My grades for Midterm

Q1

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When the time goes toward infinity, what can be said about the value of N in (a) and (b)?
(a) $\frac{dN}{dt} = -\lambda_1 N$
(b) $\frac{dN}{dt} = \lambda_2 N$

- N in (a) and (b) goes to infinity
- N in (a) and (b) goes to zero
- N goes to zero in (a) and to infinity in (b)
- ✔ It depends on the values of λ_1 and λ_2

Q2

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The ODE below has two different cases for its initial condition:

$$\frac{dx}{dt} = 3x - 10x^2$$

Case (a): x(0) =1

Case (b): x(0)=4

What would be the values of x in these cases in steady state?

- x will be 1/3 in case (a) and x will be 4/3 in case (b).
- x will be 1/10 in case (a) and x will be 4/10 in case (b).

- [x] x will be 3/10 in both cases.
- [ ] x will be 10/3 in both cases.

Q3

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Which of the four statements about the differential equation $\frac{dp}{dt} = rp(1 - \frac{p}{k})$ are true?

- [ ] k is related to the growth rate of p
- [x] r could depend on various factors, not just

one factor

- Both the growth rate and carrying capacity are equal
- Both the growth rate and carrying capacity are positive
- Getting infected repeatedly with Covid is good for your immune

system

Q4

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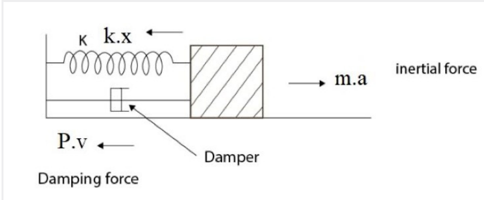
Consider a damped harmonic oscillator with mass m=2 shown in the attached figure, which can be described by the following differential equation:

$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$, with

$x(0) = 1$ and $x'(0) = 0$.

What can we say about the values of k (the spring constant) and P (the damping constant)?



- Values of P and k are equal to each other.
- Value of P is 6 but we cannot determine the value of k based on the provided information.
- Value of P will be 6.
- Value of k will be 16.

Q5

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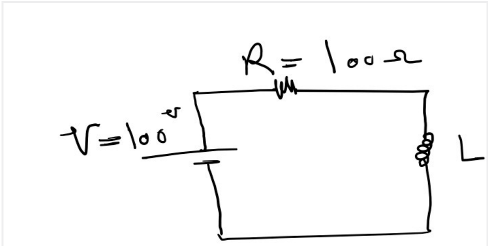
Consider the circuit in the

attached figure, which consist of an ideal voltage source (which gets turned on at time $t = 0$), ideal resistor with resistance R, and an ideal inductor with inductivity L. In case (a), the value of L is unknown, while in case (b) $L = 20mH$. What can we say about the steady-state current in cases (a) and (b)?



- [x] The steady-state current is 1A in both cases.
- [ ] The steady-state current is 1A in case (b) but it cannot be determined in case (a).
- [ ] The steady-state current is 0A in case (a) and 1A in case (b).
- [ ] The steady-state current is 1A in case (a) and 0A in case (b).

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Q6

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Consider a rational function $H(s) = \frac{s}{(s^2+1)^2}$. If we know that $H(s)$ is the Laplace transform of a convolution between two functions $f(t)$ and $g(t)$, what could be the Laplace transforms $F(s)$ and $G(s)$ of these two functions?

- ○ $F(s) = G(s) = \frac{1}{2}\frac{s}{(s^2+1)^2}$
- ✔ ● $F(s) = \frac{s}{s^2+1}$; $G(s) = \frac{1}{s^2+1}$
- ○ $F(s) = G(s) = \frac{s}{(s^2+1)^2}$
- ○ $F(s) = \frac{2s}{s^2+1}$; $G(s) = \frac{1}{s^2+1}$

Q7

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What is the inverse Laplace transform of

$F(s) = \frac{s^2+7s+12}{s^2+5s+6}$?

- $f(t) = 2e^{-2t}$
- ✔ (selected) $f(t) = 2e^{-2t}u_{-1}(t) + \delta(t)$, where where $u_{-1}(t)$ refers to the unit step function and $\delta(t)$ refers to Dirac delta function (aka unit-impulse)
- $f(t) = 2e^{-2t}\delta(t)$, where $\delta(t)$ refers to Dirac delta function (aka unit-impulse)
- $f(t) = 2(e^{-2t} - 4e^{-6t})u_{-1}(t)$, where $u_{-1}(t)$ refers to the unit step function
- none of the above

Q8

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An even function x(t) has a Laplace transform X(s) and x(0)=0. If we know that $\int_0^{+\infty}$ x(t) dt=13, then which one of the following statements is true?

- [x] $X(0) = 26$
- [ ] $lim_{s\to\infty} X(s) = 13$
- [ ] $X(0) = 13$
- [ ] $X(s)$ has a pole at s=13
- [ ] none of the above

none of the above statements are correct

Q9

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Consider the function $g(t) = x(t) + x(-2t)$, with $x(t) = 5e^{-3t}u_{-1}(t)$, where $u_{-1}(t)$ denotes the unit step function. If $G(s)$ is the Laplace transform of g(t), what would be $G(s)$ region of convergence (ROC)?

- ROC for $G(s)$ does no exist
- -3<s
- s<3
- -6<s<3
- ✔ (selected) none of the above

Q10

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What is the Laplace transform of
$x(t) = e^{-3t}u_{-1}(t-5)$?

- $X(s) = \frac{e^{-5s}}{s+3}$
- $X(s) = \frac{e^{5s}}{s+3}$
- $X(s) = \frac{e^{-5(s-3)}}{s+3}$
- ✔ (selected) $X(s) = \frac{e^{-5(s+3)}}{s+3}$
- none of the above

Q11

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Consider two functions $x(t)$ and $y(t)$, such that for $t < 0$ $x(t) = y(t) = 0$, that are related through the following differential equations:

$\frac{dx}{dt} = -3y(t) + 2\delta(t)$

$\frac{dy}{dt} = 5x(t)$

What are the Laplace transforms $X(s)$ and $Y(s)$ of $x(t)$ and $y(t)$?

- $X(s) = \frac{3s}{s^2+15}$ and $Y(s) = \frac{15}{s^2+15}$
- ✔ $X(s) = \frac{2s}{s^2+15}$ and $Y(s) = \frac{10}{s^2+15}$
- $X(s) = \frac{s^2+s}{s^2+15}$ and $Y(s) = \frac{5s+5}{s^2+15}$
- $X(s) = \frac{s^2+2s}{s^2+15}$ and $Y(s) = \frac{5s+10}{s^2+15}$
- none of the above

Q12

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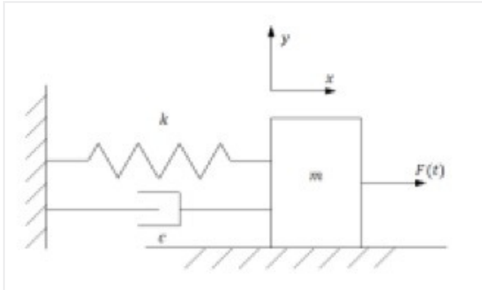
A damped harmonic oscillator consisting of a mass, spring, and damper (see attached picture) can be described by the following differential equation:
$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$,
where x(t) is the position of the mass and $x(0) = x'(0) = 0$.

What will be the position of the mass at t = 5s, if m=100g, $k = 4N/m$ and $c = 1N.s/m$?



- 5m
- 1m
- ✔ 0m (selected)
- 10m