ECE 358: Problem Set 3 Solution

Problem 1.

Suppose that you are transmitting blocks of data over a link having a known bit error rate BER. What is the probability that a received block of size L has error(s) but goes undetected? What is this probability if you can detect all single errors? Explain your assumptions. Draw the curves giving these 2 probabilities as a function of L for a given BER and as a function of BER for a given L. Comment the curves.

Let P_n denote the probability that a frame arrives with n bits in error.

a) With no error detection.

$$P_0 = \text{probability that a frame arrives with no bits in error} = (1-BER)^L$$

A frame arrives with undetected errors as soon as there is an error (since there is no detection method). Hence the probability of undetected error is

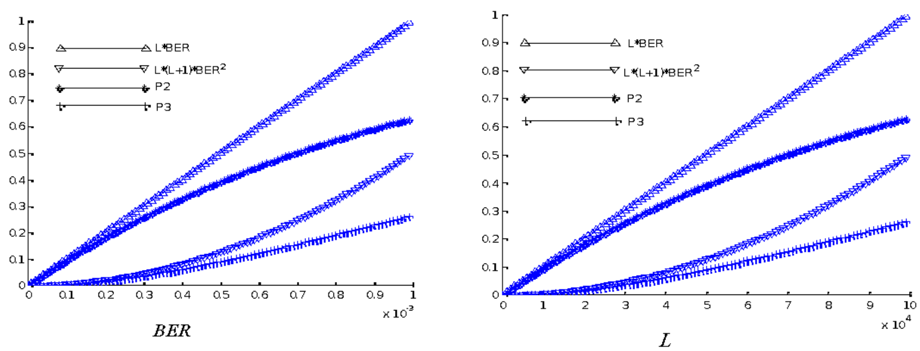
$$P_{Error} = P_2 = (1-P_0) = (1-(1-BER)^L) \approx LBER \textbf{ if } LBER \ll 1.$$

b) If we can only detect single error and if we assume that we need to add h bits to do so. Then a frame (the size is now L+h) with errors will go undetected if it contains more than one error. Hence the probability of undetected errors is one minus the probability of no error, minus the probability of one error:

$$P_{Error} = P_3 = 1 - P_0 - P_1 = 1-(1-BER)^{(L+h)} - (L+h)BER(1-BER)^{(L+h-1)}$$

If $LBER \ll 1$, $P_{Error} \approx (L+h)(L+h-1)BER^2/2$ (since $(1-x)^n = 1-nx+n(n-1)x^2/2$)

The curves are plotted with the above equations. However, note that the approximations are only valid for $LBER \ll 1$ as shown by the curves. The first curves are drawn for $L=1000$ with BER going from 10^{-3} *to* 10^{-9}. The second curves are drawn for $BER = 10^{-5}$, with L going from *100 to 100000.* The main gain of a detecting single error is obtained for a low to moderate BER and a frame size which is not too large.



Note: Stop-and-go protocol = Alternate bit protocol = Stop-and-wait protocol

<u>Problem 3.</u>

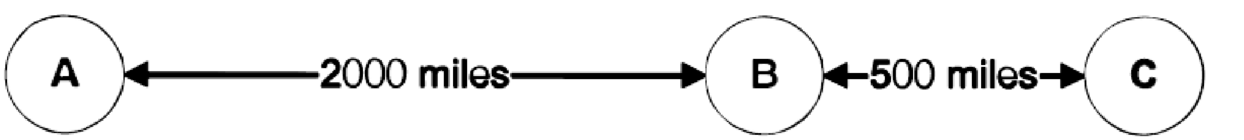


Figure 1

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a) In Figure 1, frames generated by node A are sent to node C through node B. Determine the minimum transmission rate required between B and C so that the buffers of node B are not flooded, based on the following data:

- The data rate between A and B is 300 kbps
- The propagation delay is 12µsec/mile for both lines
- There are full duplex lines between the nodes.
- All data frames are 1500 bits long; ACK frames are separate frames of negligible length
- Between A and B, a sliding-window protocol with window size of 5 is used.
- Between B and C, the stop-and-wait protocol is used.
- There are no errors

<u>For link AB</u>

C = 300kbps,

DATA frame size (L) = 1500 bits

Time to transmit a data frame (d) = L/C = 0.0050 sec

Time for the receiver to transmit an ACK frame (a) = 0 (negligible)

Propagation delay (P) = 12µsec/mile * 2000 miles = 0.0240 sec
T = d + 2P = 0.0530 sec
Window size (W) = 5
Efficiency = $\min[(\frac{W \cdot d}{T}),1]$ = 0.4717
Throughput = Efficiency * C = 141.5094 kbps = 94.3396 pkts/sec

<u>For link BC</u>
C = x bps, (to determine)
DATA frame size (L) = 1500 bits
Time to transmit a data frame (d) = L/x
Time for the receiver to transmit an ACK frame (a) = 0 (negligible)
Propagation delay (P) = 12µsec/mile * 500 miles = 0.0060 sec
T = d + 2P =1500/x + 0.0120 sec
Window size (W) = 1 (Stop and Wait)
Throughput = 1/T = 1/(d+2P) pkts/sec
The maximum throughput (when capacity is infinite) = 1/(2P) = 83.3333 pkts/sec
In other words, the link BC (operating with stop-and-wait protocol) will never achieve the throughput achieved by link AB, no matter how large is the data rate of link BC.

b) Now, imagine a scenario similar to the one in question (a) except for the fact that link AB operates with a stop and wait protocol and link BC operates with a sliding window protocol of window size 5. Find the transmission rate required for link BC so that the buffers of node B are not flooded.

<u>For link AB</u>
C = 300E3 bps,
DATA frame size (L) = 1500 bits
Time to transmit a data frame (d) = L/C = 0.0050
Time for the receiver to transmit an ACK frame (a) = 0 (negligible)
Propagation delay (P) = 12µsec/mile * 2000 miles = 0.0240 sec
T = d + 2P = 0.0530 sec
Window size (W) = 1 (Stop and wait)
Throughput = $(\frac{1}{T})$ = 18.8679 pkts/sec = 28.302kbps

<u>For link BC</u>
C = x bps, (to determine)
DATA frame size (L) = 1500 bits
Time to transmit a data frame (d) = L/x
Time for the receiver to transmit an ACK frame (a) = 0 (negligible)
Propagation delay (P) = 12µsec/mile * 500 miles = 0.0060 sec
T = d + 2P =1500/x + 0.0120 sec
Window size (W) = 5
Efficiency = min(Wd/T, 1) = min (5*1500/(x * (1500/x + 0.0120)) , 1)
= min(7500/(0.0120x +1500), 1)

First, assume $7500/(0.0120x + 1500) < 1$ is satisfied.
Then,
Throughput = Efficiency * x = $7500x/(0.0120x + 1500)$
We need to find x such that this throughput is equal to the throughput of link AB.
i.e., $7500x/(0.0120x + 1500) = 28.302E3$
Which can be solved with $x = 5.9289$ kbps
Now, we check if our assumption of $7500/(0.0120x + 1500) < 1$ is satisfied with this solution.
It turns out that this does not satisfy the assumption.
Thus, the efficiency of link BC has to be 1, which implies the required data rate of link BC is 28.302kpbs or greater to avoid buffer overflow.

Problem 5.

A link is such that the ideal window size for a maximum pipeline effect is 30 packets of 1500 bytes each (not worrying at all about the overhead).

a) If the propagation speed is 10^8 m/s, what is the length of the link if its rate is 1 Mb/s? Explain your assumptions.
b) If the length of the link is 8 km, what is the link rate? Explain your assumptions.

Assumptions: acks and processing time are negligible

$W_i = \dfrac{2\tau + L/C}{L/C} = \dfrac{2\tau C}{L} + 1$ with τ propagation delay, C the rate of the link and L the packet size.
Now if u is the length of the link and v the propagation speed, we have $v\,\tau = u$ and hence

$W_i = \dfrac{2uC}{vL} + 1$

a) Then $u = \dfrac{(W_i - 1)vL}{2C} = \dfrac{29 \times 10^8 \times 1500 \times 8}{2 \times 10^6} = 17400km$

(if you assume that ACK=L then $W_i = \dfrac{2\tau + 2L/C}{L/C} = \dfrac{2\tau C}{L} + 2$ then $W_i = \dfrac{2uC}{vL} + 2$, then $u = \dfrac{(W_i - 2)vL}{2C} = \dfrac{28 \times 10^8 \times 1500 \times 8}{2 \times 10^6} = 16800km$)

b) Then $C = \dfrac{(W_i - 1)vL}{2u} = \dfrac{29 \times 10^8 \times 1500 \times 8}{16 \times 10^3} = 2.175Gb/s$

(if you assume that ACK=L then $C = \dfrac{(W_i - 2)vL}{2u} = \dfrac{28 \times 10^8 \times 1500 \times 8}{16 \times 10^3} = 2.1Gb/s$)

Problem 7

In this problem, there is no danger in overflowing the receiver since the receiver's receive buffer can hold the entire file. Also, because there is no loss and acknowledgements are returned before timers expire, TCP congestion control does not throttle the sender. However, the process in host A will not continuously pass data to the socket because the send buffer will quickly fill up. Once the send buffer becomes full, the process will pass data at an average rate or R << S.

Problem 8

a) It takes 1 RTT to increase CongWin to 2 MSS; 2 RTTs to increase to 3 MSS; 3 RTTs to increase to 4 MSS; 4 RTTs to increase to 5 MSS; and 5 RTTs to increase to 6 MSS.

b) In the first RTT 1 MSS was sent; in the second RTT 2 MSS was sent; in the third RTT 3 MSS was sent; in the forth RTT 4 MSS was sent; in the fifth RTT, 5 MSS was sent. Thus, up to time 5 RTT, 1+2+3+4+5 = 15 MSS were sent (and acknowledged). Thus, one can say that the average throughput up to time 5 RTT was (15 MSS)/(5 RTT) = 3 MSS/RTT.