This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

Midterm

Due: Friday, March 4, 2022 4:00 pm (EST)

Assignment description

Welcome to the Math 213 midterm! This midterm is designed to take 50min to complete. However, since it is administered online, submissions are accepted until 4pm, to allow for connection glitches and Accessibility accommodations. The exam consists of eight (8) multiple-choice questions, with 5 points for each correct answer and -1 point penalty for each wrong answer. I'll be available 2:30pm-4pm on MS Teams to answer clarifying questions (chat and white board).

Additional rules:

1) This is an open-book midterm in terms of material provided by the course (lecture slides, John Thistle's lecture notes, homework assignment solution sets, solutions and your notes from the tutorials). No other sources are allowed!

2) You are not allowed to consult outside sources or people. Also, no collaboration with your classmates or discussion of the midterm until the final submission deadline at 4pm.

3) You can use a calculator.

Good luck!

Submit your assignment

Help

Q1 (5 points)

A circle with radius r (r is a positive real number) can be described as a set of points with coordinates (x, y) that have a constant distance from the origin:
$x^2 + y^2 = r^2$.
For points with coordinate x, such that $0 < x \ll r$, the coordinate y can be described as
$y(x) \approx ax^2 + bx + c$, such that:

- $a = 1, b = \frac{1}{2r}, c = 0$
- $a = -\frac{1}{2r}, b = 0, c = r$
- $a = \frac{1}{r}, b = \frac{1}{2}, c = r$
- $a = 1, b = 0, c = -\frac{1}{2r^2}$
- $a = 1, b = -\frac{2}{r^2}, c = 1$

Q2 (5 points)

Consider the following ODE with given initial condition:
$\frac{dx}{dt} = 3x - 5x^2$
x(0)=2
What would be its steady-state solution?

- 0.6
- $\frac{5}{3}$
- 2
- this ODE does not have a steady-state solution
- 0

Q3 (5 points)

^{131}I is a radioactive isotope of iodine that is formed as one of the major fission products from nuclear reactors and nuclear explosions. As a major cause of increased thyroid cancer after nuclear contamination, it is one of the health hazards from open-air atomic bomb testing (popular in the 1950s) and from disasters involving nuclear power plants. It is also used as a radioactive tracer for oil fracking and in nuclear medicine.

The number of ^{131}I atoms in a sample as a function of time, $N(t)$, can be described with a differential equation

$\frac{dN}{dt} = -\lambda N(t)$

The half-life (time after which the number of ^{131}I atoms decreases to 1/2 of the initial value) is 8 days.

How many days would it take for the number of ^{131}I atoms to decrease to 1/10 of the initial value?

- 30.8 days
- 20.1 days
- 40.3 days
- 10.4 days
- 26.6 days

Q4 (5 points)

What is the inverse Laplace transform of

$F(s) = \frac{2(s+2)}{s^2+5s+6}$, where Re{s}>-3?

- $f(t) = 2e^{-3t}$
- $f(t) = 2e^{-3t}u_{-1}(t)$, where $u_{-1}(t)$ refers to the unit step function
- $f(t) = 2e^{-3t}\delta(t)$, where $\delta(t)$ refers to Dirac delta function (aka unit-impulse)
- $f(t) = (2e^{-3t} - 4e^{-4t})u_{-1}(t)$, where $u_{-1}(t)$ refers to the unit step function
- none of the above

Q5 (5 points)

Function x(t) has a Laplace transform X(s). If we know that $\int_{-\infty}^{+\infty}$ x(t) dt=6, then which one of the following statements is true?

- $X(0) = 3$
- $lim_{s \to \infty} X(s) = 6$
- $X(0) = 6$
- $X(s)$ has a pole at s=6
- $X(s) = 6$

Q6 (5 points)

Consider the function
$g(t) = x(t) + 4x(-t)$,
with $x(t) = 3e^{-2t}u_{-1}(t)$, where $u_{-1}(t)$ denotes the unit step function. If $G(s)$ is the Laplace transform of g(t), what would be $G(s)$ region of convergence (ROC)?

- ROC for $G(s)$ does no exist
- -2<s
- s<-2
- -2<s<2
- -2>s>2

Q7 (5 points)

What is the Laplace transform of
$x(t) = e^{-5t}u_{-1}(t-3)$?

- $X(s) = \frac{e^{-3s}}{s+5}$
- $X(s) = \frac{e^{3s}}{s+5}$
- $X(s) = \frac{e^{-3(s-5)}}{s+5}$
- $X(s) = \frac{e^{-3(s+5)}}{s+5}$
- none of the above

Q8 (5 points)

Consider two functions $x(t)$ and $y(t)$, such that for $t < 0$ $x(t) = 1$ and $y(t) = 0$, that are related through the following differential equations:

$\frac{dx}{dt} = -3y(t) + 2\delta(t)$
$\frac{dy}{dt} = 5x(t)$

What are the Laplace transforms $X(s)$ and $Y(s)$ of $x(t)$ and $y(t)$?

- $X(s) = \frac{3s}{s^2+15}$ and $Y(s) = \frac{15}{s^2+15}$
- $X(s) = \frac{2s}{s^2+15}$ and $Y(s) = \frac{10}{s^2+15}$
- $X(s) = \frac{s^2+s}{s^2+15}$ and $Y(s) = \frac{5s+5}{s^2+15}$
- $X(s) = \frac{s^2+2s}{s^2+15}$ and $Y(s) = \frac{5s+10}{s^2+15}$
- none of the above