CS 486 - Assignment 4

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Contents

I used the cursor (cursor.com) code editor to help create the plots in this assignment.

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1.1 1

Variables: D (Dunetts), T (TRIMONO-HT/S), S (Sloepnea), F (Foriennditis), G (Degar)

D states: 0 (None), 1 (Mild), 2 (Severe)

T, S, F, G states: 0 (Absent), 1 (Present)

Bayesian Network: $D \rightarrow S$, $T \rightarrow S$, $D \rightarrow F$, $D \rightarrow G$

We arbitrarily choose 0.2 and 0.8 for high/low probabilities respectively in the cases where the prior probabilities aren't given.

1.2 2

```
import numpy as np
import matplotlib.pyplot as plt
import copy
from tqdm import tqdm


def add_noise(cpts, delta):
    noisy_cpts = copy.deepcopy(cpts)

    noise_d = np.random.uniform(0, delta, size=noisy_cpts['D'].shape)
    noisy_cpts['D'] = noisy_cpts['D'] + noise_d
    noisy_cpts['D'] /= noisy_cpts['D'].sum()

    # not noising t

    for node in ['F', 'G', 'S']:
        for key in noisy_cpts[node]:
            noise = np.random.uniform(0,
                                      delta,
                                      size=noisy_cpts[node][key].shape)
```

```python
            noisy_cpts[node][key] = noisy_cpts[node][key] + noise
            noisy_cpts[node][key] /= noisy_cpts[node][key].sum()

    return noisy_cpts


def get_prob(cpt_entry, state):
    return cpt_entry[state]


def e_step(data, cpts):
    num_patients = data.shape[0]
    num_states = 3
    weights = np.zeros((num_patients, num_states))

    for i in range(num_patients):
        patient = data[i]
        s_val, f_val, g_val, t_val, d_known = patient[0], patient[1], patient[
            2], patient[3], patient[4]

        if d_known != -1:
            weights[i, d_known] = 1.0
        else:
            unnormalized_probs = np.zeros(num_states)
            for d_state in range(num_states):
                prob_s_dt = get_prob(cpts['S'][(d_state, t_val)], s_val)
                prob_f_d = get_prob(cpts['F'][(d_state,)], f_val)
                prob_g_d = get_prob(cpts['G'][(d_state,)], g_val)
                prob_d = get_prob(cpts['D'], d_state)
                unnormalized_probs[
                    d_state] = prob_s_dt * prob_f_d * prob_g_d * prob_d

            total_prob = unnormalized_probs.sum()
            if total_prob > 1e-8:
                weights[i, :] = unnormalized_probs / total_prob
            else:
                weights[i, :] = 1.0 / num_states

    return weights


def m_step(data, weights):
    num_patients = data.shape[0]
    num_states = 3
    new_cpts = copy.deepcopy(initial_cpts)

    total_weights = weights.sum(axis=0)
    new_cpts['D'] = total_weights / num_patients
```

```python
t_counts = np.zeros(2)
for i in range(num_patients):
    t_val = data[i, 3]
    t_counts[t_val] += weights[i].sum()
t_present_count = np.sum(data[:, 3] == 1)
new_cpts['T'] = np.array(
    [1.0 - t_present_count / num_patients, t_present_count / num_patients])

f_counts = {}
for d_state in range(num_states):
    f_counts[(d_state,)] = np.zeros(2)

for i in range(num_patients):
    f_val = data[i, 1]
    for d_state in range(num_states):
        f_counts[(d_state,)][f_val] += weights[i, d_state]

for d_state in range(num_states):
    total_d_weight = f_counts[(d_state,)].sum()
    if total_d_weight > 1e-8:
        new_cpts['F'][(d_state,)] = f_counts[(d_state,)] / total_d_weight
    else:
        new_cpts['F'][(d_state,)] = np.array([0.5, 0.5])

g_counts = {}
for d_state in range(num_states):
    g_counts[(d_state,)] = np.zeros(2)

for i in range(num_patients):
    g_val = data[i, 2]
    for d_state in range(num_states):
        g_counts[(d_state,)][g_val] += weights[i, d_state]

for d_state in range(num_states):
    total_d_weight = g_counts[(d_state,)].sum()
    if total_d_weight > 1e-8:
        new_cpts['G'][(d_state,)] = g_counts[(d_state,)] / total_d_weight
    else:
        new_cpts['G'][(d_state,)] = np.array([0.5, 0.5])

s_counts = {}
for d_state in range(num_states):
    for t_state in range(2):
        s_counts[(d_state, t_state)] = np.zeros(2)

for i in range(num_patients):
    s_val, t_val = data[i, 0], data[i, 3]
```

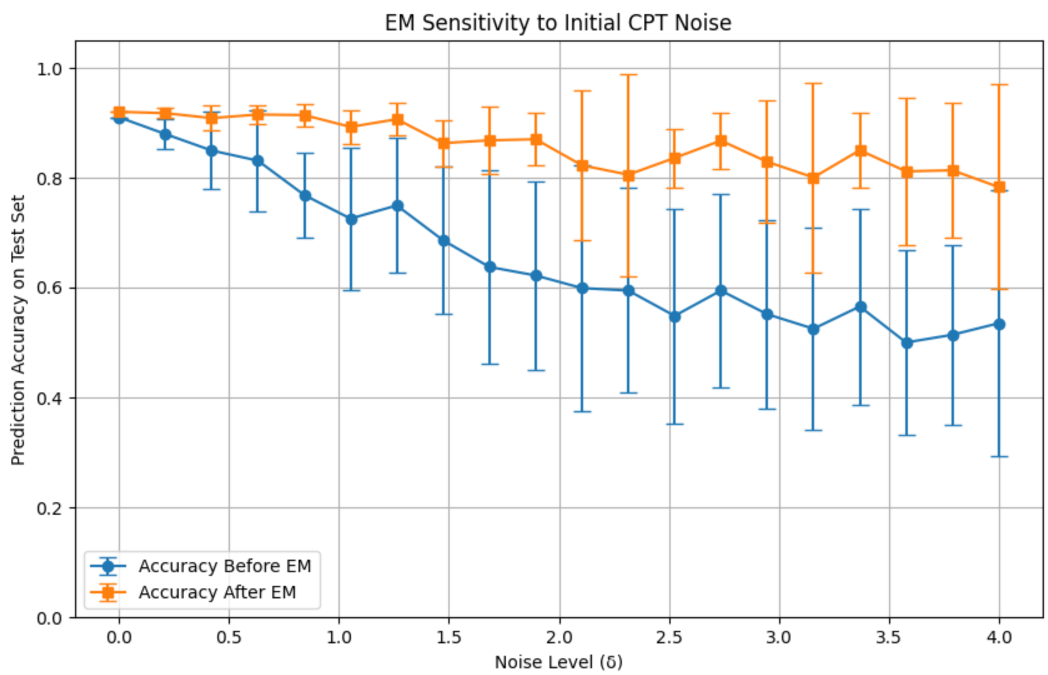
```
        for d_state in range(num_states):
            s_counts[(d_state, t_val)][s_val] += weights[i, d_state]

    for d_state in range(num_states):
        for t_state in range(2):
            total_dt_weight = s_counts[(d_state, t_state)].sum()
            if total_dt_weight > 1e-8:
                new_cpts['S'][(
                    d_state,
                    t_state)] = s_counts[(d_state, t_state)] / total_dt_weight
            else:
                new_cpts['S'][(d_state, t_state)] = np.array([0.5, 0.5])

    return new_cpts


def llh(data, weights, cpts):
    log_likelihood = 0.0
    num_patients = data.shape[0]
    num_states = 3

    log_cpts = copy.deepcopy(cpts)
    log_cpts['D'] = np.log(log_cpts['D'] + 1e-8)
    log_cpts['T'] = np.log(log_cpts['T'] + 1e-8)
    for node in ['F', 'G', 'S']:
        for key in log_cpts[node]:
            log_cpts[node][key] = np.log(log_cpts[node][key] + 1e-8)

    for i in range(num_patients):
        patient = data[i]
        s_val, f_val, g_val, t_val = patient[0], patient[1], patient[
            2], patient[3]
        patient_ll = 0.0
        for d_state in range(num_states):
            if weights[i, d_state] > 1e-8:
                log_prob_d = get_prob(log_cpts['D'], d_state)
                log_prob_t = get_prob(log_cpts['T'], t_val)
                log_prob_s_dt = get_prob(log_cpts['S'][(d_state, t_val)], s_val)
                log_prob_f_d = get_prob(log_cpts['F'][(d_state,)], f_val)
                log_prob_g_d = get_prob(log_cpts['G'][(d_state,)], g_val)

                log_joint = log_prob_s_dt + log_prob_f_d + log_prob_g_d + log_prob_t + log_prol
                patient_ll += weights[i, d_state] * log_joint

        log_likelihood += patient_ll

    return log_likelihood
```

```python
def run_em(train_data, initial_cpts, convergence_threshold, max_iterations=10):
    cpts = copy.deepcopy(initial_cpts)
    prev_llh = -np.inf
    iterations = 0

    for _ in range(max_iterations):
        weights = e_step(train_data, cpts)
        cpts = m_step(train_data, weights)

        current_llh = llh(train_data, weights, cpts)
        llh_dif = abs(current_llh - prev_llh)

        if llh_dif < convergence_threshold:
            break

        prev_llh = current_llh
        iterations += 1

    return cpts


def predict(patient_data, cpts):
    num_states = 3
    posterior_probs = np.zeros(num_states)
    s_val, f_val, g_val, t_val = patient_data[0], patient_data[1], patient_data[
        2], patient_data[3]

    for d_state in range(num_states):
        prob_s_dt = get_prob(cpts['S'][(d_state, t_val)], s_val)
        prob_f_d = get_prob(cpts['F'][(d_state,)], f_val)
        prob_g_d = get_prob(cpts['G'][(d_state,)], g_val)
        prob_d = get_prob(cpts['D'], d_state)
        posterior_probs[d_state] = prob_s_dt * prob_f_d * prob_g_d * prob_d

    return np.argmax(posterior_probs)


def calculate_acc(test_data, cpts):
    n_correct = 0
    n_total = test_data.shape[0]

    for i in range(n_total):
        patient = test_data[i]
        true_d_state = patient[4]
        predicted_d_state = predict(patient, cpts)
        if predicted_d_state == true_d_state:
            n_correct += 1
```

```python
    return n_correct / n_total


if __name__ == "__main__":
    initial_cpts = {
        'D': np.array([0.5, 0.25, 0.25]),
        'T': np.array([0.9, 0.1]),
    }

    initial_cpts_f = {  #P(F|D)
        (0,): 0.2,
        (1,): 0.8,
        (2,): 0.2,
    }

    initial_cpts_g = {  #P(G|D)
        (0,): 0.2,
        (1,): 0.2,
        (2,): 0.8,
    }

    initial_cpts_s = {  #P(S|D,T)
        (0, 0): 0.2,
        (1, 0): 0.8,
        (2, 0): 0.8,
        (0, 1): 0.01,
        (1, 1): 0.01,
        (2, 1): 0.01,
    }

    def expand_cpt(cpt_dict):
        full_cpt = {}
        for key, p_state1 in cpt_dict.items():
            full_cpt[key] = np.array([1.0 - p_state1, p_state1])
        return full_cpt

    initial_cpts['F'] = expand_cpt(initial_cpts_f)
    initial_cpts['G'] = expand_cpt(initial_cpts_g)
    initial_cpts['S'] = expand_cpt(initial_cpts_s)

    train_data = np.loadtxt('traindata.txt', dtype=int)
    test_data = np.loadtxt('testdata.txt', dtype=int)

    delta_values = np.linspace(0.0, 4.0, 20)
    num_trials = 20
    convergence_thresh = 0.05
```

```python
results_before_em = {delta: [] for delta in delta_values}
results_after_em = {delta: [] for delta in delta_values}

for delta in tqdm(delta_values):
    for trial in range(num_trials):
        noisy_initial_cpts = add_noise(initial_cpts, delta)

        accuracy_before = calculate_acc(test_data, noisy_initial_cpts)
        results_before_em[delta].append(accuracy_before)

        learned_cpts = run_em(train_data, noisy_initial_cpts,
                              convergence_thresh)

        accuracy_after = calculate_acc(test_data, learned_cpts)
        results_after_em[delta].append(accuracy_after)

mean_before = [np.mean(results_before_em[d]) for d in delta_values]
std_before = [np.std(results_before_em[d]) for d in delta_values]
mean_after = [np.mean(results_after_em[d]) for d in delta_values]
std_after = [np.std(results_after_em[d]) for d in delta_values]

plt.figure(figsize=(10, 6))
plt.errorbar(delta_values,
             mean_before,
             yerr=std_before,
             label='Accuracy Before EM',
             fmt='-o',
             capsize=5)
plt.errorbar(delta_values,
             mean_after,
             yerr=std_after,
             label='Accuracy After EM',
             fmt='-s',
             capsize=5)

plt.xlabel('Noise Level ()')
plt.ylabel('Prediction Accuracy on Test Set')
plt.title('EM Sensitivity to Initial CPT Noise')
plt.legend()
plt.grid(True)
plt.ylim(0, 1.05)
plt.savefig('em_accuracy_vs_delta.png')
```

1.3 3



2 2

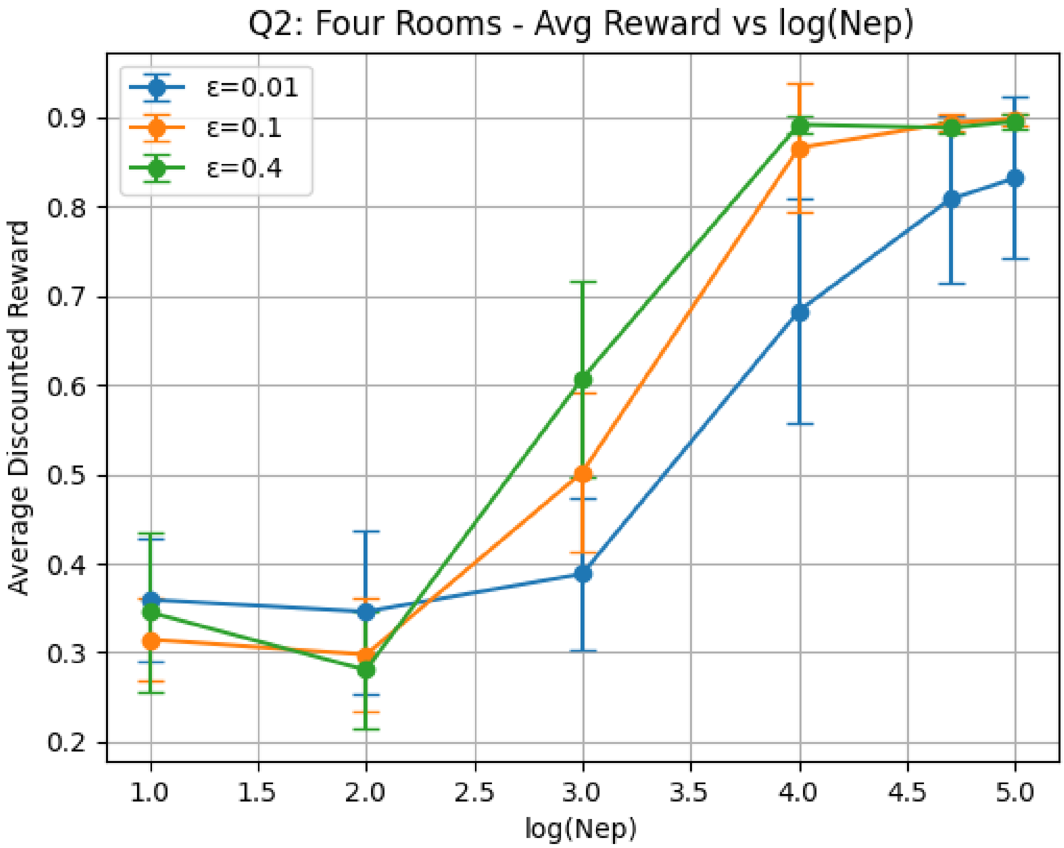
2.1 1

The optimal search strategy so it does not cross its own path is a spiral starting at the center. E.g. so numbering steps from 0-24:

$$\begin{bmatrix} 12 & 11 & 10 & 9 & 24 \\ 13 & 2 & 1 & 8 & 23 \\ 14 & 3 & 0 & 7 & 22 \\ 15 & 4 & 5 & 6 & 21 \\ 16 & 17 & 18 & 19 & 20 \end{bmatrix}$$

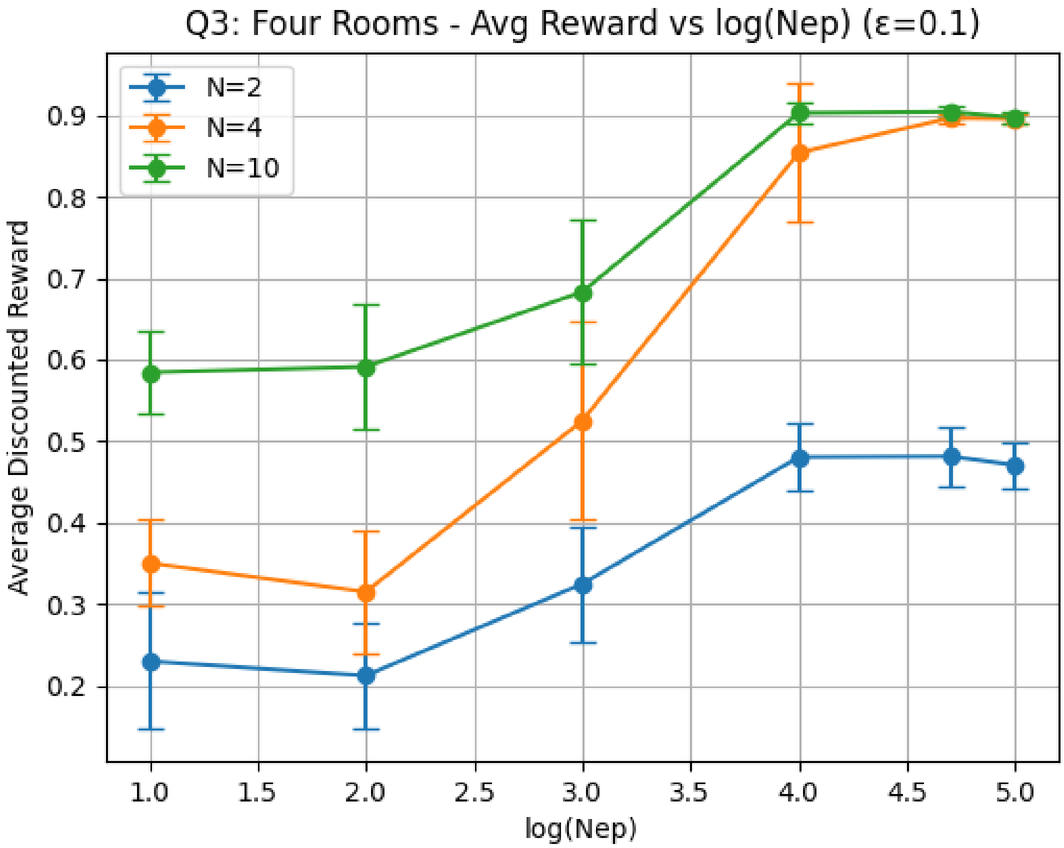
The reward is then $1/24 \sum_{i=1}^{24} 0.95^i \approx 0.56$

2.2 2

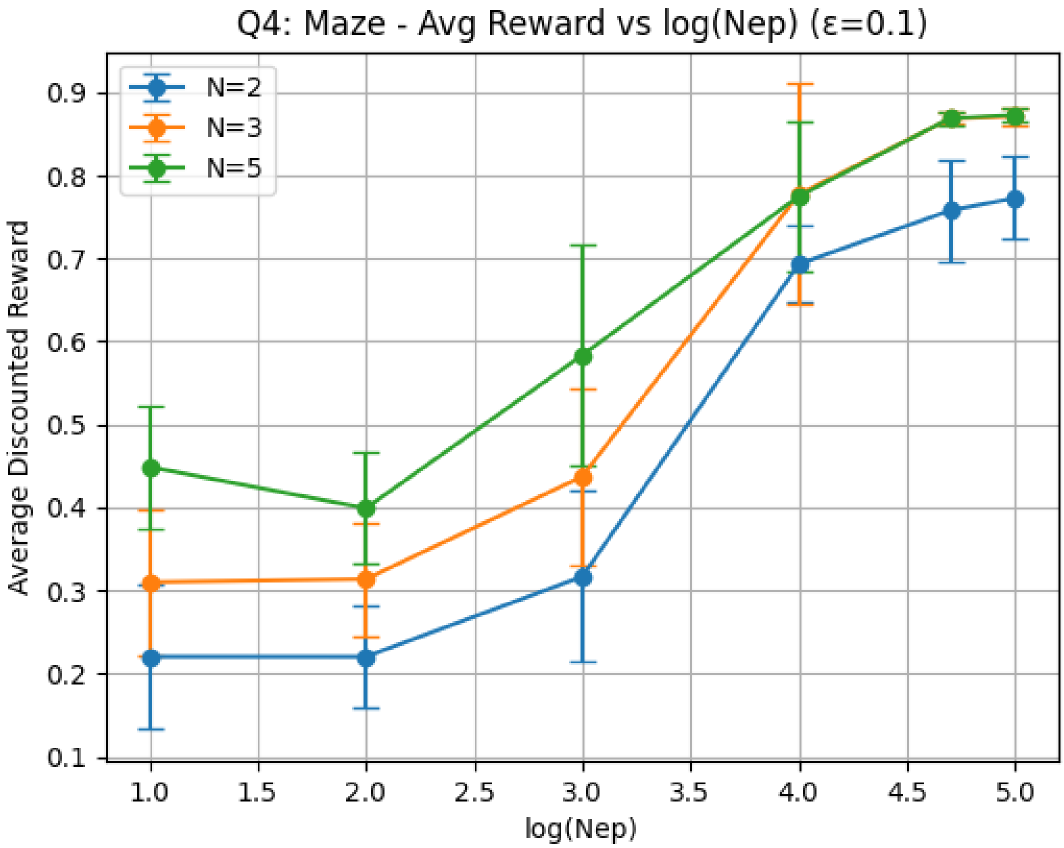


Q2: Four Rooms - Avg Reward vs log(Nep)
ε=0.01
ε=0.1
ε=0.4
Average Discounted Reward
log(Nep)
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

2.3 3

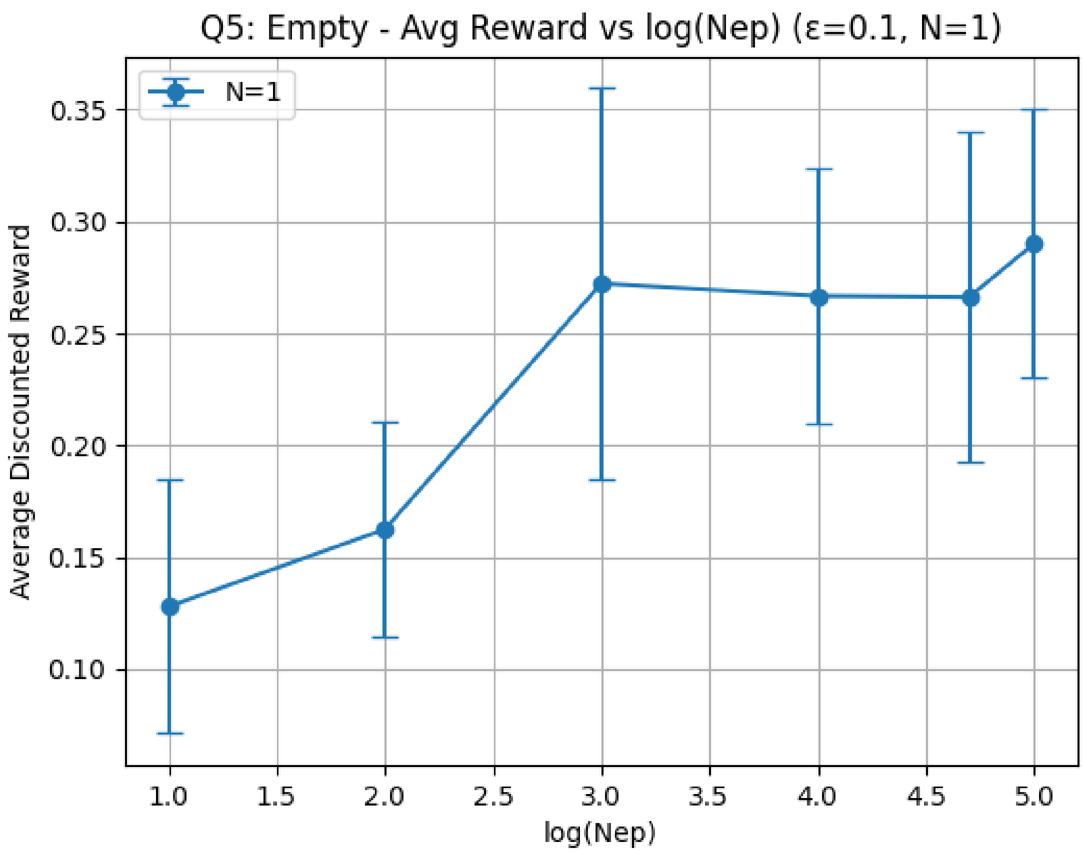

Q3: Four Rooms - Avg Reward vs log(Nep) (ε=0.1)
N=2
N=4
N=10
Average Discounted Reward
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
log(Nep)

2.4 4



Q4: Maze - Avg Reward vs log(Nep) (ε=0.1)
N=2
N=3
N=5
Average Discounted Reward
log(Nep)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

2.5 5



2.6 6

We experimentally reach an upper bound of about 0.35 at $\log n = 5$ or $100k$, so to get to 0.56 it will probably take 6-7 $\log n$ (1-10M) episodes if the trendlines hold.