ECE 311 Lab 3
Z-transform and System Response

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1 Z-Transform

Report Item:
For each transfer function below, plot the pole-zero plot and impulse response of length $N = 20$ using **pzplot** and **impz**. Also note whether each system is stable or unstable. Can you reach this conclusion by observing the impulse response? Why?

$$H_1(z) = 2 + 5z^{-2} + 4z^{-3} - 3z^{-6}$$

$$H_2(z) = 3 + 2z^{-1} - 2z^{-3}$$

$$H_3(z) = \frac{z^{-3} + z^{-6} - 2z^{-7}}{12 + z^{-1} + 4z^{-3}}$$

Answer:

I have attached three graphs below, and each graph represents one transfer function ($H_1(z)$, $H_2(z)$, $H_3(z)$ respectively).
For a particular system, we can check its stability by looking at whether its impluse response is absolutely summable or not. Also, we can check whether a system is stable by looking at whether the poles of its transfer function are located inside the unit circle (we are looking at causal signals that are right-sided). For all $H_1(z)$, $H_2(z)$, $H_3(z)$, we can find that all their poles lie in the unit circle. This means that their regions of convergence (ROC) are all contain unit circle. (If there are poles at origin, the pzplot will not include this pole, so I plot it by myself in MATLAB). Therefore, all these three systems are BIBO stable.

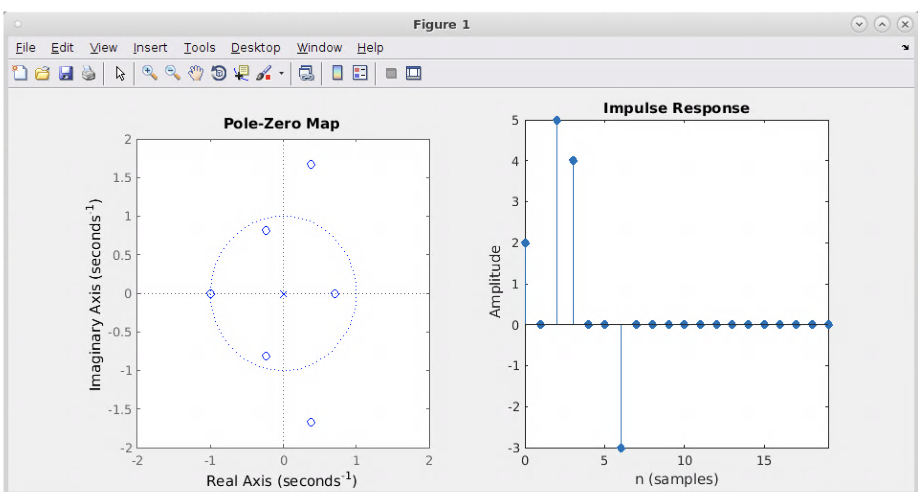

Figure 1: Pole-Zero Map and Impluse response of $H_1(z)$

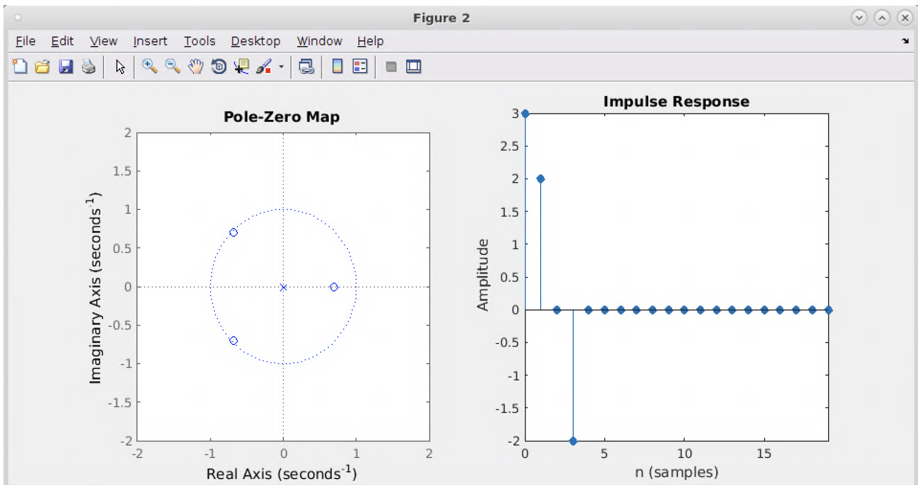

Figure 2: Pole-Zero Map and Impluse response of $H_2(z)$

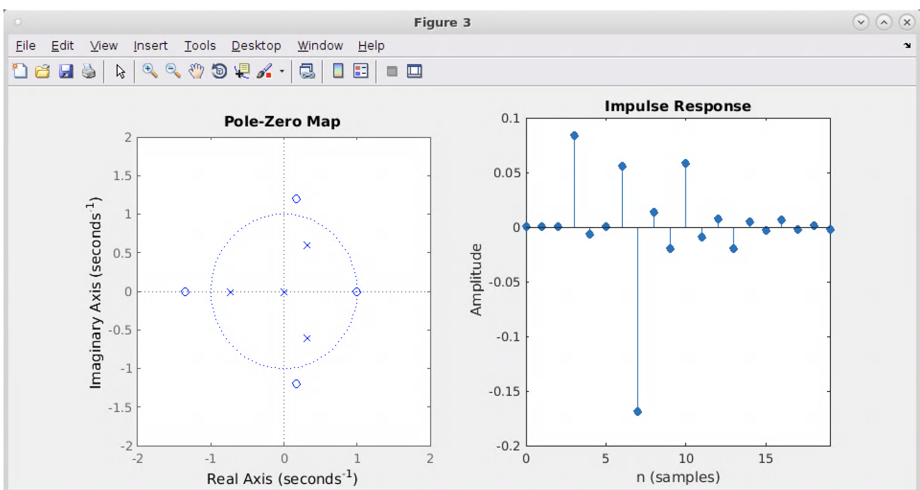

Figure 3: Pole-Zero Map and Impluse response of $H_3(z)$

Report Item:
Consider the system

$$H(z) = \frac{z}{(z + e^{\frac{-i8\pi}{10}})(z + e^{\frac{i8\pi}{10}})}$$

Plot the pole-zero plot and impulse response of this system using **pzplot** and **impz** for $N = 35$. Is this system BIBO stable? Why? For what inputs is the output unbounded? Plot the response of this system for such an input. For what inputs is the output bounded? Plot the response of this system for such an input.

Answer:

First, we try to simplify the expression for $H(z)$ to the form that we can use MATLAB to calculate. The denominator of $H(z)$ is given by $A(z) = (z + e^{\frac{-i8\pi}{10}})(z + e^{\frac{i8\pi}{10}}) = z^2 + z(e^{\frac{-i8\pi}{10}} + e^{\frac{i8\pi}{10}}) + 1 = z^2 + 2cos(\frac{4}{5}\pi)z + 1$. So $H(z) = \frac{z}{z^2+2cos(4\pi/5)z+1} = \frac{z^{-1}}{1+2cos(4\pi/5)z^{-1}+z^{-2}}$. We can set b = [0, 1, 0] and a = [1, 2cos(4π/5), 1]. Then plug them into the **pzplot** function and **impz** function to find the impulse response $H(z)$.
Details of $H(z)$ is given in the following graph.

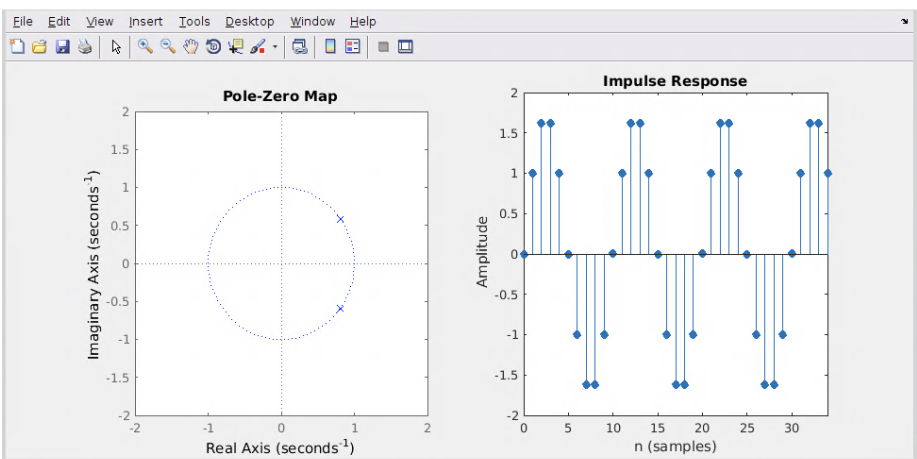

Figure 4: Pole-Zero Map and Impluse response of $H(z)$

Since the poles of $H(z)$ are located on the unit circle, this system is not BIBO stable (Also, from the impulse response we can see that the $h(z)$ is not absolutely summable). We can create an input signal that its output is not bounded. For simplicity, we can pick $x[n]$ such that its trasfer function $X(z)$ has same poles as $H(z)$. In this way, the output $Y(z)$ will have double poles on unit circle and therefore unbounded. Let $x[n] = cos(\frac{\pi n}{5})u[n]$. $X(z) = \frac{1-cos(\pi/5)z^{-1}}{1-2cos(\pi/5)z^{-1}+z^{-2}}$.

So for this input, we have the output in the following graph (I use the **filter** function in MATLAB to create the output y[n] in T-domain).

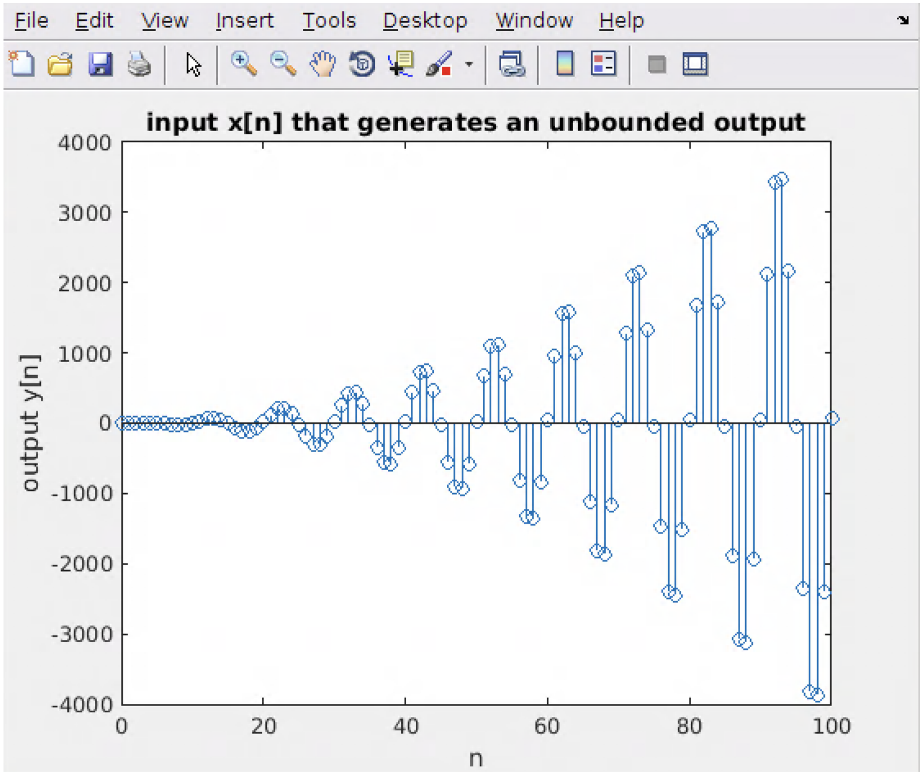

Figure 5: Unbounded output generated by $x[n] = cos(\frac{\pi n}{5})u[n]$

To generate a bounded output, we need to avoid creating same poles in $X(z)$ as in $H(z)$. So I pick $x[n] = (\frac{1}{2})^n u[n]$. $X(z) = \frac{z}{z-\frac{1}{2}}$ and it has only one pole at $z = \frac{1}{2}$. The output of this input is given in the following graph. From the graph we can see that the maximum value of this output is smaller than 2.5 (i.e. $|y[n]| < 2.5$). It is a bounded output.

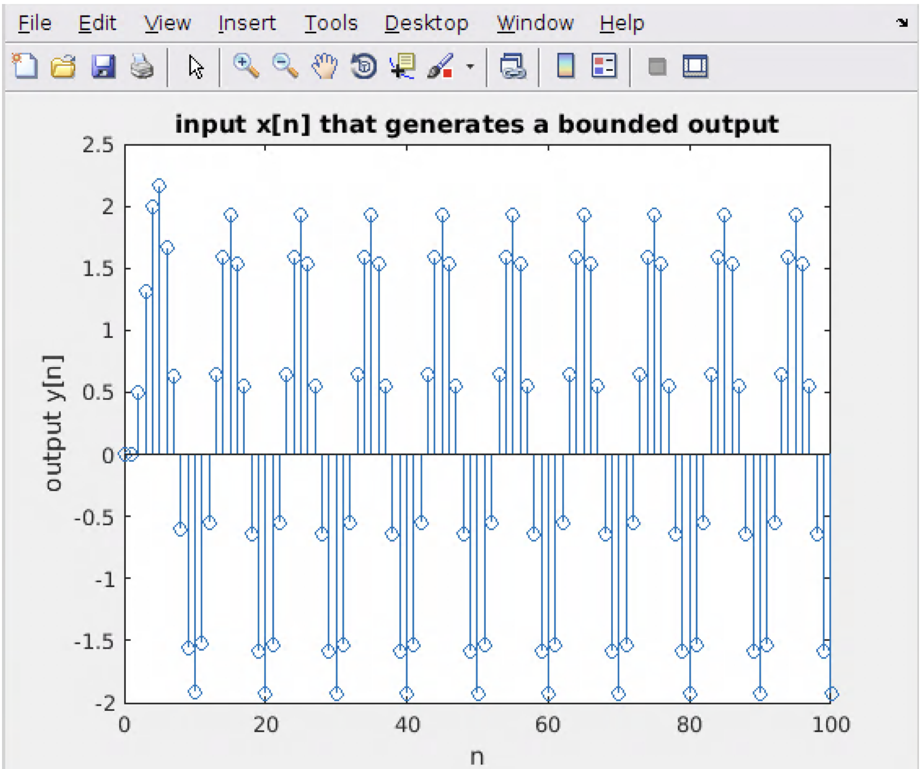

Figure 6: Bounded output generated by $x[n] = (\frac{1}{2})^n u[n]$

2 Filter function in MATLAB

Report Item:

The **filter** function in MATLAB can be used to verify the z-transform expression of a causal sequence. Let $x[n]$ be a causal sequence with a rational expression $X(z) = B(z)/A(z)$.

(a)

Given $x[n] = [(\frac{1}{2})^n + (-\frac{1}{3})^n]u[n]$, determine $X(z)$ analytically.

Answer:

Since $a^n u[n]$ has Z-Transfrom $\frac{z}{z-a}$, and Z-Trasform has linearity, we have

$$X(z) = \frac{z}{z - \frac{1}{2}} + \frac{z}{z - (-\frac{1}{3})} = \frac{2z^2 - \frac{z}{6}}{z^2 - \frac{z}{6} - \frac{1}{6}} = \frac{2 - \frac{1}{6}z^{-1}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

(b)

Verify your expression in (b) by comparing the output when using the **filter** function.

Answer:

In MATLAB, **filter** function does the convolution. In expression y = filter(b, a, x), it does the convolution that $y = h * x$ where $H(z) = \frac{b}{a}$ and a, b are the numerator and denominator of $H(z)$ respectively. In order to get $x[n]$, from $x[n] = x[n] * \delta[n]$ we can let b be the numerator coefficient of $x[n]$, a be the denominator and the last argument of **filter** function be $\delta[n]$. Specifically, let b = [2, -1/6, 0], and a = [1, -1/6, -1/6]. Graphs following are $x[n]$ generated by **filter** function, $x[n]$ plotted by expression and the difference between this two method. From the graphs we can see that these two method give us the same answer (difference is less than 10^{-16}, and can be considered as accuracy error).

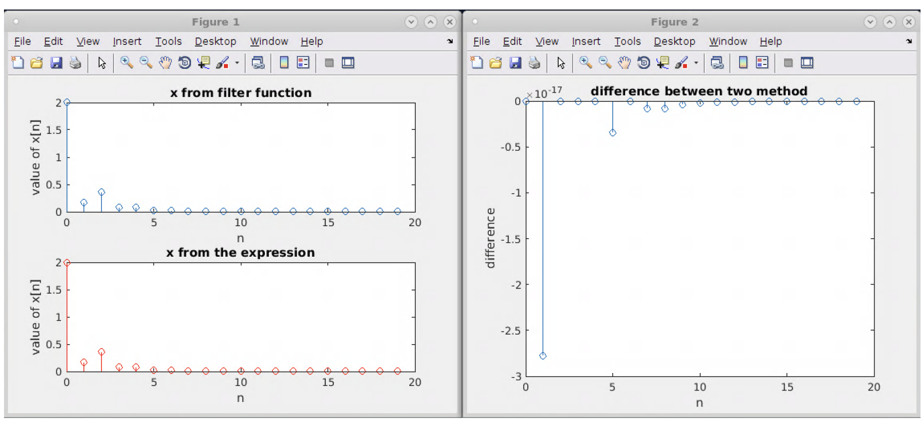

Figure 7: Vertify the expression by comparing the output using filter function

3 LSI System Response

Report Item:
The response of a LSI system to the input $x[n] = u[n]$ is $y[n] = 2(1/3)^n u[n]$.

(a)

Find the impulse response $h[n]$ of this LSI system analytically.

Answer:

For an LSI system, if the input is $\delta[n]$, the output is the impulse response. $\delta[n] = u[n] - u[n-1]$. So

$$h[n] = 2(1/3)^n u[n] - 2(1/3)^{n-1} u[n-1]$$

(b)

Use **stem** to plot out $h[n]$ for $0 \leq n \leq 19$.

Answer:

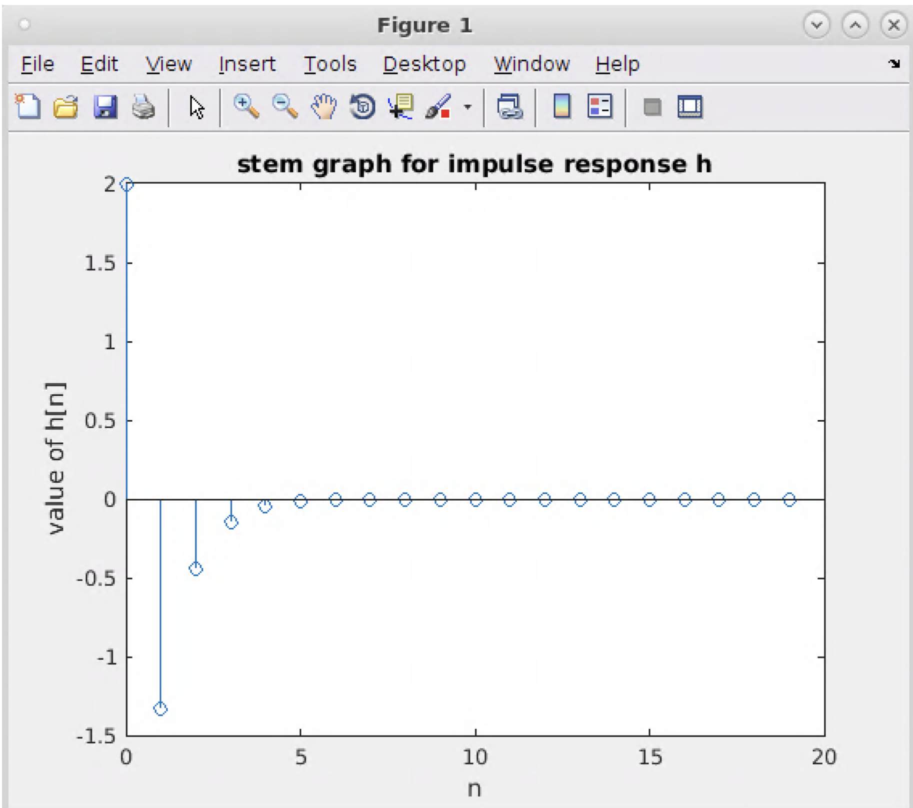

Figure 8: stem graph for h[n] for $0 \leq n \leq 19$

(c)

Given an input $x[n] = (1/2)^n u[n]$ for $0 \leq n \leq 19$. Plot with **stem** the output $y[n]$ using convolution with $h[n]$ from part (b).

Answer:

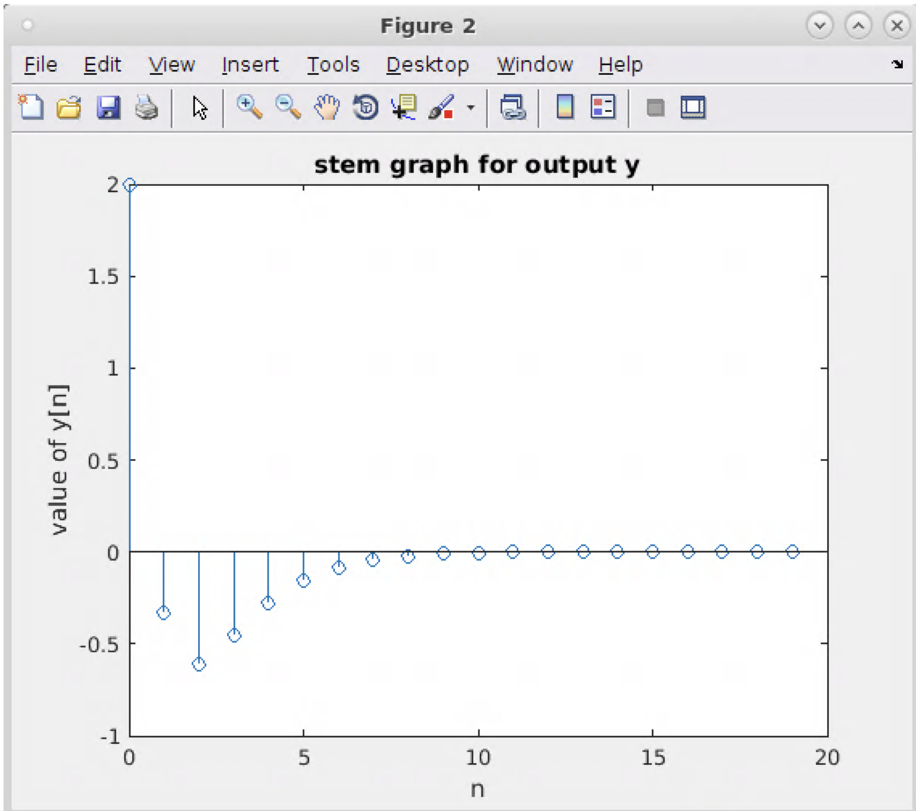

Figure 9: stem graph for y[n] for $0 \leq n \leq 19$

4 Poles and Zeros

Report Item:

Consider a causal LSI system described by the difference equation

$$y[n] = \frac{1}{2}y[n-1] + x[n] - \frac{1}{1024}x[n-10]$$

(a)

Determine the system function $H(z)$ and plot the poles and zeros using the function **pzplot**

Answer:

Do Z-Transform on both sides and get

$$Y(z)(1 - \frac{1}{2}z^{-1}) = X(z)(1 - \frac{1}{1024}z^{-10})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{1024}z^{-10}}{1 - \frac{1}{2}z^{-1}}$$

Let b = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1/1024], a = [1, -1/2, 0, 0, 0, 0, 0, 0, 0, 0, 0] and use **pzplot** to get the details of transfer function $H(z)$.

Answer:

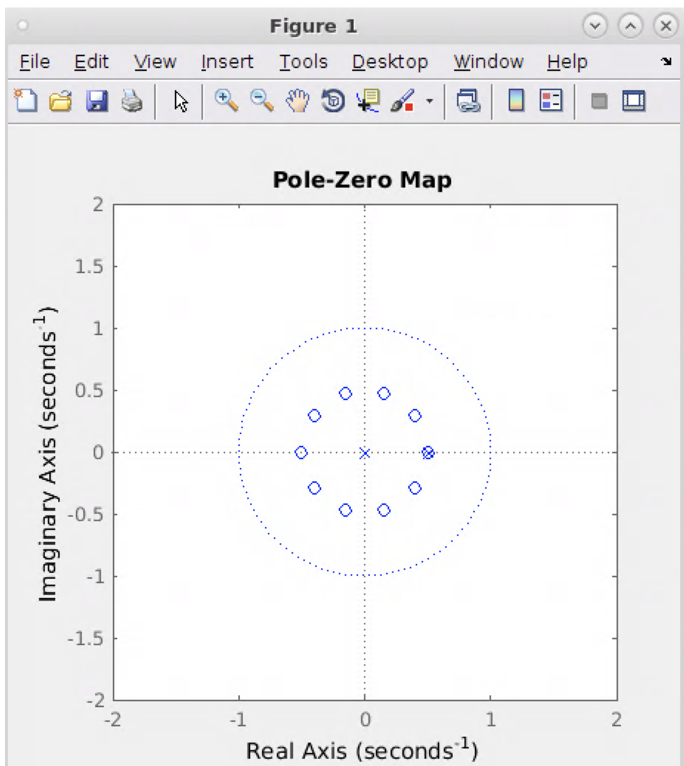

Figure 10: Pole-Zero Map of $H(z)$

(b)

Compute and plot the impulse response $h[n]$ of the system using the function **impz**. Use $N = 30$ for the number of samples.

Answer:

$$H(z) = \frac{1 - \frac{1}{1024}z^{-10}}{1 - \frac{1}{2}z^{-1}} = \frac{z}{z - \frac{1}{2}} - \frac{1}{1024}\frac{z}{z - \frac{1}{2}}z^{-10}$$

Therefore

$$h[n] = (\frac{1}{2})^n u[n] - \frac{1}{1024}(\frac{1}{2})^{n-10}u[n-10]$$

Graph below is the impulse response $h[n]$ of the system by using the function **impz**.

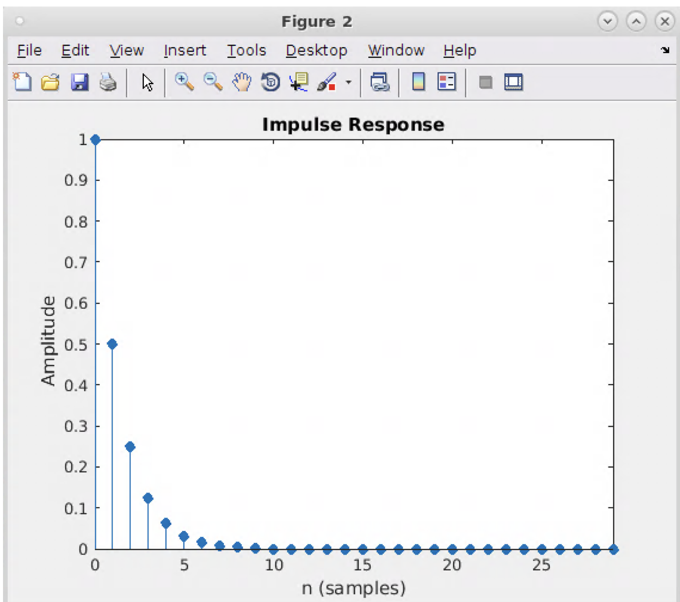

Figure 11: Impluse response of $H(z)$

5 Filter Function vs Recursive For Loop

Report Item:
A difference equation is given by

$$y[n] = x[n] - x[n-1] + 0.81y[n-2], \; n \geq 0$$

with initial conditions $y[-1] = y[-2] = 0$ and excited by the input $x[n] = (0.7)^n u[n]$.

(a)

Use the recursive for loop from last weeks lab to compute $y[n]$ for $0 \leq n \leq 49$.

Answer:

The graph is on next page.

(b)

Given the input $x[n]$ for $0 \leq n \leq 49$, compute $y[n]$ using the **filter** function

Answer:

To use the **filter** function, we must first find out the trasfer function of this system. Do Z-Transfer to the input-output relation and get

$$Y(z)(1 - 0.81z^{-2}) = X(z)(1 - z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - 0.81z^{-2}}$$

So in the **filter** function, we let b = [1, -1, 0], and a = [1, 0, -0.81].

(c)

Plot the results obtained from part (a) and part (b) on the same graph.

Answer:

The graph on the next page is the plot of output $y[n]$ by two methods mentioned in part (a) and (b). Also, I plot the difference of these two methods. From the difference, we can see that these two methods give us the same result (the error is less than 10^{-16}).

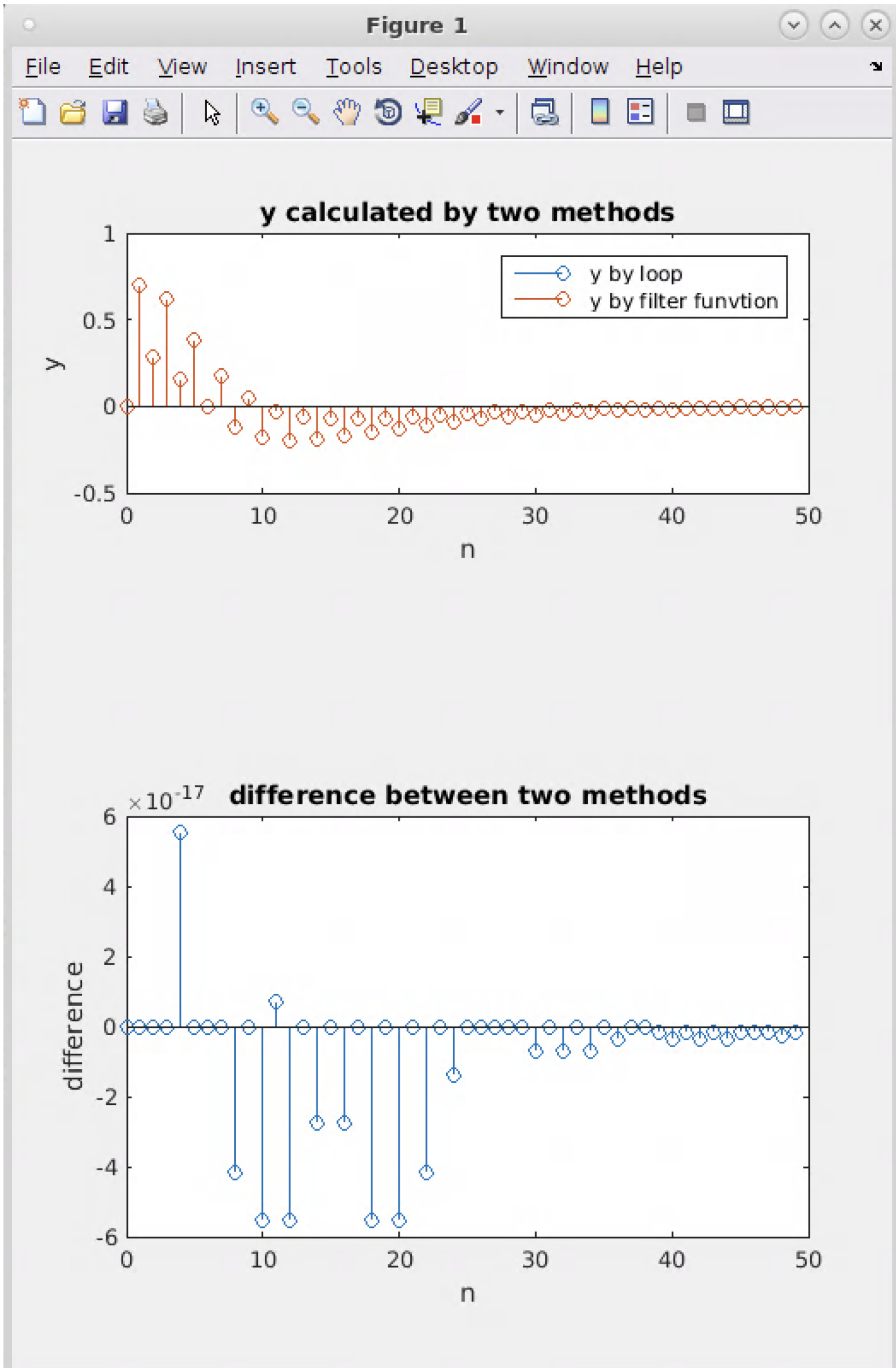

Figure 12: Output of the system and difference between two methods