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Answer 1

a.

(a)((a+c)*(b)(a+c)*(b)(a+c)*)(cc)

b.

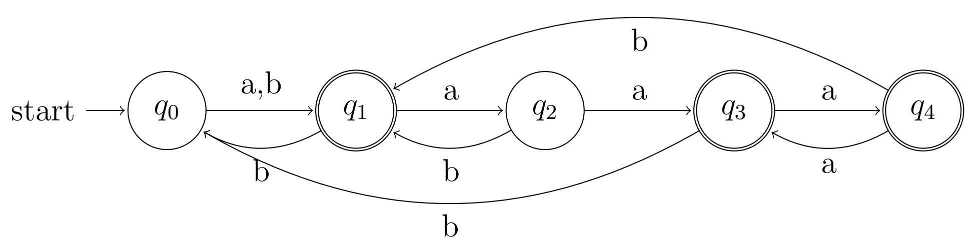
(b+c)(bb+bc+cb+cc)*(a)(bb+bc+cb+cc)* + (bb+bc+cb+cc)*(a)(b+c)(bb+bc+cb+cc)*

c.

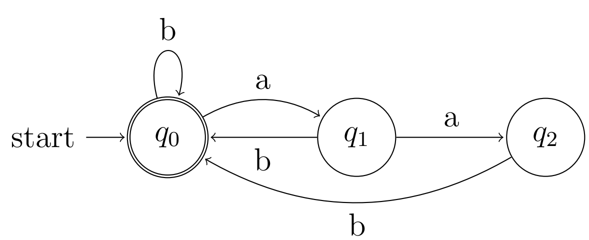
(a*)((b$^+$a$^+$b$^+$a$^+$)*)(b*)

Answer 2

a.



b.



Answer 3

It is not. {01},{0011}... are regular since they contain only a string,
however the infinite union is the set of $\{0^x1^x \mid x \geq 0\}$
and it is not regular obviously by Pumping Lemma.
Hence the infinite unions are not closed for the family of regular languages.

Answer 4

Let's say L = { $a^ib^jc^{2j} \mid i \geq 0 , j \geq 0$ }.
Since L is infinite, we can apply Pumping Lemma.
Let m be the integer in the Pumping Lemma.
Pick a string w such that $w \in L$ and length $|w| \geq m$.
We pick $w = a^mb^mc^{2m}$. Let's write $a^mb^mc^{2m} = xyz$.
From the pumping lemma, it must be length $|\ y\ z\ | \leq m$, $|\ y\ | \geq 1$

$$xyz = \overbrace{aa...a}^{m}\ \overbrace{bb...b}^{m}\ \overbrace{cc...cc...cc...cc...c}^{2m}$$
$$xyz = \underbrace{aa...abb...bcc...cc...cc}_{x}\ \underbrace{c...c}_{y}\ \underbrace{c...c}_{z}$$

Thus $y=c^k$, $k \geq 1$.
From the Pumping Lemma, $xy^iz = xz \in L$, i=0,1,2... So, $xy^0z = xz \in L$.

$$xz = \overbrace{aa...a}^{m}\ \overbrace{bb...b}^{m}\ \overbrace{cc...cc...cc...c}^{2m\text{-}k}$$
$$xz = \underbrace{aa...abb...bcc...cc...cc}_{x}\ \underbrace{c...c}_{z}$$

$a^m\ b^m\ c^{2m-k} \in L$, $k \geq 1$
But, L={ $a^ib^jc^{2j} \mid i \geq 0 , j \geq 0$ } ,therefore $a^m\ b^m\ c^{2m-k} \in L$ is not true!
And so L is not regular.

Answer 5

Proof1]

Regular languages are closed under set difference.
Because $L1 - L2 = L1 \cap \overline{L2} = \overline{\overline{L1} \cup L2}$.
And regular languages are closed under union(ii) and complementation(i).
$\overline{L1}$ and L2 is regular.
$\overline{\overline{L1} \cup L2}$ is also regular.
Hence, L1 - L2 is regular. For every pair of regular languages L1 and L2, L1 - L2 is also regular.
And the family of regular languages is closed under set difference.

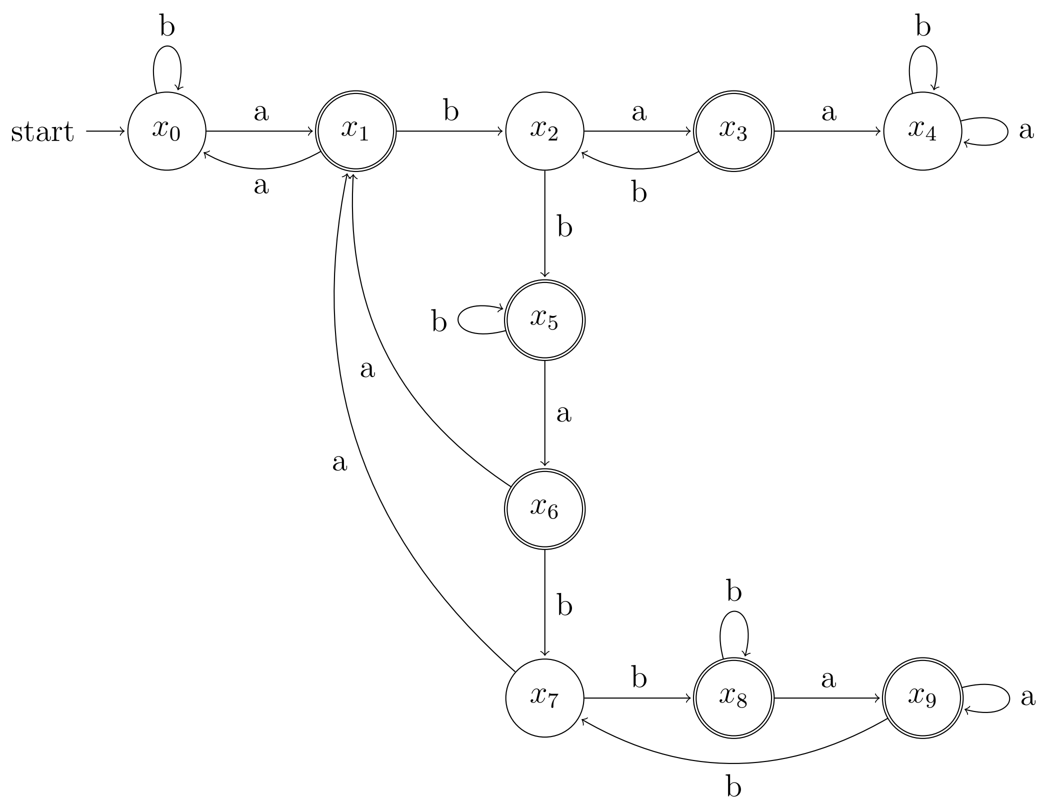
i) Complement of a language L is $\sum^*$ – L (for an alphabet $\sum$. And $\sum^*$ contains L)
Since $\sum^*$ is regular, the complement of a regular language is always regular.

ii)Let Lx and Ly be the languages of regular expressions R1 and R2,respectively.
Then R1 + R2 is a regular expression whose language is Lx ∪ Ly.
If Lx and Ly are regular, Lx ∪ Ly is also regular. So regular languages are closed under union.

Proof2]

Let L1 and L2 be DFA's languages.
Let L3 = L1 - L2
The final states of L3 will be the pairs where L1-state is final but L2-state is not.
And it is also a DFA language, so it is a regular language always.

Answer 6



x0 = q_0
x1 = q_1,q_2
x2 = q_3
x3 = q_2

x4 = - (trap)
x5 = q_1,q_3
x6 = q_0,q_2
x7 = q_0,q_3
x8 = q_0,q_1,q_3
x9 = q_0,q_1,q_2

There is no unreachable state, it can be seen on the graph clearly.
And also there is no non-distinguishable state, it can be seen in the table below.
Hence, this is the minimal DFA. It cannot be minimized anymore.

.	a	b
x0:	x1	x0
x1:	x0	x2
x2:	x3	x5
x3:	x4	x2
x4:	x4	x4
x5:	x6	x5
x6:	x1	x7
x7:	x1	x8
x8:	x9	x8
x9:	x9	x7