University of Waterloo
CS 341 Fall 2022
Written Assignment 3

Due Date: Monday Nov. 7th at Midnight (11:59pm), to Crowdmark
All work submitted must be the student's own.

Question 1 [12 marks] Greedy

Suppose there are n houses built along an east-west road. The position of each house is specified by its distance from the western endpoint; they are given in an array $P[1...n]$, where $P[i] = x$ denotes the distance of house i from the western endpoint. A number of communication towers are to be built. Each tower can cover the houses placed in distance of at most R from it. The goal is to place towers on the road so that every house is covered by at least one tower.

Give a greedy algorithm to find the minimum number of towers required. Provide

- Pseudocode for the algorithm,
- a proof that it is correct (i.e., produces the minimum possible number of towers), and
- give the asymptotic runtime.

Question 2 [8 marks] Greedy vs. Dynamic Programming

In the topic of greedy algorithms, we solved a problem named *Scheduling to minimize lateness*. Suppose we want to solve this problem using dynamic programming.

Give a definition of the subproblems, provide the base cases, derive and justify a recursive relation, analyze the runtime of your algorithm (if implemented - you do not need to give the implementation) and state a conclusion on the feasibility of using dynamic programming to solve this problem.

Question 3 [12 marks] Graphs

The input is an **undirected unweighted** graph $G = (V, E)$ and two nodes w and v in G. The desired output is a single number: the number of shortest v-w paths in G. (Here, "shortest" means the minimal number of edges.) There could be multiple shortest paths; we are looking for the number of shortest paths, not the list of all such paths.

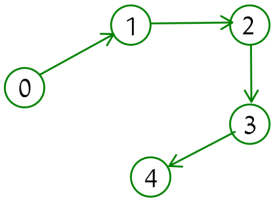
Provide an $O(n + m)$ algorithm to solve this problem.

Give pseudocode for your algorithm, justify its correctness, and analyze its runtime (i.e., explain why it has the desired complexity).

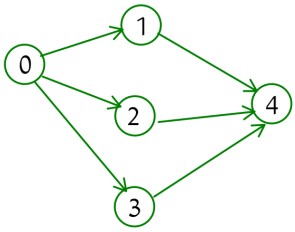
Question 4 [12 marks] Graphs

The input is a **acyclic directed unweighted** graph $G = (V, E)$ containing $m = |E|$ edges and $n = |V|$ nodes. The edges in the graph define a dependency relationship between the nodes. We define that a node v *depends* on node $u \neq v$ if there is a directed path from u to v; that is, if either edge (u, v) exists in the graph, or there is another node z so that (z, v) is an edge in the graph and z depends on u. A node u is an *ultimate node* if every other node v in the graph, either u depends on v or v depends on u.

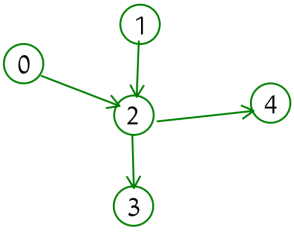
Give an algorithm that outputs a list of all ultimate nodes in the graph. Give pseudocode, provide a correctness argument, and analyze the runtime of your algorithm. A few examples are shown in the following figure:



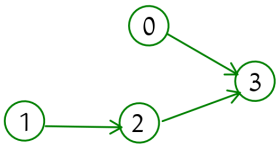
ultimate nodes: 0, 1, 2, 3 ,4



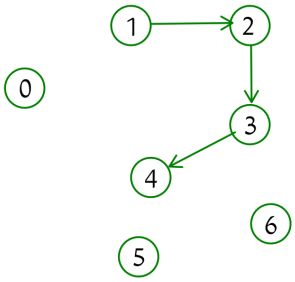
ultimate nodes: 0, 4



ultimate nodes: 2



ultimate nodes: 3



ultimate nodes: None