University of Waterloo
Department of Electrical & Computer Engineering

SE 380 : Introduction to feedback control

Midterm Exam

February 29 2012

Name: ____________________

Student Number: ____________________

Notes

- 12 total pages including this one.
- Aids: non-programmable (i.e. non-graphing) calculator, ruler.
- Total marks available: 80.
- You may write on the backs of pages.
- Only exams written in ink will be considered for re-marking.
- Answers given without justification will not be considered.
- Useful formulae on page 12.

Problem	Mark	Available Marks
1		20
2		30
3		10
4		20
Total		80

Problem 1 [20 Marks = 5+15]

(a) Is the system

$$G(s) = \frac{1}{s^2 + 1}$$

BIBO stable? If not, find a bounded input that produces a bounded output.

(b) Draw the piece-wise linear approximate magnitude and phase Bode plots for the system

$$G(s) = \frac{40s^2(s - 100)}{(s + 1000)(s^2 + 20s + 400)}.$$

Be sure to indicate the slopes on your final plots. For your convenience, log paper is provided in Figure 1.

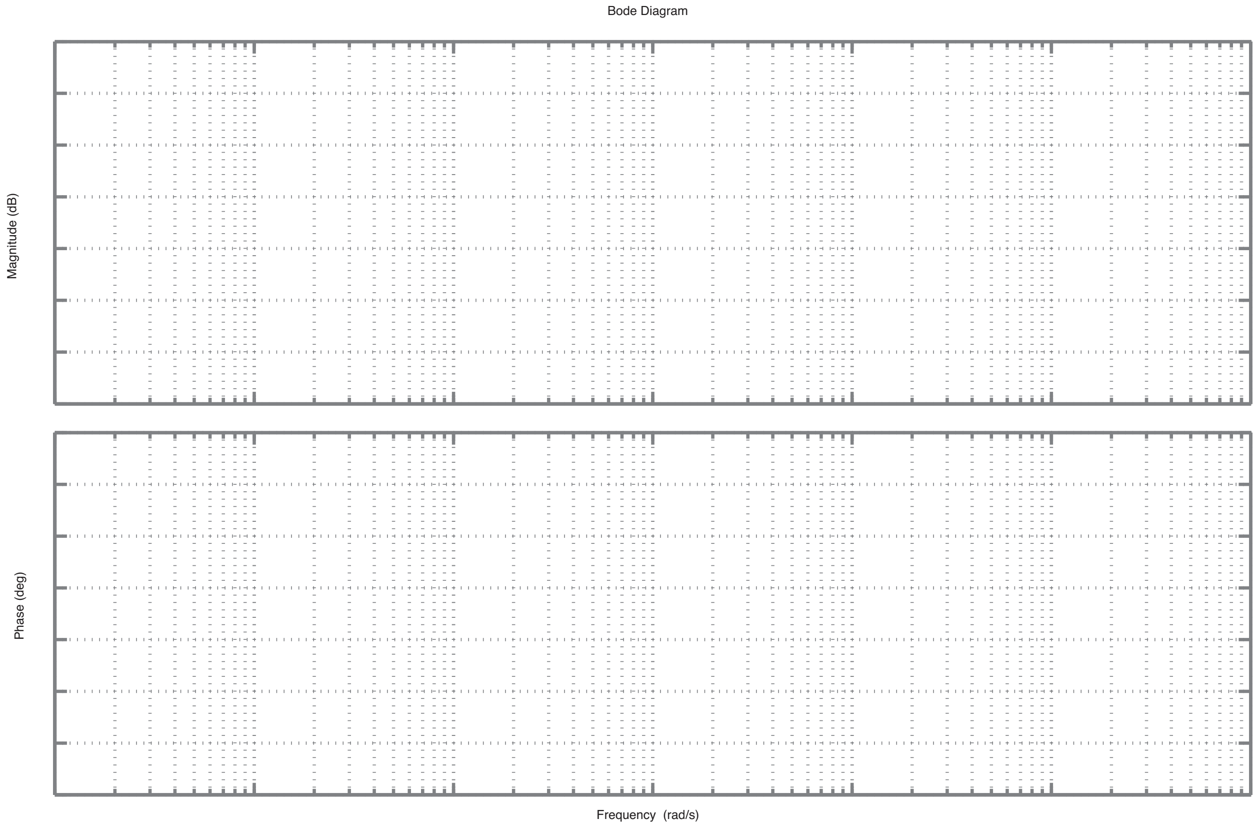

Figure 1: Bode plot.

Problem 2 [30 Marks = 5+10+5+10]

Suppose that we want to suspend a metal ball in the air by adjusting the current in an electromagnet as shown in Figure 2. In Figure 2 u is the input voltage; y is the position of the ball; i is the current

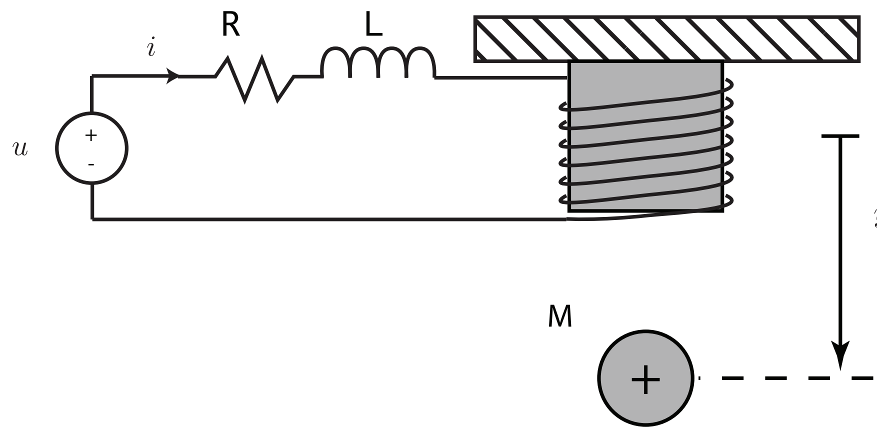

Figure 2: Electromagnetic levitation system.

in the windings, M is the mass of the ball; g is the acceleration due to gravity; L and R are the inductance and resistance of the coil. The magnet exerts an attractive force on the ball according to

$$F_{ball}(i, y) = \frac{Ki^2}{y}$$

where K is a positive constant.

(a) Assume that $L = 0$. Introduce the state variables $x_1 = y$, $x_2 = \dot{y}$. Find a nonlinear state model

$$\dot{x} = f(x, u)$$
$$y = h(x)$$

of the system. Note: we take the downward direction to be positive y as indicated in the figure.

(b) Suppose that we want the ball to be suspended at $y = 0.5$. Find the equilibrium values corresponding to the ball being at this desired position.

(c) Linearize the system about the equilibrium point from part (b).

(d) Let $K = 1$, $R = 1$, $M = 1$, $g = 9.8$. Find the transfer function from δu to δy for the linearized system.

Problem 3 [10 Marks]

Consider the system in Figure 3. Find the transfer function from R to Y.

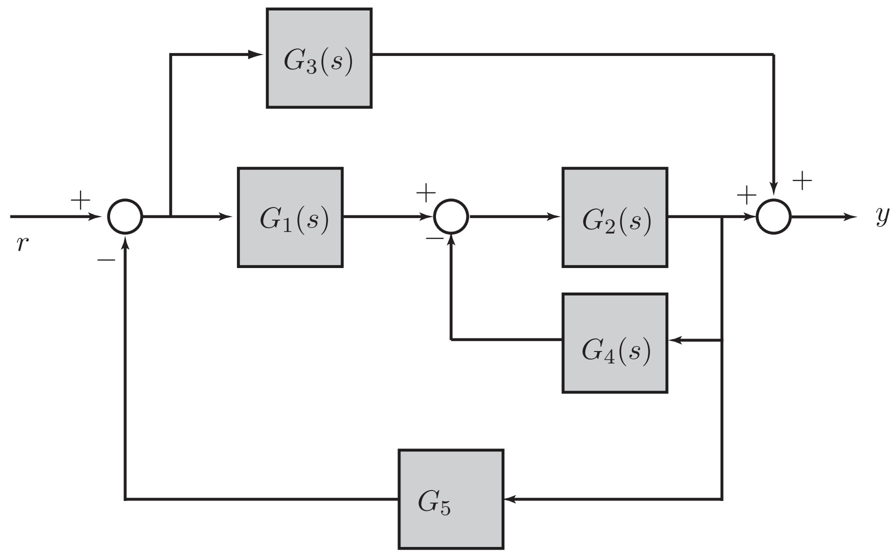

Figure 3: Block diagram for problem 3(a).

Problem 4 [20 Marks = 5+5+5+5]

Consider the feedback system in Figure 4 with

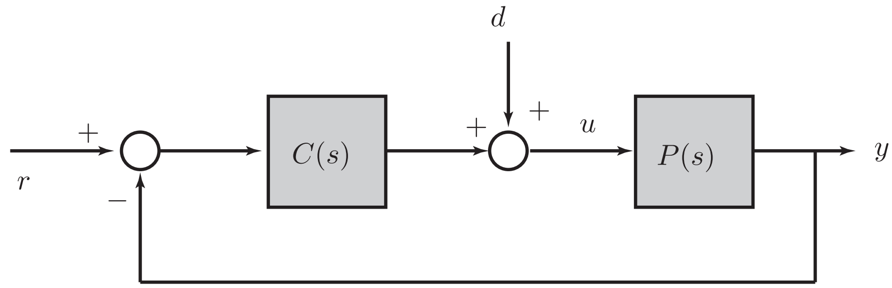

Figure 4: Feedback system for problem 4.

$$C(s) = K_P + \frac{K_I}{s}, \qquad P(s) = \frac{1}{2s+1}$$

where K_p and K_I are real, constant, controller gains.

(a) Find conditions on K_P and K_I so that the output response, due to the reference input r, is under damped.

(b) Suppose that we are given the following specifications for the transfer function from R to Y.

- BIBO stability
- $T_s \leq 3$ for a step input.
- Less than 10 percent overshoot for a step input.

Treating the closed-loop system as a prototype second order system, draw the region of allowable s-plane pole locations so that the system meets these specifications.

(c) Choose values of K_P and K_I so that the specifications from part (b) are met. Comment on how the actual closed-loop system will respond given that it isn't actually a prototype second order system.

(d) Using the values of K_I and K_P from part (c), find the steady-state output $y_{ss}(t)$ due to a unit step disturbance $d(t) = \mathbf{1}(t)$ when $r(t) = 0$.

Useful formulae

$$T_s \approx \frac{4}{\zeta\omega_n} \text{ (2\% settling time)}, \qquad \%OS = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \text{ (overshoot)} \Leftrightarrow \zeta = -\frac{\ln \%OS}{\sqrt{\pi^2 + (\ln \%OS)^2}}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \text{ (time-to-peak)}, \qquad T_r \approx \frac{2.16\zeta + 0.6}{\omega_n} \text{ (rise time for } 0.3 < \zeta < 0.8)$$

Prototype first and second order systems

$$G(s) = \frac{K}{1+\tau s}, \qquad G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Table 1: Important (one-sided) Laplace transforms.

Description	Time domain $x(t)$, $t \geq 0$	s-Domain $X(s)$
Unit step	$\mathbf{1}(t)$	$\frac{1}{s}$
Impulse	$\delta(t)$	1
Ramp	t	$\frac{1}{s^2}$
Exponential	e^{at}	$\frac{1}{s-a}$
Sine	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
Cosine	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
General exponential	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
Growing / decaying sine	$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$
Growing / decaying cosine	$e^{at}\cos(\omega t)$	$\frac{s}{(s-a)^2+\omega^2}$
Sine with linear growth	$t\sin(\omega t)$	$\frac{2s\omega}{(s^2+\omega^2)^2}$
Cosine with linear growth	$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$