SE 380 Introduction to Feedback Control
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HOMEWORK 2
Due date: September 29, 2023

1 Consider the state-space model

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^2$, $u \in \mathbb{R}$, $y \in \mathbb{R}$.

a Given $\mu, \tau \in \mathbb{R}$, find values for A, B, C, and D such that the corresponding transfer function is

$$G(s) = \frac{\mu}{1 + \tau s}. \tag{2}$$

b Is the choice of A, B, C, and D unique? Motivate your answer.

2 Consider a first-order system with transfer function given by (2), with $\mu > 0$ and $\tau > 0$.

a Compute the response $y_1(t)$ to a step input $u_1(t) = H(t)$ and the response $y_2(t)$ to a ramp input $u_2(t) = tH(t)$.

b Find the values of μ and τ for which

(i) $\lim_{t\to\infty} |y_1(t) - u_1(t)| = 0$ (constant reference tracking)

(ii) $\lim_{t\to\infty} |y_2(t) - u_2(t)| = 0$ (ramp reference tracking)

Discuss your answers.

3 Consider the system described by the input-output equation $y(t) = u(t - \tau)$, where $\tau > 0$.

a Compute its transfer function.

b Sketch the corresponding Bode plot.

c Compute the response to the sinusoidal input $u(t) = 0.15 \sin(2\pi t)$.

d Does the result of the "Fundamental theorem of frequency response" hold in this case?