SE 380 Introduction to Feedback Control

Gennaro Notomista

HOMEWORK 1

Due date: September 20, 2023

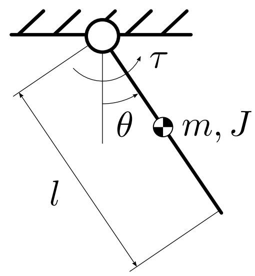

Figure 1: Pendulum

1 The equations of motion of the pendulum depicted in Fig. 1 are as follows:

$$J\ddot{\theta} + \frac{mgl}{2}\sin\theta = \tau,$$

where J, m, and l are the rotational inertia, the mass, and the length of the pendulum, respectively, g is the acceleration due to gravity, and τ is a torque applied at the hinge of the pendulum.

a Choosing the state $x = \begin{bmatrix}\theta & \dot{\theta}\end{bmatrix}^T$ and the input $u = \tau$, write a state space representation of the pendulum.

b Find all equilibrium configurations.

c Linearize the pendulum model around the configuration

$$x = \begin{bmatrix}0 & 0\end{bmatrix}^T, \quad u = 0,$$

and write down the expressions of the matrices A and B.

d Linearize the pendulum model around the configuration

$$x = \begin{bmatrix}\frac{\pi}{4} & 0\end{bmatrix}^T, \quad u = \frac{mgl}{2\sqrt{2}},$$

and write down the expressions of the matrices A and B.

e When would you use the linearized models found in **c** and **d**, respectively, to describe the motion of the pendulum?

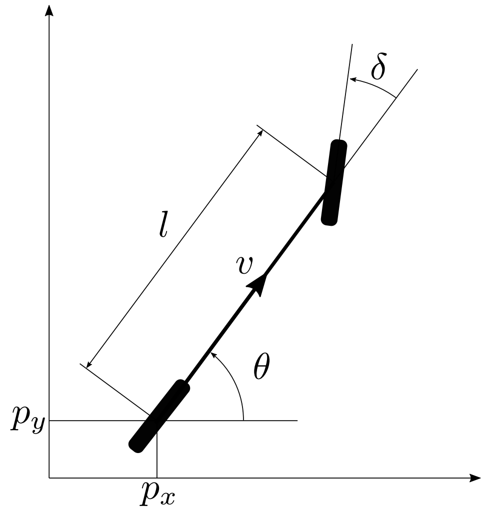

Figure 2: Kinematic bicycle model

2 The equations of motion of the kinematic bicycle model—describing the motion of an autovehicle with Ackermann steering geometry—depicted in Fig. 2 are as follows:

$$\dot{x} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \frac{v}{l}\tan\delta \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u. \tag{1}$$

The state vector is defined as $x = \begin{bmatrix} p_x & p_y & \theta & v & \delta \end{bmatrix}^T$, where $\begin{bmatrix} p_x & p_y \end{bmatrix}^T \in \mathbb{R}^2$ is the position of the midpoint of the rear axle of the vehicle in a reference system fixed on the ground where the vehicle moves, $v \in \mathbb{R}$ is the longitudinal velocity of the vehicle, θ is its heading (yaw angle), l is the wheelbase (distance between front and rear axles), and δ is the steering angle measured at the front wheel. The input vector is $u = \begin{bmatrix} a & \omega \end{bmatrix}^T$, where a is the longitudinal acceleration of the vehicle and ω is the steering angle speed.

a Based on Algorithm 1, write a Python script to simulate the dynamics of the kinematic bicycle model.

Algorithm 1 Dynamic simulation of the kinematic bicycle model

Initialize simulation time step Δt, simulation time T, wheelbase l, state x, and set $t = 0$
while $t \leq T$ **do**
  Compute $\dot{x}$ ▷ according to (1)
  $x \leftarrow x + \dot{x}\Delta t$ ▷ evolution of the state forward in time using forward Euler integration
  $t \leftarrow t + \Delta t$
end while

b Given the simulation time step $\Delta t = 0.01$s, simulation time $T = 10$s, wheelbase $l = 2$, initial condition $x = 0$, input signal $u(t) = \begin{bmatrix} 0.1 & \cos t \end{bmatrix}^T$, plot the trajectory of the vehicle—i.e. of the point $\begin{bmatrix} p_x & p_y \end{bmatrix}^T$ on the plane—over the time interval $[0, T]$.