SE 380 — Final Exam

Examination Start Window	December 11, 2021 – December 12, 2021
Duration of Exam	2.5 Hours
Number of Test Pages	10
Total Possible Marks	90
Additional Materials Allowed	See Instructions.

Instructions

1. This is an open-book examination which allows you to use,
   - Written Sources: textbooks, course notes, lecture notes, problem set solutions.
   - Some Online Sources: static web content, videos.
   - Tools: symbolic and/or numerical computation software, calculators, graphing software.

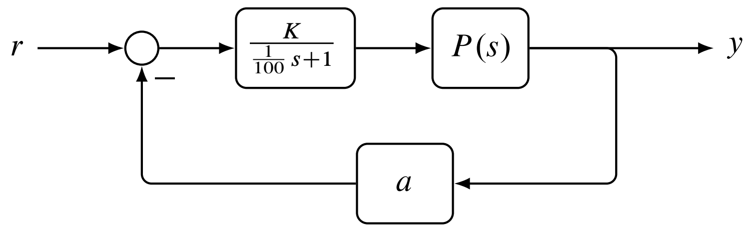
   However, you are **not allowed to discuss** what is on this examination itself with *anyone* besides the instructional team (via a private post on Piazza).
2. Upload **clearly, legibly written solutions** for each question to the Crowdmark assessment associated with this examination. You *may* also typeset your solution, if you like.
3. If you used symbolic or numerical computation packages, please **upload your code or commands**, printed as a `PDF`, to the pertinent question.
4. Please confirm, before the end of your exam, that you have **uploaded all your solutions correctly and in-order**.
5. Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions.
6. Use exact values where possible.

Marking Scheme

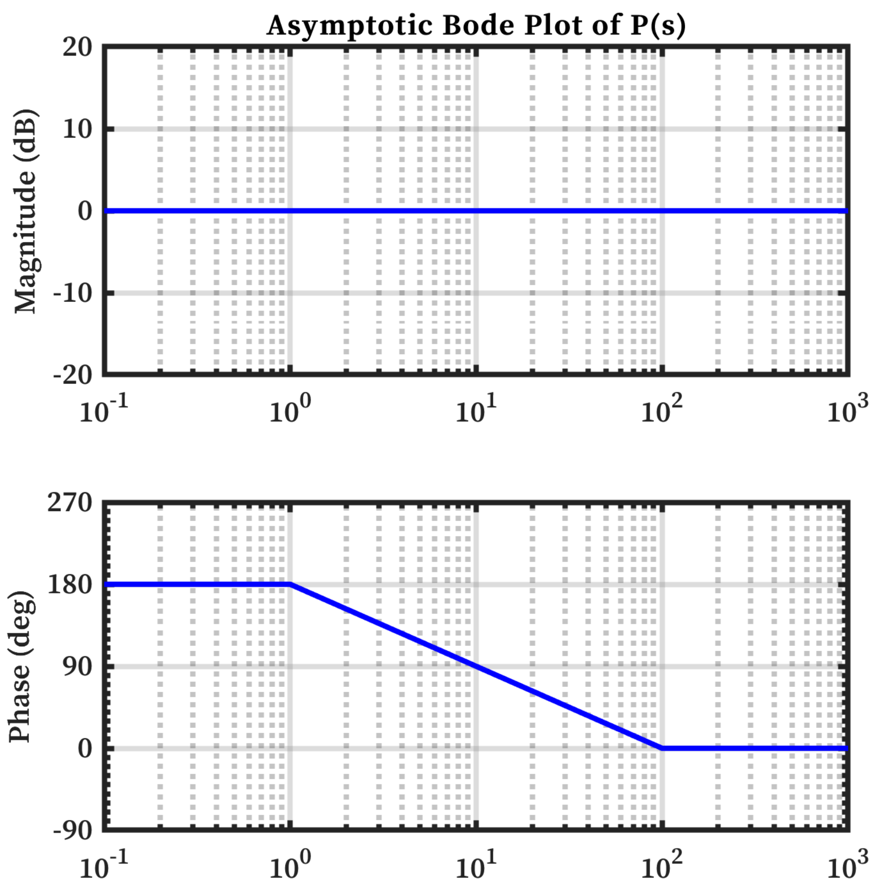
	Graded Out Of
Question 1	10
Question 2	10
Question 3	10
Question 4	10
Question 5	10
Question 6	10
Question 7	10
Question 8	10
Question 9	10
Total	90

Question 1

Consider the negative feedback loop



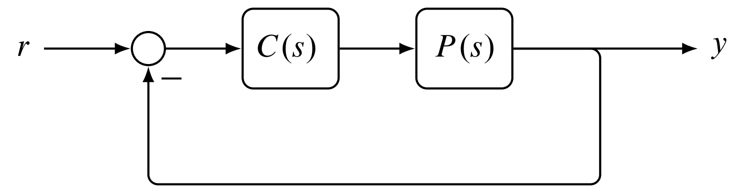
where $a > 0$ is an unknown parameter and the Bode plot of $P(s)$ is



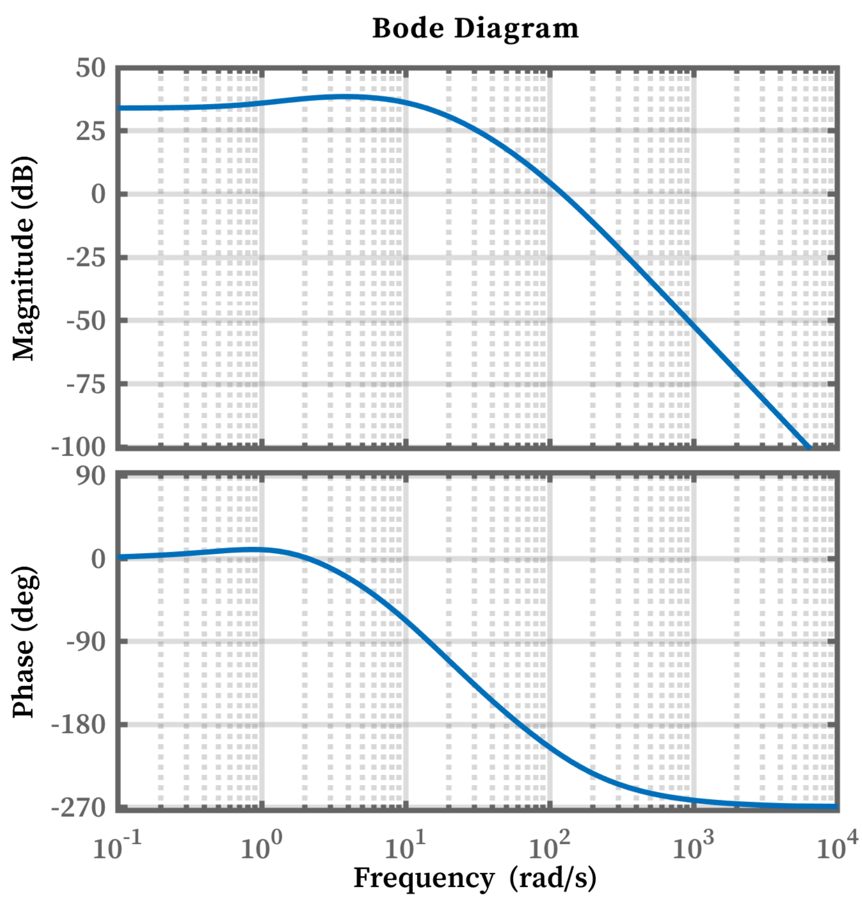
Sketch (by hand) the Nyquist plot for the loop transfer function of the feedback loop and find necessary and sufficient conditions on the gain $K \in \mathbb{R}$ so that the closed-loop system is IO stable. Assume $P(s)$ is a BIBO stable transfer function. [/10]

Question 2

Consider the standard unity feedback loop



Suppose the Bode plot of the BIBO stable plant, $P(s)$, is,



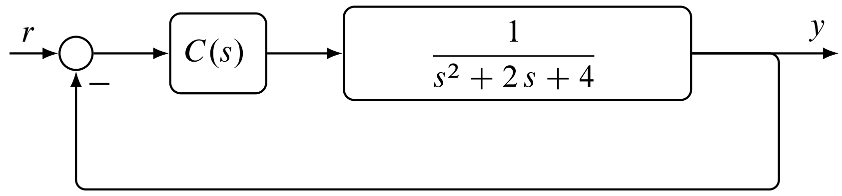
(A) Design a controller $C(s)$ that ensures the gain margin of the loop transfer function is at least 10 dB. [/5]

(B) Determine the steady-state error e_{ss} to an input $r(t) = (11)\,\mathbf{1}(t)$. [/2]

(C) Does there exist a controller that can increase the gain margin without changing the steady-state gain? Explain your answer. [/3]

Question 3

Consider the standard unity feedback loop



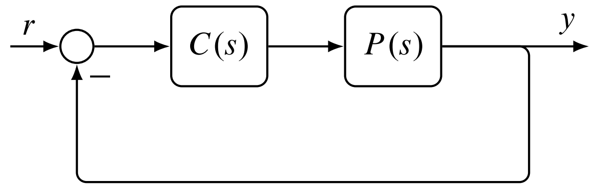
Design a *non-ideal* (implementable) P.I.D. compensator $C(s)$ so that the closed-loop system,

- perfectly tracks step references,
- has a settling time faster than 4 s, and
- has an overshoot larger than $100e^{-2}\%$.

[/10]

Question 4

Consider the standard unity feedback loop,

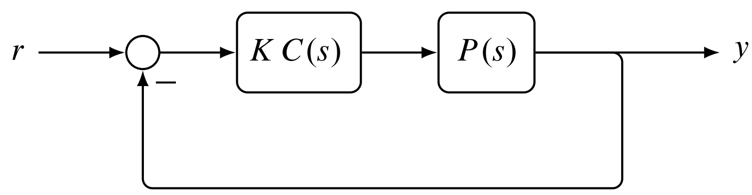


where the plant $P(s)$ is,

$$P(s) = \frac{1}{s+3}. \tag{1}$$

(A) Design a controller $C(s)$ so that the closed-loop system perfectly tracks sinusoidal references of frequency 4 rad s^{-1}. [/3]

(B) Consider the modified feedback loop,

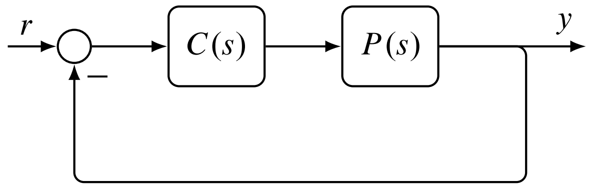


where your $C(s)$ is chosen as in Part A, $P(s)$ is as defined in (1), and $K > 0$ is a newly introduced variable parameter. Sketch (by hand) the root locus treating $K > 0$ as the variable parameter. [/5]

(C) Describe how you would pick the gain $K > 0$ using the root locus to improve the response characteristics of your closed-loop system. [/2]

Question 5

Consider the standard unity feedback loop,



where the plant is,

$$P(s) = \frac{100}{s^2}.$$

(A) Sketch (by hand) the piecewise-linear approximation of the Bode plot of $P(s)$. Compute the gain crossover and the phase margin. `[   /4]`

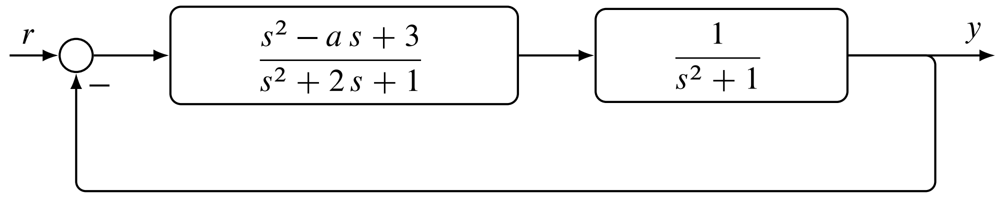
(B) Using your Bode plot in Part A, design a controller $C(s)$ so that,

- the closed-loop system tracks reference *ramps* perfectly, i.e. tracks signals of the form $r(t) = A\,t\,\mathbf{1}(t)$,
- the loop transfer function has a phase margin greater than 10°, and
- the loop transfer function has a gain crossover of 1 rad s^{-1}.

`[   /6]`

Question 6

Consider the standard unity feedback control system,



Find necessary and sufficient conditions on the parameter $a \in \mathbb{R}$ to ensure the system is IO stable. `[   /10]`

Question 7

Consider the plant modelling some physical system with unknown parameters $m, b > 0$,

$$P(s) = \frac{1}{s\,(m\,s + b)}.$$

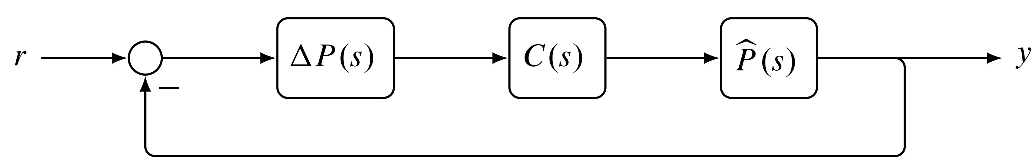
In practice, we acquire an estimate $\hat{m}$ and $\hat{b}$ of m and b respectively, and design a compensator $C(s)$ so that the closed-loop system with the plant

$$\widehat{P}(s) = \frac{1}{s\,(\hat{m}\,s + \hat{b})}$$

is IO stable. Suppose this was done and the closed-loop system with this estimated plant enjoyed IO stability. But, our compensator $C(s)$, in reality, is connected to the real plant $P(s) = \widehat{P}(s)\,\Delta P(s)$ where

$$\Delta P(s) = \frac{\hat{b}}{b}\,\frac{\frac{\hat{m}}{\hat{b}}\,s + 1}{\frac{m}{b}\,s + 1}.$$

One can take the view that this is equivalent to adding an additional compensator in the closed-loop system,



(A) Express $\Delta P(s)$ in the form

$$\Delta P(s) = K\frac{\alpha\,T\,s + 1}{T\,s + 1}.$$

[/2]

(B) Define the relative error of the estimated parameters by

$$\delta m := \frac{\hat{m} - m}{m},$$

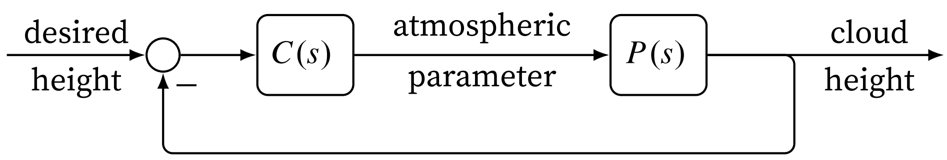
$$\delta b := \frac{\hat{b} - b}{b},$$

and write α only in terms of the relative errors. [/2]

(C) Assume $\delta b = 0$ and $\delta m > 0$, i.e. we over-estimated the inertia but we know $b = \hat{b}$ exactly. Using a sketch of the Bode plot of $\Delta P(s)$, describe the effect of the large, positive, relative error $\delta m > 0$ on the closed-loop stability of the system. [/6]

Question 8

In "Feedback Control of Cumuliform Cloud Formation based on Computational Fluid Dynamics" by Dobashi et. al[1], the authors present a technique that leverages a feedback control loop to regulate the numerical solution to a cloud-generating, fluid dynamics solver towards a "desired cloud shape." Such a software can be used by an artist to digitally design realistic clouds that conform to a desired formation that is artistically relevant to the scene they are designing for a movie or video game. At a basic level, each *column of air* in the simulation grid executes the feedback loop



where $P(s)$ is the numerical simulation running in real-time with our control law $C(s)$. The plant $P(s)$ takes in an atmospheric parameter (not relevant to this discussion) and spits out the current cloud height at the particular column of air in question. Carefully inspect the figure below along with the caption, taken from the paper,

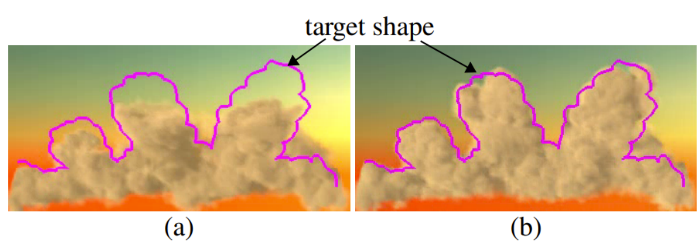


Figure 5: *Comparison of different control. (a) uses proportional (P) control only while (b) uses proportional-integral (PI) control. PI controller can fill the gaps between the clouds and the target shape.*

(A) Assuming only that $P(s)$ is BIBO stable and that the loop is IO stable, explain, using theory in this course, why the P.I. controller worked but the P. controller did not. `[  /5]`

(B) Suppose we now wanted to animate the cloud height in a slow, sinusoidal manner with a frequency and amplitude of the artist's choice. How could we modify this loop to achieve this end? Is it possible to perfectly track the sinusoid? Justify your answer. List all the assumptions we must make for your proposed method to work. `[  /5]`

[1] Dobashi, Y., Kusumoto, K., Nishita, T., Yamamoto, T. 2008. Feedback Control of Cumuliform Cloud Formation based on Computational Fluid Dynamics. ACM Trans. Graph. 27, 3, Article 94 (August 2008), 8 pages. DOI = 10.1145/1360612.1360693 `http://doi.acm.org/10.1145/1360612.1360693`.

Question 9

Your work on Labs 3–5 accelerated your career as a control engineer at VenX. You are now a lead engineer for a new team, tasked with prototyping a new courier strategy that has the potential to disrupt the postal service and courier service space and reap fortunes for your company. You are tasked with a mission to design the software that controls a large quadcopter that can autonomously:

- hover over a loading bay at a fixed point so that payload (the package) may be attached,
- travel point-to-point in space without swinging the payload in a substantial way,
- hover over the destination so the payload may be detached.

Your software must also adequately handle disturbances such as light, variable winds or strong, consistent winds. Describe in your own words the steps you must take to design such a system and, for each step you provide, relate at least one useful strategy learned in this course that could be employed. You may refer to any content in this course (lectures, notes, labs), or related to this course (prerequisite courses). `[ /10]`