University of Waterloo
Department of Electrical & Computer Engineering

SE 380 : Introduction to feedback control

Midterm Exam

February 28 2013

Name: ______________________________

Student Number: ______________________________

Notes

- 13 total pages including this one.
- Aids: non-programmable (i.e. non-graphing) calculator, ruler.
- Total marks available: 50.
- You may write on the backs of pages.
- Only exams written in ink will be considered for re-marking.
- **Answers given without justification will not be considered.**
- Useful formulae on page 13.

Problem	Mark	Available Marks
1		10
2		10
3		10
4		10
5		10
Total		50

Problem 1 [10 Marks = 4 + 4 + 2]

(a) A single-input, single-output system has impulse response

$$g(t) = te^{-2t} + \cos(t), \qquad t \geq 0.$$

Is this system BIBO stable?

(b) The transfer function of an LC circuit is

$$G(s) = \frac{1}{LCs^2 + 1}.$$

Is the output bounded if the input is the unit step? Assume that $L > 0$, $C > 0$.

(c) The transfer function of an LC circuit is

$$G(s) = \frac{1}{LCs^2 + 1}.$$

Prove that the circuit is not BIBO stable. Assume that $L > 0$, $C > 0$.

Problem 2 [10 Marks]

Draw the piece-wise linear approximate magnitude and phase Bode plots for the system

$$G(s) = \frac{80(s + 1000)}{s^2(s + 10)(s - 100)}.$$

Be sure to indicate the slopes on your final plots. For your convenience, log paper is provided in Figure 1.

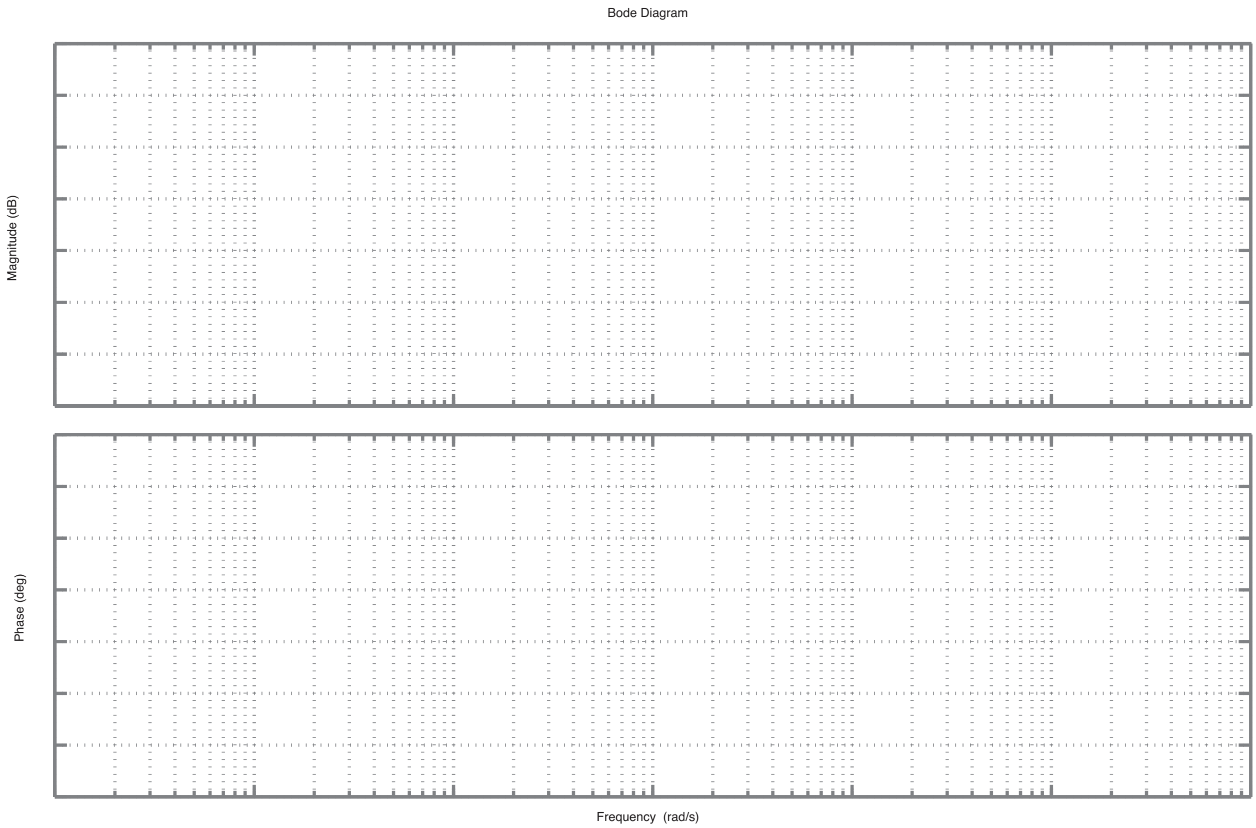

Figure 1: Bode plot.

Problem 3 [10 Marks = 1 + 1 + 4 + 4]

Consider the queueing system illustrated in Figure 2. A simplified continuous-time state space

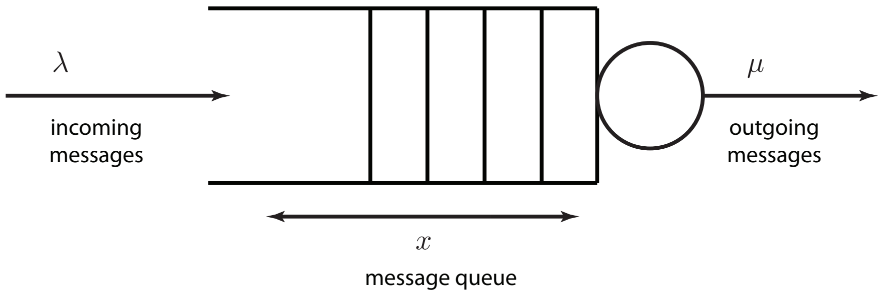

Figure 2: Schematic diagram of a queuing system. Messages arrive at rate λ and are stored in a queue. Messages are processed and removed from the queue at rate μ. The average size of the queue is given by $x \in \mathbb{R}$.

model for this system, using the notation in Figure 2, is given by

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= \lambda u - \mu_{\max}\frac{x}{1+x} \\ y &= x. \end{aligned}$$

Here $x \in \mathbb{R}$ is the average queue size, $u \in \mathbb{R}$ is an admission control policy that accepts incoming requests when the queue can service them quickly and rejects them when the queue is full so that they can try another server. Assume throughout this question that the arrival rate λ is constant and that the maximum service rate $\mu_{\max}$ is constant.

(a) Suppose that there is no admission control, i.e., suppose that $u = 1$, so that every incoming request is accepted. Find the equilibrium of the system.

(b) Now suppose that we want to control the queue system with an average queue size of 4, i.e., $y = y_0 = 4$. Find the corresponding equilibrium (x_0, u_0) associated with y_0.

(c) Linearize the queueing system about the equilibrium point from part (b). Write down the matrices A, B, C, D in the linearized model.

(d) Find the transfer function, the bandwidth and the steady-state gain of the linearized system from part (c).

Problem 4 [10 Marks]

Consider the system in Figure 3. Find the four transfer functions $\frac{Y}{R}$, $\frac{Y}{D}$, $\frac{U}{R}$, $\frac{U}{D}$.

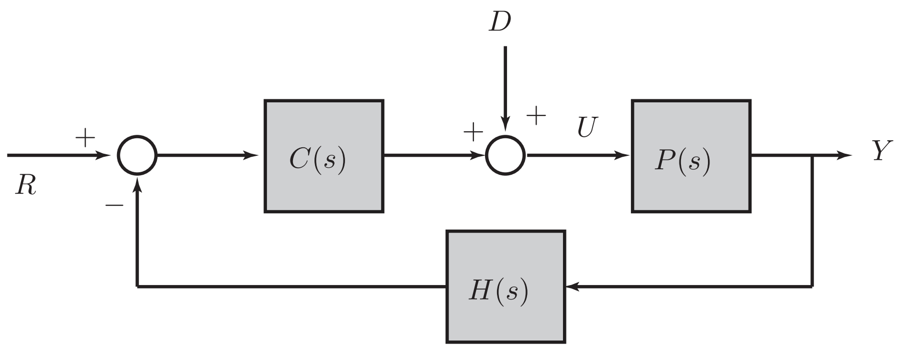

Figure 3: Block diagram for problem 4.

Problem 5 [10 Marks = 2+2+2+2+2]

Consider the feedback system in Figure 4.

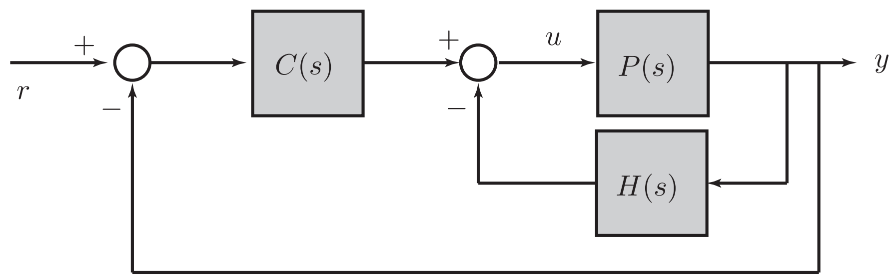

Figure 4: Feedback system for problem 5.

This is a control system for the plant

$$P(s) = \frac{1}{s^2}$$

with an inner-loop feedback controller

$$H(s) = K_D s$$

and an outer-loop feedback proportional controller

$$C(s) = K_P$$

where K_P and K_D are real, constant, controller gains.

(a) Find the transfer function from the reference input r to the output y.

(b) Find conditions on K_P and K_D so that the output response, due to the reference input r, is under damped.

(c) Suppose that we are given the following specifications for the transfer function from R to Y.

- BIBO stability
- $T_s \leq 3$ for a step input.
- Less than 10 percent overshoot for a step input.

Draw the region of allowable s-plane pole locations so that the closed-loop system meets these specifications.

(d) Choose values of K_P and K_D so that the specifications from part (c) are met.

(e) Using the values of K_P and K_D from part (d), find the steady-state tracking error $e_{ss} = \lim_{t\to\infty}(r(t) - y(t))$ due to a unit ramp reference input $r(t) = t$, $t \geq 0$.

Useful formulae

$$T_\mathrm{s} \approx \frac{4}{\zeta\omega_n} \text{ (2\% settling time)}, \qquad \%\mathrm{OS} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \text{ (overshoot)} \Leftrightarrow \zeta = -\frac{\ln\%\mathrm{OS}}{\sqrt{\pi^2 + (\ln\%\mathrm{OS})^2}}$$

$$T_\mathrm{p} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \text{ (time-to-peak)}, \qquad T_\mathrm{r} \approx \frac{2.16\zeta + 0.6}{\omega_n} \text{ (rise time for } 0.3 < \zeta < 0.8)$$

Prototype first and second order systems

$$G(s) = \frac{K}{1+\tau s}, \qquad G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Table 1: Important (one-sided) Laplace transforms.

Description	Time domain $x(t)$, $t \geq 0$	s-Domain $X(s)$
Unit step	$\mathbf{1}(t)$	$\frac{1}{s}$
Impulse	$\delta(t)$	1
Ramp	t	$\frac{1}{s^2}$
Exponential	e^{at}	$\frac{1}{s-a}$
Sine	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
General exponential	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
Growing / decaying sine	$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$
Growing / decaying cosine	$e^{at}\cos(\omega t)$	$\frac{s}{(s-a)^2+\omega^2}$
Sine with linear growth	$t\sin(\omega t)$	$\frac{2s\omega}{(s^2+\omega^2)^2}$
Cosine with linear growth	$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$