SE 380 — HW 2

Bilal Khan
bilal2vec@gmail.com

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Consider the following model of a DC motor:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K_m}{s(R_a(Js+b) + K_bK_m)}$$

where the value of all parameters is positive, the input $u(t) = \mathcal{L}^{-1}(U(s))$ is the voltage supplied to the motor, and the output $y(t) = \mathcal{L}^{-1}(Y(s))$ is the angle of the motor axle. Assume a unit step input voltage is supplied.

1.1 (a)

Find the transfer function betwen the angular speed - time derivative of the angle - and the input voltage.

$$\omega(t) = y'(t)$$

$$G_\omega(s) = sG(s) = \frac{K_m}{R_a(Js+b) + K_bK_m}$$

1.2 (b)

Find the steady-state angular speed of the motor axle

$$
\begin{aligned}
\lim_{t\to\infty} \omega(t) &= \lim_{s\to 0} sG_\omega(s)U(s) \\
&= \lim_{s\to 0} s\frac{K_m}{R_a(Js+b)+K_bK_m}\frac{1}{s} \\
&= \lim_{s\to 0} \frac{K_m}{R_a(Js+b)+K_bK_m} = \frac{K_m}{R_ab+K_bK_m}
\end{aligned}
$$

1.3 (c)

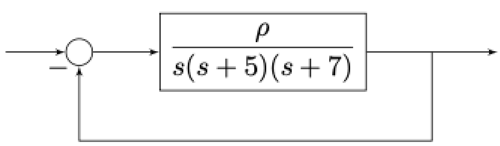
How long does the motor take to reach 99% of its steady-state speed?

$$
\begin{aligned}
G_\omega(s) &= \frac{K_m}{R_a(Js+b)+K_bK_m} \\
&= \frac{K_m}{(R_aJ)s+(R_ab+K_bK_m)} \\
&= \frac{\frac{K_m}{R_ab+K_bK_m}}{\frac{R_aJ}{R_ab+K_bK_m}s+1}
\end{aligned}
$$

$\tau = \dfrac{R_aJ}{R_ab+K_bK_m}$, we have that settling time @ 99% is approximately $5\tau = 5\dfrac{R_aJ}{R_ab+K_bK_m}$.

2 2

Consider the following feedback control system.



Find the values of ρ for which the closed-loop system is stable.

$L(s) = \dfrac{\rho}{s(s+5)(s+7)}$. We need to check if the poles of $1 + L(s)$ are in the left half of the plane.

$$
\begin{aligned}
0 = 1 + L(s) &= 1 + \frac{\rho}{s(s^2+12s+35)} \\
&= 1 + \frac{\rho}{s^3+12s^2+35s} \\
0 &= s^3 + 12s + 35s + \rho
\end{aligned}
$$

None of the given coefficients are negative, so we don't know if it is Hurwitz or not, all we can know is that for the system to be stable, $\rho > 0$.

For all elements in the first column to be positive $\rho > 0$ and $\frac{\rho}{12} - 35 > 0$.

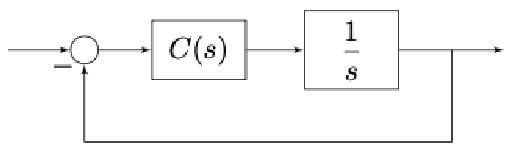
s^3	1	35
s^2	12	ρ
s^1	$\frac{\rho}{12} - 35$	0
s^0	ρ	0

$$\begin{aligned} \frac{\rho}{12} - 35 &> 0 \\ \frac{\rho}{12} &> 35 \\ \rho &> 420 \end{aligned}$$

The system is stable for $\rho > 420$.

3 3

Consider the following feedback control system.



Design a proportional controller a controller iwth transfer function $C(s) = K$ for some $K \in \mathbb{R}$ so that the closed-loop system satisfies the following specifications: It is stable, the steady state gain is 1, and the settling time is less than $100ms$.

$L(s) = \dfrac{K}{s}$

$T(s) = \dfrac{L(s)}{1 + L(s)} = \dfrac{\frac{K}{s}}{1 + \frac{K}{s}} = \dfrac{K}{s + K}$

For the system to be stable, its poles are negative and so $K > 0$.

The steady-state gain is given by $\lim_{s \to 0} sT(s)U(s) = \lim_{s \to 0} \dfrac{K}{s + K} = \dfrac{K}{K} = 1$. So the steady-state gain is 1.

The settling time to 99% is given by $5\tau = 5\dfrac{1}{K} < 0.1$. so $K > 50$.

This means that $C(s) = K$ for any $K > 50$