CS 370 - A1

Bilal Khan
bilal2vec@gmail.com

September 24, 2023

Contents

1 1

1.1 a

The smallest value is given by $(0.10000)_4 \times 4^{-10}$. The largest value is given by $(0.33333)_4 \times 4^{10}$.

1.2 b

$(0.0321223)_4/(10)_4 = (0.0321223)_4 \times 4^{-1} = (0.321223)_4 \times 4^{-2}$. The mantissa here has 6 digits and has to be rounded to 5 digits. Since the 6th digit (3) is larger than half the maximum significand (2), we round up. The result is $(0.32123)_4 \times 4^{-2}$.

1.3 c

Machine epsilon is given by $\frac{1}{2}\beta^{-(m-1)}$. In this case, $\beta = 4$ and $m = 5$ and machine epsilon is given by $\frac{1}{2} \times 4^{-4}$.

1.4 d

All values in this number system where the exponent $p \leq 0$ are smaller than one. There are 21 possible exponents in $[-10, 10]$ and so 11/21 of the numbers are smaller than one.

2 2

Given a machine epsilon E, $f(\bar{x} \ominus \bar{y}) = (\bar{x} - \bar{y})(1 + E)$, and $f(\bar{x} \otimes \bar{y}) = (\bar{x} \times \bar{y})(1 + E)$, We can find the relative error of the whole expression.

$$\begin{aligned}
f(y \ominus 1) &= (y - 1)(1 + E) \\
f(y \oplus 1) &= (y + 1)(1 + E) \\
f(x \otimes y) &= (x \times y)(1 + E)
\end{aligned}$$

$$\begin{aligned}
f((y \ominus 1) \otimes (y \oplus 1)) &= (((y - 1)(1 + E)) \times ((y + 1)(1 + E)))(1 + E) \\
&= ((y - 1)(1 + E)(y + 1)(1 + E))(1 + E) \\
&= ((y - 1)(y + 1)(1 + E)(1 + E))(1 + E) \\
&= ((y^2 - y + y - 1)(1^2 + 2E + E^2))(1 + E) \\
&= (y^2 - 1)(1 + 2E + E^2)(1 + E) \\
&= (y^2 - 1)(1 + 2E + E^2 + E + 2E^2 + E^3) \\
&= (y^2 - 1)(1 + 3E + 3E^2 + E^3)
\end{aligned}$$

The bound on the relative error is then $3E + 3E^2 + E^3$.

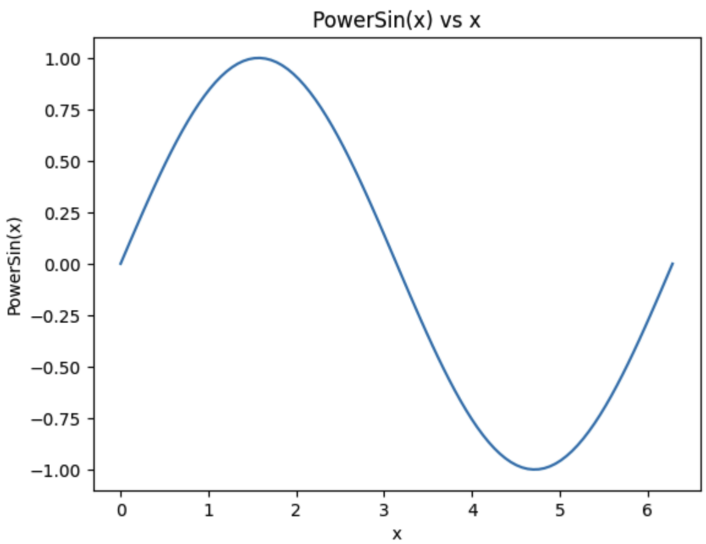
3 3

3.1 a

```
import math

def PowerSin(x):
  idx = 1
  exp = 3
  sum = 0
  term = x

  while sum + term != sum:
```

```
        sum = sum + term
        term = ((-1)**idx) * (x**exp) / math.factorial(exp)

        idx += 1
        exp += 2

    return sum
```

3.2 b



3.3 c

x	PowerSin(x)	sin(x)	Error	Number of Terms
$\pi/2$	1.0000000000000002	1.0	0.0000000000000002	11
$11\pi/2$	−1.000000000155901	−1.0	0.000000000155901	37
$21\pi/2$	1.0046249045393962	1.0	0.0046249045393962	59
$31\pi/2$	17863.02585515233	−1.0	17864.02585515233	77

3.4 d

At sufficiently large values, the floating point errors become large enough that the power series no longer converges to the correct value. This can be fixed in a way for this example by always computing `PowerSin(x % (2 * math.pi))` since the sin function is periodic.

4 4

5 5

6 6

7 7