)$, prove that $a^2 \equiv b^2(\text{mod } 4n)$

Question 5

Let h_1, h_2, g_1, and g_2 be functions from set of positive integer numbers to positive real numbers. **Prove** or **disprove** that if $h_1(n)$ is $O(g_1(n))$ and $h_2(n)$ is $O(g_2(n))$ then $h_1(n)/h_2(n)$ is $O(g_1(n)/g_2(n))$ for all n.

Question 6

For any three positive integers x, y, and z, prove that lcm(x.y, x.z) = x.lcm(y,z)

1   Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.

2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.

3. **Late Submission: Not allowed**

4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.

6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2   Submission

Submission will be done via COW. Download the given template file, "the2.tex", when you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the2.tex
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