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Answer 1

By Induction

Basis: k=2 , $2^k = 4$ vertices. A Hamilton cycle can be clearly seen [00, 01, 11, 10]

Inductive: We must show how to produce A_{k+1} graph from A_k graph.
Assume that $k > 1$, k $\in Z^+$, and A_k has a Hamilton cycle.
Let [c_1, c_2,...c_x] are the binary digit vertices for x = 2^k
Then [0c_1, 0c_2,...0c_x,1c_1, 1c_2,...1c_x] are the binary digit vertices for 2x = 2^{k+1} ,because $2^{k+1} = 2^k \cdot 2$ and we can produce it by adding each vertex '0' and '1'.
So, A_{k+1} has a Hamilton cycle if A_k has a Hamilton cycle.

Answer 2

The chromatic number of the graph is 4
It is at least 3, because of the triangle of [E,F,G]
Let's say E='green' , F='red' , G='blue' .
C is different than F and G, then C='green'.
A is different than G and C, then A='red'.
H is different than F and C, then H='blue'.
D is different than A,C,F; then D='blue'.
I is different than D,E,F; then I must take a new colour.
Because D='blue', E='green', F='red', we cannot color I with these three ones, we need a new colour. Let's say I='purple'.
Note that chromatic number increased to 4.
Lastly, B is different than A and I. B can be either green or blue, so it doesn't change the number.
Hence, the chromatic number of the graph is exactly 4 [green,red,blue,purple].

Answer 3

For T_1, The number of nodes for depths are:
at height 0: 1
at height 1: 1!

at height 2: 2·1=2!
at height 3: 3·2·1=3!
at height h: h·(h-1)·(h-2)...·1=h!

For T_2, The number of nodes for depths are:
at height 0: 1
at height 1: h
at height 2: h·(h-1)
at height 3: h·(h-1)·(h-2)
at height h: h·(h-1)·(h-2)...·1=h!

There is 1 node at height 0 in both T_1 and T_2
There are h! nodes at height h in both T_1 and T_2

For $h > n > 0$, there are n! nodes at height n in T_1
For $h > n > 0$, there are $\dfrac{h!}{(h-n)!}$ nodes at height n in T_2
$\dfrac{h!}{(h-n)!}$ is larger than (n!) as long as $h > n > 0$; because $h > n$, $(h-1)$>$(n-1)$,...$(h-n+1)$>1
So, number of nodes in T_1 is smaller than number of nodes in T_2 at any height between 0 and h

For h=0, number of nodes are equal (it's 1)
For h=1, number of nodes are equal (it's 2)
Otherwise(h≥2), number of nodes in T_1 are smaller than number of nodes in T_2
Consequently, the number of nodes in T_1 is smaller or equal to the number of nodes in T_2.

Answer 4

If T has n nodes, it has (n-1) edges, because it is a tree.
There are $\binom{n}{2}$ possible pairs for a graph having n nodes.
Complement of a graph with (n-1) edges and n nodes, has $\binom{n}{2}$ - (n-1) edges.
If the complement of T is also a tree, it gives $\binom{n}{2}$ - (n-1) = (n-1)
So, $\binom{n}{2}$ = 2(n-1)
$\dfrac{n(n-1)}{2}$ = 2(n-1)
$\dfrac{n}{2} = 2$
n=4

Answer 5

Answer 6