SE 380
Midterm review

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Lectures 1–2 — 1 (DB)

- Introduction to control systems
- Open loop VS closed loop, nomenclature, examples, control design cycle

Lecture 3 — 2, 3 (DB)

- Modeling systems using differential equations
- The concept of state
- State-space representation

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases}$$

Lecture 4 — 2 (DB)

- Linearization
  - Linearizing a nonlinear system

$$\begin{cases} \dot{x} = f(\bar{x},\bar{u}) + \left.\dfrac{\partial f}{\partial x}\right|_{\substack{x=\bar{x}\\u=\bar{u}}} (x-\bar{x}) + \left.\dfrac{\partial f}{\partial u}\right|_{\substack{x=\bar{x}\\u=\bar{u}}} (u-\bar{u}) + h.o.t. \\ y = h(\bar{x},\bar{u}) + \left.\dfrac{\partial h}{\partial x}\right|_{\substack{x=\bar{x}\\u=\bar{u}}} (x-\bar{x}) + \left.\dfrac{\partial h}{\partial u}\right|_{\substack{x=\bar{x}\\u=\bar{u}}} (u-\bar{u}) + h.o.t. \end{cases}$$

  - Definition of the matrices A, B, C, D
  - LTI systems
- Equilibrium
  - configuration: $(\bar{x},\bar{u})$ s.t. $f(\bar{x},\bar{u}) = 0$
  - point: $\bar{x}$ s.t. $f(\bar{x},\bar{u}) = 0$
- Linearization about an equilibrium configuration

Lecture 5 — 2 (DB)

- Laplace transform (LT)

- LT pairs, properties, and theorems

Lecture 6 — 2 (DB)

- Transfer functions (TF): $G(s) = \dfrac{Y(s)}{U(s)}$
- Proper/strictly proper/improper TF, order, poles and zeros
- Computation of the output response $y(t)$ of a system to an input $u(t)$ using LT and TF

Lecture 7 — 2 (DB)

- Examples of TFs
- TF of LTI systems

Lecture 8 — 8 (DB)

- Response of systems to sinusoidal inputs
- ♡ "Fundamental theorem of frequency response": $G(s)$ BIBO + no pole/zero cancelations, input signal $u(t) = U\sin(\omega t)$. The steady state output signal is

$$y_{ss}(t) = |G(j\omega)|U\sin(\omega t + \angle G(j\omega))$$

Lecture 9 — 8 (DB)

- TF representation

$$G(s) = \frac{\mu}{s^\rho} \frac{\prod_i (1 + T_i s) \prod_i \left(1 + \dfrac{2\xi_i}{\alpha_{n,i}} s + \dfrac{s^2}{\alpha_{n,i}^2}\right)}{\prod_i (1 + \tau_i s) \prod_i \left(1 + \dfrac{2\zeta_i}{\omega_{n,i}} s + \dfrac{s^2}{\omega_{n,i}^2}\right)}$$

- Bode plots: meaning, interpretation, and usage
- Sketch of asymptotic Bode plots for the following TF
  - Constant
  - Zeros/poles at the origin
  - Real zeros/poles
  - Complex conjugate zeros/poles

Lecture 10 — 8 (DB)

- Bode plots of low-pass filters

Lecture 11 — 3, 6 (DB)

- Matrix exponential
- Solution of LTI system
  - Zero-input response: $x_{zi}(t) = e^{A(t-t_0)}x(t_0)$

  - Zero-state response: $x_{zs}(t) = \int_{t_0}^{t} e^{A(t-\tau)} Bu(\tau) \mathrm{d}\tau$
  - Complete response: $x(t) = x_{zi}(t) + x_{zs}(t)$
  - Output response: $y(t) = Cx(t) + Du(t)$

- State transition matrix and its LT: $e^{At} = \mathcal{L}^{-1}\left\{(sI - A)^{-1}\right\}$

- Stability of LTI systems

  - Stable system: $\forall x(t_0) \in \mathbb{R}^n$, $x_{zi}(t)$ $\forall t \geq t_0$ is uniformly bounded
  - Asymptotically stable: stable and $x_{zi}(t) \xrightarrow{t\to\infty} 0$ (converges to 0)
  - Exponentially stable: asymptotically stable and $\exists c, \lambda > 0$ s.t. $\|x(t)\| \leq ce^{\lambda(t-t_0)}\|x(t_0)\|$ $\forall t \geq t_0$ (converges to 0 exponentially fast)
  - Unstable: not stable

Lecture 12 — 3, 5, 6 (DB)

- BIBO stable: Every bounded input produces a bounded output ($\exists c < \infty$ s.t. $\sup_{t\geq 0} \|y(t)\| \leq c \sup_{t\geq 0} \|u(t)\|$)

- Characterization of stability

  - Asymptotic stability $\Leftrightarrow$ All eigenvalues of A have negative real part
  - BIBO stability $\Leftrightarrow$ All poles of the TF have negative real part

- Asymptotic stability $\implies$ BIBO stability (because the poles of a TF are a subset of the eigenvalues of the matrix A)

- BIBO stability $\not\Longrightarrow$ Asymptotic stability (because of cancellations of poles with non-negative real part)

- Performance metrics

  - Steady-state gain
  - Rise time
  - Peak time
  - Overshoot
  - Settling time

Lecture 13 — 5 (DB)

- First-order system $G(s) = \dfrac{\mu}{1 + \tau s}$

  - Performance
    * Steady-state gain $= \mu$
    * Rise time $\approx 2\tau$
    * Settling time at 5% $\approx 3\tau$
    * Settling time at 2% $\approx 4\tau$
    * Settling time at 1% $\approx 5\tau$

- Second-order system $G(s) = \dfrac{\mu\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
  - Overdamped, critically damped, underdamped, undamped, unstable
  - Performance (complex poles)
    - Steady-state gain $= \mu$
    - % O.S. $= 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$
    - Settling time at $\epsilon\% \approx -\dfrac{1}{\zeta\omega_n}\ln 0.01\epsilon$

Lecture 14 — 5 (DB)

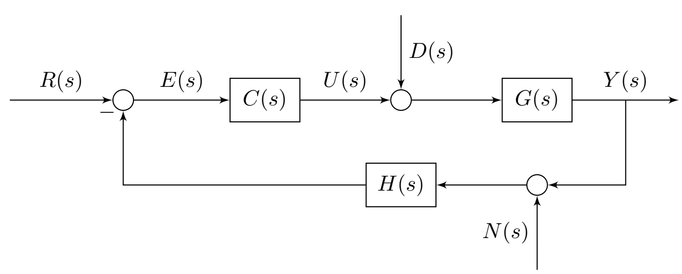
- Dominant poles
- Fast and slow dynamics

Lecture 15 — Additional reading

- Time-domain system identification
- Least-squares estimation of the system parameters Θ
  - Least-squares problem: $\text{minimize}_\Theta \|Y - D\Theta\|^2$
  - Solution: $\Theta^\star = D^\dagger Y = (D^T D)^{-1} D^T Y$

Lecture 16 — 2 (DB)

- Block diagrams manipulation
- Negative feedback control loop



- Transfer function relations between $Y(s)$, $U(s)$, $E(s)$—output, input, and error signals, respectively—and $R(s)$, $D(s)$, $N(s)$—reference, disturbance, and noise signals, respectively, of the negative feedback control loop

Lecture 17 — 4 (DB)

- Stability of series, parallel, and feedback interconnections
  - Series/parallel interconnection of stable systems is stable
  - Feedback interconnection of stable systems might not be stable

Lecture 18 — 6 (DB)

- Routh-Hurwitz stability criterion
  - # roots of a polynomial with real part ≥ 0 = # sign changes in the first column Routh table

Lecture 19 — 9 (DB)

- Nyquist plot: meaning and interpretation
- Nyquist stability criterion

Lecture 20 — 9 (DB)

- Gain margin k_m and phase margin φ_m

Lecture 21 — 10 (DB)

- Loop shaping: from specifications for the closed-loop TF $F(s)$ to constraints on the Bode plot of the open loop TF $L(s)$
  1. Stability
  2. Robust stability (k_m and φ_m)
  3. Static precision (steady-state error in step response)
  4. Dynamic precision (tracking non-constant reference signals)
  5. Disturbance rejection
  6. Noise attenuation
  7. Realizability of the controller $C(s)$

Lectures 22–23 — 7 (DB)

- Control design using loop shaping
- Without and with integral control

Lecture 24 — 10 (DB)

- Lead compensators
  - Transfer function
    $$C(s) = \mu \frac{1+Ts}{1+\alpha Ts}$$
    $\mu > 0$, $T > 0$, $0 < \alpha < 1$.
  - Typical design choices
    * Set $\alpha = 0.1$ ($55°$ phase lead in the midpoint between the zero and the pole)
    * Choose T s.t. $\frac{1}{\sqrt{\alpha}T} \approx \omega_c$
    * Don't increase ω_c too much through the gain
  - $\alpha = 0$: PD controller
- Lag compensators
  - Transfer function
    $$C(s) = \mu \frac{1+Ts}{1+\alpha Ts}$$
    $\mu > 0$, $T > 0$, $\alpha > 1$.

- Typical design choices
  - Set $\alpha = 10$
  - $T \geq \frac{10}{\omega_c}$ ($\leq 6^\circ$ phase lag at ω_c)
  - Don't increase ω_c too much through the gain
  - $\mu = \alpha$ not to change the gain at high frequency
  - $\mu = 1$ to decrease the ω_c with the goal of increasing φ_m
- $\alpha \to \infty$: PI controller

Lecture 25 — 7 (DB)

- Lead-lag compensators
- PID
  - Frequency domain interpretation: Realizable PID = lead-lag controller where the pole of the lag is at 0
  - Time domain interpretation: $u(t) = K_P e(t) + K_I \int_0^t e(\tau)\mathrm{d}\tau + K_D \frac{\mathrm{d}e(t)}{\mathrm{d}t}$
  - Practical considerations
    - Limitation of the derivative action in presence of discontinuous reference signals
    - Limitation of the integral action in presence of input saturation
    - Gain tuning

Lecture 26 — 7 (DB)

- Root locus: meaning and interpretation
- Rules to sketch the root locus for positive loop gain

Lecture 27 — 7 (DB)

- Control design using root locus

Lectures 28–29 — 11 (DB)

- Pole placement via state-feedback controller
  - Controllable canonical form of the state-space representation of $G(s)$
  - $u = -Kx$, $K^T \in \mathbb{R}^n$
  - Design of K s.t. $\dot{x} = (A - BK)x$ meets desired specifications

Lecture 30 — 11 (DB)

- Control design using state feedback