UNIVERSITY OF GUELPH
SCHOOL OF ENGINEERING ENGG*3410

Instructor:

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Lab 5: Designing PID Controllers

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1.0- Introdution:

Through class, it was observed that the root locus graphically displayed both transient response and stability information.The locus can be sketched quickly to get a general idea of the changes in transient response generated by changes in gain. Specific points on the locus also can be found accurately to give quantitative design information. The root locus typically allows to choose the proper loop gain to meet a transient response specification. As the gain is varied, different regions of response is produced. Setting the gain at a particular value yields the transient response dictated by the poles at that point on the root locus. Thus, designers are limited to those responses that exist along the root locus.
The main objective of this lab is to explore different aspects and procedures of designing various controllers. Controllers include P controllers, PD controllers, and PID controllers. The transfer function described in figure 1 will be used to design and implemnet the controllers. Controllers will be designed using calculation techiques used in class and verified using MATLAB. Controller will be simulated to ensure objective requirements are met. After verification, controllers will be implemented on the system from lab 4 and tested accordingly.

$$H(s) = \frac{10}{s(0.02s + 1)}$$

Figure 1- Transfer Function used to design controllers

Using the transfer function described in figure 1, a pure proportional controller(P), a proportional derrivative (PD), and a proportional integrator derrivative (PID) will be designed to perform the requirements outlined in the lab manual.

1.1 - Design Overview:

Controller	Specifications	Procedure to complete task
Pure Proportional Controller(P)	- Satisfies closed loop stability - Perfect tracking - Overshoot less than 20% - Settling time less than 8/0.9τ	Requirements will be achived by manipulating the Kp gain.
Proportional Derrivative Controller (PD)	- Satisfies closed loop stability - Perfect tracking - No overshoot - Settling time as small as possible	Requirements will be achived by manipulating the Kp gain and the Kd gain.
Proportional Integrator Derrivative controller (PID)	- Perfect Ramp tracking - Closed Loop Stability	Requirements will be achived by manipulating the Kp gain, Ki gain and the Kd gain.

Table 1: The table below describes the controllers designed and the specifications required

2.0- Experimental Setup:

Using control theory concepts learned in class, controllers were designed to meet specific requirements highlighted in table 1. To design the controller and ensure accuracy is achieved, 5 procedures were followed to evaluate performance of design on the real motor control. Procedures are highlighted below:

2.1- Procedure 1:

The first procedure involves theoredical calculations. The transfer funtion (figure 1) was used to find the specific variables needed to achieve the requirements highlighted in table 1. These variables will be highlighted further in the Theoredical Design analysis section as the first procedure differs for each controller. The main objective of this procedure is to find a K value and a frequency value that can be used to obtain the required system properties.

2.2- Procedure 2:

Procedure 2 involves analysing the results from procedure 1 and implement the designed controller in MATLAB simulation. Using the simulation, the gains that meet the given specifications were identified. A more detailed description is presented in the simulation result section for the 3 different controllers designed. The main objective of this procedure is to find P and/or D gain that satisfy the specifications required.

2.3- Procedure 3:

Gains from procedure 2 where used to test the system designed. The test was conducted on the real motor system using the Graphical user interface provided by the teacher assistants. Results from this procedure will be discussed in the discussion section.

2.4- Procedure 4:

Data from procedure 3 was imported into MATLAB to analize results. Using MATLAB, Motor Output Vs time and motor input vs time graphs were generated. These graphs were used to analize the designed controller. Graphs can be found in the discussion section and the appendix.

2.5- Procedure 5:

The designed controller was tested by changing the setpoint values and running the motor for longer than 30 seconds. The main goal of this procedure is to ensure accuracy and requiments and met.

3.0- <u>Theoretical Design Analysis:</u>

The theoretical Design Analysis section will expand on the 1st procedure described in the experimental setup section. In lab 5 a P controller, PD controller and a PID controller was designed. This section will expand on the theoredical approach of the 3 different controllers. To achieve the desired design, a set of requirements must be fulfilled. Each Controller must achieve closed loop stability. To fulfill this requirement Both ς and T_s must be calculated to verify if true. The damping ratio was calculted for all different types of controllers. The settling time was calculed using the following formula:

$$T_s = \frac{4}{\zeta w_n} < \frac{8}{0.9}\tau = \frac{8}{0.9}(0.02) = 0.177s$$

Figure 2- Ts Theoretical Calculations

3.1- Pure Porpotional Controller:

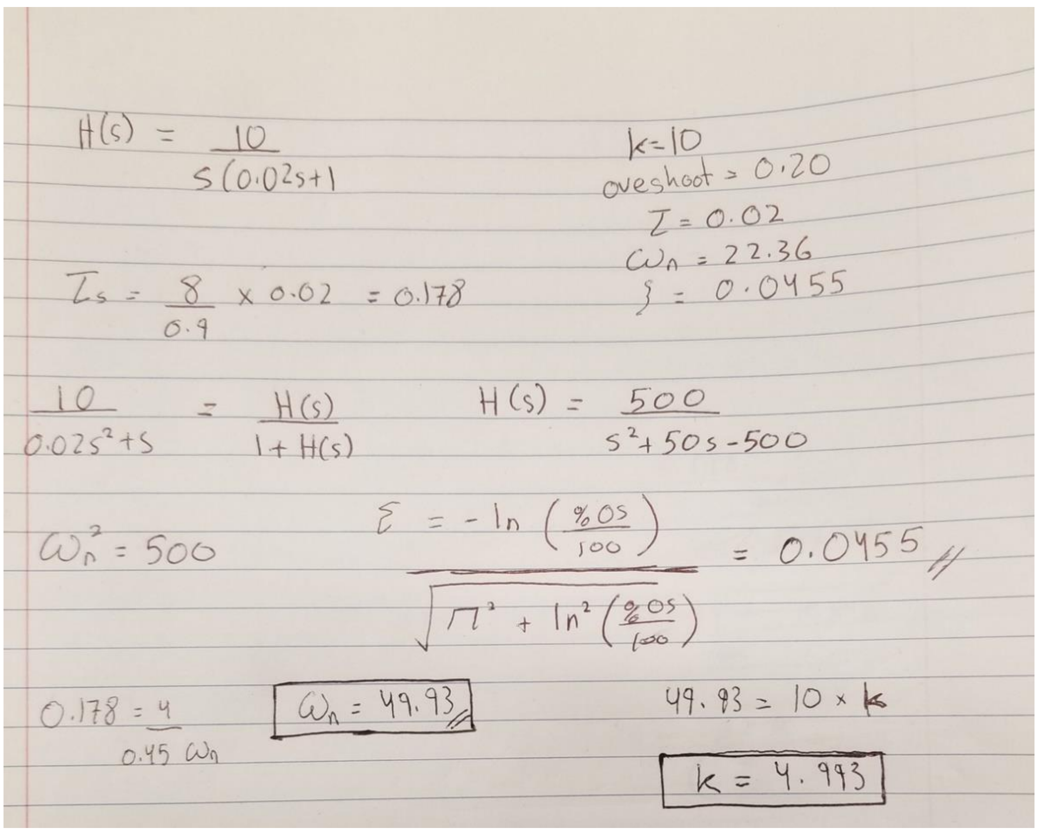
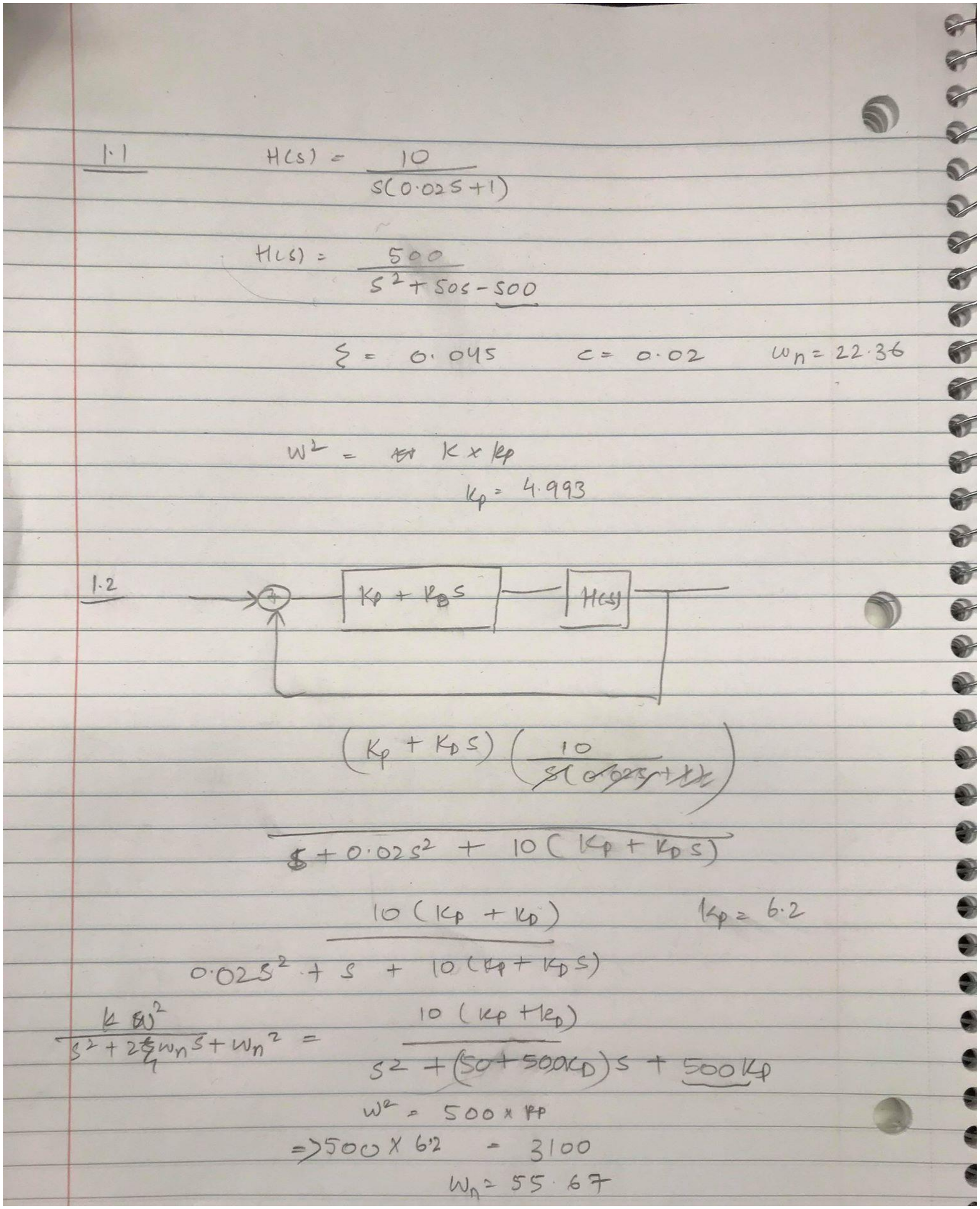


Figure 3- Pure Porpotional Controller Calculations (correction: damping = 0.455)

Figure 3 presents the Theoretical calculations used to find the K value needed when testing the designed controller on the Real motor system. The damping ratio was calculated to be 0.455, and the frequency required to meet the requirements was calculated to be 49,93. This results in a K value of 4.993.

3.2- Porpotional Derrivative Controller:



1.1

$$H(s) = \frac{10}{s(0.02s+1)}$$

$$H(s) = \frac{500}{s^2+50s-500}$$

$\xi = 0.045$ $\quad c = 0.02$ $\quad \omega_n = 22.36$

$$w^2 = K \times K_p$$

$$K_p = 4.993$$

1.2

$$\frac{(K_p + K_D s)\left(\frac{10}{s(0.02s+1)}\right)}{s+0.02s^2+10(K_p+K_D s)}$$

$$\frac{10(K_p+K_D)}{0.02s^2+s+10(K_p+K_D s)} \qquad K_p = 6.2$$

$$\frac{K\omega^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{10(K_p+K_D)}{s^2+(50+500K_D)s+500K_p}$$

$$w^2 = 500 \times K_p$$

$$\Rightarrow 500 \times 6.2 = 3100$$

$$W_n = 55.67$$

Figure 4- PD Theoretical Calculations part 1

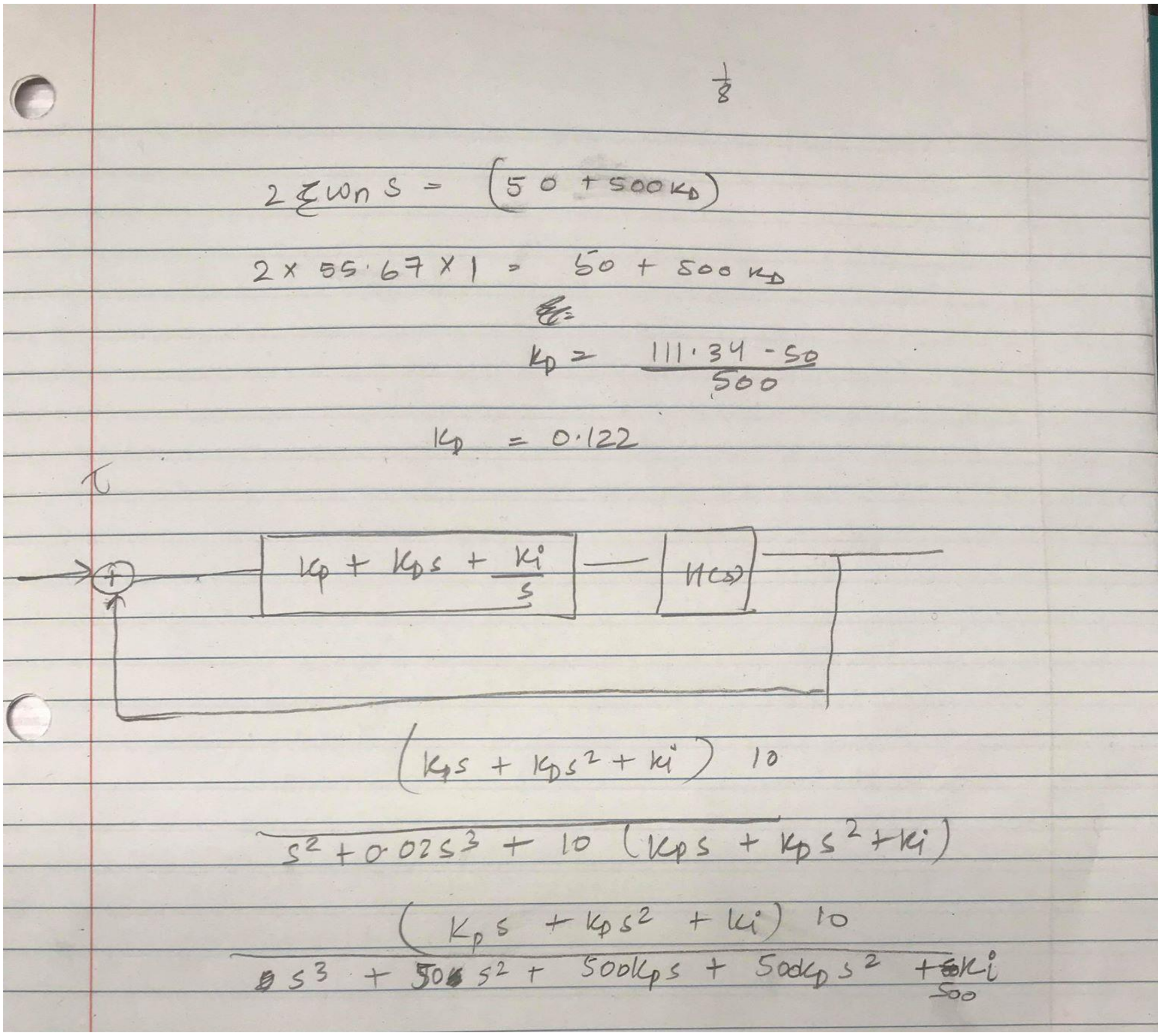
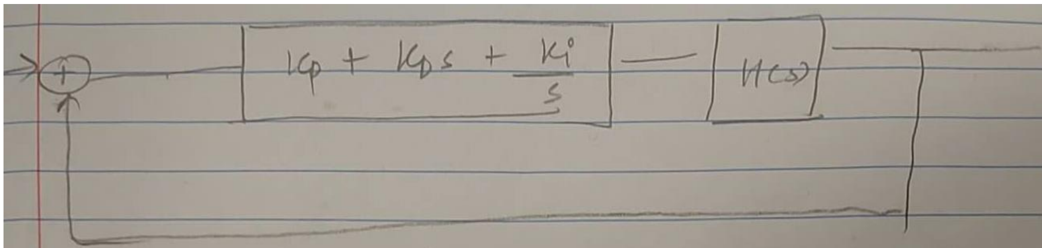


Figure 5- PD theoretical Calculation part 2

The calculations in figure 4 and 5 suggest that a Kd value of 0.122 and a Kp value of 6.2 should be used to meet the required criteria.

3.3- Porpotional Integrator Derrivative Controller:

In order to create a perfect ramp tracking, a PID controller was designed. The PID controller was used due to the effects of utilizing a Kd gain. This is benificial as it provides a better stability and a faster speed. This ensures that the system meets the closed loop staility specification required as part of the criteria listed in table 1. The calculations were completed using MATLAB. Code is posted in appendix



$$\frac{(K_p s + K_D s^2 + K_i)\,10}{s^2 + 0.02s^3 + 10(K_p s + K_D s^2 + K_i)}$$

$$\frac{(K_p s + K_D s^2 + K_i)\,10}{s^3 + 500 s^2 + 500K_p s + 500K_D s^2 + 500K_i}$$

$$0.02S^3 + 2S^2 + 62S + 10K_i$$

S^3	0.02	62	0
S^2	2	$10K_i$	0
S^1	b_1	0	
S^0	C_1		

$$b_1 = -\frac{\begin{vmatrix} 0.02 & 62 \\ 2 & 10K_i \end{vmatrix}}{2} = \frac{-0.2K_i + 124}{2}$$

$$-0.1K_i + 62$$

$$C_1 = 10K_i$$

$$0 < K < 620$$

6- Theoretical Calculations For PID Controller

4.0- Simulation Results:

4.1- Pure Porpotional Controller:

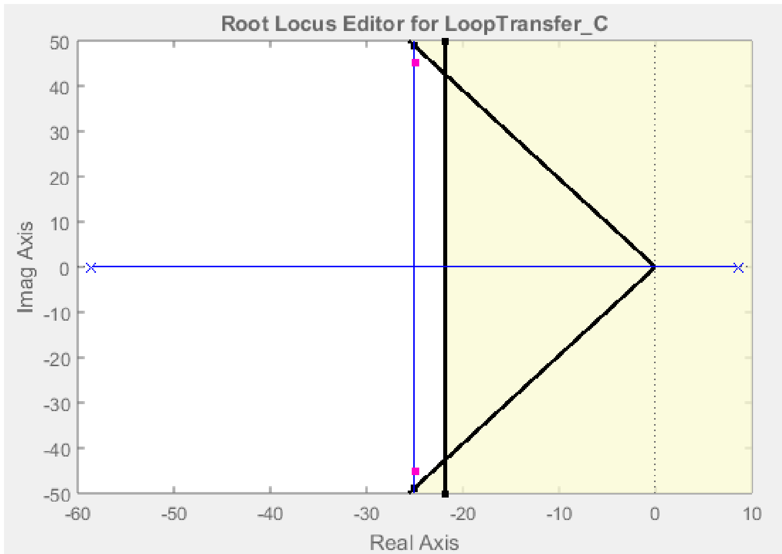


Figure 7- Region Of Specified Transfer Function

The root locus diagram was used to simulate the pure porpotional controller. The region of the specified transfer function was determined using the settling time and the maximum overshoot defined in the introduction section. Figure 6 represents that region that allowed for a basepoint of proper poles location for the system to meet the requirements and gain selection.

Using the region above, a P gain of 5.74 was used to design the P controller. This value was based on the root locus analysis above. This value allows the system to obtain a closed loop stabilit, an overshoot less than 20%, and perfect tracking. The overshoot was calculated to be 19.12%.

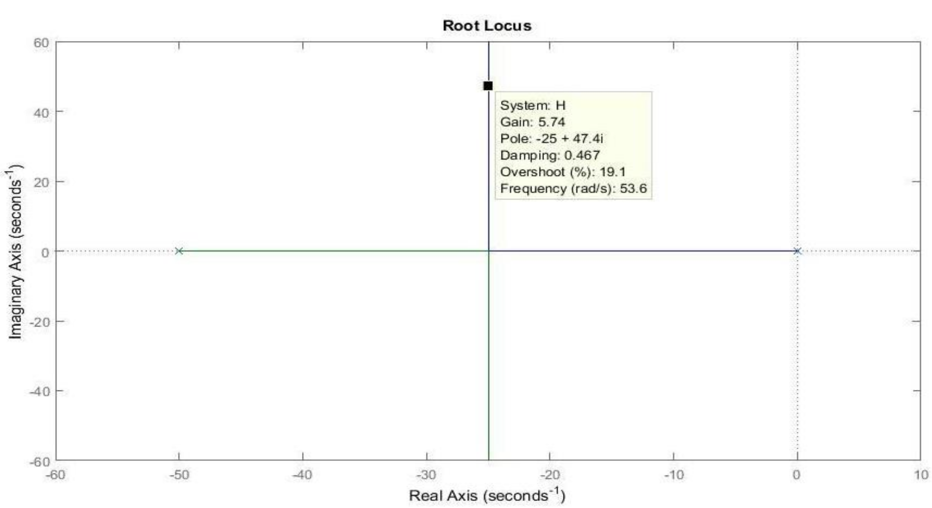


Figure 8- Root Locus Of The Transfer Function Showing Kp Gain And 20% Overshoot

The P controller had a gain of 5.74, damping ratio of 0.467, and a frequency of 53.6 rad/s.

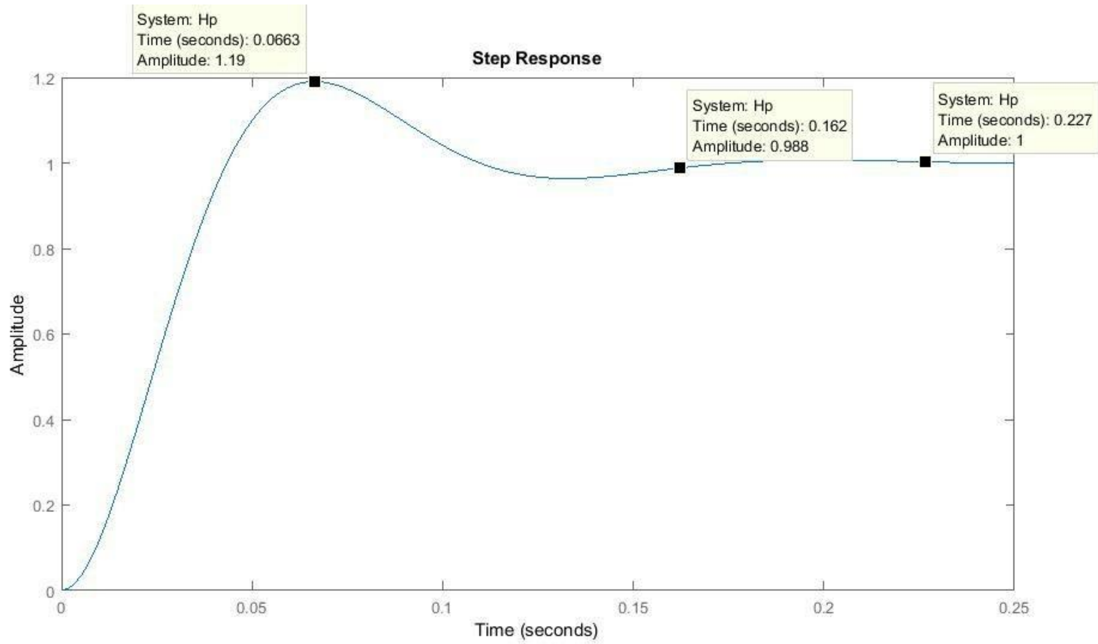


Figure 9- Step of the Transfer Function Showing Settling Time

Figure 8 represents the step response of the controller designed. Using the step response, the settling time was approximated to be 0.162 seconds with an amplitude of 0.988. This satisfies the condition of being less than 0.177 seconds.

4.2- Porpotional Derrivative Controller:

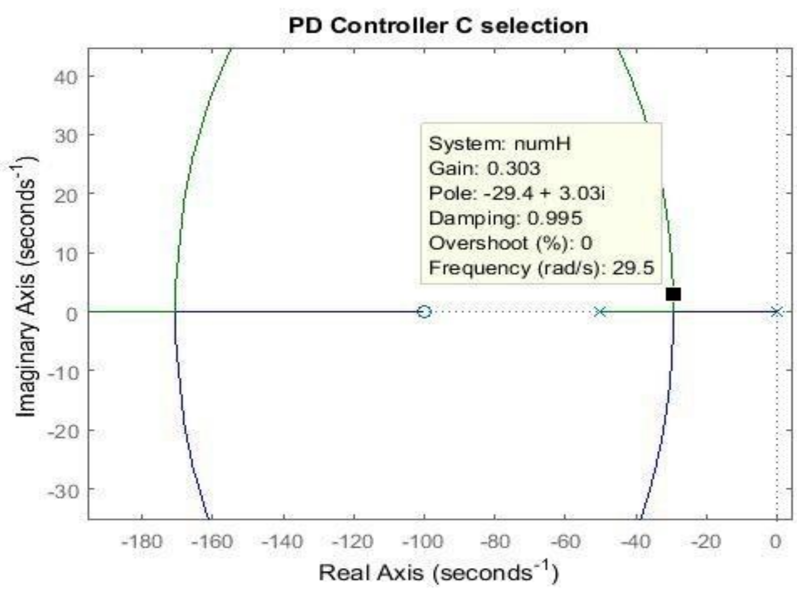


Figure 10- PD Controller Design With New Specifications

For the PD Controller, Kp was directly related to Kd. The Kp was kept the same as the P controller at a value of 5.74, and the value of Kd was determined using the formula : Kp/100. This ensures that the effects of the Kd gain would be present yet minimal in comparison to the Kp gain. As seen in figure 9, the root locus of the designed controller shows that the opershoot was 0% and the overall gain is 0.303. This value can be calculated using the following formula: K= Kp + Kd * S. Figure 9 proves that the PD controller requirements were all met. Matlab analysis was used to ensure appropriate selection was taken.

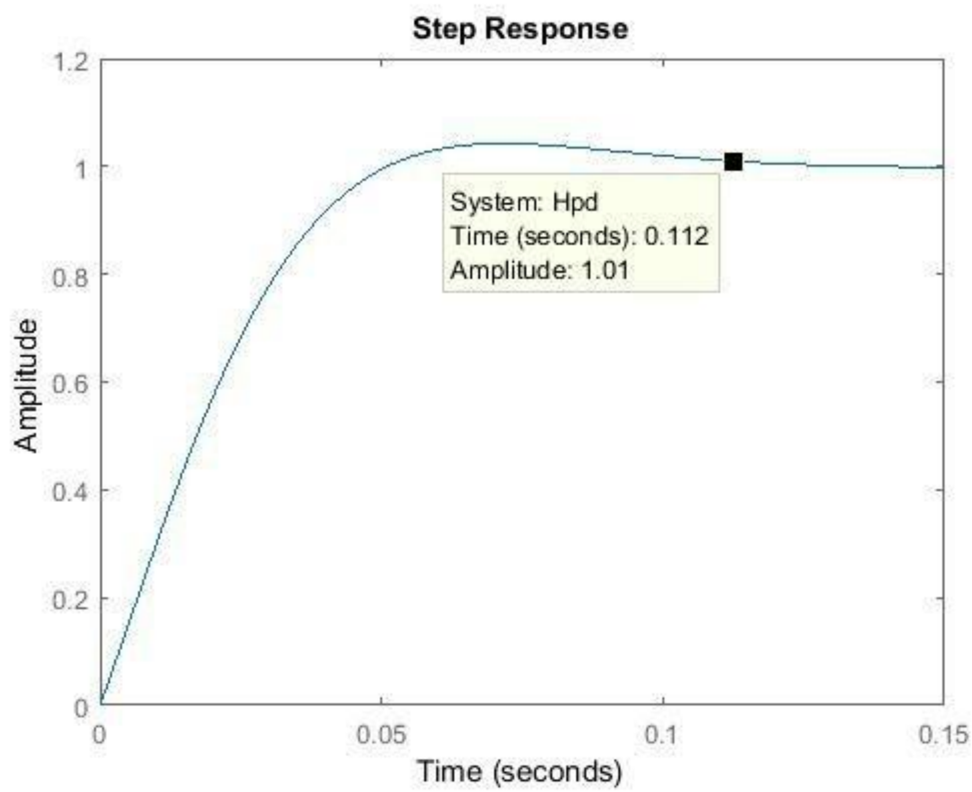


Figure 11- Step Response of the PD Controller Design With New Specifications

Figure 10 proves that the system designed meets the requirements highlighted in table 1. The settling time was reached at 0.112 seconds which is less than 0.177 seconds. The step response also proves that the overshoot of the system was approximately 0.

4.3- Porpotional Integrator Derrivative Controller:

The final requirement of the lab is to design a controller that achieves perfect ramp tracking and a closed loop stability. To design such controller, a PID controller design was used. Ki value was set to 1 The Kp and Kd values from the PD controller designed previously was used when designing the PID controller. Figure 11 represents the step response of the system. The step response proves that the system is a perfect ramp as the slope of the graph is linear.

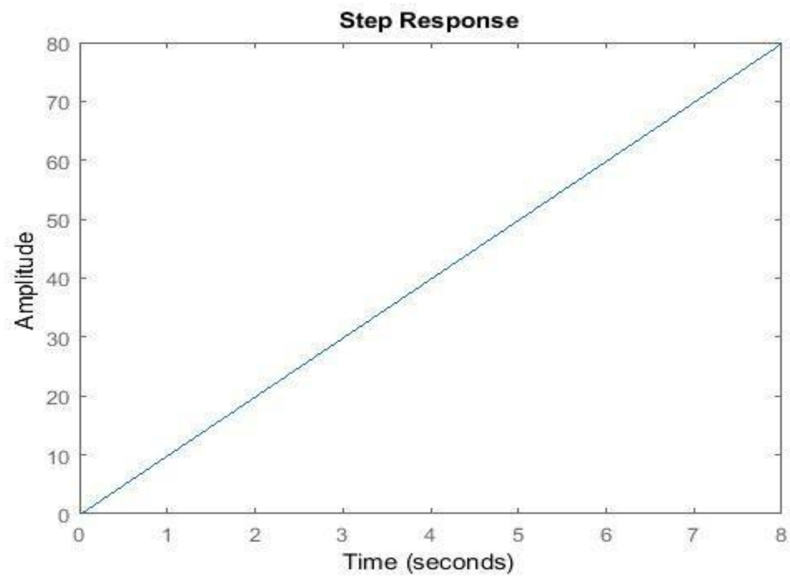


Figure 12- Step Response of the PID Controller Showing Perfect Ramp Tracking

5.0- <u>Experimental Results:</u>

5.1- Pure Porpotional Controller:

A setpoint of 50 was chosen as a reference for the experiment to stay consistent with the previous labs. The Kp values from root locus were used in real motor system. Using the root locus from MATLAB, overshoot of the system was calculated to be around 15% percent, which satisfy the condition in lab to be less than 20 percent. Motor output vs time data was obtained and graph was plotted using MATLAB. Different values of Kp were plugged in the system and value at which motor saturates was obtained. Kp value at which motor saturated was found to be 6.2 and was used in pd controller. Results of the theoretical approach are represented in figure 12 and 13. The motor input vs time graph is represented in figure 14.

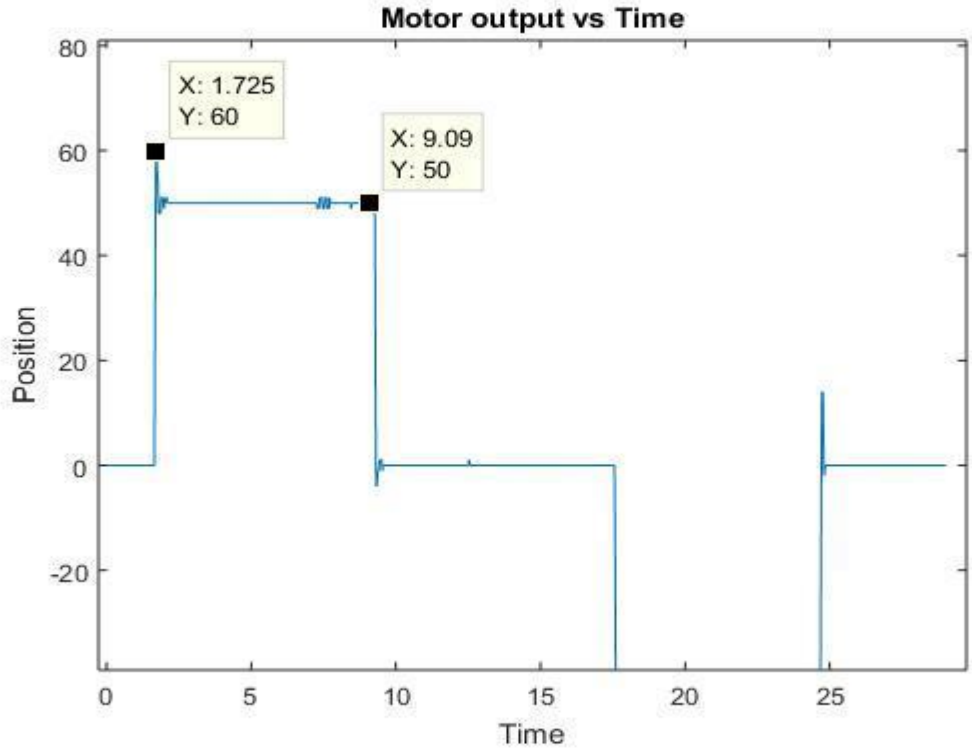


Figure 13- Motor Output Vs Time For The Real Motor Control Using theoretical calculations

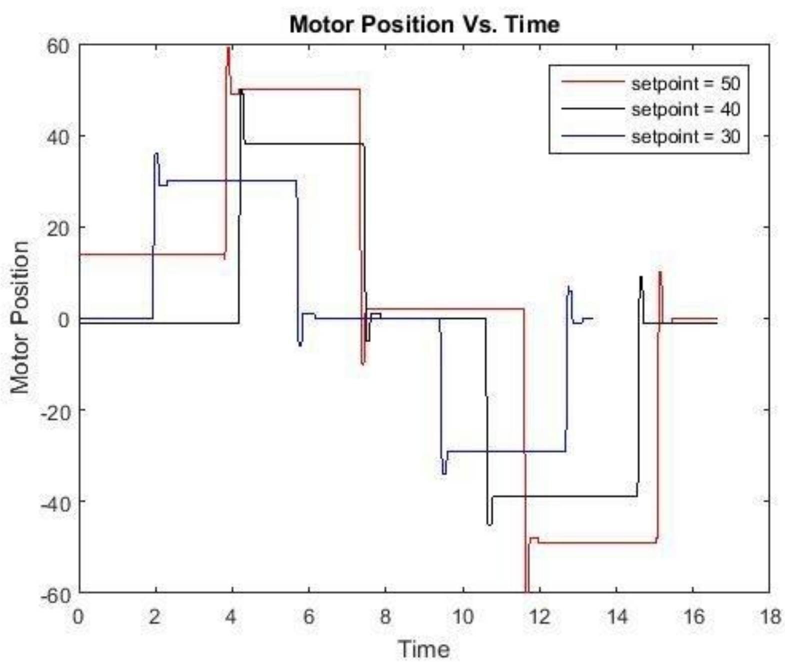


Figure 14- Motor Position Vs. Time For the Real Motor Control of the Designed P Controller

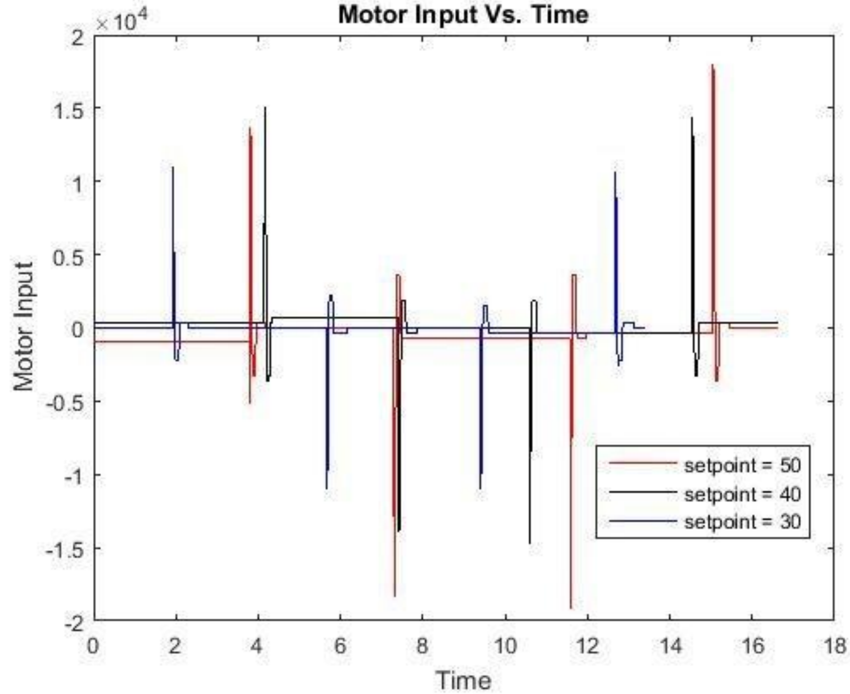


Figure 15- Motor Input Vs. Time For the Real Motor Control of the Designed P Controller

5.2- Porpotional Derrivative Controller:

A setpoint of 50 was used initially for implementing the PD controller. The figure below shows there's no overshoot in the system with the calculated value of Kp and Kd. The value of Kd was obtained from the calculations shown above while the Kp value was obtained from root locus in part 5.1. Perfect tracking can also be seen on graph as theatrical and experimental graphs overlap as shown in figure 19.

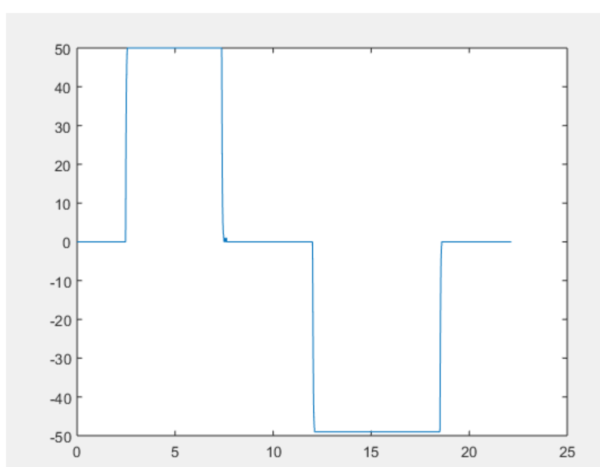


Figure 16- Motor position Vs Time plot of a the designed PD controller

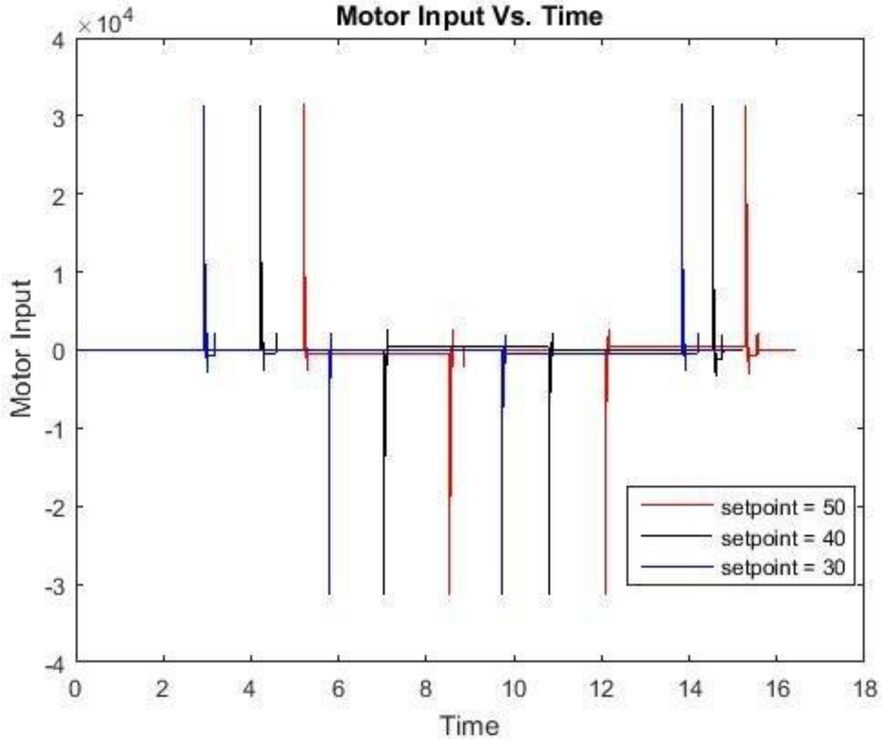


Figure 17- Motor Input Vs. Time For the Real Motor Control of the Designed PD Controller

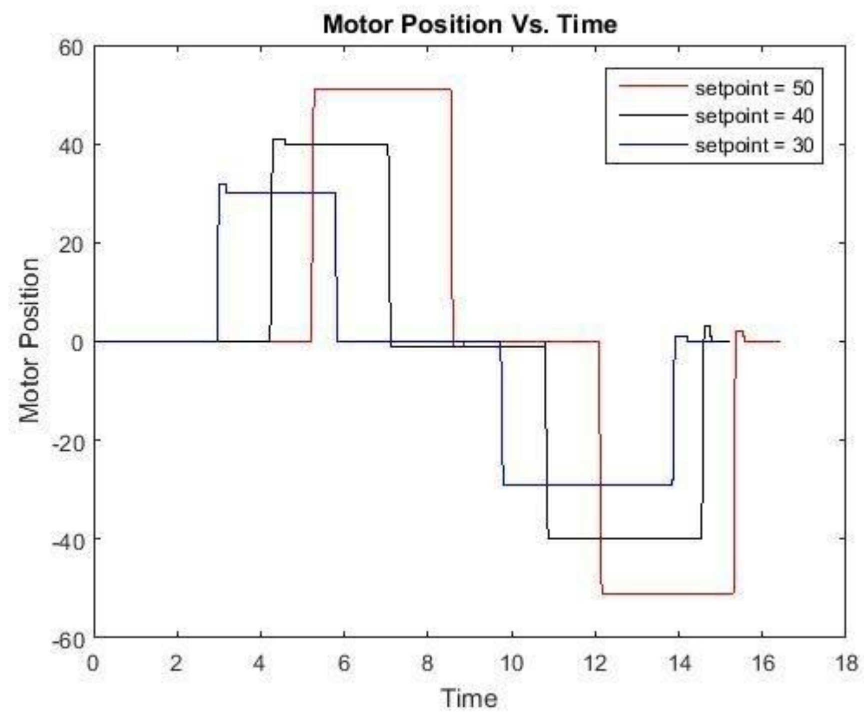


Figure 18- Motor Position Vs. Time For the Real Motor Control of the Designed PD Controller

5.3- Porpotional Integrator Derrivative Controller:

An integral was added to pd controller in parallel and the feedback loop was added because
It doesn't work with open loop system. Equation was formed and RH table was formed to get the range of KI values. Zeros and poles were plotted on maltab along with integral block and zeros were adjusted so a minimum or zero ramp input was obtained. When the simulation was performed it is concluded that actual ramp and simution result overlapped which means that system follows perfect ramping and closed loop stability.

6.0 Discussion

When designing using simulation, the controllers are represented under ideal conditions an operate as expected. To ensure controller design is designed properly and meets the requirements, the simulation result is compare to the theoredical result. Comparing the theoretical and simulation results ensures accuracy. Comparison was executed by overlaying both results on the same graph to make both systems easier to analize. Figure 18 represent the theoretical versus real output of the P controller designed. The blue graph represents the simulation results and the red graph represents the theoretical results. Through analizing figure 18, 19, and 20 the theoretical results seem to have a higher overshoot in all 3 different types of controllers designed in this lab.

Although the overshoot differs in both graphs, controllers still ssatisfy the requirements presented in table 1. For the P controller, the overshoot of the theoretical results was exactly 20%. The overshoot of the simulation results for the P controller was 19.32%. This proves that the system does meet the requirements of the lab. Similar behavior can be seen for the PD and the PID graphs. The theoretical results does not behave exactly as expected compared to the simulation results. These plots indicate that the real overshoot is lower than the theoretical values. This can be due to rounding errors/ calculation errors committed during the theoretical approach. This does not mean that the theoretical results are wrong, but this comparison tells that the simulation results can not be trusted as there is no ideal system in the real world.

The settling points for the simulation and theoretical values appear to line up. The settling point of both results are approximately the same. By looking back at the results section, both results reach steaty state at the same rate and same point of time. Both results meet the required settling time.

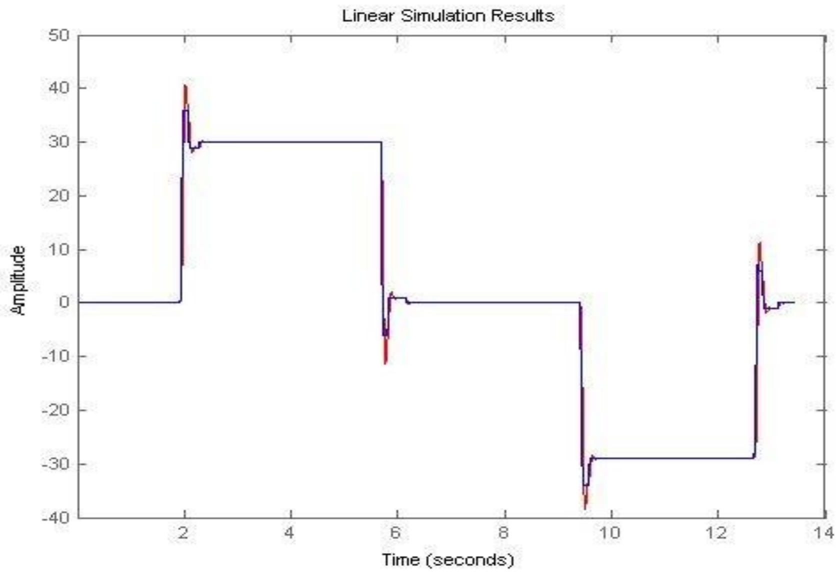


Figure 19- Theoretical versus real output of the P controller

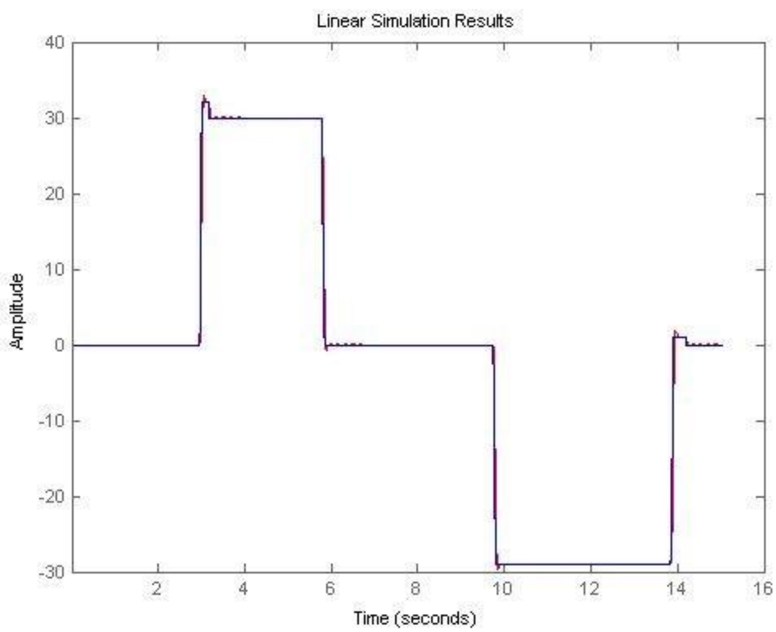


Figure 20- Theoretical versus real output of the PD controller

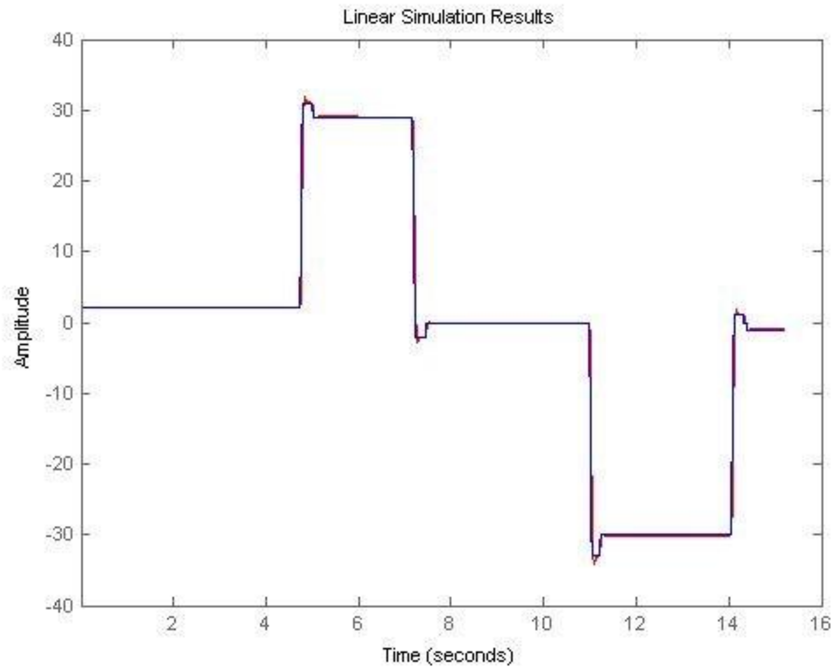


Figure 21- Theoretical versus real output of the PID controller

7.0- Conclusion:

The designed pure porpotional controller met all the required criteria highlighted in table 1. The closed loop stability along with the perfect tracking was met and maintained using the theoretical approach as described in section 3. The root locus was also used to prove that the controller met all criteria. The root locus simulation can be found in section 4. A Kp gain of 5.74 was calculaed using selected settling time of 0.177 sec and an overshoot of 20% based on the transfer function that can be found in figure 1. The controller was applied on the real motor system, and verified using MATLAB, the root locus and the step unit approach.

The porpotional derivative controller was similarly designed using the root locus to select a Kd gain. The Kd gain of 0.303 eliminated overshoot while maintaining perfect tracking and minial settline time. For the porpotional derivative controller, MATLAB and the step response method was used to verify results. Result were applied on the real controller. The designed porpotional derivative controller met all the required criteria as proven by the results section above.

To achieve perfect ramp tracking, a porotional Integral derivative controller was designed. A PID controller was chosen because a PID controller provides ptimal control of the system compared to a PD or a P controller. To design a PID controller, a Ki gain of 1 was selected to ensure effectivity of controller on the system. Through the result section, it was observed that the step response of the system was linear. Linearity proves that the system achieved perfect ramp tracking. The controller was not tested further as the step response test was enough to prove that the main objective was achieved.

This project was very helpful in expanding the knowledge of controllers, and helped in understanding the relationship between performance of controller and having different gain values. Through this lab, MATLAB was a great tool that was used heavily in the design process. MATLAB was used to verify theoredical calculations and results from applying the controllers on the real motor controller. These results were converted to graphs to help in understanding how the controllers behave. The output Vs time and Input vs time graphs showed the relationship of having different gain values. MATLAB provides a great assortment of tools to ensure the theoretical application of a controller.

For future imlementation, MATLAB will be used to plot input vs time and output vs time graphs using theoretical values to analize how the system will behave before jumping into the design process. This will give an overview of the main objective and provide the designer with a basic idea of how his/her design should look like.