# ECE 380 — Final Exam

**Last Name** ______________________________

**First Name** ______________________________

**UWaterloo Email** ______________________________

**Student ID** ______________________________

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|---|---|
| Examination Date | April 09, 2022 |
| Duration of Exam | 2.5 Hours |
| Number of Test Pages | 32 (16 Double Sided) |
| Total Possible Marks | 90 |
| Additional Materials Allowed | See Instructions. |

**Instructions**

1. You are permitted to use:
   - one “cheat sheet” (US Letter or A4 sized), double sided, and
   - a standard or graphing calculator.
2. Write **clear and legible solutions**.
3. Your grade is influenced by how clearly you express your ideas, and how well you organize your solutions.
4. The weight of a question does not indicate the required effort to solve the question, nor how much work must be shown.
5. Numerical answers should be **in exact values unless otherwise specified**.

**Marking Scheme**

| | **Graded Out Of** |
|---|---|
| Question 1 | 15 |
| Question 2 | 15 |
| Question 3 | 15 |
| Question 4 | 15 |
| Question 5 | 10 |
| Question 6 | 10 |
| Question 7 | 10 |
| Total | 90 |

# Question 1

Consider a plant

$$P(s) = \frac{10}{s^2 - 5s - 10}$$

**(A)** Design an implementable (non-ideal) PID controller $C(s)$ so that the system in standard unity negative feedback satisfies the following specifications:

(i) a settling time of at most 4 seconds,

(ii) a time-to-peak approximately equal to $\pi$ seconds and

(iii) step disturbances are rejected.

[ /10]

**(B)** Consider your closed loop system. Which of the following statements are true?

(i) The closed-loop transfer function from $R(s)$ to $Y(s)$ does not have a zero.

(ii) A ramp disturbance results in non-zero steady-state error.

(iii) The state-space model of the controller is asymptotically stable.

(iv) The closed-loop system is IO stable.

(v) The closed-loop transfer function from $D(s)$ to $E(s)$ has two zeros at the origin.

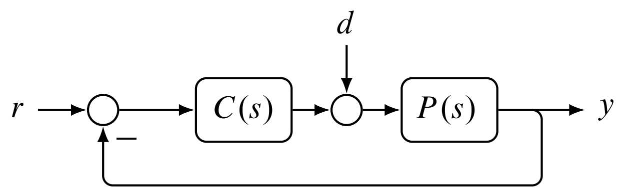
[ /5]

# Question 2

Consider the plant

$$P(s) = \frac{s+2}{s^2-1},$$

placed in standard unity negative feedback with a controller $C(s)$,




**(A)** Design a controller $C(s)$ so that the closed loop system tracks step references perfectly. `[  /10]`

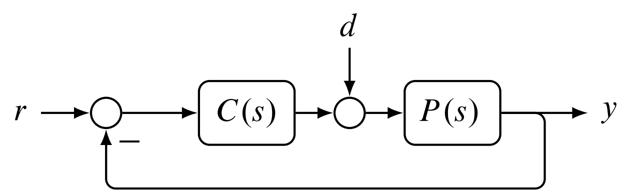
**(B)** Suppose instead that the plant

$$P(s) = \frac{s}{s^2-1}.$$

Can you design a controller $C(s)$ so that the closed loop system tracks step references perfectly? If so, produce the controller. If not, explain why. `[ /5]`

# Question 3

Consider the closed loop system,




**(A)** Let

$$C(s) = \frac{a_1 s^6 + a_2 s^5 + s^4 + s^3 + s^2 + s^1}{s^{10} - s^9 - s^7 + s}, \qquad P(s) = \frac{1}{s^5}.$$

Find conditions, if possible, on the parameters $a_1, a_2 \in \mathbb{R}$ so that the closed loop system is IO stable. If it is not possible to make the loop IO stable, explain why. `[ /5]`

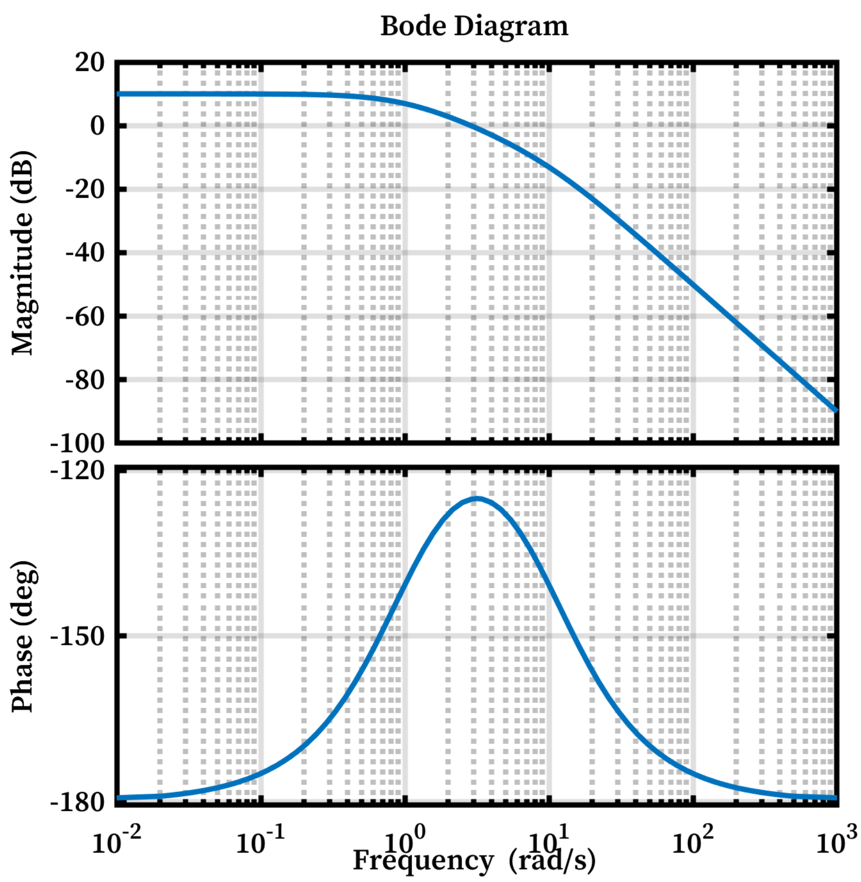
**(B)** Let

$$C(s) = \frac{a_3 s^2 + a_2 s + a_1}{s^2 + s}, \qquad P(s) = \frac{1}{s+1}.$$

Find conditions, if possible, on the parameters $a_1, a_2, a_3 \in \mathbb{R}$ so that the closed loop system is IO stable. If it is not possible to make the loop IO stable, explain why. `[ /10]`

# Question 4

Consider the Bode plot of the plant $P(s)$ given below. The plant $P(s)$ has a pole with positive real part, no poles on the imaginary axis, and an unknown number of poles with negative real part.




**(A)** Design a lead or lag compensator $C(s)$ so that the closed loop system in standard unity negative feedback achieves

- the steady-state error to a unit step reference is less than, or equal to $1/100$, and
- the phase margin is equal to $45°$.

[ /10]

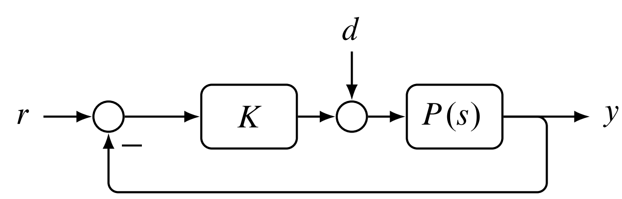
**(B)** You made a choice between either lead or lag compensation in your design for Part (A). Explain whether or not the other design methodology could work. Which compensator would you choose if we modified the first specification to read:

> the magnitude of the steady-state error to low frequency signals (i.e. $10\,\mathrm{rad\,s^{-1}}$) is less than, or equal to $1/100$ of the input amplitude.

[ /5]

# Question 5

Let $P(s)$ be strictly proper and BIBO stable. Consider the closed-loop system for $K > 0$,




**(A)** Using the root locus, graphically justify why the closed loop system is IO stable for sufficiently small $K > 0$. `[ /3]`

**(B)** In addition to being strictly proper and BIBO stable, suppose we can write $P(s)$ in the form,

$$P(s) = \frac{A(s)}{(s - p_1)(s - p_2)(s - p_3)(s - p_4)},$$

where $A(s)$ is a monic[1] polynomial. Propose necessary conditions on $P(s)$ so that the closed-loop system is IO stable for all $K > 0$. Justify your conditions using the root locus. *Hint: These conditions should place constraints on:*

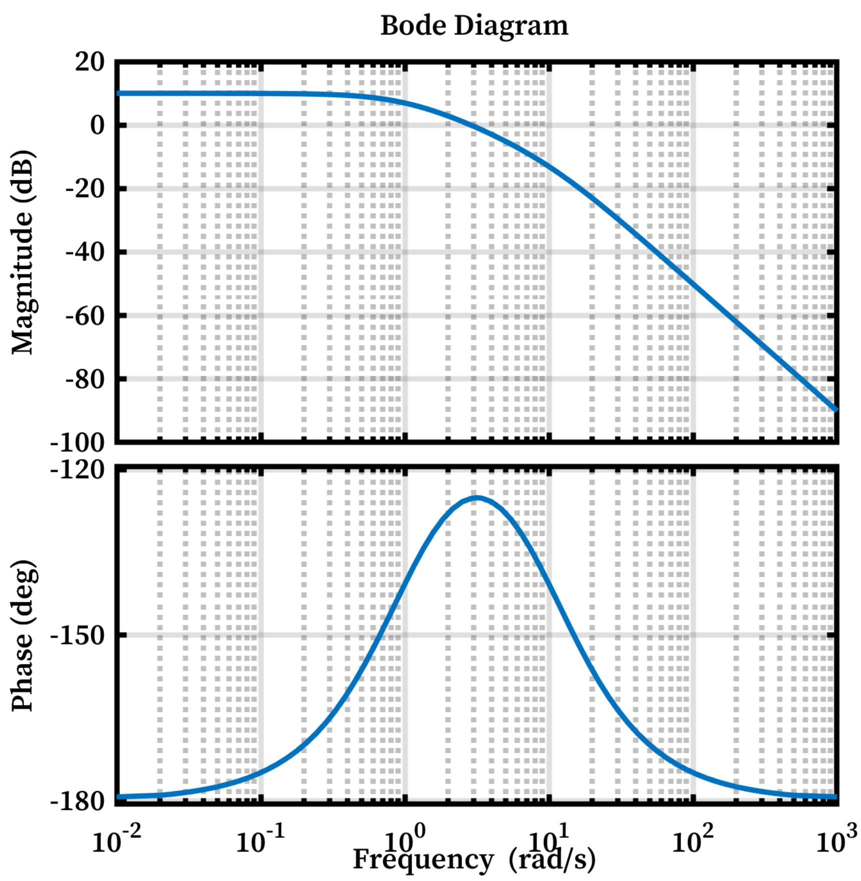
- *the number of roots of $A(s)$ and their locations in $\mathbb{C}$, and*
- *the relative location of $p_1, p_2, p_3, p_4$.*

`[ /7]`

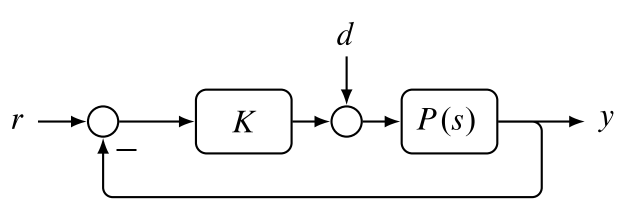
[1]Leading coefficient is 1.

## Question 6

The plant $P(s)$ with exactly one pole with strictly positive real part has the Bode plot:




Sketch the Nyquist Plot and use it to find conditions on the gain $K \neq 0$ so that the closed-loop system,

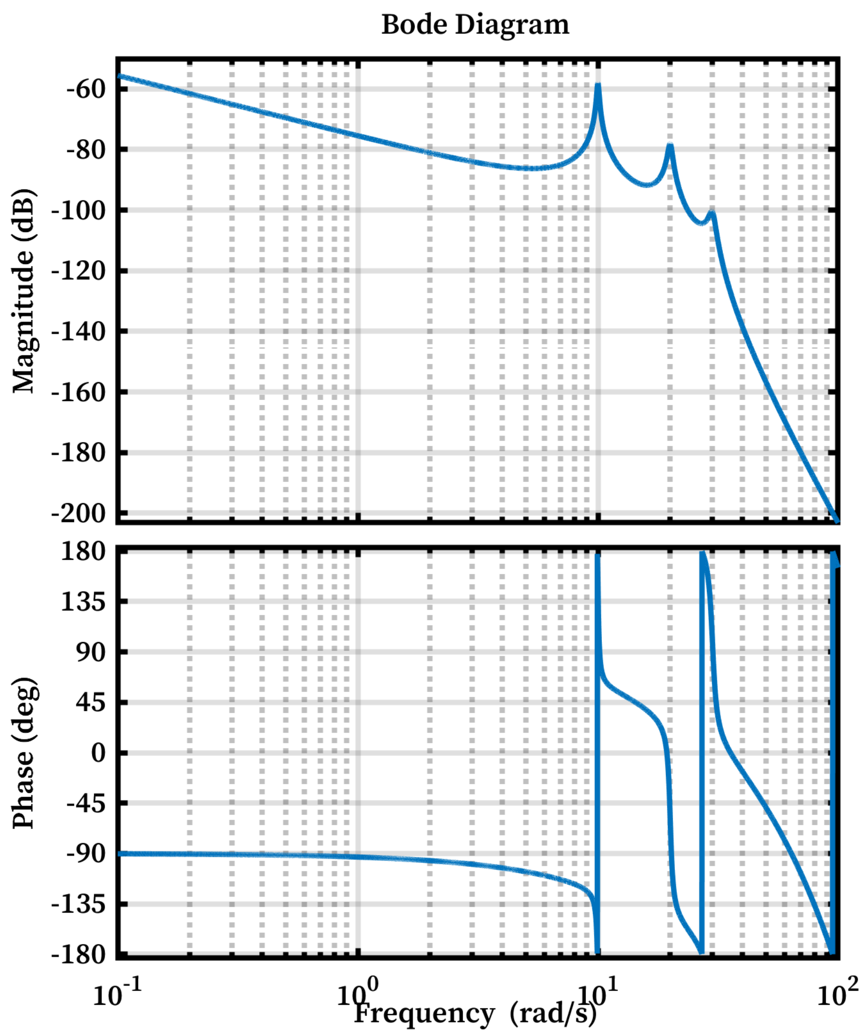



is IO stable. [ /10]

# Question 7

You are an engineer designing a motor control system for a long, rotating beam that is deployed on the Orbiter that travels to and from the International Space Station (ISS). The rotating beam is one component of the robotic arm used to manipulate and draw out components within the Orbiter and bring them towards the ISS. Engineering constraints demand that the beam be flexible. As a result, the end of the beam, lags behind the motion of the motor and, worse still, can oscillate even when the motor head is not rotating.

The applied input is a motor torque $u(t) \in \mathbb{R}$ and the output is the angular position $\theta(t) \in \mathbb{R}$ of the *end* of the beam. Engineers on your team acquire a Bode plot of the plant (below).




Consider the following specifications, listed in order of most important to least important,

(i) the controller implementation (state-space model) must be asymptotically stable,

(ii) the closed-loop system must be stable even in the presence of errors in the plant model,

(iii) perfectly track ramp references (constant motor head velocity),

(iv) perfectly reject step disturbances (constant unknown force),

Propose a controller design that can meet the specifications. If you cannot meet all the specifications, use your engineering judgement to suggest a compromise. Then explain how you would achieve this compromised specification. State the pros and cons of your design.

*In this question, I am looking for your thought process. It is less important to specify precise numbers, and more important to state your assumptions, and the procedures that must be performed precisely.* `[A+ = 10, A = 9, A- = 8, B = 7, C = 6, D = 5, E, = 2.5, F = 0]`

# Miscellaneous Formulas

**First Order Systems.**

$$G(s) = \frac{K}{\tau\, s + 1} \qquad T_{2\%} = 4\tau \qquad \omega_{\mathrm{BW}} = \frac{1}{\tau}$$

**Second Order Systems.**

$$G(s) = \frac{K\,\omega_n^2}{s^2 + 2\,\zeta\,\omega_n\, s + \omega_n^2} \qquad T_{2\%} \approx \frac{4}{\zeta\,\omega_n} \qquad T_p = \frac{\pi}{\omega_n\,\sqrt{1-\zeta^2}}$$

$$\%\mathrm{OS} = \exp\left(-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}\right) \iff \zeta = -\frac{\ln \%\mathrm{OS}}{\sqrt{\pi^2 + \ln(\%\mathrm{OS})^2}}.$$

**State-Space Models.**

$$\begin{aligned} \dot{x} &= A\,x + B\,u \\ y &= C\,x + D\,u \end{aligned} \qquad Y(s)/U(s) = C\,(sI - A)^{-1}\,B + D$$

$$\exp(t\,A) = \sum_{k=0}^{\infty} \frac{t^k}{k!}\,A^k = \mathfrak{L}^{-1}\{(sI - A)^{-1}\}$$

**Common Laplace Transforms.** All signals are assumed to be one-sided.

| Name | $f(t)$ | $F(s)$ |
|---|---|---|
| Impulse | $\delta(t)$ | $1$ |
| Unit Step | $\mathbf{1}(t)$ | $1/s$ |
| Unit Ramp | $t$ | $1/s^2$ |
| Exponential | $\exp(a\,t)$ | $1/(s-a)$ |
| Sine | $\sin(\omega\,t)$ | $\omega/(s^2+\omega^2)$ |
| Cosine | $\cos(\omega\,t)$ | $s/(s^2+\omega^2)$ |
| General Exponential | $t^n\ \exp(a\,t)$ | $n!/(s-a)^{n+1}$ |
| Growing/Decaying Sine | $\exp(a\,t)\ \sin(\omega\,t)$ | $\frac{\omega}{(s-a)^2+\omega^2}$ |
| Growing/Decaying Cosine | $\exp(a\,t)\ \cos(\omega\,t)$ | $\frac{s-a}{(s-a)^2+\omega^2}$ |

Time delay of $T$ seconds: $G(s) = \exp(-s\,T)$.

**Standard Unity Negative Feedback System**