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Answer 1

Let P(k) be the proposition that (C^k-1) is divisible by (C-1) for k≥1
Basis: P(1) is true, because (C-1) is divisible by (C-1)
Inductive: Assume that P(k) is true for an arbitrary integer x with x≥1 and (C^x-1) is divisible by (C-1). Also P(k+1) must be true and (C^{x+1}-1) must be divisible by (C-1).
(C^{x+1}-1) = C^{x+1} - C^x + C^x -1 = (C^x(C-1)) + (C^x-1)
(C^x(C-1)) is clearly divisible by (C-1) and (C^x-1) is also divisible if P(k) is true . Then, if ((C^x-1) is divisible by (C-1), (C^{x+1}-1) is also divisible by (C-1).
That means P(k+1) is true when P(k) is true.

Answer 2

Let P(n) be the proposition that $(1-\frac{1}{1+2})\cdot(1-\frac{1}{1+2+3})\cdot\ldots\cdot(1-\frac{1}{1+2+3+...+n}) = \frac{n+2}{3n}$ for n≥2.
Basis:P(2)=$(1-\frac{1}{1+2}) = \frac{2}{3} = \frac{4}{6}$, it is true.
Inductive: Assume that P(k) is true for an arbitrary integer k with k≥2. Then P(k+1) must also be true.
P(k) = $(1-\frac{1}{1+2})\cdot(1-\frac{1}{1+2+3})\cdot\ldots\cdot(1-\frac{1}{1+2+3+...+k}) = \frac{k+2}{3k}$
P(k+1) = $(1-\frac{1}{1+2})\cdot(1-\frac{1}{1+2+3})\cdot\ldots\cdot(1-\frac{1}{1+2+3+...+k+(k+1)}) = $ P(k)$\cdot(1-\frac{1}{1+2+3+...+(k+1)})$
$1-\frac{1}{1+2+3+...+(k+1)} = 1- \frac{2}{(k+1)(k+2)} = \frac{k^2+3k}{k^2+3k+2}$, because $\sum_{n=1}^{k+1}$ n = $\frac{(k+1)(k+2)}{2}$
P(k) = $\frac{k+2}{3k}$
P(k+1) = $(\frac{k+2}{3k})\cdot(\frac{k^2+3k}{k^2+3k+2}) = \frac{(k+2)(k+3)(k)}{3(k)(k+1)(k+2)} = \frac{(k+3)}{3(k+1)}$
That is true. P(k+1) is true when P(k) is true.

Answer 3

Without constraints we have C(12,4) ways to make a quadrilateral.
No three vertices can be collinear : C(7,1)·C(5,3)·3

No four vertices can be collinear : C(5,4)·3
Number of quadrilaterals that can be formed : C(12,4) - C(7,1)·C(5,3)·3 - C(5,4)·3 = 270

Answer 4

If we subtract 1 from odd positive numbers and 2 from even positive numbers, we can convert them non-negative even integers.
(x_1-1) + (x_2-2) + (x_3-1) + (x_4-2) + (x_5-1) =60
(x_1-1) = 2a
(x_2-2) = 2b
(x_3-1) = 2c
(x_4-2) = 2d
(x_5-1) = 2e
a+b+c+d+e=30, then the formula is C(5+30-1, 30) = C(34,30) = C(34,4)=46376

Answer 5

Let a_n denote the number of ways to arrange these courses without three consecutive elective courses.
Initial case : $a_1 = 2$, $a_2 = 5$, $a_3 = 4$
Ending with a must course : (...,must,must) : any schedule of length (n-2) without three consecutive elective courses $\rightarrow$ (a_{n-2})
Ending with a free elective course : (...,must,must,free) $\rightarrow$ (a_{n-3})
: (...,must,must,free,free) $\rightarrow$ (a_{n-4})
: (...,must,must,technical,free) $\rightarrow$ (a_{n-4})
total $\rightarrow$ 2(a_{n-4}) + (a_{n-3})
Ending with a technical elective course : (...,must,must,technical) $\rightarrow$ (a_{n-3})
: (...,must,must,free,technical) $\rightarrow$ (a_{n-4})
: (...,must,must,technical,technical) $\rightarrow$ (a_{n-4})
total $\rightarrow$ 2(a_{n-4}) + (a_{n-3})
All the ways : (a_{n-2}) + 2(a_{n-3}) + 4(a_{n-4})
a_n = (a_{n-2}) + 2(a_{n-3}) + 4(a_{n-4})

Answer 6