My grades for Assignment 4

Q1

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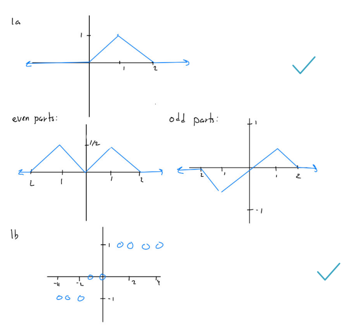
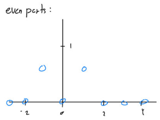
Sketch the following sigals and then determine and sketch their even and odd parts (make sure to label the sketches carefully):
a) $x(t) = t(u_{-1}(t) - u_{-1}(t-1)) + (2-t)(u_{-1}(t-1) - u_{-1}(t-2))$
b) $x[n] = u_{-1}[n] - u_{-1}[-n-1]$, where n corresponds to the discrete time.
Hint: $x(t) = x_e(t) + x_o(t)$, where $x_e(t) = \frac{x(t)+x(-t)}{2}$ and $x_o(t) = \frac{x(t)-x(-t)}{2}$.

Question 1

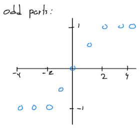


2

even parts:



odd parts:



Q2

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As discussed in class, a system may or may not be:
1) Memoryless
2) Time invariant
3) Linear
4) Causal
5) Stable

Determine if each of the continuous-time systems described below have or do not have these properties (make sure to justify your answers). In each case, $x(t)$ denotes the system input and $y(t)$ the system output.

a) $y(t) = x(t-2) + x(2-t)$ b) $y(t) = cos(3t)x(t)$
c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$ d) $y(t) = (x(t) + x(t-2))u_{-1}(t)$

Question 2

(a)

$$y(t) = x(t-2) + x(2-t)$$

1. Memoryless: False, it depends on the value of the input at time $t-2$ and $2-t$.
2. Time Invariant: False, shifting the input and output by a constant τ will result in different outputs as the signs of t in $x(t-2)$ and $x(2-t)$ are different and shifting the input vs output will result in different values (i.e. $2-(t+\tau)$ vs $2-t+\tau$).
3. Linear: True, the relationship between the input and output is a linear mapping.
4. Causal: False, for $t=0$, the output $y(t)$ depends on the inputs $x(t-2) = x(-2)$ and $x(2-t) = x(2)$ so it can depend on future values of the input.
5. Stable: True, the output of the system is bounded by the input.

(b)

$$y(t) = \cos(3t)x(t)$$

1. Memoryless: True, it depends only on the current value of the input.
2. Time Invariant: False, cos is not time invariant.
3. Linear: True, multiplying $x(t)$ by a constant will result in the overall output being multiplied by the same constant.
4. Causal: True, it depends only on the current value of the input.
5. Stable: True, cos is bounded in $[0, 1]$ and so the output is bounded by the input.

(c)

$$y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$$

1. Memoryless: False, it depends on all past values of the input.
2. Time Invariant: False, time shifting the input $x(\tau+\delta)$ will result in a different output ($y(t) = \int_{-\infty}^{2t} x(\tau+\delta)d\tau$) than time shifting the output $y(t+\delta) = \int_{-\infty}^{2(t+\delta)} x(\tau)dt$.
3. Linear: True, multiplying $x(\tau)$ by a constant will result in the overall integral begin multiplied by the same constant.
4. Causal: False, it depends on the input at time $2t$ and so it can depend on future values of the input.
5. Stable: False, even if the input $x(\tau)$ is bounded, the value of the integral will scale with t, and is not bounded.

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(d)

$$y(t) = (x(t) + x(t-2))u_{-1}(t)$$

1. Memoryless: False, it depends on a past value of the input $(x(t-2))$.
2. Time Invariant: False, time shifting the input and time shifting the output are not equivalent.
3. Linear: True: The function is linear in $x(t)$.
4. Causal: True: The function is only dependent on current and past values of the input.
5. Stable: True: The transfer function of each $x(t), x(t-2)$ is 1.

Q3

10 / 10

Determine which of the properties listed in Problem 2 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $x[n]$ denotes the system input and $y[n]$ the system output.
a) $y[n] = x[-n]$ b) $y[n] = x[n-2] - 2x[n-8]$
c) $y[n] = nx[n]$ d) $y[n] = x[4n+1]$

Question 3

(a)

$$y[n] = x[-n]$$

1. Memoryless: False, it depends on the value of the input at time $-n$.
2. Time Invariant: False, shifting the input is not equivalent to shifting the output. For example, shifting the input by $n-\tau$, $x[-(n-\tau)]$ will result in the output being shifted by $n+\tau$, $y[n+\tau]$. Shifting the output by $n-\tau$, $y[n-\tau]$ will result in the input being shifted by $n-\tau$, $x[-n-\tau]$. These are not equivalent.
3. Linear: True, this is a linear function.
4. Causal: False, at negative times, the output depends on positive values of the input from the

future.

5. Stable: True, the transfer function of the input is 1.

(b)

$$y[n] = x[n-2] - 2x[n-8]$$

1. Memoryless: False, it depends on the value of the input at time $n-2$ and $n-8$.
2. Time Invariant: True, the function is linear in time.
3. Linear: True, this is a linear function.
4. Causal: True, it depends only on past values of the input.
5. Stable: True, the transfer function of the input is 1 for the first input and -2 for the second input.

(c)

$$y[n] = nx[n]$$

1. Memoryless: True, it depends only on the current value of the input.
2. Time Invariant: False, a delay of the input $x[n+\tau]$ ($y[n] = nx[n+\tau]$) is not equivalent to a delay of the output ($y[n+\tau] = (n+\tau)x[n+\tau]$).
3. Linear: True, multiplying the input $x[n]$ by a constant c will result in the output being multiplied by c.
4. Causal: True, it depends only on current values of the input.
5. Stable: False, the transfer function of the input depends on the value of n, which is not bounded given a bounded input $x[n]$.

(d)

$$y[n] = x[4n + 1]$$

1. Memoryless: False, it depends on the value of the input at time $4n + 1$.
2. Time Invariant: False, shifting the input by τ will result in the output being shifted by 4τ, $y[n+4\tau+1]$. Shifting the output by τ will result in the input being shifted by τ, $x[4n+1+\tau]$. These are not equivalent.
3. Linear: True, the output is a linear function of $x[4n + 1]$.
4. Causal: False, it depends on the value of the input at time $4n + 1$, which is in the future.
5. Stable: True, the transfer function of the input is 1.

Q4

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Q4

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Consider a linear time-invariant (LTI) system whose response to the signal $x_1(t) = u_{-1}(t) - u_{-1}(t-2)$ is the signal $y_1(t) = 2t(u_{-1}(t) - u_{-1}(t-1)) + (4-2t)(u_{-1}(t-1) - u_{-1}(t-2))$. Determine and carefully sketch the response of the system to the input $x_2(t) = u_{-1}(t) - 2u_{-1}(t-2) + u_{-1}(t-4)$.

Question 4

$$x_1(t) = u_{-1}(t) - u_{-1}(t-2)$$
$$y_1(t) = 2t(u_{-1}(t) - u_{-1}(t-1)) + (4-2t)(u_{-1}(t-1) - u_{-1}(t-2))$$

We can simplify $y_1(t)$

$$y_1(t) = 2tu_{-1}(t) - 2tu_{-1}(t-1) + 4u_{-1}(t-1) - 4u_{-1}(t-2) - 2tu_{-1}(t-1) + 2tu_{-1}(t-2)$$
$$y_1(t) = 2tu_{-1}(t) - 4tu_{-1}(t-1) + 4u_{-1}(t-1) - 4u_{-1}(t-2) + 2tu_{-1}(t-2)$$

Taking the Laplace transform of both $x_1(t)$ and $y_1(t)$, we get

$$\begin{aligned} X_1(s) &= \frac{1}{s} - \frac{e^{-2s}}{s} = \frac{1-e^{-2s}}{s} \\ Y_1(s) &= \frac{2}{s^2} - \frac{4e^{-s}}{s^2} + \frac{4e^{-s}}{s} - \frac{4e^{-2s}}{s} + \frac{2e^{-2s}}{s^2} \\ &= \frac{2-4e^{-s}+2e^{-2s}}{s^2} + \frac{4e^{-s}-4e^{-2s}}{s} \end{aligned}$$

We can find the transfer function $H(s)$

$$\begin{aligned} Y_1(s) &= H(s)X_1(s) \\ H(s) &= \left(\frac{2-4e^{-s}+2e^{-2s}}{s^2} + \frac{4e^{-s}-4e^{-2s}}{s}\right)\left(\frac{s}{1-e^{-2s}}\right) \\ H(s) &= \frac{2-4e^{-s}+2e^{-2s}}{s(1-e^{-2s})} + \frac{4e^{-s}-4e^{-2s}}{1-e^{-2s}} \end{aligned}$$

To find the output of the system under a new input $x_2(t) = u_{-1}(t) - 2u_{-1}(t-2) + u_{-1}(t-4)$, we can compute its Laplace transform, and then use the transfer function $H(s)$ to find the output $Y_2(s)$, then take the inverse Laplace transform of $Y_2(s)$ to find $y_2(t)$.

$$\begin{aligned} X_2(s) &= \frac{1}{s} - \frac{2e^{-2s}}{s} + \frac{e^{-4s}}{s} = \frac{1-2e^{-2s}+e^{-4s}}{s} = \frac{(1-e^{-2s})^2}{s} \\ Y_2(s) &= H(s)X_2(s) \\ &= \left(\frac{2-4e^{-s}+2e^{-2s}}{s(1-e^{-2s})} + \frac{4e^{-s}-4e^{-2s}}{1-e^{-2s}}\right)\left(\frac{(1-e^{-2s})^2}{s}\right) \\ &= \frac{(2-4e^{-s}+2e^{-2s})(1-e^{-2s})}{s^2} + \frac{(4e^{-s}-4e^{-2s})(1-e^{-2s})}{s} \\ &= \frac{2-4e^{-s}+2e^{-2s}-2e^{-2s}+4e^{-3s}-2e^{-4s}}{s^2} + \frac{4e^{-s}-4e^{-2s}-4e^{-3s}+4e^{-4s}}{s} \\ &= \frac{2-4e^{-s}+4e^{-3s}-2e^{-4s}}{s^2} + \frac{4e^{-s}-4e^{-2s}-4e^{-3s}+4e^{-4s}}{s} \\ &= \frac{2}{s^2} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2} - \frac{2e^{-4s}}{s^2} + \frac{4e^{-s}}{s} - \frac{4e^{-2s}}{s} - \frac{4e^{-3s}}{s} + \frac{4e^{-4s}}{s} \\ y_2(t) &= 2tu_{-1}(t) - 4tu_{-1}(t-1) + 4tu_{-1}(t-3) - 2tu_{-1}(t-4) + 4u_{-1}(t-1) \\ &\quad - 4u_{-1}(t-2) - 4u_{-1}(t-3) + 4u_{-1}(t-4) \end{aligned}$$

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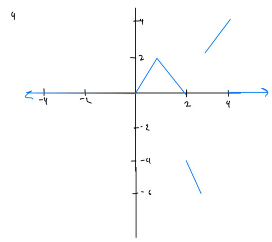

Wrong graph, please refer to the solution set.

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Q5

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Consider a discrete-time system S with input $x[n]$ and output $y[n]$ related by $y[n] = x[n](g[n] + g[n-2])$.
a) If $g[n] = 1$ for all n, show that S is time invariant.
b) If $g[n] = n$, show that S is not time invariant.

Question 5

A system is time-invariant if applying a time delay on the input is equivalent to applying a time delay on the output.

(a)

$$\begin{gathered} y[n] = x[n](g[n] + g[n-2]) \\ g[n] = 1 \end{gathered}$$

Applying a time delay on the input:

$$\begin{aligned} y[n] &= x[n](g[n] + g[n-2]) \\ &= x[n+\tau](g[n] + g[n-2]) \\ &= x[n+\tau](1+1) \\ &= 2x[n+\tau] \end{aligned}$$

Applying a time delay on the output:

$$\begin{aligned} y[n] &= x[n](g[n] + g[n-2]) \\ y[n+\tau] &= x[n+\tau](g[n+\tau] + g[n+\tau-2]) \\ &= x[n+\tau](1+1) \\ &= 2x[n+\tau] \end{aligned}$$

The system is time-invariant.

(b)

$$\begin{gathered} y[n] = x[n](g[n] + g[n-2]) \\ g[n] = n \end{gathered}$$

Applying a time delay on the input:

$$\begin{aligned} y[n] &= x[n](g[n] + g[n-2]) \\ &= x[n+\tau](g[n] + g[n-2]) \\ &= x[n+\tau](n+n-2) \\ &= x[n+\tau](2n-2) \end{aligned}$$

Applying a time delay on the output:

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$$\begin{aligned} y[n] &= x[n](g[n] + g[n-2]) \\ y[n+\tau] &= x[n+\tau](g[n+\tau] + g[n+\tau-2]) \\ &= x[n+\tau](n+\tau+n+\tau-2) \\ &= x[n+\tau](2n+2\tau-2) \end{aligned}$$

The two expressions are not equal, so the system is not time-invariant.

Q6

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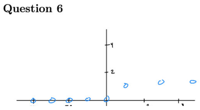
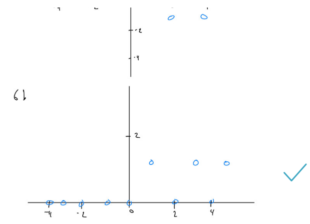
Consider a discrete-time feedback system, in which we label its output as $y[n]$, its external input as $x[n]$, and an intermediate signal in the system $e[n]$. In this feedback system $e[n] = x[n] - y[n]$ and $y[n] = e[n-1]$. Assume $y[n] = 0$ for $n < 0$.

a) Sketch the output of the system when $x[n] = \delta[n]$
b) Sketch the output of the system when $x[n] = u_{-1}[n]$

1 Question 6

6a





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Q7

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Compute and sketch the convolution $y[n] = x[n] * h[n]$ of the following pairs of discrete-time signals.
a) $x[n] = \alpha^n u_{-1}[n]$ and $h[n] = \beta^n u_{-1}[n]$, where $\alpha \neq \beta$
b) $x[n] = h[n] = \alpha^n u_{-1}[n]$

Question 7

(a)

$$x[n] = \alpha^n u_{-1}(n)$$
$$h[n] = \beta^n u_{-1}(n)$$

$$\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \sum_{k=-\infty}^{\infty} \alpha^k u_{-1}(k)\beta^{n-k}u_{-1}(n-k) \\
&= \sum_{k=0}^{n} \alpha^k \beta^{n-k} \\
&= \beta^n \times \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k \\
&= \frac{\beta^n \left(1 - \left(\frac{\alpha}{\beta}\right)^{n+1}\right)}{1 - \frac{\alpha}{\beta}} \\
&= \frac{\beta^n \left(1 - \left(\frac{\alpha}{\beta}\right)^{n+1}\right)}{\frac{\beta-\alpha}{\beta}} \\
&= \frac{\beta^{n+1} \left(1 - \left(\frac{\alpha}{\beta}\right)^{n+1}\right)}{\beta - \alpha} \\
&= \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}
\end{aligned}$$

(b)

$$x[n] = \alpha^n u_{-1}(n)$$
$$h[n] = \alpha^n u_{-1}(n)$$

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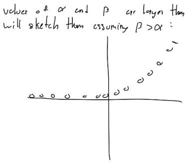
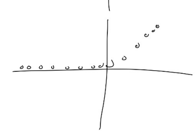
$$
\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \sum_{k=-\infty}^{\infty} \alpha^k u_{-1}(k)\alpha^{n-k}u_{-1}(n-k) \\
&= \sum_{k=0}^{n} \alpha^k \alpha^{n-k} \\
&= \sum_{k=0}^{n} \alpha^k \frac{\alpha^n}{\alpha^k} \\
&= \alpha^n \times \sum_{k=0}^{n} \left(\frac{\alpha}{\alpha}\right)^k \\
&= \alpha^n \times \sum_{k=0}^{n} 1^k \\
&= \alpha^n \times n
\end{aligned}
$$

7a

The shape of the graph will differ depending on if the values of α and β are larger than each other, we will sketch them assuming $\beta > \alpha$:



7b



Q8

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Use the convolution integral to find the response $y(t)$ of an LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results.
a) $x(t) = e^{-\alpha t}u_{-1}(t)$ and $h(t) = e^{-\beta t}u_{-1}(t)$. (do this both for when $\alpha \neq \beta$ and when $\alpha = \beta$
b) $x(t) = u_{-1}(t) - 2u_{-1}(t-2) + u_{-1}(t-5)$ and $h(t) = e^{2t}u_{-1}(1-t)$.

Question 8

(a)

$$x(t) = e^{-\alpha t}u_{-1}(t)$$
$$h(t) = e^{-\beta t}u_{-1}(t)$$

First, assuming that $\alpha \neq \beta$:

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(t) * h(t)dt \\
&= \int_{-\infty}^{\infty} e^{-\alpha t}u_{-1}(t)e^{-\beta(n-t)}u_{-1}(n-t)dt \\
&= \int_{0}^{n} e^{-\alpha t}e^{-\beta(n-t)}dt \\
&= \int_{0}^{n} e^{-\alpha t}e^{-\beta n+\beta t}dt \\
&= \int_{0}^{n} e^{-\alpha t}e^{\beta t}e^{-\beta n}dt \\
&= e^{-\beta n} \times \int_{0}^{n} e^{(\beta-\alpha)t}dt \\
&= e^{-\beta n} \times \frac{e^{(\beta-\alpha)n} - 1}{\beta - \alpha} \\
&= \frac{e^{-\beta n}e^{\beta n}e^{-\alpha n} - e^{-\beta n}}{\beta - \alpha} \\
&= \frac{e^{-\alpha n} - e^{-\beta n}}{\beta - \alpha}
\end{aligned}$$

Now, if $\alpha = \beta$:

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(t) * h(t)dt \\
&= \int_{-\infty}^{\infty} e^{-\alpha t}u_{-1}(t)e^{-\alpha(n-t)}u_{-1}(n-t)dt \\
&= \int_{0}^{n} e^{-\alpha t}e^{-\alpha(n-t)}dt \\
&= \int_{0}^{n} e^{-\alpha t}e^{-\alpha n+\alpha t}dt \\
&= \int_{0}^{n} e^{-\alpha t}e^{\alpha t}e^{-\alpha n}dt \\
&= \int_{0}^{n} e^{-\alpha n}dt = e^{-\alpha n} \times \int_{0}^{n} dt = ne^{-\alpha n}
\end{aligned}$$

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(b)

$$x(t) = u_{-1}(t) - 2u_{-1}(t-2) + u_{-1}(t-5)$$
$$h(t) = e^{2t}u_{-1}(1-t)$$

$$\begin{aligned}
y(n) &= \int_{-\infty}^{\infty} x(t) * h(t)dt \\
&= \int_{-\infty}^{\infty} (u_{-1}(t) - 2u_{-1}(t-2) + u_{-1}(t-5))e^{2(n-t)}u_{-1}(1-(n-t))dt \\
&= e^{2n}\int_{-\infty}^{\infty} (u_{-1}(t) - 2u_{-1}(t-2) + u_{-1}(t-5))e^{-2t}u_{-1}(1-(n-t))dt \\
&= e^{2n}(\int_{-\infty}^{\infty} u_{-1}(t)e^{-2t}u_{-1}(1-(n-t))dt \\
&- 2\int_{-\infty}^{\infty} u_{-1}(t-2)e^{-2t}u_{-1}(1-(n-t))dt \\
&+ \int_{-\infty}^{\infty} u_{-1}(t-5)e^{-2t}u_{-1}(1-(n-t))dt)
\end{aligned}$$

The first integral is has two unit step functions on it, one to ensure that t is positive, and the other to ensure that $1-(n-t)$ is positive. The second unit step function will have a value of one when $t \geq n-1$ and we can use that to bound the integral's lower and upper limits to $[n-1, \infty]$. Note that in the case where $n = 0$, the the lower limit of the integral is -1, which is not a valid value to start at due to the first unit step function, which means that we want to bound our integral to start at $\min(1, n-1)$. A similar thing happens with the second and third integrals, where we want to set the lower limit of integration to $\min(2, n-1)$ and $\min(5, n-1)$ respectively. To do this, we will set the lower limit of integration for all three integrals to $n-1$ but add an appropriately shifted unit step function to each integral to ensure that the integral is only evaluated when the lower limit is valid. The end result will look like a piecewise function.

$$\int_{-\infty}^{\infty} u_{-1}(t)e^{-2t}u_{-1}(1-(n-t))dt = u_{-1}(n-1)\int_{[illegible]}^{\infty} e^{-2t}dt = \frac{e^{2-2n}}{[illegible]}u_{-1}(n-1)$$

$$\int_{-\infty}^{\infty} u_{-1}(t-2)e^{-2t}u_{-1}(1-(n-t))dt = u_{-1}(n-2)\int_{n-1}^{\infty} e^{-2t}dt = \frac{e^{2-2n}}{2}u_{-1}(n-2)$$
$$\int_{-\infty}^{\infty} u_{-1}(t-5)e^{-2t}u_{-1}(1-(n-t))dt = u_{-1}(n-5)\int_{n-1}^{\infty} e^{-2t}dt = \frac{e^{2-2n}}{2}u_{-1}(n-5)$$

Combining this all together:

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$$\begin{aligned} y(n) &= e^{2n}\left(\frac{e^{2-2n}}{2}u_{-1}(n-1) - 2\frac{e^{2-2n}}{2}u_{-1}(n-2) + \frac{e^{2-2n}}{2}u_{-1}(n-5)\right) \\ &= \frac{e^2}{2}u_{-1}(n-1) - \frac{e^2}{2}u_{-1}(n-2) + \frac{e^2}{2}u_{-1}(n-5) \\ &= \frac{e^2}{2}\left(u_{-1}(n-1) - 2u_{-1}(n-2) + u_{-1}(n-5)\right) \end{aligned}$$

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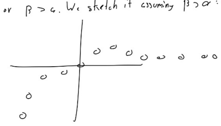
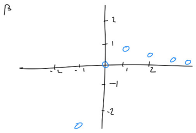
8a

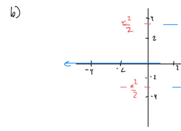
The shape of the plots will differ depending on if $\alpha > \beta$ or $\beta > \alpha$. We sketch it assuming $\beta > \alpha$:

$\alpha \neq \beta$



$\alpha = \beta$





Incomplete graph, please refer to the solution set.

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Q9

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Let $h(t)$ be a triangular pulse that can be described as $h(t) = (t+1)(u_{-1}(t+1) - u_{-1}(t)) + (1-t)(u_{-1}(t) - u_{-1}(t-1))$ and let $x(t)$ be an impulse train, such that
$x(t) = \Sigma_{k=-\infty}^{+\infty} \delta(t-kT)$.
Determine and sketch $y(t) = x(t) * h(t)$ for the following values of T:
a) $T = 4$ b) $T = 2$ c) $T = 3/2$ d) $T = 1$.

Question 9

Note that the input is a sum of discrete-time impulses, the convolution of the input with the impulse response of the system is then the response of the system (which is the input applied to the impulse response) to each impulse. We can then apply the sifting property to get that the output of the system is the sum of the triangular impulse response shifted by the time of each impulse.

$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \left(\sum_{k=-\infty}^{\infty} \delta(t-kT)\right) * h(t) \\
&= \sum_{k=-\infty}^{\infty} (\delta(t-kT) * h(t)) \\
&= \sum_{k=-\infty}^{\infty} h(t-kT)
\end{aligned}
$$

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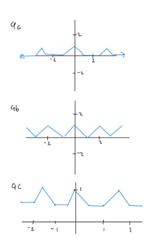

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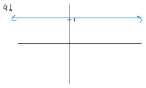

Q10

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The following are the impulse responses of continuous-time LTI systems. Determine if each system is causal and/or stable. Justify your answers.
a) $h(t) = e^{-4t}u_{-1}(t-2)$ b) $h(t) = e^{-6t}u_{-1}(3-t)$

Question 10

(a)

This system is causal. If the impulse response is triggered at $t = 0$, the $u_{-1}(t-2)$ term ensures that the system does not use any input from the future since $t-2$ will be negative at that point and the unit step function will be zero.

Taking the Laplace transform of the impulse response:

$$\begin{aligned} h(t) &= e^{-4t}u_{-1}(t-2) \\ &= e^{-4t+8-8}u_{-1}(t-2) \\ &= e^{-4(t-2)-8}u_{-1}(t-2) \\ &= e^{-8}e^{-4(t-2)}u_{-1}(t-2) \\ H(s) &= \frac{e^{-8}e^{-2s}}{s+4} \\ H(s) &= \frac{e^{-2(s+4)}}{s+4} \\ H(s) &= \frac{e^{-2(s+4)}}{s-(-4)} \end{aligned}$$

The real part of the pole of the rational transfer function of this system is negative and so it is stable.

(b)

This system is non-causal. If the impulse response is triggered at $t = 0$, the $u_{-1}(2-t)$ term will be active at all times $t \geq -2$, which would mean that the impulse response of the input could use input from the future at those times.

Similarly to the previous part, we can see that

$$\begin{aligned} h(t) &= e^{-6t}u_{-1}(-t+3) \\ h(t) &= -e^{-6t}u_{-1}(t+3) \\ h(t) &= -e^{-6t-18+18}u_{-1}(t+3) \\ h(t) &= -e^{-6(t+3)+18}u_{-1}(t+3) \\ h(t) &= -e^{18}e^{-6(t+3)}u_{-1}(t+3) \\ h(t) &= -e^{18}e^{3s}e^{-6t}u_{-1}(t) \\ h(t) &= -e^{18}e^{3s}\frac{1}{s-(-6)} \\ h(t) &= -\frac{e^{3(s+6)}}{s-(-6)} \end{aligned}$$

The real part of the pole of the rational transfer function of this system is negative and so it is stable.

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not stable, please refer to the solution set.

Q11

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Consider the first-order difference equation
$y[n] + 2y[n-1] = x[n]$.
Assuming the condition of initial rest (i.e., if $x[n] = 0$ for $n < n_0$, then y[n]=0 for $n < n_0$), find the impulse response of a system whose input and output are related by this difference equation. You may solve the problem by rearranging the difference equation so as to express $y[n]$ in terms of $y[n-1]$ and $x[n]$ and generating the values of $y[0]$, $y[+1]$, $y[+2]$, ... in that order.

Question 11

We are given that $x[n] = 0$ and $y[n] = 0$ for $n < n_0$.

$$y[n] + 2y[n-1] = x[n]$$
$$y[n] = x[n] - 2y[n-1]$$

To get the impulse response of the system, we can set $x[0] = \delta[0] = 1$ and solve for $y[n]$ from there.

$$y[0] = x[0] - 2y[n-1] = 1 - 0 = 1$$
$$y[1] = x[1] - 2y[1-1] = 0 - 2 * 1 = -2$$
$$y[2] = x[2] - 2y[2-1] = 0 - 2 * -2 = 4$$
$$y[3] = x[3] - 2y[3-1] = 0 - 2 * 4 = -8$$

We can see that the response of the system is in the form of powers of -2 as the system alternates between a postive and negative power of 2. We also have that the system is zero for $n < 0$. Finding the impulse response of the system, we get:

$$h[n] = (-2)^n u{-}1[n]$$