Tutorial -4.

Q.1 There are 3 black and 2 white Balls in Box. 2 balls are taken from it. Find expected no. of white ball

Ans. $x : 0, 1, 2$

3 black, 2 - white

(i) $P(x=0) = \frac{3c_2}{5c_2} = \frac{3}{10}$

(ii) $P(x=1) = \frac{3c_1 \cdot 2c_1}{5c_2} = \frac{6}{10}$

(iii) $P(x=2) = \frac{1}{10}$

$E(x) = f(0) + f(1) + f(2)$

$= (0)\frac{6}{10} + (1)\frac{3}{10} + (2)\frac{1}{10}$

$= \frac{6}{10} + \frac{2}{10}$

$= \frac{8}{10}$

$= 0.8$

Q.2 The probability of occurence of disease to a worker of chemical factory is $\frac{1}{4} = 0.25$. Find probability that 2 out of 5 worker chosen at a random will

Suffer from disease

Ans

$n = 5$

$x = 2$

$P(\text{Success}) = \frac{1}{4}$

$q = 1 - \frac{1}{4}$

$= 3/4$

$f(x) = \binom{n}{x} p^x q^{n-x}$

$= {}^5C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{5-2}$

$= 0.2637$

Q.3. Mean and Variance of Binomial distribution are 15 and 6 respectively. Find value of n and p.

Ans

$E(x) = n \cdot p$

$V(x) = n \cdot p \cdot q$

$\therefore E(x) = 15$

$V(x) = 6$

$\therefore np = 15$

$npq = 6$

$q = \frac{6}{15} = 0.4$

$p = 1 - q$

$= 1 - 0.4$

$\boxed{\therefore p = 0.6}$

$$npq = 6$$

$$n = \frac{6}{p \cdot q}$$

$$= \frac{6}{(0.6)(0.4)}$$

$$\boxed{n = 25}$$

8.4. Probability that a blade manufactured by factory is defective is $\frac{1}{500}$. ~~Lets~~ Blades are packed in packet of 10 blades. Find expected number of packets containing

1) no defective blades
2) 1 defective blade
3) 2 defective blade

in consignment of 10000 packets

ans

$$E(x) = n \cdot p$$

$$= \frac{10}{500}$$

$$\lambda = \frac{1}{50}$$

1) $$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$f(0) = \frac{e^{-1/50} \lambda^0}{0!} = e^{-1/50} = 0.9802$$

From 10000 packets there are 10000×0.9802
, Approx = 9802

2) $f(1) = \frac{e^{-1/50} \, d^1}{1!}$

$= e^{-1/50} \cdot \frac{1}{50}$

$= 0.0196$

From 10000 packets there are approximately $10000 \times 0.0196 = 196$ packets have one defective blade

3) $f(2) = \frac{e^{-1/50} \left(\frac{1}{50}\right)^2}{2!}$

$= \frac{0.0004}{2}$

$= 0.0002$

From 10000 packets there are approx $10000 \times 0.0002 = .2$ packets which have 2 defective blades

Q5 Between hours of 2 pm to 4 pm average no of phone calls per minute coming into switch board of company is 2.5. Find probability

that during I particular minute there will be no phone calls and ~~Exactly~~ three calls.

Ans $d = 2.5$

$$f(x=0) = \frac{e^{-2.5}(2.5)^0}{0!}$$

$$= 0.0821$$

$$f(x=3) = \frac{e^{(-2.5)} \cdot (2.5)^3}{3!}$$

$$= \frac{(0.0821)(15.6250)}{6}$$

$$= \frac{1.2826}{6}$$

$$= 0.2138.$$

Q6 The ~~everest~~ average height of group of soldier is 68.22 and the variance of high is 10.89. Out of 1000 soldier how many soldier do you expected to be 6 ft tall.

Ans $E(\bar{x}_{he}) = 68.22 = \mu$

$V(x) = 10.89 = \sigma^2$

$x \geq 72''$ (6 ft)

$P(x \geq 72) = P(z \geq 1.1459)$

$$z = \frac{72 - 68.22}{3.3000}$$

$$= \frac{3.78}{3.3}$$

$$= 1.1455$$

$$= 1.15 \quad \text{Round off}$$

$$= P(z \geq 1.1455)$$

$$= P = 0.5 - P(0 \leq z \leq 1.15)$$

$$= 0.5 - 0.3749$$

$$= 0.1251$$

Q.7 A coin is tossed 900 times. Find probability that no. of heads is between 434.35 and 465

ans

$$\text{Mean} = np$$

$$= 900 \cdot \frac{1}{2} \quad (\text{Heads})$$

$$= 450$$

$$\text{Variance} = \sqrt{\frac{900}{4}}$$

$$= \sqrt{225} = 15$$

$$Z = \frac{700 - 435}{}$$

$$Z = \frac{435 - 450}{15}$$

$$= -1$$

$$Z = \frac{465 - 450}{15}$$

$$= 1$$

$$P(-1 \leq Z \leq 1) = 2(P(Z \leq 1)$$

$$= 2 \cdot (0.3413)$$

$$= 0.6826$$

Q8 A bag contains 20 balls out of which 15 balls are of red colour and 5 balls are of black colour. A random sample of 5 balls is taken. Find the probability that sample contain 2 ~~black~~ red balls

ans 15 Red 5 Black

$$n = 5$$

$$p = \frac{5}{20} = \frac{1}{4}$$

$$q = 1 - \frac{1}{4}$$

$$= \frac{3}{4}$$

$$f(x=2) = {}^5C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^3$$

$$= \frac{5 \times 4^2}{2} \cdot \frac{1}{16} \cdot \frac{27}{64}$$

$$= \frac{10}{16} \cdot \frac{27}{64} = \frac{270}{1024} = 0.263$$

Q.9 There are 10 electronic bulbs in a box out of which 3 bulbs are defective if 3 bulbs are selected at random from the box. Find the expected number of defective bulbs

Ans

$$E(x) = n \cdot p$$

$$= 3 \cdot \frac{3}{10}$$

$$= \frac{9}{10}$$

$$= 0.9$$

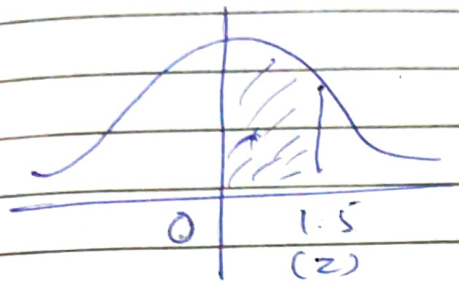
p = defective bulbs

$$= \frac{3}{10}$$

Q20 Find probability that standard normal variant lies between 0-1.5 it means

$P(0 \leq z \leq 1.5) = 0.4332$

(According to Normal distribution Table)



Q.17 (1) A weekly wage of 1000 work men are normally distributed around mean of 70 and S.D of Rs 5. Estimate no. of workers whose weekly wage will be

1) Between Rs 70 to 72
2) Between Rs 69 to 72
3) more than 75
4) more than 80
5) less than 60

Ans 1) $x = 70$ $x = 72$ $\sigma = 5$, $\mu = 70$

$z = \frac{70-70}{5}$ $z = \frac{72-70}{5}$

$= 0$ $= \frac{2}{5} = 0.4$

$f(z>0) + f(z<0.4)$

$P(0<z<0.4) = 0.1554$

2) $x = 69$ $\quad\quad$ $x = 72$

$\frac{69-70}{5}$ $\quad\quad$ $\frac{72-70}{5}$

$z = \frac{-1}{5}$ $\quad\quad$ $= \frac{2}{5}$

$= -0.2$ $\quad\quad$ $= 0.4$

$\therefore P(0<z<0.2) + P(0<z<0.4) = 0.2347$

3) $z = \frac{75-70}{5} = 1$ $\quad$ $P(z>1) = 0.5 - P(0 \leq z \leq 1)$

$= 0.5 - 0.3413$

$= 0.1587$

4) $z = \frac{80-70}{5} = 2$ $\quad$ $P(z>2) = 0.5 - P(0 \leq z \leq 2)$

$= 0.5 - 0.4772$

$= 0.0228$

5) $z = \frac{60-70}{5} = -2$ $\quad$ $P(z<-2) = 0.5 - P(0 \leq z \leq 2)$

$= 0.0228$

Q. 12. The customer account of certain department store have an average balance Rs. 120 and standard deviation of Rs 40. Assume that account balance are normally distributed

1) what probability that account has Rs > 150
2) Probability account has balance between Rs 100 and Rs 150
3) what is a probability that account has balance between 60 and 90

(1) $z = \frac{150 - 120}{40}$ $\left(\frac{x - \mu}{\sigma}\right)$

$= 0.75$

$\therefore P(z > 0.75) = 0.5 - P(0 \le z \le 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$

(2) for $x = 100$

$z = \frac{100 - 120}{40}$

$= -0.5$

for $x = 150$

$= \frac{150 - 120}{40}$

$= 0.75$

$\therefore P(-0.5 \le z \le 0.75)$

$= P(0 \le z \le 0.5) + P(0 \le z \le 0.75)$

$= 0.1915 + 0.2734$

$= 0.4649$

(3) for $x = 60$

$z = \frac{60 - 120}{40}$

$= \frac{-60}{40}$

$= -1.5$

for $x = 90$

$z = \frac{90 - 120}{40}$

$= \frac{-30}{40}$

$= -0.75$

$\therefore P(0 \le z \le 1.5) + P(0 \le z \le 0.75)$

$= 0.4332 + 0.2734$

$= 0.7066$