Math 239 Final Exam Prep

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1 Final 2009 Winter

1.1 Question 1

[5 marks] Give a combinatorial proof that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

If a k-element subset of a set of n-elements includes the first element then you have to choose $k-1$ more elements from the remaining $n-1$ elements in the set which you can do in $\binom{n-1}{k-1}$ ways. If they do not include the first element then you have to choose k elements from the $n-1$ elements in the set that do not include the first element which you can do in $\binom{n-1}{k}$ ways. There are $\binom{n}{k}$ ways to choose a k-element subsets of a set of n elements, and every one of these will either include the first element or not, and so the total number of ways to do so is the sum of the ways to do it with the first element included and the ways to do it without it included: $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$.

1.2 Question 2

[4 marks] Let n be an even positive integer, and m a positive integer. Find an expression for the following coefficient, involving no summation notation. Justify your answer.

$$[x^n](1-3x^2)^{-m}(1+x)^2$$