Student Information

Full Name : Bilal Özlü
Id Number : 1942614

Answer 1

a.

{ $x^a y^a z^b$: a,b $\geq$ 0 } $\cup$ { $x^a y^b z^a$: a,b $\geq$ 0 }

b.

U = (x,y)* \$V (x,y)* : left and right side of \$V are reverse of each other.
V = (x,y)*
So the language is: UV : $\underbrace{(x,y)}_{\mathrm{w}}$* \$(x,y)* $\underbrace{(x,y)}_{\mathrm{w}^R}$* (x,y)*
.

c.

The regular expression is: $x(x^9)^* + ((z,y)(z,y))^*$

Answer 2

a.

Assume that L_1 is a context free language.
Since L_1 is infinite, we can apply Pumping Lemma.
Let a = $2^m 1^m$. Then, $L_1 = aa^R 2^{|a|} = 2^m 1^m 1^m 2^m 2^{2m}$
By Pumping Lemma, a can be decomposed as a = uvxyz with |vxz| $\leq$ m and |vy| $\geq$ 1 such that $uv^i xy^i z \in L_1$ for i $\geq$ 0.

•Case 1
for i = 0
$\underbrace{22...211...}_{\mathrm{uvxy}}\ \underbrace{11...122...2}_{\mathrm{z}}$
|a| = 2m - |vy| is less than | a^R |, so $uv^0xy^0z \notin L_1$
•Case 2
for i = 0

$\underbrace{22...211...1}_{u}\ \underbrace{11...122...}_{vxy}\ \underbrace{22...2}_{z}$

$|\ a^R\ | = 2m - |vy|$ is less than $|a|$, so $uv^0xy^0z \notin L_1$

•Case 3

for i=0

$\underbrace{22...211...1122...2}_{u}\ \underbrace{22...2}_{vxyz}$

$|\ 2^{|a|}\ | = 2m - |vy|$ is less than $|a|$. So, $uv^0xy^0z \notin L_1$

•Case 4

for $i = 0$

$\underbrace{22...1}_{u}\ \underbrace{11...1}_{vxy}\ \underbrace{122...22...2}_{z}$

$|\ a|\ 2^{|a|}\ |\ | = 4m - |vy|$

$(4m - |vy|)/2 < |\ 2^{|a|}\ | = 2m$. So, $uv^0xy^0z \notin L_1$

•Case 5

for $i = 0$

$\underbrace{22...11...12}_{u}\ \underbrace{22...2}_{vxy}\ \underbrace{22...2}_{z}$

$|\ a^R 2^{|a|}\ | = 4m - |vy|$

$(4m - |vy|)/2 < |a| = 2m$. So, $uv^0xy^0z \notin L_1$

Contradiction. Then, this assumption is false.

Hence, L_1 is not context-free.

b.

L_2 is a CFL.

•A is context free.

$S \rightarrow xSz \mid T$

$T \rightarrow yTz \mid \varepsilon$

•B is a regular language.

$S \rightarrow XY$

$X \rightarrow x \mid xx \mid xxx \mid ... \mid x^{100} \mid \varepsilon$

$Y \rightarrow y \mid yy \mid yyy \mid ... \mid y^{100} \mid \varepsilon$

Note that if B is a RL, also $\overline{B}$ is a RL.

$L_2 = A \setminus B = A \cap \overline{B}$ is context-free.

Intersection of a CFL and a RL is a context-free language because we can construct a new NPDA machine that accepts the NPDA of that context free language and the DFA of that regular language.

c.

Assume that L_3 is a context free language.

Since L_3 is infinite, we can apply Pumping Lemma.

Pick a string w = $a^k b^k c^{k^2}$, w $\in L_3$.
By Pumping Lemma w can be decomposed as w = uvxyz with $|vxz| \leq k$ and $|vy| \geq 1$ such that $uv^ixy^iz \in L_3$ for $i \geq 0$.
Now, We will examine all the possible locations of string vxz in w.

•Case 1

for i=0

$\underbrace{aaa...a}_{uvxy}\ \underbrace{aabb...bcc...c}_{z}$

$uv^0xy^0z = a^{k-|vy|}b^kc^{k^2} \notin L_3$

•Case 2

for i=0

$\underbrace{aaa...a}_{uv}\ \underbrace{ab}_{x}\ \underbrace{b...b}_{y}\ \underbrace{bcc...b}_{z}$

$uv^0xy^0z = a^{k-|v|}b^{k-|y|}c^{k^2} \notin L_3$

•Case 3

for i=0

$\underbrace{aaa...ab}_{u}\ \underbrace{b...b}_{vxy}\ \underbrace{bcc...c}_{z}$

$uv^0xy^0z = a^kb^{k-|vy|}c^{k^2} \notin L_3$

•Case 4

for i=0

$\underbrace{aaa...ab}_{u}\ \underbrace{b...b}_{v}\ \underbrace{bc}_{x}\ \underbrace{cc...c}_{yz}$

$uv^0xy^0z = a^kb^{k-|v|}c^{k^2-|y|} \notin L_3$

•Case 5

for i=0

$\underbrace{aaa...abb...bc}_{u}\ \underbrace{c...c}_{vxyz}$

$uv^0xy^0z = a^kb^kc^{k^2-|vy|} \notin L_3$

•Case 6

v or y containing "ab" or "bc"
If i>0, uv^ixy^iz would be a…ab…bc…cb…c or a…ab…ba…ab…b…c…c. And it would not be in L_3.
There are no other cases to examine, and we got contradiction in all cases above.
So, L_3 is not context-free.

d.

$L_4 = \overline{L}$ = (1,2)* - L
L_4 = {(1,2)* - 1*2*} $\cup$ { 1^n2^m : n>m } $\cup$ { 1^n2^m : m>2n }
If each of the three parts of L_4 is context-free, we can see that L_4 is context-free, since union operator is closed for CFLs.
• {(1,2)* - 1*2*} is a regular language. Because (1,2)* and 1*2* are regular clearly and set differ-

ence is closed for RLs. And also every RL is a CFL.

• { 1^n2^m : n>m } is a context free language.

S → 1A

A → 1A | 1A2 | ε

• { 1^n2^m : m>2n } is a context free language.

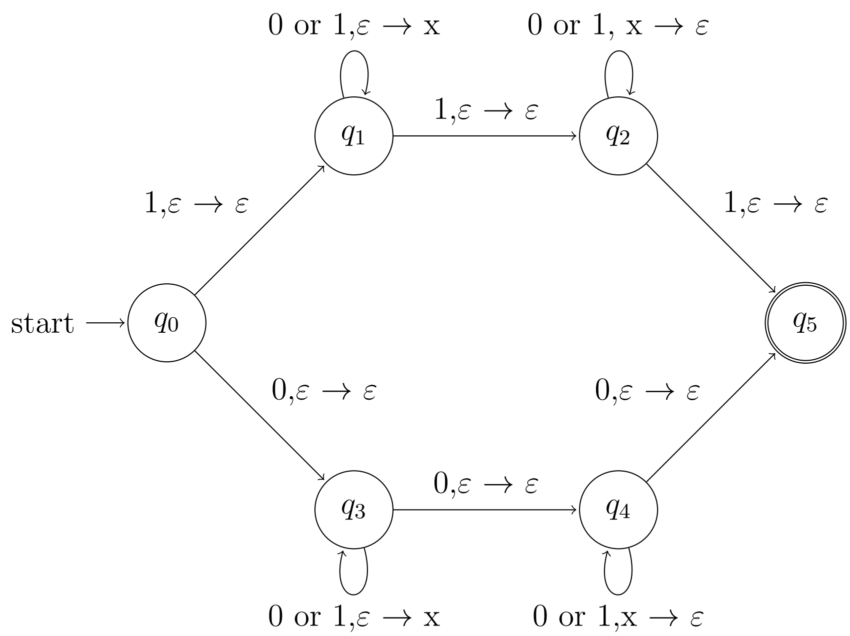
S → A2

A → A2 | 1A22 | ε

So, L_4 is context-free.

Answer 3

a.



b.

State	String	Stack
[q_0,	01000,	λ]
[q_3,	1000,	λ]
[q_3,	000,	x]
[q_4,	00,	x]
[q_4,	0,	λ]
[q_5,	λ,	λ]

"01000" is accepted.

c.

Firstly, we need to trace all computations of the string "00100"

1

$[q_0, 00100, \lambda]$
$[q_3, 0100, \lambda]$
$[q_3, 100, x]$
$[q_3, 00, xx]$
$[q_3, 0, xxx]$
$[q_3, \lambda, xxxx]$
Rejected.

2

$[q_0, 00100, \lambda]$
$[q_3, 0100, \lambda]$
$[q_3, 100, x]$
$[q_3, 00, xx]$
$[q_3, 0, xxx]$
$[q_4, \lambda, xxx]$
Rejected.

3

$[q_0, 00100, \lambda]$
$[q_3, 0100, \lambda]$
$[q_3, 100, x]$
$[q_3, 00, xx]$
$[q_4, 0, xx]$
$[q_4, \lambda, x]$
Rejected.

4

$[q_0, 00100, \lambda]$
$[q_3, 0100, \lambda]$
$[q_3, 100, x]$
$[q_3, 00, xx]$
$[q_4, 0, xx]$
$[q_5, \lambda, xx]$
Rejected.

5

$[q_0, 00100, \lambda]$
$[q_3, 0100, \lambda]$
$[q_4, 100, \lambda]$
Rejected.

There is no accepted one. Hence, $00100 \notin L(M)$.

Answer 4

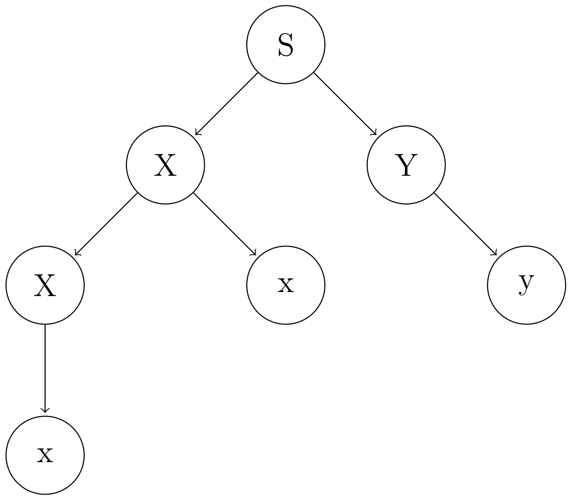
a.

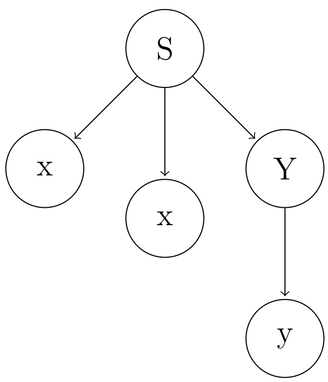
• $L_1 \cup (L_2 \setminus R)$ is context free.
Because, $L_2 \setminus R = L_2 \cap \overline{R}$ is context free.
If R is a regular language, $\overline{R}$ is also regular, since regular languages are closed under complementation.
And intersection of a CFL and a RL is a context-free language, because we can construct a new NPDA machine that accepts the NPDA of that context free language and the DFA of that regular language.
Now we know that $(L_2 \setminus R)$ is a CFL, also L_1 is a CFL.
Since context free languages are closed under union operation, $L_1 \cup (L_2 \setminus R)$ is context free.

• $R \setminus (L_1 \cup L_2)$ may or may not be context free.
As context free languages are closed under union operation, $(L_1 \cup L_2)$ is a context free language.
Let's say $L_1 \cup L_2 = L$ (a CFL).
$(R \setminus L)$ does not have to be context free.
$(R \setminus L) = (R \cap \overline{L})$, but $\overline{L}$ may not be context free, since the context-free languages are not closed under complementation.

b.

In order to show that G is ambiguous, we need to show a string that can be given by two different parse trees.
The string "xxy" has two different left-most derivations:
$S \Rightarrow XY \Rightarrow XxY \Rightarrow xxY \Rightarrow xxy$
$S \Rightarrow xxY \Rightarrow xxy$





Since "xxy" has two different left-most derivations, G is ambiguous.

c.

L = { $x^i y^i z^i$: $i \geq 0$ } is can be recognized by M, but not recognized by N.
Because it is well known that L is not context free, it cannot be recognized by N(1 stack PDA).
However L can be recognized by M(2 stack PDA) and it can be seen below.
⇒ Push all initial "x"s to a stack. Push the following "y"s to the other stack.
When a "z" is read, pop one "x" and one "y" from the stacks.
Since there is a "x" and a "y" for each "z", when the inputs end and the two stacks be empty, it will be accepted.

It can be seen that M is at least powerful as N, because a 2 stack PDA can simulate a 1 stack PDA, by not using it's second stack.
And, above, we saw that L = { $x^i y^i z^i$: $i \geq 0$ } is recognized by M, but not recognized by N.
That means, M(a 2 stack PDA) can recognize every language which N(a 1 stack PDA) can recognize, also M can recognize at least one different language(L) that cannot be recognized by N.
So, M is more powerful than N.