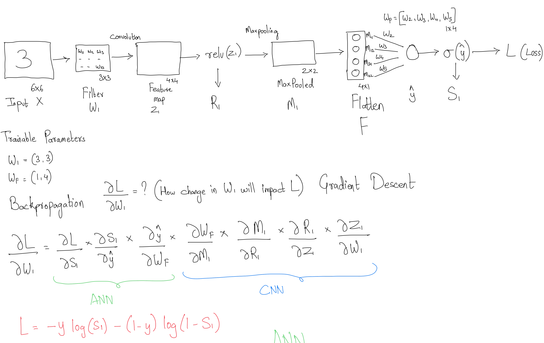

Trainable Parameters

$W_1 = (3,3)$

$W_F = (1,4)$

Backpropagation $\quad \frac{\partial L}{\partial W_1} = ?$ (How change in W_1 will impact L) Gradient Descent

$$\frac{\partial L}{\partial W_1} = \underbrace{\frac{\partial L}{\partial S_1} \times \frac{\partial S_1}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial W_F}}_{\text{ANN}} \times \underbrace{\frac{\partial W_F}{\partial M_1} \times \frac{\partial M_1}{\partial R_1} \times \frac{\partial R_1}{\partial Z} \times \frac{\partial Z_1}{\partial W_1}}_{\text{CNN}}$$

$$L = -y \log(S_1) - (1-y)\log(1-S_1)$$

ANN

$$\frac{\partial L}{\partial S_1} = -\frac{y}{S_1} + \frac{(1-y)}{(1-S_1)} = \frac{-y(1-S_1)+(1-y)S_1}{S_1(1-S_1)} = \frac{-y+\cancel{yS_1}+S_1-\cancel{yS_1}}{S_1(1-S_1)} = \frac{(S_1-y)}{S_1(1-S_1)} = \frac{\partial L}{\partial S_1} = d_1$$

$$\frac{\partial S_1}{\partial \hat{y}} = \frac{1}{1+e^{-\hat{y}}} = \sigma(\hat{y}) = \sigma(\hat{y})(1-\sigma(\hat{y})) = S_1(1-S_1) = \frac{\partial L}{\partial S_1} = d_2$$

$$\frac{\partial \hat{y}}{\partial W_F} = ? \qquad \hat{y} = FW_F \text{ (We can add bias)}$$

$$\frac{\partial \hat{y}}{\partial W_F} = F = \frac{\partial L}{\partial \hat{y}} = \frac{\partial L}{\partial S_1} \times \frac{\partial S_1}{\partial \hat{y}} \times \frac{\partial \hat{y}}{\partial W_F}$$

$$= \frac{(S_1-y)}{\cancel{S_1(1-S_1)}} \times \cancel{S_1(1-S_1)} \times F$$

$$= \underset{(1,1)}{(S_1-y)}\,\underset{(1,1)}{F} = \underset{(4,1)}{d_3}$$

$$W_F = (1,4)^T_{\text{shape}}$$

CNN

$$\frac{\partial W_F}{\partial M_1} = (S_1-y)F = \text{Reshape}(M_1.\text{shape}) = d_4 = \frac{\partial L}{\partial M_1}$$

$(1,4) \longrightarrow (2,2)$ Non-trainable

$$\frac{\partial M_1}{\partial R_1} = ? \qquad M_1 = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Non-trainable

Convert 2x2 to 4x4 R_1

$$R_1 = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & m_{11} & 0 & 0 \\ 0 & 0 & m_{12} & 0 \\ 0 & 0 & 0 & m_{12} \\ 0 & m_{21} & 0 & 0 \end{bmatrix}$$

M_1: $m_{11}, m_{12}, m_{21}, m_{22}$

$$\Downarrow$$

$$\frac{\partial L}{\partial R_1} = \begin{cases} \frac{\partial L}{\partial M_{1(x,y)}}, & \text{if } M \text{ is the max} \\ 0, & \text{otherwise} \end{cases} = d_5$$

$$\frac{\partial R_1}{\partial Z_1} = \text{relu}(Z_1) = \begin{cases} 1 & \text{if } Z_1 > 0 \\ 0 & \text{if } Z_1 < 0 \end{cases} = \frac{\partial L}{\partial Z_1} = d_6$$

Non-trainable $\qquad$ 4x4

$$\frac{\partial Z_1}{\partial W_1} = ? \text{ (Convolution Filter } W_1\text{)}$$

Trainable

$$\underset{(3,3)}{X} \longrightarrow \underset{(2,2)}{W_1} \longrightarrow \underset{(2,2)}{Z_1}$$

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \qquad W_1 = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \qquad Z_1 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\frac{\partial L}{\partial Z_1} = \begin{bmatrix} \frac{\partial L}{\partial Z_{11}} & \frac{\partial L}{\partial Z_{12}} \\ \frac{\partial L}{\partial Z_{21}} & \frac{\partial L}{\partial Z_{22}} \end{bmatrix}$$

Weight Sharing

$$Z_{11} = X_{11}W_{11} + X_{12}W_{12} + X_{21}W_{21} + X_{22}W_{22}$$

$$Z_{12} = X_{12}W_{11} + X_{13}W_{12} + X_{22}W_{21} + X_{23}W_{22}$$

$$Z_{21} = X_{21}W_{11} + X_{22}W_{12} + X_{31}W_{21} + X_{32}W_{22}$$

$$Z_{22} = X_{22}W_{11} + X_{23}W_{12} + X_{32}W_{21} + X_{33}W_{22}$$

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial Z_1} \times \frac{\partial Z_1}{\partial W_1}$$

$$\frac{\partial L}{\partial W_1} = \begin{bmatrix} \frac{\partial L}{\partial W_{11}} & \frac{\partial L}{\partial W_{12}} \\ \frac{\partial L}{\partial W_{21}} & \frac{\partial L}{\partial W_{22}} \end{bmatrix}$$

$$\frac{\partial L}{\partial W_{11}} = \frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial W_{11}} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial W_{11}} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial W_{11}} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{11}}$$

$$\frac{\partial L}{\partial W_{12}} = \frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial W_{12}} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial W_{12}} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial W_{12}} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{12}}$$

$$\frac{\partial L}{\partial W_{21}} = \frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial W_{21}} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial W_{21}} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial W_{21}} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{21}}$$

$$\frac{\partial L}{\partial W_{22}} = \frac{\partial L}{\partial Z_{11}} \times \frac{\partial Z_{11}}{\partial W_{22}} + \frac{\partial L}{\partial Z_{12}} \times \frac{\partial Z_{12}}{\partial W_{22}} + \frac{\partial L}{\partial Z_{21}} \times \frac{\partial Z_{21}}{\partial W_{22}} + \frac{\partial L}{\partial Z_{22}} \times \frac{\partial Z_{22}}{\partial W_{22}}$$

$$\frac{\partial Z_{11}}{\partial W_{11}} = X_{11} \qquad \frac{\partial Z_{12}}{\partial W_{12}} = X_{12} \qquad \frac{\partial Z_{21}}{\partial W_{21}} = X_{21} \qquad \frac{\partial Z_{22}}{\partial W_{22}} = X_{22}$$

$$\frac{\partial L}{\partial W_{11}} = \frac{\partial L}{\partial Z_{11}} X_{11} + \frac{\partial L}{\partial Z_{12}} X_{12} + \frac{\partial L}{\partial Z_{21}} X_{21} + \frac{\partial L}{\partial Z_{22}} X_{22}$$

$$\frac{\partial L}{\partial W_{12}} = \frac{\partial L}{\partial Z_{11}} X_{12} + \frac{\partial L}{\partial Z_{12}} X_{13} + \frac{\partial L}{\partial Z_{21}} X_{22} + \frac{\partial L}{\partial Z_{22}} X_{23}$$

$$\frac{\partial L}{\partial W_{21}} = \frac{\partial L}{\partial Z_{11}} X_{21} + \frac{\partial L}{\partial Z_{12}} X_{22} + \frac{\partial L}{\partial Z_{21}} X_{31} + \frac{\partial L}{\partial Z_{22}} X_{32}$$

$$\frac{\partial L}{\partial W_{22}} = \frac{\partial L}{\partial Z_{11}} X_{22} + \frac{\partial L}{\partial Z_{12}} X_{23} + \frac{\partial L}{\partial Z_{21}} X_{32} + \frac{\partial L}{\partial Z_{22}} X_{33}$$

$$\frac{\partial L}{\partial W_1} = \text{conv}\left(X, \frac{\partial L}{\partial Z_1}\right)$$

Backpropagation in CNN