SE 380 P3

1st Bilal Khan
b54khan@uwaterloo.ca

2nd Bohdan Hrotovytskyy
bhrotovy@uwaterloo.ca

I. Item 1

We can find the controllable canonical state space form of this transfer function by reading off from the transfer function:

$$\begin{aligned}
\dot{x_1} &= \begin{bmatrix} 0 & 1 \\ -0.005 & -1.05 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 \\
y_1 &= \begin{bmatrix} 0.53 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \end{bmatrix} u_1 \\
\dot{x_2} &= \begin{bmatrix} 0 & 1 \\ -0.005 & -1.05 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 \\
y_2 &= \begin{bmatrix} 0.53 & 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \end{bmatrix} u_2
\end{aligned}$$

II. Item 2

Solving the overshoot equation for ζ using wolfram alpha gives us $\zeta \approx 0.78$. This gives us $\omega_n = 1.28$ that satisfies the settling time. The desired closed loop poles are $-0.78 \pm 1.28i$. The desired characteristic polynomial is:

$$\begin{aligned}
P(\lambda) &= (\lambda - (-0.78 + 1.28i))(\lambda - (-0.78 - 1.28i)) \\
&= (\lambda + 0.78 - 1.28i)(\lambda + 0.78 + 1.28i) \\
&= \lambda^2 + 1.56\lambda + 2.24
\end{aligned}$$

Using Thm 3 from the notes, the K_1 and K_2 values are then given by:

$$\begin{aligned}
1.05 + a &= 1.56 \\
0.005 + b &= 2.24
\end{aligned}$$

$$K_1 = K_2 = [2.235, 0.51]^T$$

III. Item 3

```
import numpy as np
class Controller:
    def __init__(self, delta_t):
        self.y = np.zeros((2, 1))
        self.delta_t = delta_t
        self.K_1 = self.K_2 =
         ↪ np.array([2.235, 0.51])
    def calculate_acceleration(self, y):
        y_dot = (y - self.y) /
         ↪ self.delta_t
        self.y = y
        x_1 = np.array([y[0], y_dot[0]])
        x_2 = np.array([y[1], y_dot[1]])
        u_1 = -np.dot(self.K_1, x_1)
        u_2 = -np.dot(self.K_2, x_2)
        return np.array([u_1,
         ↪ u_2]).reshape(2, 1)
```

IV. Item 4

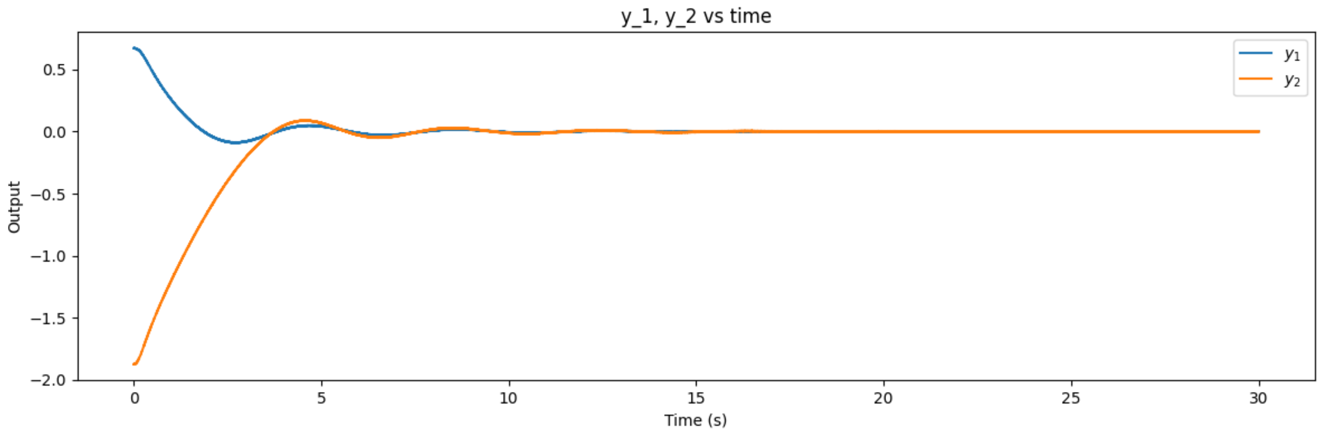
Computing the controllable canonical state space form of the filter gives us:

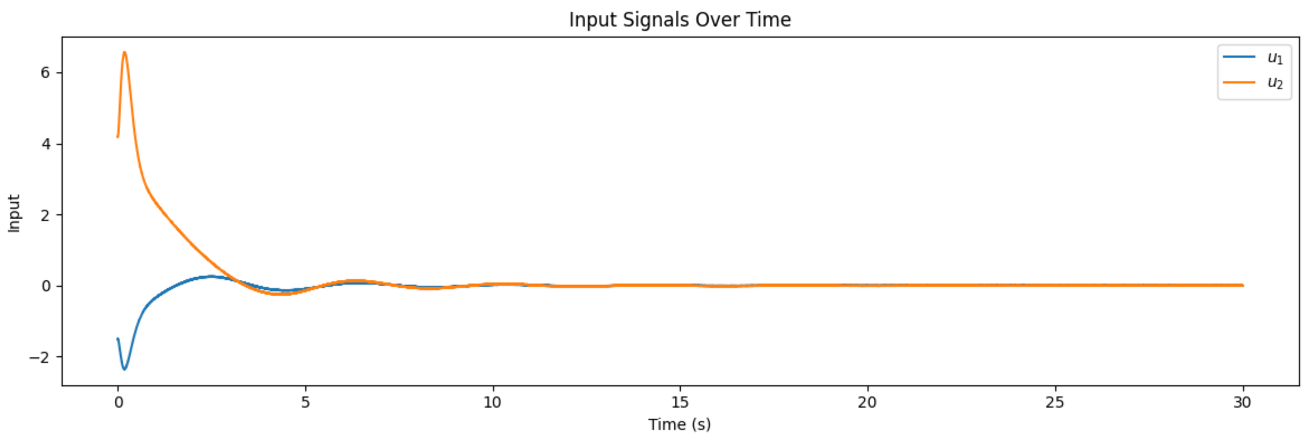
$$\begin{aligned}
A_f &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1000 & -300 & -30 \end{bmatrix} \\
B_f &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\
C_f &= \begin{bmatrix} 1000 & 0 & 0 \end{bmatrix} \\
D_f &= 0
\end{aligned}$$

We then implement this in code in controller.py (lines 20-40) following the hint

V. Item 5

The code for this can be found in main.py. The plots are shown below:


y_1, y_2 vs time
Output
Time (s)
y_1
y_2
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
5
10
15
20
25
30


Input Signals Over Time
Input
Time (s)
u_1
u_2
6
4
2
0
−2
0
5
10
15
20
25
30