Joint lecture MTE 544 – SE 380

The separation principle

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Logistics

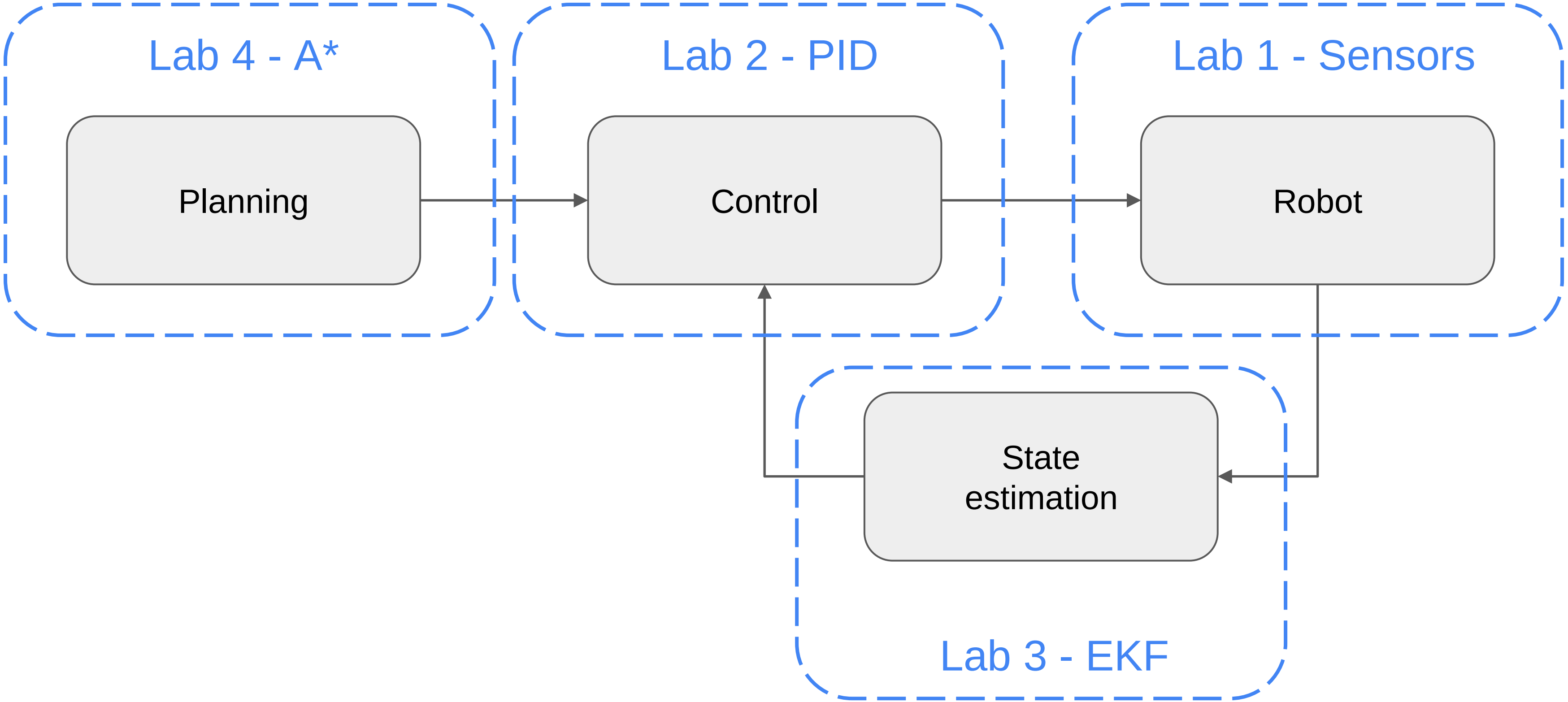
4:30 – 5:15: Lecture

5:15 – 6:00: Live demo in E7 2nd floor event space with TurtleBot & Robomaster

Overview of MTE544 - Autonomous Mobile Robots

- Mobile robot modeling
- Sensors
- Localization
- Planning

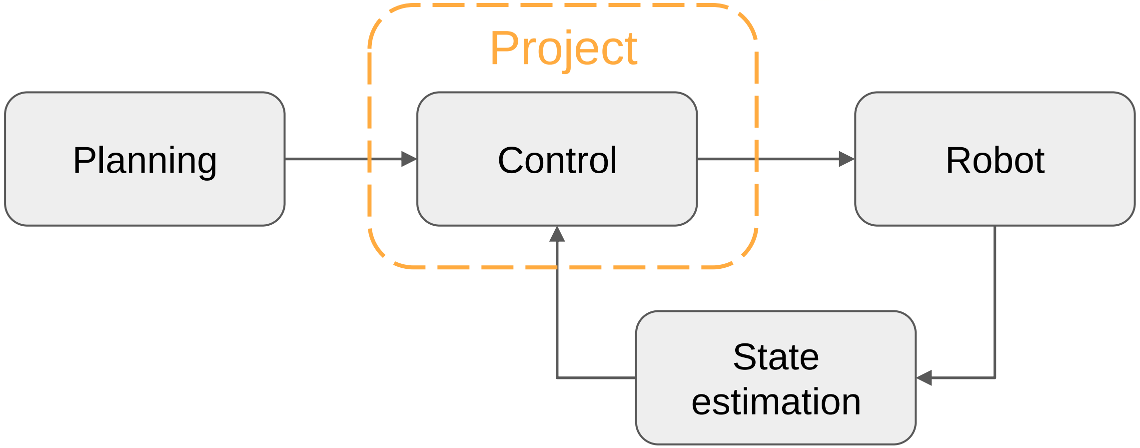
Overview of MTE544 - Autonomous Mobile Robots



Overview of SE380 - Introduction to Feedback Control

(Similar to MTE360 - Automatic Control Systems)

- Linear systems modeling and analysis
- Control design
  - Loop shaping
  - Root locus
  - State feedback



State feedback control design

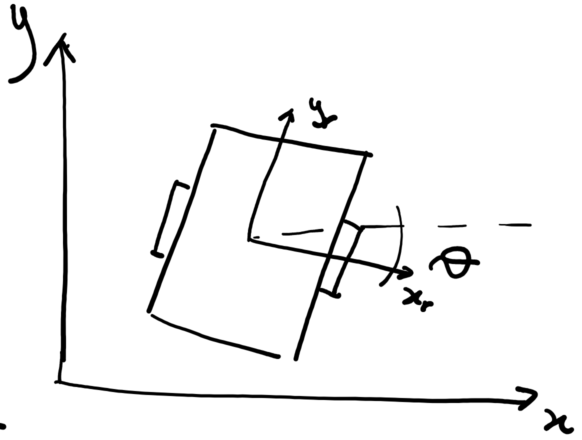
$$\begin{cases} \dot{x} = \boxed{A}x + \boxed{B}u \\ y = \boxed{C}x + \boxed{D}u \end{cases}$$

$x \in \mathbb{R}^n$

$u \in \mathbb{R}^m$

$y \in \mathbb{R}^p$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\vartheta} \end{bmatrix} = \begin{bmatrix} \cos\vartheta & 0 \\ \sin\vartheta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$



$x_{,} \equiv p$

$$\begin{cases} \dot{p} = v \\ \dot{\vartheta} = \omega \\ y = \begin{bmatrix} p \\ u \end{bmatrix} \end{cases}$$

$\vartheta \to$ orientation of the robot

State feedback control design

$(A, B) \Rightarrow$ controllable

$\forall \{\lambda_1^*, \ldots, \lambda_n^*\} \; \exists$ state feedback controller

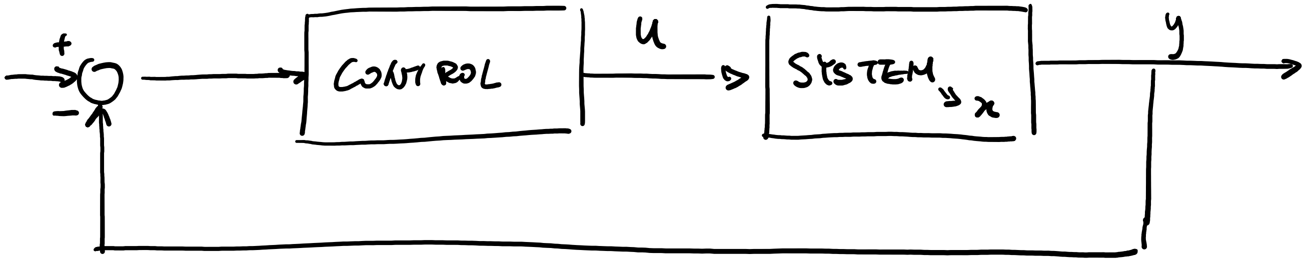
$u = -Kx \qquad K \in \mathbb{R}^{m \times n}$

feedback gain matrix

such that $\dot{x} = Ax + Bu =$

$$= \underbrace{(A - BK)}_{\text{eigenvalues of this matrix are at } \{\lambda_1^*, \ldots, \lambda_n^*\}} u$$

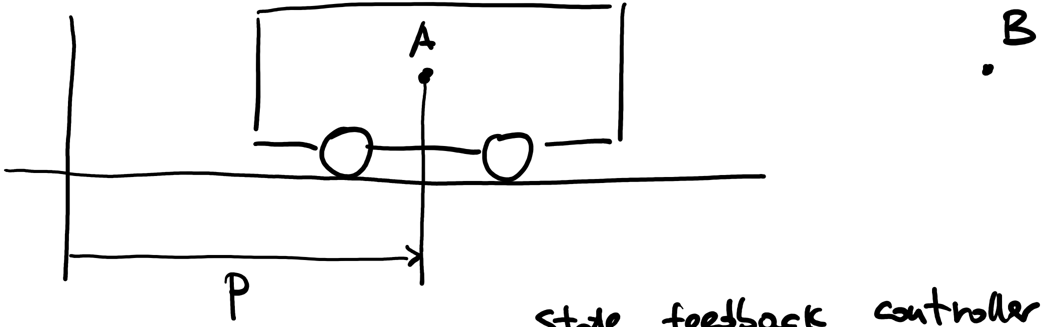
State feedback control design



If only y can be measured $\Rightarrow$ we cannot implement the control $u = -Kx$

State feedback control design

Example



$x = \begin{bmatrix} p \\ \dot{p} \end{bmatrix}$ $y = p$

$u = \boxed{\text{acceleration}}$

state feedback controller

$a = -k_1 p - \underbrace{k_2 \dot{p}}$

$\dot{p}$?

State estimator design

$$u = -Kx$$

Estimate of x : $\boxed{\hat{x}}$

$$\dot{\hat{x}} = A\hat{x} + Bu$$

$$e = x - \hat{x}$$

$$\begin{aligned}\dot{e} &= \dot{x} - \dot{\hat{x}} \\ &= Ax + \cancel{Bu} \\ &\quad - A\hat{x} - \cancel{Bu} \\ &= Ae\end{aligned}$$

State estimator design

$$\dot{e} = Ae \qquad e \longrightarrow 0 \; ?$$

Works if $\quad A$ is Hurwitz

$$\left(\mathrm{Re}(\lambda(A)) < 0 \right)$$

State estimator design

$$\dot{\hat{x}} = \boxed{\underbrace{A\hat{x} + Bu}_{\text{REPLICA OF SYS}}}\ \boxed{\underbrace{-L(\hat{y} - y)}_{\text{CLOSED-LOOP BASED ON MEAS.}}}$$

$$\hat{y} = C\hat{x} + Du \qquad L \in \mathbb{R}^{n \times p}$$

State estimator design

$$\dot{e} = \dot{\hat{x}} - \dot{x}$$
$$= A\hat{x} + \cancel{Bu} - L(\hat{y} - y) - Ax - \cancel{Bu}$$
$$= A(\hat{x} - x) - L(C\hat{x} + \cancel{Du} - Cx - \cancel{Du})$$
$$= Ae - LC(\hat{x} - x)$$
$$= \underbrace{(A - LC)}\, e$$

CONTROLLED ERROR DYNAMICS

State estimator design

(A, C) is OBSERVABLE

$$\Downarrow$$

$$\forall \{ \lambda_1^* \ldots \lambda_n^* \}$$

$$\exists \; L :$$

STATE OBS. $\dot{\hat{x}} = A\hat{x} + Bu - L(\hat{y} - y)$

$$\ell_{ij} \; \boxed{(A - LC)} = \{ \lambda_1^* \ldots \lambda_n^* \}$$

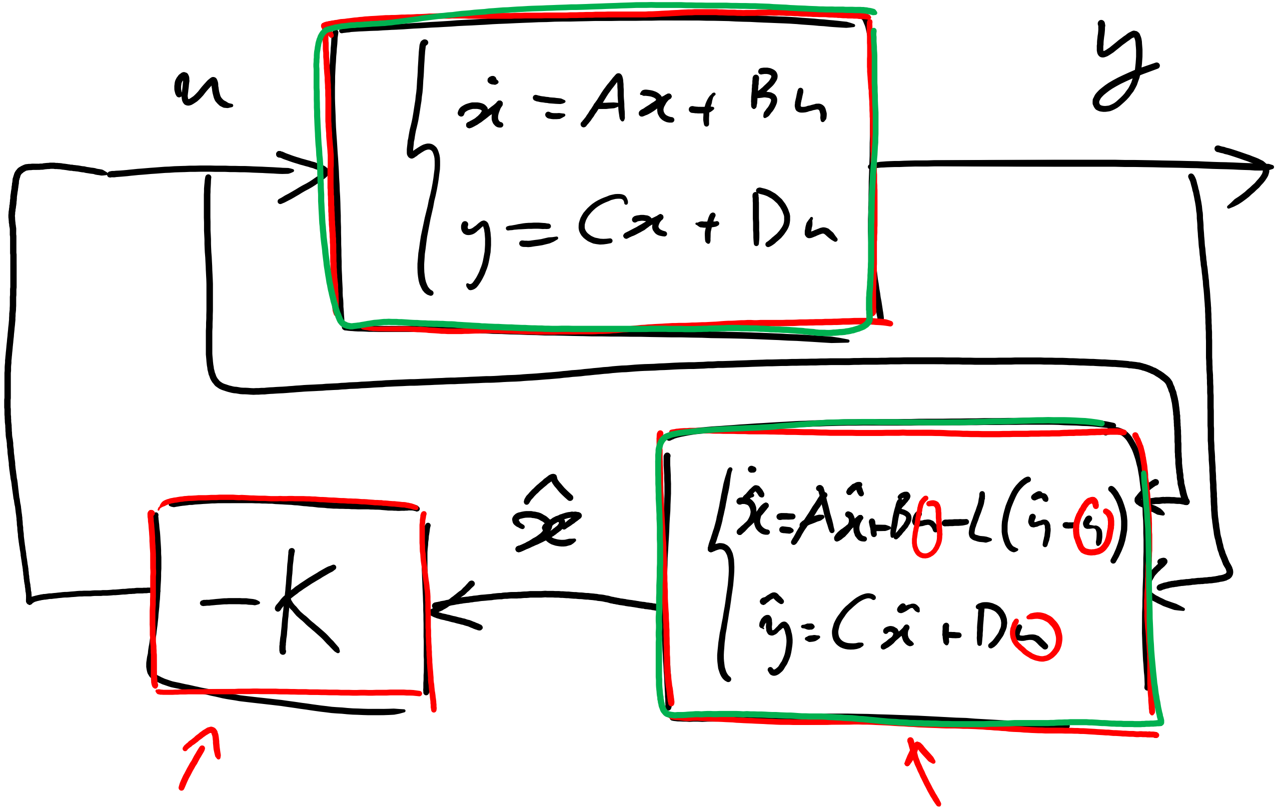
State estimator design

$$\dot{e} = (A - LC)\, e \implies e \longrightarrow 0$$

$$\hat{x} - x \longrightarrow 0$$

$$\hat{x} \longrightarrow x$$

Putting estimation & control together



Putting estimation & control together

$$\dot{x} = Ax + Bu$$

$$= Ax - BK\hat{x}$$

$$= Ax - BK(x + e)$$

$$= (\underline{A - BK})x \; \underline{- BKe}$$

$$\dot{e} = \underline{(A - LC)}e$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A-BK & \cancel{+BK} \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

$$\begin{bmatrix} \boxed{\cdot} & \cdot \\ 0 & \boxed{\cdot} \end{bmatrix}$$

Putting estimation & control together

SYSTEM

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

STATE ESTIMATOR

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu - L(\hat{y} - y) \\ \hat{y} = C\hat{x} + Du \end{cases}$$

STATE FEEDBACK CONTROLLER

$$u = -K\hat{x}$$

eigenvalue are union of

$$\underbrace{A-BK}_{\substack{\text{SE380} \\ \text{design this}}} \quad \text{and} \quad \underbrace{A-LC}_{\substack{\text{MTE544} \\ \text{design this}}}$$

$\boxed{\text{INDEPENDENTLY}}$

Putting estimation & control together

MTE 544 students

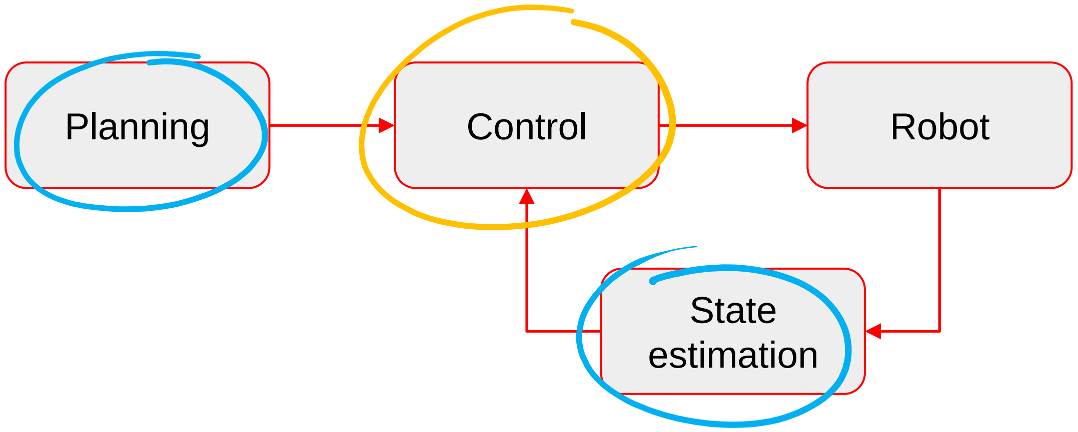
You can choose your <u>favourite planner</u> and your <u>favourite eigenvalues</u> to design a state estimator

SE 380 students

You can choose your <u>favourite eigenvalues</u> for state feedback control design

The <u>separation principle</u> tells us that:

You can close the loop and it just works!



Summary

The separation principle:

- Allows us to design a state feedback controller and implement it using an estimate of the state, instead of the actual one
- Allows us to design a state estimator, plug it in in feedback with the real system controlled using the state feedback controller
- Allows us **not to be friend** with SE380 students (for MTE544 students) and with MTE544 students (for SE380 students)
- Does **not** allow us to **blame** MTE544 students and SE380 students if our state estimator or state feedback controller, respectively, does not work

Demo - Event space 2nd floor