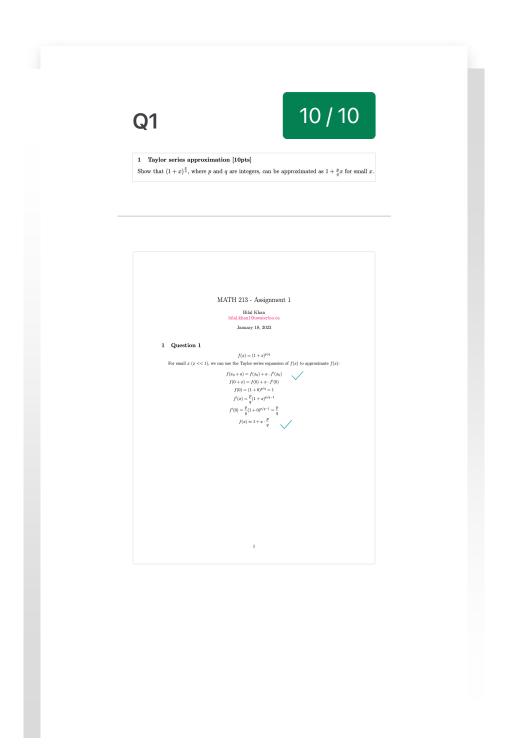
My grades for **Assignment 1**



Q2

20 / 20

2 Oscillating electron [20pts]
An electron is placed in the middle of an imaginary line connecting two fixed negative charges, each with magnitude Q, positioned at a distance 2a from each other.
3) Witte down the differential equation that would describe the displacement x(t) of the electron from its initial position as a function of time. Assume the displacement is along the line connecting the two fixed charges Q.
b) Show that for small enough displacements (specify what you mean by small), the force on the electron will scale linearly with its displacement and thus the electron would behave like a harmonic oscillator (mass each of the place of the electron).
(i) What would be the angular frequency of the electron's oscillation in part b)?

2 Question 2

2.1 (a)

2.1 (a) Let Q_i be the charge of the electron, and we know that the force on the electron from a charge is given by the Coulomb's law: $F = \frac{Q_i Q_i}{Q_i}$, where c is the Coulomb constant. Assume that the electron is placed beation g(t) due thistense from each charge is then $\alpha + c/Q_i$ and $\alpha - x(t)$, respectively. We know that the forces on the electron from each charge set in opposite directions, and then after one given by $F = m_0$ so we can write:

$$F = ma = \frac{cQQ_e}{(a + x(t))^2} - \frac{cQQ_e}{(a - x(t))^2}$$

$$a = \frac{cQQ_e}{(a + x(t))^2} - \frac{cQQ_e}{(a - x(t))^2}$$

We can see that $a=\frac{d^2x}{dt^2},$ so we can write our differential equation as:

o we can write our differential equation as:
$$\frac{d^2x}{dt^2} = \frac{cQQ_c}{(a+x(t))^2} - \frac{cQQ_c}{(a-x(t))^2}$$

For small displacements, i.e. where t << 1, and assuming that y = x(t) << 1 for small t, we can approximate $(a \pm y)^{-2}$ with its Taylor series expansion:

$$\begin{split} f(y) &= (a+y)^{-2} \\ f(0) &= (a+0)^{-2} = a^{-2} \\ f'(y) &= -2(a+y)^{-3} \\ f'(0) &= -2(a+y)^{-3} = -2a^{-3} \\ f(0+x(t)) &\approx f(0) + x(t) \cdot f'(0) = a^{-2} - 2a^{-3}x(t) \end{split}$$

$$\begin{split} g(y) &= (a-y)^{-2} \\ g(0) &= (a-0)^{-2} = a^{-2} \\ g'(y) &= -2(a-y)^{-3}(-1) \\ g'(y) &= -2(a+q)^{-3}(-1) = 2a^{-3} \\ g(0+x(t)) \approx g(0) + x(t) \cdot g'(0) = a^{-2} + 2a^{-3}x(t) \end{split}$$

 $k=4cQQ_ca^{-3}m$, which is a constant, so we can see that the force will scale linearly with the displacement and the electron behaves like a harmonic oscillator. 2.3 (c) write: $\omega = \sqrt{-(4\epsilon QQ_c a^{-3})m} = \sqrt{-4\epsilon QQ_c a^{-3}}$ Where Q is the magnitude of the charge, Q_c is the charge of the electron, a is one half of the distance between the charges, and c is the Coulomb constant.

Q3

20 / 20

3 Circumference of a circle [20 pts] Use the formula for path length introduced in class to calculate the circumference of a circle with radius r=1 in. Hint: A circle can be described as a set of points that have a constant distance from the origin: $x^2 + y^2 = r^2$

$$\begin{split} S &= -\int_{1}^{-1} \sqrt{1+(f'(x))^{2}} dx + \int_{-1}^{1} \sqrt{1+(f'(x))^{2}} dx \\ & r^{2} = x^{2} + p^{2} \\ & 1 = x^{2} + p^{2} \\ & y = \pm \sqrt{1-x^{2}} \end{split}$$

$$y' = \frac{dy}{dx} \pm (1 - x^2)^{\frac{1}{2}} = \frac{\pm x}{\sqrt{1 - x^2}}$$

$$\begin{split} \int \sqrt{1+(f'(x))^2} dx &= \int \sqrt{1+\left(\frac{\pm x}{\sqrt{1-x^2}}\right)^2} dx \\ &= \int \sqrt{1+\frac{x^2}{1-x^2}} dx \\ &= \int \sqrt{\frac{1-x^2}{1-x^2}} + \frac{x^2}{1-x^2} dx \\ &= \int \sqrt{\frac{1-x^2}{1-x^2}} dx \\ &= \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \lim^+ (z) + C \end{split}$$

$$\begin{split} S &= - \int_1^{-1} \frac{1}{\sqrt{1-x^2}} dx + \int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} dx \\ &= -\sin^{-1}(x) |_1^{-1} + \sin^{-1}(x)|_1^{-1} \\ &= -\left(\sin^{-1}(x) - \sin^{-1}(1)\right) + \left(\sin^{-1}(1) - \sin^{-1}(-1)\right) \\ &= -\left(\frac{\pi}{2} - \frac{\pi}{2}\right) + \left(\frac{\pi}{2} - \frac{\pi}{2}\right) \\ &= 2\pi \end{split}$$

Q4

25/30

4 Carbon dating [30 pts]

4 Carbon dating [30 pts]
Radiocarbon dating (alto referred to as carbon dating) is a method for determining the age of an object containing organic material by using the properties of a radioactive isotope of carbon, ¹⁴C, It is based on the fact that ¹⁴C is constantly being created in the Earth's atmosphere by the interaction of cosmic rays with atmosphere introgen. The resulting ¹⁴C combines with atmosphere coygen to four radioactive earbon with atmosphere coygen to four radioactive carbon with atmosphere coygen to four radioactive carbon with atmosphere coygen to four radioactive carbon of the control of ¹⁴C it contains begins to decrease as the ¹⁴C undergoes radioactive decay. Measuring the amount of ¹⁴C in a sample from a dead plant or animal, such as a piece of wood or a fragment of bone, provides self-Ctree is to be detected because the half-life of ¹⁴C (the period of time after which half of the ¹⁴C linside a given sample will have decayed) is about 5.73 oyars. (Source "withoption.org" as ample is the control of ¹⁴C being 5.750 years. (Source "withoption" of (nakes are to specify the units) based on the half-life of ¹⁴C being 5.750 years.

b) The abundance of ¹⁴C in the atmosphere is about 1 atom of ¹⁴C per 10¹⁹ carbon atoms. Modify the differential equation from part a, so that it describes the number of ¹⁴C actons in the atmosphere (as opposed to in a sample). Describe the role of the new terms(s) added to the original equation and estimate the value(s) of the constants in these terms (males are to specify the units). You can assume that out of every 1,000,000 on lockcules of air, roughly 781,000 are N₂, 290,000 are O₂, and (at least historically) 300 are CO₂.

Estimate the submedance of ¹⁴C (how many ¹⁴C per 10¹⁹ carbon atoms) in a parchiment from 3rd century BCE, such as the Dead Sea Secols.

4 Question 4

$$\begin{split} \frac{dN}{dt} &= -\lambda N(t) \\ \frac{dN}{dt} &= -\lambda \delta(t) \\ \frac{dN}{N} &= -\lambda dt \\ \ln N &= -\int \lambda dt \\ \ln N &= -\Delta t + C_2 \\ \ln N &= -\lambda t + C_3 \\ N &= e^{-\lambda t + C_5} \\ N(t) &= Ce^{-\lambda t} \\ N(0) &= ce^{-\lambda t} = c \end{split}$$

Using our values for the half-life of Carbon-14, we have that if we originally have 1 unit of carbon-14,

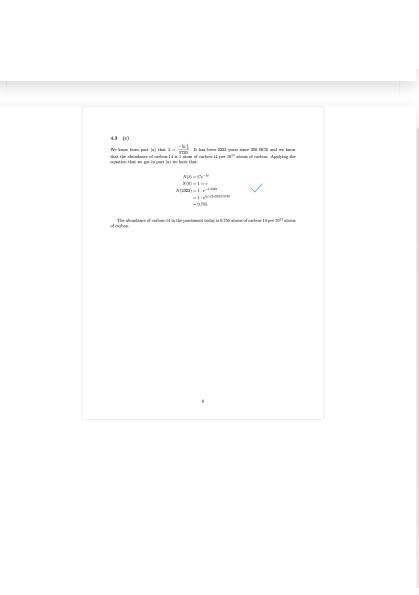
$$\begin{split} N(0) &= 1 = c \\ N(5730) &= 0.5 \\ \frac{1}{2} &= 1 \cdot e^{-\lambda \cdot 5730} \\ \ln \frac{1}{2} &= -\lambda \cdot 5730 \\ \lambda &= \frac{-\ln \frac{1}{2}}{5730} \end{split}$$

The value of λ is $\frac{-\ln\frac{1}{2}}{5730}$ with units of years.

4.2 (b)

4.2 (0) We know that the abundance of carbon atoms in the atmosphere has remained constant (until recent times writ global warming and such) at about 500 atoms of carbon (in the form of CO2) of every million molecules of air and integor being at about 781 × 10° molecules per 1 million molecules of air. We also know that the abundance of carbon 14 in the atmosphere is 1 atom of acrobs-14 per 10° atoms of authors. We are given that carbon 14 is formed in the simosphere is 1 atom of 11° at 10° atoms of authors. We are given that carbon 14 is formed in the simosphere considerable values of authors. We can use the fact that the abundance of carbon 14 in the atmosphere is constant to seame that the differential equation describing the abundance of ordon 14 in at steady state with the decay of carbon 14 being balanced by the formation 14 being balance

$$\frac{dN}{dt} = -\lambda N(t) + b = 0$$



Q5

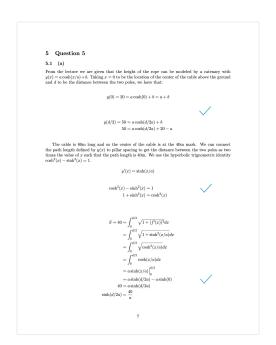
20 / 20

The amazon problem
The two ends of a cable that is
80 meters long are anchored
on two poles sticking out from

a lake (each end on a separate pole). Each anchoring point is 50 meters above the surface of the water. What is the distance between the two poles (to one

decimal point) if the centre of the cable is

- (a) 20 meters off the ground and
- (b) 10 meters off the ground? Solve this problem using the formulas derived in lecture 3 (you do not have to re-derive and solve the whole differential equation but can just start with the result that the lecture arrived at).



y(0) = 20 = a + b b = 20 - a $y(d/2) = 50 - a \cosh(d/2a) + b$ $50 - a \cosh(d/2a) + 20 - a$ $30 + a + a \cosh(d/2a)$ $\cosh(d/2a) = \frac{30 + a}{a}$ $\left(\frac{3(3 + a)^2}{a^2}\right) - \left(\frac{40}{a^2}\right)^2 - 1$ $\left(\frac{3^2 + 60a + 300}{a^2}\right) - \left(\frac{60}{a^2}\right) - 1$ $a^2 + 60a - 700 - a^2$ 60a = 700 a = 35/3We get that a = 35/3 and b = 20 - 35/3 = 25/3. We can then use these values to find the distance between the two poles. $y(d/2) = 50 = (35/3) \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 25/7 = \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 25/7 = \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 25/7 = \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 25/7 = \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 25/7 = \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 25/7 = \cosh(d/2a/3a) + \frac{25}{3}$ $\frac{50 - 36/3a}{3} = 35/7 = \cosh(d/2a/3a) + \frac{25}$

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\sin k(d/2a) = \frac{40}{a}
\sin k(d/2a) = \frac{40}{a}
y(d/2) = 50 - a \cosh(d/2a) + b
50 - a \cosh(d/2a) + 10 - a
60 + a - a \cosh(d/2a)
\cosh(d/2a) = \frac{60}{a} \pm a
\sinh(d/2a) = \frac{1}{a} \pm a
\sinh(d/2a) = \frac{1
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Q6

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6 Extra credit: Swiss-cheese car [20 pts]
Make a list of measures and technologies layered in a modern car to prevent injury of its human occupant
from a frontal collision. Group these measures and technologies according to their locations in the "Hierarchy
of Controls" pyramid described in Lecture 3.

6 Question 6 6 Question 6
Elimination of hazard
1. Drive less often
2. Avoid crowded highways
3. Obey speed limits Install rumble strips
 Install guardrails
 Install median barriers 4. Build cars with fewer blind spots Administratice controls

1. Avoid driving at night

2. Avoid driving in bad weather

3. Avoid driving when tired 4. Avoid texting and driving Provide PPE

1. Seathelts
2. Airbags
3. Automatic emergency braking

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