Lab 4
October 10, 2017
Z to Fourier: The Tale of Two Transforms

INSTRUCTIONS:
All lab submissions include a written report and source code in the form of an m-file. The report contains all plots, images, and figures specified within the lab. All figures should be labeled appropriately. Answers to questions given in the lab document should be answered in the written report. ***The written report must be in PDF format***. Submissions are done electronically through Compass 2G. This lab will be due October 24,2017 at 4:59 PM Central Time.

1 An Introduction

In this lab, we will draw connections between the system response in lab 3 and the notion of frequency response. This lab will also make clear the connection between the z-transform and the Discrete-Time Fourier Transform. Lastly, the lab will end with an application problem dealing with Nyquist sampling theorem. In the next lab, we'll extend these concepts to some cool audio applications.

2 Frequency Response

In this section, we will use the function freqz() to find the frequency response of a system. Please follow the tutorial starterkit1.m and starterkit2.m in the unzipped folder.

Report Item 1: Consider the system $y[n] = ay[n-1] + bx[n-1]$

(1) Compute the output sequence $y_1[n]$ and $y_2[n]$ for the input $x_1[n] = 3cos(\pi n/2)$ and $x_2[n] = 3sin(\pi n/4)$ for $b = .5$ and $a = .5$. Do this in MATLAB. Hint: Use $H_d(\omega)$!

(2) Provide plots of $y_1[n]$ and $y_2[n]$ for the two input signals $x_1[n]$ and $x_2[n]$. A two panel subplot should suffice.

(3) In (1), you probably calculated $H(z) = \frac{B(z)}{A(z)}$. Go ahead and use this information to obtain the output $y[n]$ using the filter function. Plot these outputs using a two panel plot. Do the outputs look different from 2?

(4) Use freqz() to plot the frequency response of the system above with 8 equally spaced points between 0 and π

(5) Rinse and repeat for $a = .8$ and $b = .2$.

Report Item 2:
Suppose now we are given the following sequence $h[n] = 1, -2, 3, -4, 0, 4, -3, 2, -1$. Now, let's perform the following tasks.

(1) By hand,compute the Z-transform of $h[n]$, note that the first entry starts at the 0^{th} index.

(2) In MATLAB,evaluate this result in (1), $H(z)$ with $z = e^{j\omega}$. Let ω be an equally spaced vector of 1000 points between $-\pi$ and π.

(3) Plot the result in (2) (Magnitude and Phase). Note this is the DTFT of sequence $h[n]$.

(4) Verify the result in (3) using freqz() by plotting the magnitude and phase of the output of freqz()

3 An FM Channel Problem

In communications, a channel is defined as a transmission media like a wire or even free space. In this problem, we will model a channel as the following.

Report Item 3:
A communications channel is modeled as the following $y[n] = 0.8y[n-1] + 0.2x[n]$. The channel is excited by

$$x[n] = cos((\pi B/F_s/N)n^2)$$

where $B = 10Hz$,$F_s = 100Hz$,$\tau = N/F_s = 10s$

(1) Plot the DTFT of system described by the above difference equation. Do this in MATLAB using freqz() and output the magnitude and phase of the frequency response.
(2) Plot the DTFT of the system for the portion of the spectrum between DC and $10Hz$. Make your spacing for ω a 1000 length vector.
(3) Plot $x[n]$ for the first $10s$
(4) Run the sequence in (3) through the system described above. Use the filter () function.
(5) Verify the result the output in 4 is correctly attenuated by comparing the frequency response in (1) and the output in (4). You can do this by overlaying output in 4 with output in 2.

4 The Inverse Filter

Sometimes you'll hear of something called an inverse filter. It exists sometimes. This is one of those rare, maybe somewhat exciting cases.

Report Item 4:
Consider the single echo system $y[n] = x[n] + 0.1x[n-5]$.
(1) Plot the frequency response and impulse response of the system above. You may use impz() and freqz().
(2) Plot the frequency response and impulse response of the inverse system. Suppose the system described above has Z-transform $H(z)$. The inverse system is $\hat{H}(z)$ such that $H(z)\hat{H}(z) = 1$. You may use impz() and freqz().
(3) Suppose we define a sequence

$$x[n] = \sum_{k=1}^{10} \frac{1}{k} sin(0.01k^2\pi n)$$

Plot $x[n]$. Do this from $n = 0$ to 50.
(4) Run $x[n]$ through $H(z)$, store this into $v[n]$. Use filter().
(5) Run $v[n]$ through $\hat{H}(z)$, store this into $y[n]$ Use filter.
(6) In a 3-panel subplot, plot $x[n], v[n], y[n]$. Comment on the result.

5 Tweet-Tweet (No, Not A Problem About Twitter)

One waveform that we use a lot in signal processing is called the chirp. It's a signal whose frequency ramps up in time. The chirp signal has a wide use of applications from RADAR to WiFi. We will simply listen to the chirp to better our understanding of the Nyquist Rate.

Report Item 5:

Consider the continuous time signal :

$$x(t) = sin(\Omega_o t + \frac{1}{2}\beta t^2)$$

Taking the derivative of the argument of sin(), we find the instantaneous frequency to be

$$\Omega_{inst} = \Omega_o + \beta t$$

Suppose we have sampling rate $\Omega_{samp} = 2\pi(8192) rad/s$.

(1) For the parameters $\Omega_o = 2\pi(3000) rad/s$ and $\beta = 2000 rad^2/s$, plot $x[n]$, the sampled version of $x(t)$ above. Do this for $1s$

(2) Use the sound function and listen to x. Use the documentation if you don't know the sound function. If the documentation is still confusing, ask one of the two TAs in the room. Write down what you hear.

(3) Plot $x[n]$

(4) Determine at which time sample $x[n]$ is aliasing.

(5) Verify this by plotting out $x[n]$ to $1s$ passed the aliasing point. Also listen to the signal using sound(). Can you hear the aliasing?