# University of Waterloo
# Department of Computer Science

## CS370 Midterm Examination: Fall 2002

Monday Nov 4, 2002 Instructors: Profs R B Simpson and D Mohaplova

The Aids allowed are:

Printed Course Notes
one text book
Hand Calculators

There are 7 questions - do all 7. There are a total of 57 marks

*There are some Matlab help notes in the Appendix*

1 Three small independent questions:

a) (2 marks) Consider the matrix $A$:

$$A = \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 2 & 0.1 \end{pmatrix} \tag{1}$$

Find the permutation matrix $P$, so that $PA$ can be factored into the product $LU$, where $L$ is lower triangular with 1$s$ on the diagonal and $U$ is upper triangular.

b) (2 marks) It is known that the condition number of A, $cond(A)$, is about $10^4$. Estimate the largest machine epsilon such that each component of $x = A\backslash b$ has at least five significant digits of accuracy?

c) (2 marks) Let $W_9 = e^{i2\pi/9}$ be a 9th root of unity. How many different complex numbers are there in the complex vector $w(3)$ of 9 components $w(3)_k = (W_9^3)^{k-1}$ for $k = 1 \ldots 9$.

2 Let $x_1 = 0,\ x_2 = 2/3,\ x_3 = 1$ be a partition of the interval $0 \le x \le 1$.
Let $q(x)$ be a piecewise quadratic polynomial that satisfies the following conditions:

i) $q(0) = 0,\ \ dq(0)/dx = 1$

ii) $q(1) = 1,\ \ dq(1)/dx = 0$

iii) $q(x)$ is continuous for $0 < x < 1$

iv) $dq(x)/dx$ is continuous for $0 < x < 1$

Note: $q(x_2)$ is not specified; i.e. $q(x)$ does not interpolate a given value at $x_2 = 2/3$.
In fact, these specifications determine the value of $q(x_2)$.
The following is a representation for $q(x)$ that contains two coefficients $a_1$ and $a_2$.

$$\text{for } 0 \le x \le 2/3\ ,\ q(x) = P_1(x) \text{ where } P_1(x) = x + a_1x^2$$
$$\text{for } 2/3 \le x \le 1,\ q(x) = P_2(x) \text{ where } P_2(x) = a_2(x-1)^2 + 1$$

a) (4 marks) Show that this representation ensures that $q(x)$ satisfies conditions i) and ii) for any $a_1,\ a_2$.

b) (2 marks) What condition does iii) place on $a_1,\ a_2$?

c) (3 marks) What condition does iv) place on $a_1,\ a_2$?

iv) *(repeated)* $dq(x)/dx$ is continuous for $0 < x < 1$

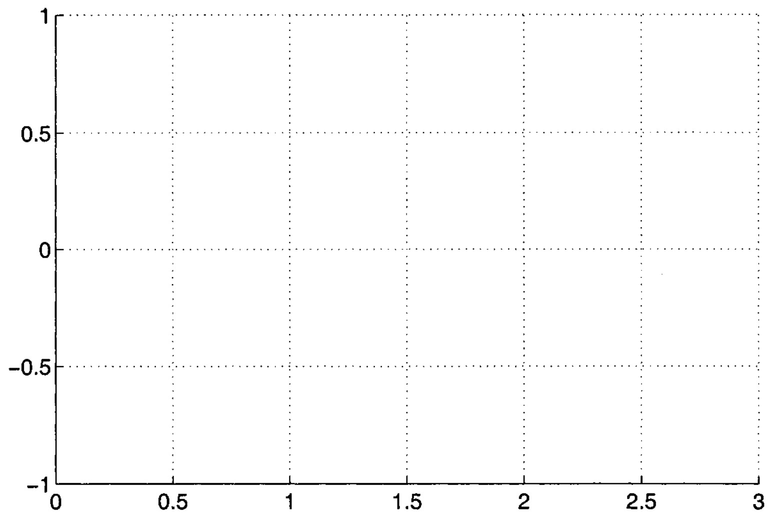
d) (2 marks) Compute $a_1,\ a_2$

e) (2 marks) Compute $q(x_2)$

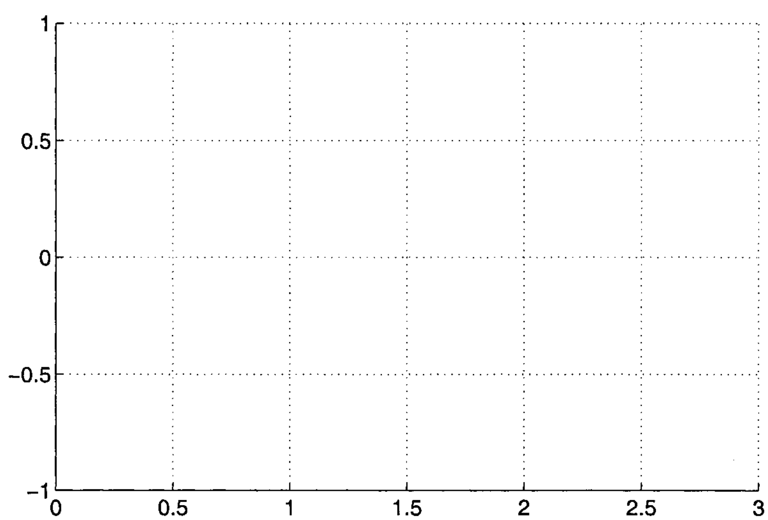
3 Consider the following table of data

| k | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $xc(k)$ | 1.5 | 0.5 | 0.5 | 1.5 | 2.5 | 1.5 | 0.5 |
| $yc(k)$ | -1.0 | -0.5 | 0 | 0.75 | .25 | -.25 | 0 |

a) (3 marks) Draw the piecewise linear interpolating parametric curve for the data




b) (2 marks) Sketch the cubic spline interpolating parametric curve for the data using your best guess at its shape. Assume natural spline boundary conditions (i.e. $d^2S(t)/dt^2 = 0$ at the end points )




*Continued ...*

c) (3 marks) Write the Matlab commands to

i. initialize arrays t=1:7 and x with $x_k$, $k = 1 : 7$

ii. compute the ppform for an interpolating cubic spline for the x coordinate data of part i.
Use the default spline command boundary conditions.

d) (3 marks) Write the Matlab commands to

i. initialize array tref=1:0.5:7

ii. use the ppform of part c) to evaluate the interpolating cubic spline for the x coordinate data at parameter points tref.

(5 marks) Consider a floating point number system with parameters $t = 4$, $\beta = 10$, $L = -3$, $U = 3$.

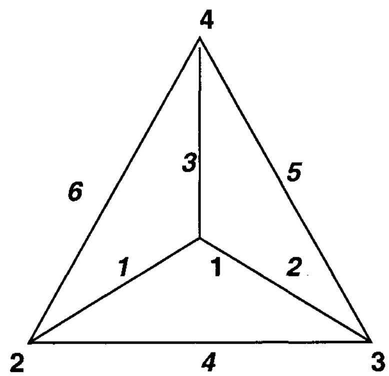
(a) Give the size of machine epsilon

(b) What is the smallest positive number in this FPNS?

(c) What number in the FPNS is $fl(12.837)$?

(d) What number is $9.123 \oplus 3.714$ ?

5 Consider a circuit with the graph representation and general equations




$$Isum = \bar{A}\bar{V}$$

for

$$\bar{A} = \begin{pmatrix} 1.5 & -.5 & -.75 & -.25 \\ -.5 & 0.9 & -.1 & -.3 \\ -.75 & -.1 & 1.05 & -.2 \\ -.25 & -.3 & -.2 & .75 \end{pmatrix}$$

Consider the scenario in which

- node 4 has voltage 5 volts and node 3 has voltage 0 volts
- nodes 1 and 2 have $ISum_k = 0\ amps\ ,\ k = 1, 2$

a) (3 marks) Write out the coefficient matrix for the equations for unknown voltages $V_1$ and $V_2$.

b) (3 marks) Compute the right hand side vector for equations in part a). Show your work.

c) (4 marks) Compute the net current supplied to the network at node 4, $ISum_4$, for this scenario, Show your work.

6 Let $A$ be a $10 \times 10$ matrix.

a) (1 mark) Circle the number which is the best estimate of the number of flops required to factor $A$

100      700      1500

Let $b^{(k)}$ for $k = 1, 2, \ldots Q$, be a sequence of column vectors of length 10.
Let $x^{(k)}$ for $k = 1, 2, \ldots Q$, be the sequence of solutions of $Ax^{(k)} = b^{(k)}$

b) (3 marks) Describe an efficient computation for computing $x^{(k)}$ for $k = 1, 2, \ldots Q$

c) (2 marks) Write a few lines of Matlab code that would accomplish b). You may assume that

- $A$ and $b^{(k)}$ have been initialized
- $b^{(k)}$ is stored in column $k$ of a $10 \times Q$ matrix , $B$

d) (3 marks) Estimate the number of flops required to carry out the computation of part b), as a function of $Q$.

7 Consider the matrix $A$:

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 5 & 3 \\ 0 & 0 & 4 \end{pmatrix} \tag{2}$$

a) (3 marks) Compute a lower triangular unit diagonal matrix M, such that $MA = \mathit{upper\ triangular}$

b) (3 marks) Compute a lower triangular unit diagonal matrix $L$ and upper triangular matrix $U$ such that

$$A = LU$$

# Appendix: Some Matlab help documentation

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help spline

 SPLINE Cubic spline data interpolation.
    YY = SPLINE(X,Y,XX) uses cubic spline interpolation to find YY, the values
    of the underlying function Y at the points in the vector XX.  The vector X
    specifies the points at which the data Y is given.  If Y is a matrix, then
    the data is taken to be vector-valued and interpolation is performed for
    each column of Y and YY will be length(XX)-by-size(Y,2).

    PP = SPLINE(X,Y) returns the piecewise polynomial form of the cubic spline
    interpolant for later use with PPVAL and the spline utility UNMKPP.

    Ordinarily, the not-a-knot end conditions are used. However, if Y contains
    two more values than X has entries, then the first and last value in Y are
    used as the endslopes for the cubic spline.  Namely:
       f(X) = Y(:,2:end-1),   df(min(X)) = Y(:,1),   df(max(X)) = Y(:,end)

    Example:
    This generates a sine curve, then samples the spline over a finer mesh:
        x = 0:10;  y = sin(x);
        xx = 0:.25:10;
        yy = spline(x,y,xx);
        plot(x,y,'o',xx,yy)

    Example:
    This illustrates the use of clamped or complete spline interpolation where
    end slopes are prescribed. Zero slopes at the ends of an interpolant to the
    values of a certain distribution are enforced:
       x = -4:4; y = [0 .15 1.12 2.36 2.36 1.46 .49 .06 0];
       cs = spline(x,[0 y 0]);
       xx = linspace(-4,4,101);
       plot(x,y,'o',xx,ppval(cs,xx),'-');

    See also INTERP1, PPVAL, SPLINES (The Spline Toolbox).
```

```
help ppval

 PPVAL  Evaluate piecewise polynomial.
    V = PPVAL(PP,XX) returns the value at the points XX of the piecewise
    polynomial contained in PP, as constructed by SPLINE or the spline utility
    MKPP.

    V = PPVAL(XX,PP) is also acceptable, and of use in conjunction with
    FMINBND, FZERO, QUAD, and other function functions.

    Example:
    Compare the results of integrating the function cos and this spline:
      a = 0; b = 10;
      int1 = quad(@cos,a,b,[],[]);
      x = a : b; y = cos(x); pp = spline(x,y);
      int2 = quad(@ppval,a,b,[],[],pp);

    int1 provides the integral of the cosine function over the interval [a,b]
    while int2 provides the integral over the same interval of the piecewise
    polynomial pp which approximates the cosine function by interpolating the
    computed x,y values.

    See also SPLINE, MKPP, UNMKPP.
```