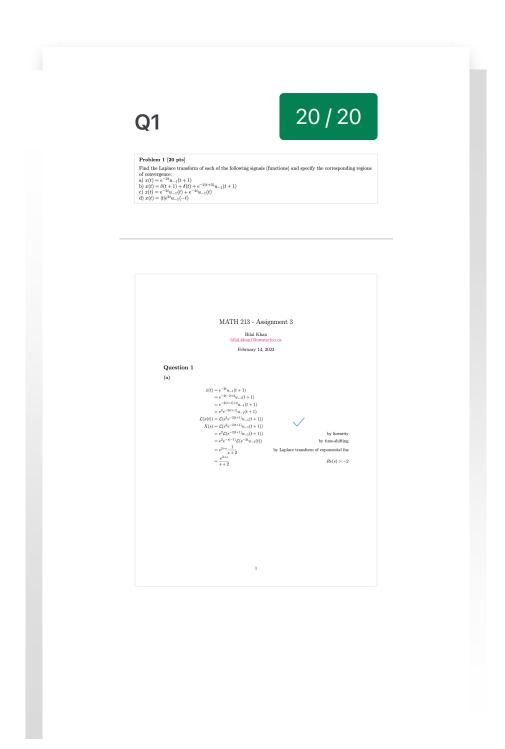
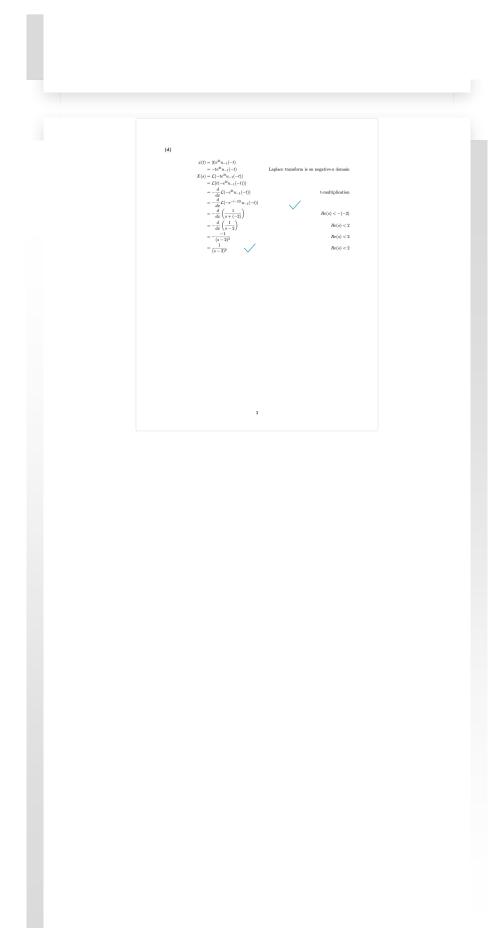
My grades for **Assignment 3**



(b) $x(t) = \delta(t+1) + \delta(t) + e^{-2(t+3)}u_{-1}(t+1) \\ (\delta(t+1)) + e^{-2(t+3)}u_{-1}(t+1) \\ (\delta(t+1$

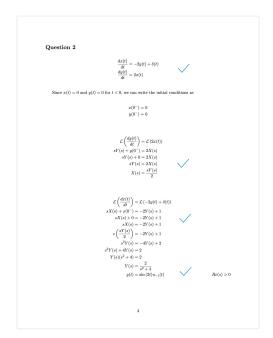


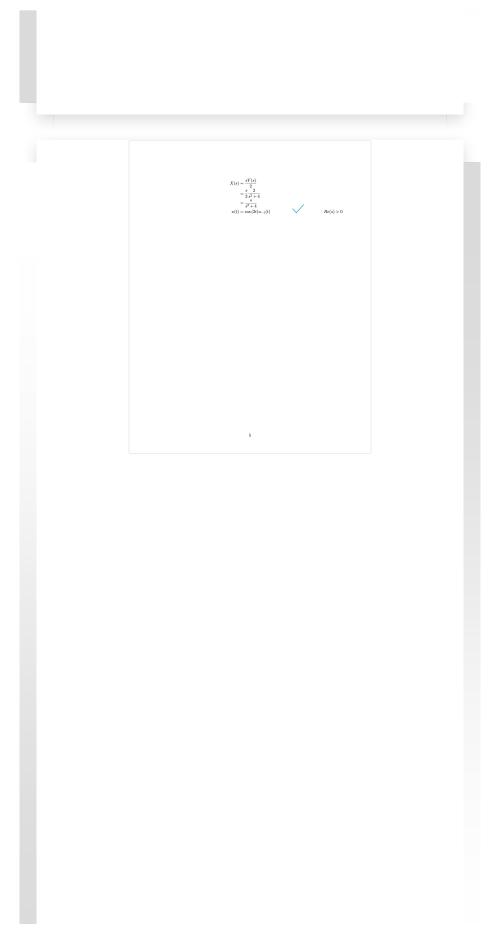
Q2

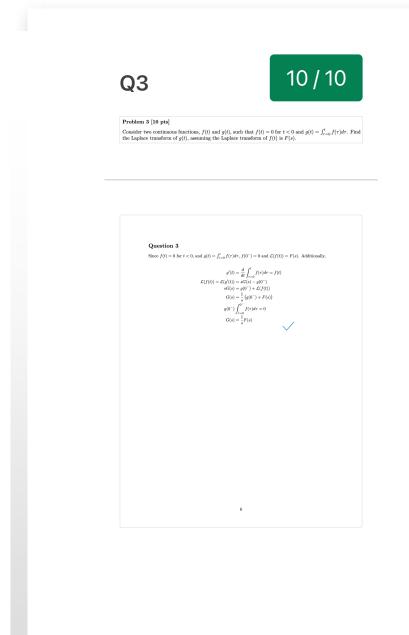
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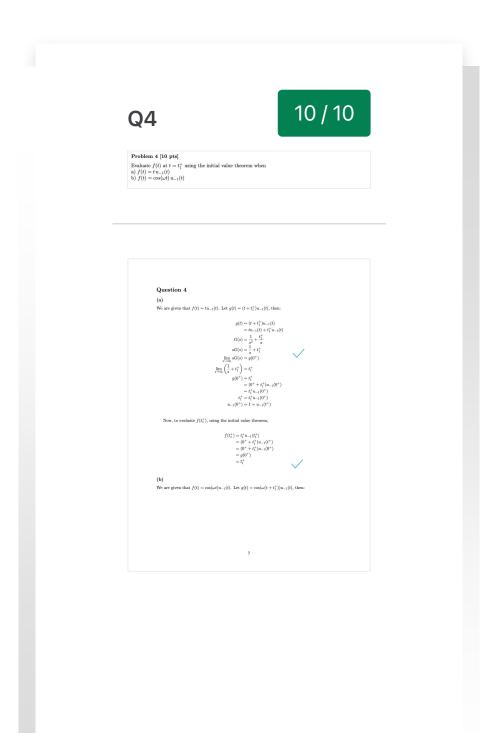
Problem 2 [15pts] Consider two signals (functions) x(t) and y(t), such that both x(t) = 0 and y(t) = 0 for t < 0, that are related through the following differential equations:

$$\begin{split} \frac{dx(t)}{dt} &= -2y(t) + \delta(t) \\ \frac{dy(t)}{dt} &= 2x(t) \end{split} \tag{2}$$









```
\begin{split} g(t) &= \cos(\omega(t+t_1^+))u_{-1}(t) \\ &= \left(\frac{\omega^{2}u^{2}+t_{1}^{2}}{s^{2}} + \frac{\omega^{2}u^{2}+t_{1}^{2}}{s^{2}}\right)u_{-1}(t) \\ &= \left(\frac{2}{s^{2}}e^{-t_{1}^{2}}\frac{\delta^{2}u^{2}}{s^{2}} + \frac{1}{s^{2}}e^{-s_{1}^{2}}e^{-s_{2}^{2}}\right)u_{-1}(t) \\ G(s) &= \frac{1}{2}e^{\delta u_{1}^{2}}\frac{\delta^{2}u^{2}}{s^{2}} + \frac{1}{2}e^{-s_{1}^{2}}\frac{\delta^{2}u^{2}}{s^{2}} + \frac{1}{s^{2}}\frac{\delta^{2}u^{2}}{s^{2}} + \frac{1}{s^{2}}\frac{\delta^{2}u^{2}}
Now, to evaluate f(t_1^+), using the initial value theorem,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                \begin{split} & \text{sing ture ass....} \\ & f(t_1^+) = \cos(\omega t_1^+) u_{-1}(t_1^+) \\ & = \cos(\omega(0^+ + t_1^+)) u_{-1}(t^+) \\ & = \cos(\omega(0^+ + t_1^+)) u_{-1}(0^+) \\ & = g(0^+) \\ & = \cos(\omega t_1^+) \end{split}
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Q5

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Problem 5 [20 pts] Consider a 10 resistor and a 1H inductor connected in parallel to an ideal current source. Let's define current in the unit of ampere produced by this ideal current source to be the input x(t) and the current through the inductor to be the system's output (alsa response) y(t). a) Determine the zero-state response of this circuit when the input is $x(t) = x_0e^{-2t}u_{-1}(t)$, where $x_0 = 1A$. b) Determine the system's output response of the circuit for $t > 0^-$, given that $y(t)^- > 1A$. b) Determine the system's output when the input current is $x(t) = x_0e^{-2t}u_{-1}(t)$, where $x_0 = 1A$, and the initial condition is the same as specified in part b).

Question 5

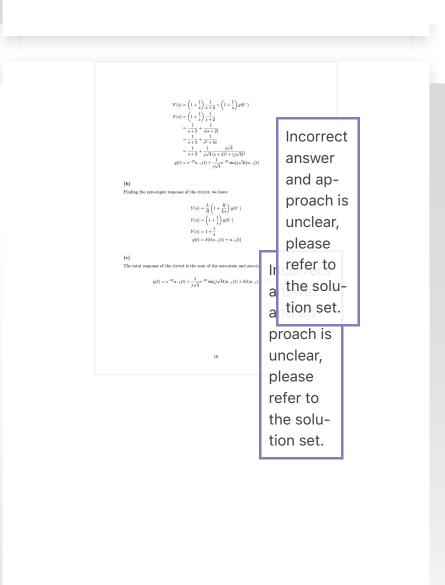
Let $x(t) = x_0 e^{-2t} u_{-1}(t)$, with $x_0 = 1A$, $R = 1\Omega$ and L = 1H. We have that the current through the inductor $i_1(t) = y(t)$. Since the resistor and inductor are connected in parallel, the voltage drop across them are oquivalent and $V_t = V_t = V$. Applying the equations for the voltage drop across the resistor and inductor, we have:

$$\begin{split} V_l &= L \frac{dy(t)}{dt} \\ V &= L \frac{dy(t)}{dt} \\ V_r &= Ri_r(t) \\ i_r(t) &= \frac{V_r}{R} = \frac{V}{R} = \frac{L}{R} \frac{dy(t)}{dt} \end{split}$$

$$\begin{split} X(s) &= \mathcal{L} \left(x_0 e^{-2t} u_{-1}(t) \right) \\ &= \mathcal{L} \left(e^{-2t} u_{-1}(t) \right) \\ &= \frac{1}{s+2} \end{split}$$

From KCL and KVL, we know that the current from the current source is equivalent to the unbined currents through the resistor and inductor. Thus,

$$\begin{split} x(t) &= y(t) + i_r(t) - y(t) + \frac{L}{R} \frac{dy(t)}{dt} \\ X(s) &= Y(s) + \frac{L}{R} (sY(s) - y(0^-)) \\ &= Y(s) + \frac{L}{R} Y(s) - \frac{L}{R} y(0^-) \\ X(s) + \frac{L}{R} y(0^-) &= Y(s) \left(1 + \frac{L}{R}\right) \\ Y(s) &= \left(1 + \frac{R}{Ls}\right) X(s) + \frac{L}{R} \left(1 + \frac{R}{Ls}\right) y(0^-) \end{split}$$





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Problem 6 [20 pts] Suppose the following facts are given about the signal (function) x(t) with Laplace transform X(s): 1. x(t) is real and even 2. X(s) has four (finite) poles and no (finite) zeros. 3. X(s) has $s = \frac{1}{2} e^{j \frac{\pi}{4}}$. 4. $\int_{-\infty}^{\infty} x(t) dt = 4$.

Question 0

We are given that X(t) has no zeros and four finite poles. From this we can assume that it is a rational function with a degree ever polynomial in the numerator and a degree four polynomial in the demonitance. We are also given that X(t) is read, so all ordinates of the polynomials must be real. Since we know at least one of the roots of the denominator is complex, we must have that its conjugate be a root to that the demonitantser is real.

Note that Euler's Identity $e^{it} = -1$ is useful in this context. Taking the fourth root of both contexts, we have $e^{it} = -1$ in Since we know the sum of the fourth root of the sero is $\frac{1}{2}\sqrt{f}$ and work from there to find the other three roots.

$$(s - 1/2\sqrt{j})(s + 1/2\sqrt{j}) = s^2 - 1/4j$$

$$\begin{aligned} z) &= \int_{-\infty}^{\infty} x(t)e^{-st}dt \\ z) &= \int_{-\infty}^{\infty} x(t)e^{-0t}dt \\ z) &= \int_{-\infty}^{\infty} x(t)dt \end{aligned}$$

$$X(s) = \frac{t}{s^4 + 1/16}$$

$$X(0) = \frac{t}{0^4 + 1/16}$$

$$4 = 16t$$

$$t = 1/4$$

We now have that $X(s)=\frac{1}{4(s^4+1/16)}$. We can see that this satisfies all our conditions. The region of convergence is R(s)>0.

-sqrt(2)/4 < Re(S)<sqrt(2)/4, please refer to the solution set.

Q7

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Problem 7 [20 pts]

Problem 7 [20 pts]
A classroom is 15 feet wide, 53 feet long, 13 feet high in the front, and 8 feet in the back. During the first lecture of the term with 130 students present, the CO₂ level in the room eventually stabilized at 1400ppm. Assume instant uniform mixing of gases in the room and a model of ventilation in which HVAC system removes air from the room at rate R_cent and replaces it at the same rate with fresh air from the outside with CO₂ level of 400ppm.
a) Estimate R_{cons} in unities of cfim and in units of cubic meters per hour. How many of air changes per hour would such R_{cons} correspond to in this classroom? [15pts]
b) Estimate how many students attended the second fecture of the term, if the CO₂ level in the room eventually stabilized at 1000ppm (assuming the same R_{cons} at both relectures). [5pts]
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Question 7

We model amount of CO2 in the room as a first-order differential equation, where y(t) is the contract volume of CO2 particles in the room and y'(t) is the change in the volume of CO2 in the room per unit time (minute). We know that at some point, the CO2 level well stabilize to a particular value and the rate of change will be zero. These are three contributing factors that a particular value and the rate of change will be zero. These are three contributing factors that constain rate, the ventilation system brings in rotation at the constaint a fixed amount of CO2 at the same constant rate, and the people in the room produce CO2 also at a constant rate. We assume $R_{\rm rest}$ is in units of cubic feet per minute (effect).

$$\begin{split} y'(t) &= -X R_{vent} y(t) + Y R_{vent} + Z \\ y(\infty) &= A \\ y'(\infty) &= 0 \end{split}$$

We can calculate the volume of the room: $V = (8 * 53 + 1/2 * (13 - 8) * 53) * 51 = 28381.5 ft^3$

We know that the final concentration of CO2 in the room is 1400 ppm (parts per million), the actual volume of CO2 in the room is:

 $y(\infty) = 1400/1,000,000,000 * 28881.5 = 39.73 ft^3$ The total volume of CO2 removed by the ventilation system is percent of the total amount of air in the room removed times the current volume of CO2 in the room:

 $(R_{\rm cent}/V)*y(\infty) = (R_{\rm sent}/28381.5)*39.73 = 0.00139*R_{\rm cent}$

 $(R_{cost}V) \circ \chi(\infty) = (R_{cost}/2856.1) \circ 30.73 = 0.00139 \cdot R_{cost}$ The total volume of CO2 added by the ventilation system is the concentration of CO2 in the outside air (400 PPM) times the number of cubic forc of outside air brought in per minute: $400/1,000,000,000 \circ R_{cost} = 0.0004 \cdot R_{cost}$ $400/1,000,000,000 \circ R_{cost} = 0.0004 \cdot R_{cost}$ The total volume of CO2 added by the people in the room in the number of people in the room times the volume of CO2 geodecode pre person per minute given that the concentration of CO2 in the calculad air a $h_0 \times 100 PPM$. $130 people *12 breaths/min* 0.5 litres/breath* 0.035 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000,000 = 1.092 ft^3 cubic feet in a litre* 40,000/1,000 = 1.092 ft^3 cubic feet i$

 $0 = -0.00139*R_{vent} + 0.0004*R_{vent} + 1.092$ $-0.00099 * R_{vent} + 1.092 = 0$

 $R_{\rm cent} = 1103.03$ cubic ft/min

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