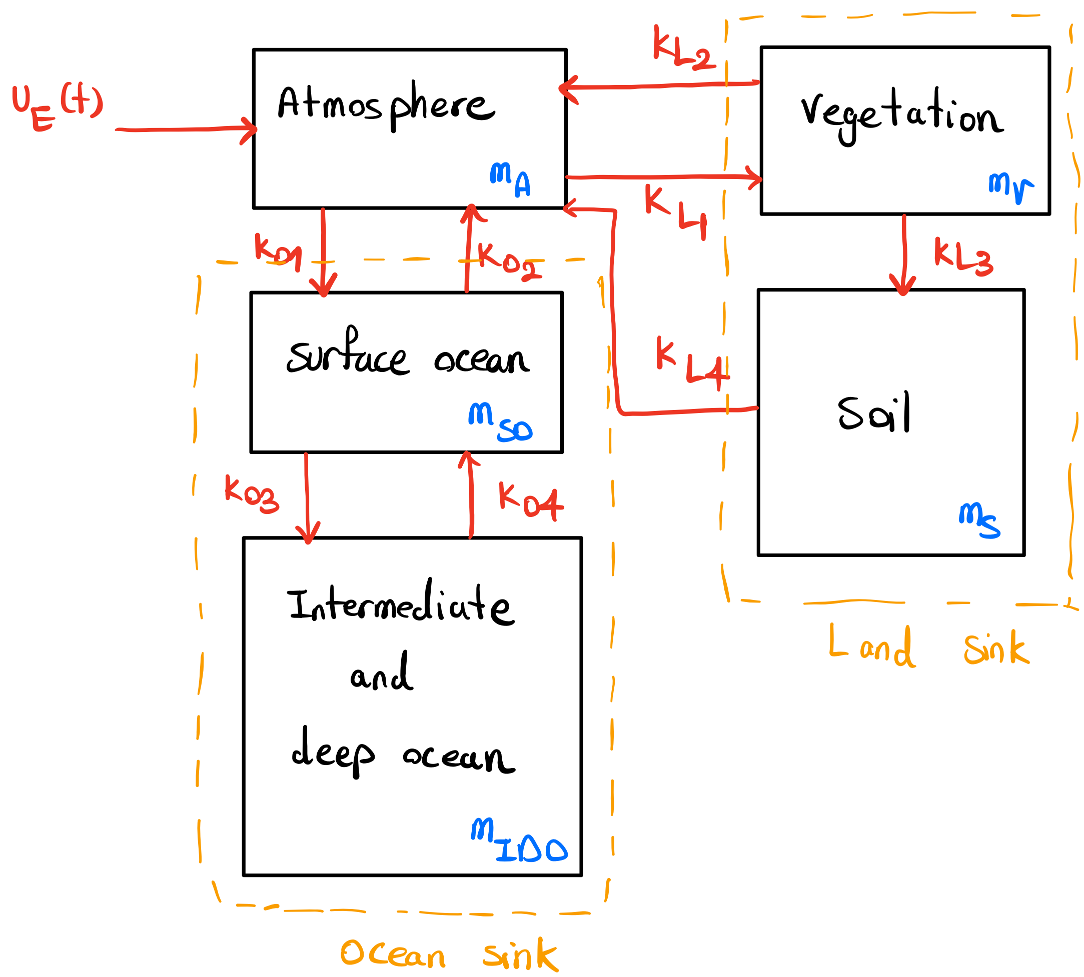
Tutorial 1_ SE 380.

The following figure shows a generic structure for global carbon cycle (GCC) :



In this figure, $m_A(t)$ is the amount of carbon in the atmosphere
$m_V(t)$ is the amount of carbon in the vegetation
$m_S(t)$ " " " soil.
$m_{SO}(t)$ " " " surface ocean.
$m_{IDO}(t)$ " " " intermediate and deep ocean.

The arrows represent the net carbon fluxes between boxes as governed by the exchange coefficient k.

$u_E(t)$ is the human generated CO_2 emissions.

The atmospheric mass balance in the atmosphere can be expressed as:

$$\frac{dm_A(t)}{dt} = u_E(t) - (k_{O1} + k_{L1})m_A(t) + k_{L2}\, m_V(t)$$

$$+ k_{O2}\, m_{SO}(t) + k_{L4}\, m_S(t)$$

a) write the remaining reservoir mass balances.

$$\frac{dm_V(t)}{dt} = k_{L1}\, m_A(t) - (k_{L2} + k_{L3})\, m_V(t)$$

$$\frac{dm_S(t)}{dt} = k_{L3}\, m_V(t) - k_{L4}\, m_S(t)$$

$$\frac{dm_{SO}(t)}{dt} = k_{O1}\, m_A(t) - (k_{O2} + k_{O3}) m_{SO}(t) + k_{O4}\, m_{IDO}(t)$$

$$\frac{dm_{IDO}(t)}{dt} = k_{O3}\, m_{SO}(t) - k_{O4}\, m_{IDO}(t)$$

b) Express the system in the state-space form.

$$\dot{x} = Ax + Bu.$$

$$x = \begin{bmatrix} m_A(t) \\ m_r(t) \\ m_S(t) \\ m_{SO}(t) \\ m_{IDO}(t) \end{bmatrix}_{4\times 1} \qquad \underset{1\times 1}{u} = u_E(t)$$

$$\underbrace{\begin{bmatrix} \frac{dm_A(t)}{dt} \\ \frac{dm_r(t)}{dt} \\ \frac{dm_S(t)}{dt} \\ \frac{dm_{SO}(t)}{dt} \\ \frac{dm_{IDO}(t)}{dt} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -(k_{01}+k_{L1}) & k_{L2} & k_{L4} & k_{02} & 0 \\ k_{L1} & -(k_{L2}+k_{L3}) & 0 & 0 & 0 \\ 0 & k_{L3} & -k_{L4} & 0 & 0 \\ k_{01} & 0 & 0 & -(k_{02}+k_{03}) & k_{04} \\ 0 & 0 & 0 & k_{03} & -k_{04} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} m_A(t) \\ m_r(t) \\ m_S(t) \\ m_{SO}(t) \\ m_{IDO}(t) \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{B} u$$