Tutorial 2_ SE 380

$$\dot{x} = \underbrace{\begin{bmatrix} -0.1 & 0 & 0_{2\times2} \\ 0 & -2 & \\ & & \begin{matrix} -1 & 1 \\ -1 & -1 \end{matrix} \end{bmatrix}}_{4\times4} x_{4\times1} + \underbrace{\begin{bmatrix} I_{2\times2} \\ I_{2\times2} \end{bmatrix}}_{4\times2} u_{2\times1}$$

$$y_{2\times1} = \underbrace{\begin{bmatrix} I_{2\times2} & I_{2\times2} \end{bmatrix}}_{2\times4} x_{4\times1} \qquad x \in \mathbb{R}^4, \quad u, y \in \mathbb{R}^2$$

$$G(s) = C(sI - A)^{-1}B$$

$$= \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} s+0.1 & 0 & 0 \\ 0 & s+2 & \\ & & \begin{matrix} s+1 & -1 \\ 1 & s+1 \end{matrix} \end{bmatrix}^{-1} \begin{bmatrix} I \\ I \end{bmatrix}$$

$$= \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} \frac{1}{s+0.1} & 0 & 0 \\ 0 & \frac{1}{s+2} & \\ & & \frac{1}{s^2+2s+2}\begin{bmatrix} s+1 & 1 \\ -1 & s+1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix}$$

$$= \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} \frac{1}{s+0.1} & 0 \\ 0 & \frac{1}{s+2} \\ \frac{1}{s^2+2s+2} \begin{bmatrix} s+1 & 1 \\ -1 & s+1 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+0.1} + \frac{s+1}{s^2+2s+2} & \frac{1}{s^2+2s+2} \\ -\frac{1}{s^2+2s+2} & \frac{1}{s+2} + \frac{s+1}{s^2+2s+2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2s^2+3.1s+2.1}{(s+0.1)(s^2+2s+2)} & \frac{1}{s^2+2s+2} \\ \frac{-1}{s^2+2s+2} & \frac{2s^2+5s+4}{(s+2)(s^2+2s+2)} \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

$$G_{11}(s) = \frac{Y_1(s)}{U_1(s)} = \frac{2s^2+3.1s+2.1}{(s+0.1)(s^2+2s+2)}$$

$$= \frac{2.1\left(1+\frac{3.1}{2.1}s+\frac{2}{2.1}s^2\right)}{0.1(1+10s)2\left(1+s+\frac{s^2}{2}\right)}$$

$$= \frac{\textcircled{1}\ 10.5\ \left(1+\frac{3.1}{2.1}s+\frac{2}{2.1}s^2\right)\ \textcircled{3}}{(1+10s)\ \left(1+s+\frac{s^2}{2}\right)}$$

$\textcircled{2}$ $\textcircled{4}$

② $\tau = 10 \longrightarrow \omega_{p_1} = \frac{1}{\tau} = 0.1$

③ $\alpha_n = \sqrt{\frac{2.1}{2}} \simeq 1.02 \qquad \frac{2\xi}{\alpha_n} = \frac{3.1}{2.1} \longrightarrow \xi \simeq 0.76$

$$z = -\xi\alpha_n \pm j\,\alpha_n\sqrt{1-\xi^2} \quad \leftarrow \text{zeros}$$
$$= -0.78 \pm j\,0.66$$

④ $\omega_n = \sqrt{2} = 1.44 \qquad \frac{2\xi}{\omega_n} = 1 \longrightarrow \xi = \frac{\sqrt{2}}{2} \simeq 0.74$

Bode Plots — Final sketch with this color that is the sum (approximation)