Lab 3
September 26, 2017
Z-transform and System Response

INSTRUCTIONS:
All lab submissions include a written report and source code in the form of an m-file. The report contains all plots, images, and figures specified within the lab. All figures should be labeled appropriately. Answers to questions given in the lab document should be answered in the written report. ***The written report must be in PDF format***. Submissions are done electronically through **Compass 2g**.

1 Z-Transform

For LSI systems, causality and stability as well as other useful properties can be determined from the z-transform of the system. The z-transform can be expressed as

$$X(z) = \sum_{n=0}^{N-1} x(n)z^{-n} \tag{1}$$

where $z = \sigma + i\omega$ is complex.
Any LSI system can be expressed as a difference equation of the form

$$y(n) + \sum_{k=1}^{M-1} a_k y(n-k) = \sum_{l=0}^{N-1} b_l x(n-l) \tag{2}$$

The z-transform of (2) is

$$Y(z) + \sum_{k=1}^{M-1} a_k Y(z)z^{-k} = \sum_{l=0}^{N-1} b_l X(z)z^{-l} \tag{3}$$

from which the transfer function $H(z) = Y(z)/X(z)$ can be written as

$$H(z) = \frac{\sum_{l=0}^{N-1} b_l z^{-l}}{1 + \sum_{k=1}^{M-1} a_k z^{-k}} \tag{4}$$

The zeros and poles of $H(z)$ are the solutions to $\sum_{l=0}^{N-1} b_l z^{-l} = 0$ and $1 + \sum_{k=1}^{M-1} a_k z^{-k} = 0$, respectively. Since both the numerator and denominator are polynomials, they can be written be factorized into

$$H(z) = \frac{\prod_{l=0}^{N-1}(1 - z_l z^{-1})}{\prod_{k=1}^{M-1}(1 - p_k z^{-1})} \tag{5}$$

where p_k denotes a pole and z_l, a zero. Using (5), the magnitude can be expressed as

$$|H(z)| = \frac{\prod_{l=0}^{N-1} |1 - z_l z^{-1}|}{\prod_{k=1}^{M-1} |1 - p_k z^{-1}|} \tag{6}$$

where $|1 - z_l z^{-1}| \equiv |z - z_l|$ is the distance between z and z_l. A useful representation of the z-transform is the pole-zero plot. For $h(n) = \{1, 3, 4, 5, 3, 1\}$, the pole-zero plot is

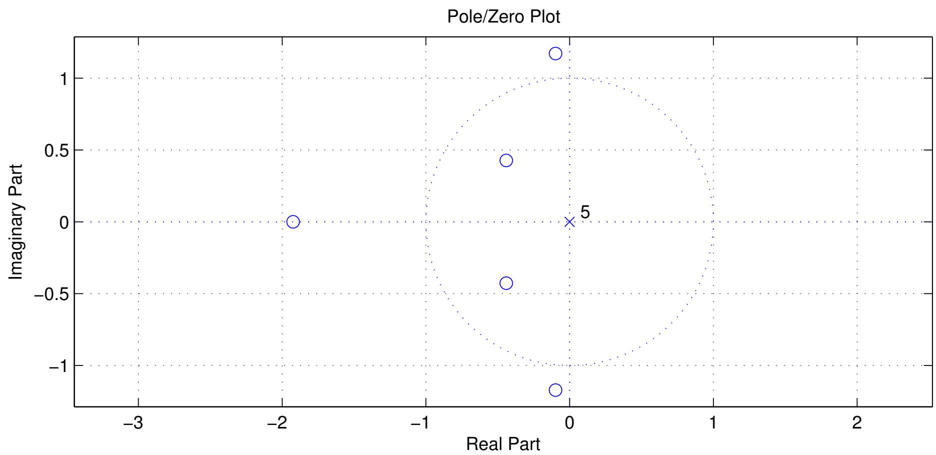


Figure 1: A pole-zero plot. **The circles indicate zeros and the crosses indicate poles.** For any system, there is an equal number of poles and zeros.

In general, the magnitude response of a system becomes larger when the observation point is closer to a pole or far from a zero. The magnitude response becomes smaller when an observation point is farther from a pole or closer to a zero. This enables one to guess the behavior of system from its pole-zero plot.

The stability of a system can be ascertained from its pole-zero plot. For a causal system, **if *all* poles are contained within unit circle, then the system is stable.** If even one pole lies on or outside the unit circle, then the system is unstable.

Transfer functions can be defined in MATLAB using $\mathbf{tf}(b, a)$ where $b = [b_0, b_1, \ldots, b_{N-1}]$ are the numerator coefficients and $a = [1, a_1, \ldots, a_{M-1}]$ are the denominator coefficients corresponding to

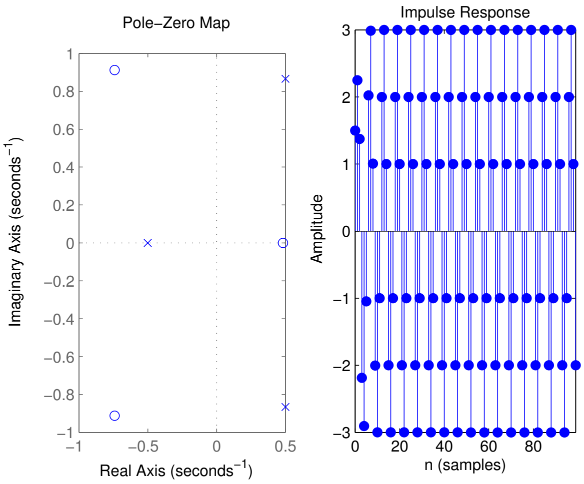
$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_{N-1} z^{-(N-1)}}{1 + a_1 z^{-1} + \cdots + a_{M-1} z^{-(M-1)}} \tag{7}$$

The object returned from **tf** can be used in **pzplot** to obtain the pole-zero plot of the transfer function. The impulse response of $h(z)$ can be found using $\mathbf{impz}(b, a)$.

```
b = [3,3,2,-2];
a = [2,-1,1,1];
S = tf(b,a);
N = 100;

figure;
subplot(121);
pzplot(S);
subplot(122);
impz(b,a,N);
```

(a) Example code using **pzplot** and **impz**.



(b) Pole-zero and impulse response plots.

Report Item: For each transfer function below, plot the pole-zero plot and impulse response of length $N = 20$ using **pzplot** and **impz**. Also note whether each system is stable or unstable. Can you reach this conclusion by observing the impulse response? Why?

$$H_1(z) = 2 + 5z^{-2} + 4z^{-3} - 3z^{-6} \tag{8}$$

$$H_2(z) = 3 + 2z^{-1} - 2z^{-3} \tag{9}$$

$$H_3(z) = \frac{z^{-3} + z^{-6} - 2z^{-7}}{12 + z^{-1} + 4z^{-3}} \tag{10}$$

Report Item: Consider the system

$$H(z) = \frac{z}{(z + e^{\frac{-i8\pi}{10}})(z + e^{\frac{i8\pi}{10}})} \tag{11}$$

Plot the pole-zero plot and impulse response of this system using **pzplot** and **impz** for N = 35. Is this system BIBO stable? Why? For what inputs is the output unbounded? Plot the response of this system for such an input. For what inputs is the output bounded? Plot the response of this system for such an input.

2 Filter function in Matlab

Report Item: The **filter** function in Matlab can be used to verify the z-transform expression of a causal sequence. Let $x[n]$ be a causal sequence with a rational expression $X(z) = B(z)/A(z)$.

(a) Given $x[n] = [(\frac{1}{2})^n + (-\frac{1}{3})^n]u[n]$, determine $X(z)$ analytically.
(b) Verify your expression in (b) by comparing the output when using the **filter** function

3 LSI System Response

Report Item: The response of a LSI system to the input $x[n] = u[n]$ is $y[n] = 2(1/3)^n u[n]$.

(a) Find the impulse response $h[n]$ of this LSI system analytically.
(b) Use **stem** to plot out $h[n]$ for $0 \leq n \leq 19$
(c) Given an input $x[n] = (1/2)^n u[n]$ for $0 \leq n \leq 19$. Plot with **stem** the output $y[n]$ using convolution with $h[n]$ from part (b).

4 Poles and Zeros

Report Item: Consider a causal LSI system described by the difference equation

$$y[n] = \frac{1}{2}y[n-1] + x[n] - \frac{1}{1024}x[n-10]$$

(a) Determine the system function $H(z)$ and plot the poles and zeros using the function **pzplot**
(b) Compute and plot the impulse response $h[n]$ of the system using the function **impz**. Use N = 30 for the number of samples.

5 Filter Function vs Recursive For Loop

Report Item: A difference equation is given by

$$y[n] = x[n] - x[n-1] + 0.81y[n-2], \quad n \geq 0$$

with initial conditions $y[-1] = y[-2] = 0$ and excited by the input $x[n] = (0.7)^n u[n]$

(a) Use the recursive for loop from last week's lab to compute $y[n]$ for $0 \leq n \leq 49$

(b)Given the input $x[n]$ for $0 \leq n \leq 49$, compute $y[n]$ using the **filter** function.

(c) Plot the results obtained from part (a) and part (b) on the same graph.