Final exam

Due: Saturday, April 9, 2022 10:05 pm (EDT)

Assignment description

This is the final exam for Math 213 (W2022). It consists of ten questions overall: six multiple-choice questions, two short-answer questions (Q3 and Q4), and two long-answer questions (Q8 and Q9). The exam is *'open book'*, i.e. you may use course materials (your notes, lecture slides, assignment solutions -- both yours and the ones posted on Learn, John Thistle's notes, Chen's book). **Please do not use other materials!** You may use a calculator. Collaboration and external help are not allowed. The nominal time allocated for the exam is 120min, plus a 30min grace period to upload answers to Q3, Q4, Q8, and Q9.

Make sure you hit 'submit' before 10:05pm

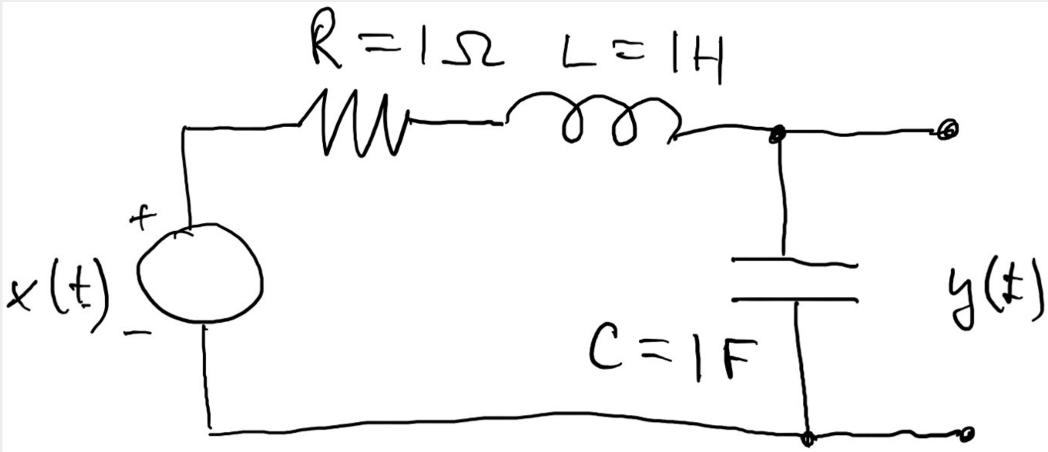
Late submissions will incur a 5% per minute penalty (unless you can prove technical difficulties).

Please refrain from discussing your answers to the exam questions until ~11pm (some students were granted extra time via AAS)

Submit your assignment

Help

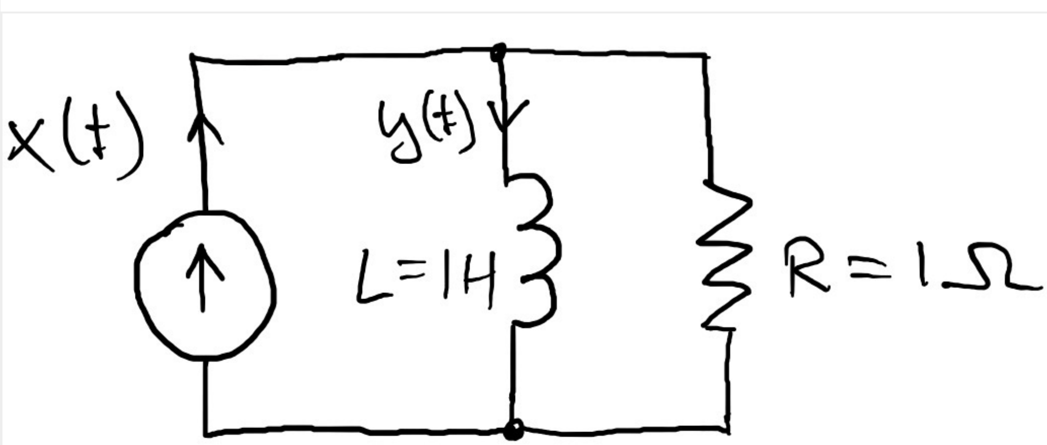
Q1 (10 points)



Consider the RLC circuit shown in the figure. Here, $x(t)$ is the input voltage. The voltage $y(t)$ across the capacitor is considered the system output. What is the differential equation relating $x(t)$ and $y(t)$?

- $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y(t) = x(t)$
- $\frac{d^2y}{dt^2} + \frac{dy}{dt} - y(t) = x(t)$
- $\frac{d^2y}{dt^2} - \frac{dy}{dt} + y(t) = x(t)$
- $\frac{d^2y}{dt^2} - \frac{dy}{dt} - y(t) = x(t)$
- other

Q2 (10 points)



A current source connected in the RL circuit in the figure produces an input current $x(t)$ and the system output is considered to be the current $y(t)$ flowing through the inductor. What is the frequency response $H(j\omega)$ of this system?

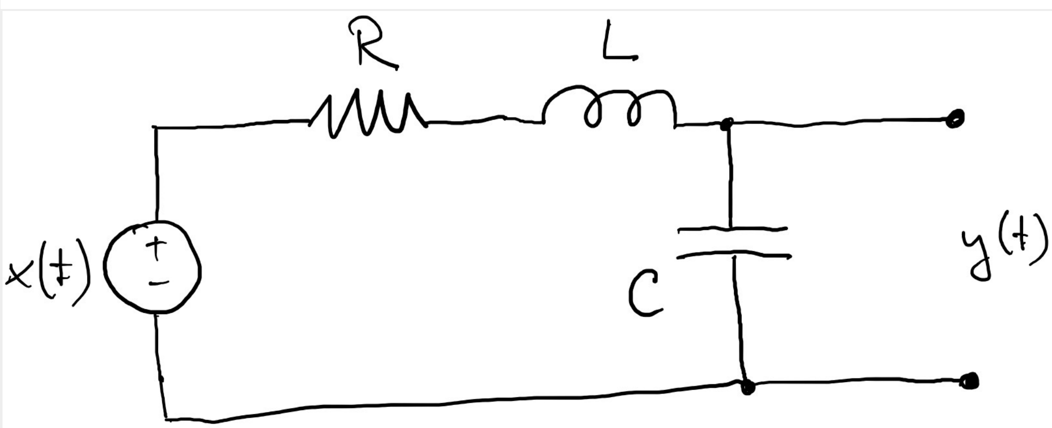
- $\frac{1}{j\omega}$
- $\frac{1}{1-j\omega}$
- $\frac{1}{1+j\omega}$
- $\frac{j\omega}{1+j\omega}$
- $1 + j\omega$

Q3 (10 points)

The frequency response of an LTI system S is $H(j\omega) = 1$, when $|\omega| \leq 100$, and $H(j\omega) = 0$ when $|\omega| > 100$. The input into S is a signal $x(t)$ with fundamental period $T = \pi/6$, which results in output signal $y(t) = x(t)$.
If we represent $x(t)$ as Fourier series with coefficients c_n, for what values of n is it guaranteed that $c_n = 0$?

(upload answer only; must fit onto a single line)

Q4 (10 points)



A system is implemented with an RLC circuit shown in the figure, with $x(t)$ voltage being the input signal and voltage $y(t)$ across the capacitor the system output. How should R, L, and C be related so that there is no oscillation in the output when the input to the system 'at rest' is a step function?

(upload answer and a brief justification only; must fit into two lines)

Q5 (10 points)

The input $x(t)$ and output $y(t)$ of a causal, stable LTI system are related by the differential equation
$\frac{dy}{dt} + 7y(t) = 3x(t)$.
What is the final value of the step response of this system?

- not possible to tell since the initial conditions are not provided
- 0
- 3
- 3/7
- other

Q6 (10 points)

The Fourier transform of the impulse response $h(t)$ of a causal LTI system is found to be $H(j\omega) = \frac{1}{4+j\omega}$. We observe an output signal $y(t) = e^{-4t}u_{-1}(t) - e^{-5t}u_{-1}(t)$, where $u_{-1}(t)$ is the unit step function, that results from an input signal $x(t)$. What is $x(t)$?

- $e^{-5t}u_{-1}(t)$
- $e^{-4t}u_{-1}(t)$
- $e^{-4t}u_{-1}(t) - e^{-5t}u_{-1}(t)$
- $e^{-3t}u_{-1}(t)$
- e^{-4t}

Q7 (10 points)

A causal LTI system is described by the following constant-coefficient linear differential equation:

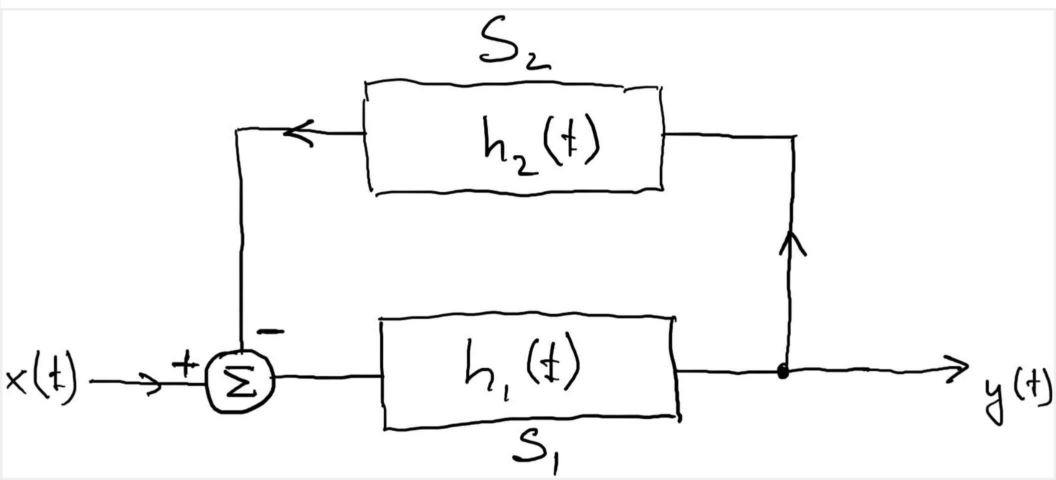
$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y(t) = x(t)$,

where $x(t)$ is the input, $y(t)$ is the output.

Is this system BIBO stable?

- yes
- no
- depends on the initial conditions
- depends on the input signal
- 42

Q8 (20 points)



Consider the system shown in the diagram, where $x(t)$ is the input, $y(t)$ is the output, $h_1(t) = \frac{1}{3}(e^t - e^{-2t})u_{-1}(t)$ is the impulse response of the subsystem S_1, $h_2(t) = K\delta(t)$ is the impulse response of the subsystem S_2, and Σ denotes signal addition (but note the signs).

For what values of K is the overall system BIBO stable?

(explain your reasoning; the provided answer must fit into one page or less)

Notice how adding a 'negative feedback' can turn an unstable system into a stable one.

Q9 (20 points)

A discrete-time system S can be described through the following recursive relation between its input $x[n]$ and its output $y[n]$:

$y[n] = \alpha y[n-1] + (1-\alpha)x[n]$,

where the integer n represents the discrete time and α is a real number, such that $0 < \alpha < 1$, which serves as a system parameter that can be set before the system starts operating.

a) [10 points] Write an explicit expression for $y[n]$ in terms of past and present input values $\{x[n], x[n-1], x[n-2], \ldots\}$.

b) [10 points] What is the impulse response $h[n]$ of this system?

(the provided answer must fit into one page or less)

Q10 (10 points)

Mark the systems described below using their responses to a complex exponential input e^{j5t}, which are **definitely not** LTI:

- ○ $S_1 \ : \ e^{j5t} \rightarrow te^{j5t}$
- ○ $S_2 \ : \ e^{j5t} \rightarrow e^{j5(t-1)}$
- ○ $S_3 \ : \ e^{j5t} \rightarrow cos(5t)$
- ○ $S_4 \ : \ e^{j5t} \rightarrow e^{j5t}u_{-1}(t)$
- ○ $S_5 \ : \ e^{j5t} \rightarrow 3e^{j5t}$