UNIVERSITY OF GUELPH

SCHOOL OF ENGINEERING

ENGG*3410

Instructor:
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Lab 3: Modeling

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1.0 Introduction

The main objective of lab 3 is to identify the transfer function of a given DC motor. A step input was applied to the system using different proportional gains. Collected data was analyzed and plotted using MATLAB. Key characteristics such as percent overshoot, settling time, and peak time were obtained through the analyzed data. Obtained characteristics where used to calculate the transfer function. The transfer function was later validated using two other transfer functions with different Kp values

2.0 Results

2.1 Question One

Different gains were recorded during the lab. These results were used to choose an appropriate proportional gain that best represented the system. The best data set was determined by comparing the overshoot of different Kp gains. The system response data set is considered to be a “good” data set when overshoot percentage is between 15-40%. After experimenting, the kp value of 7 recorded the best data set. **Figure 1** shows a plot of motor output vs time. The reference signal was constructed using the error signal and the motor output signal.

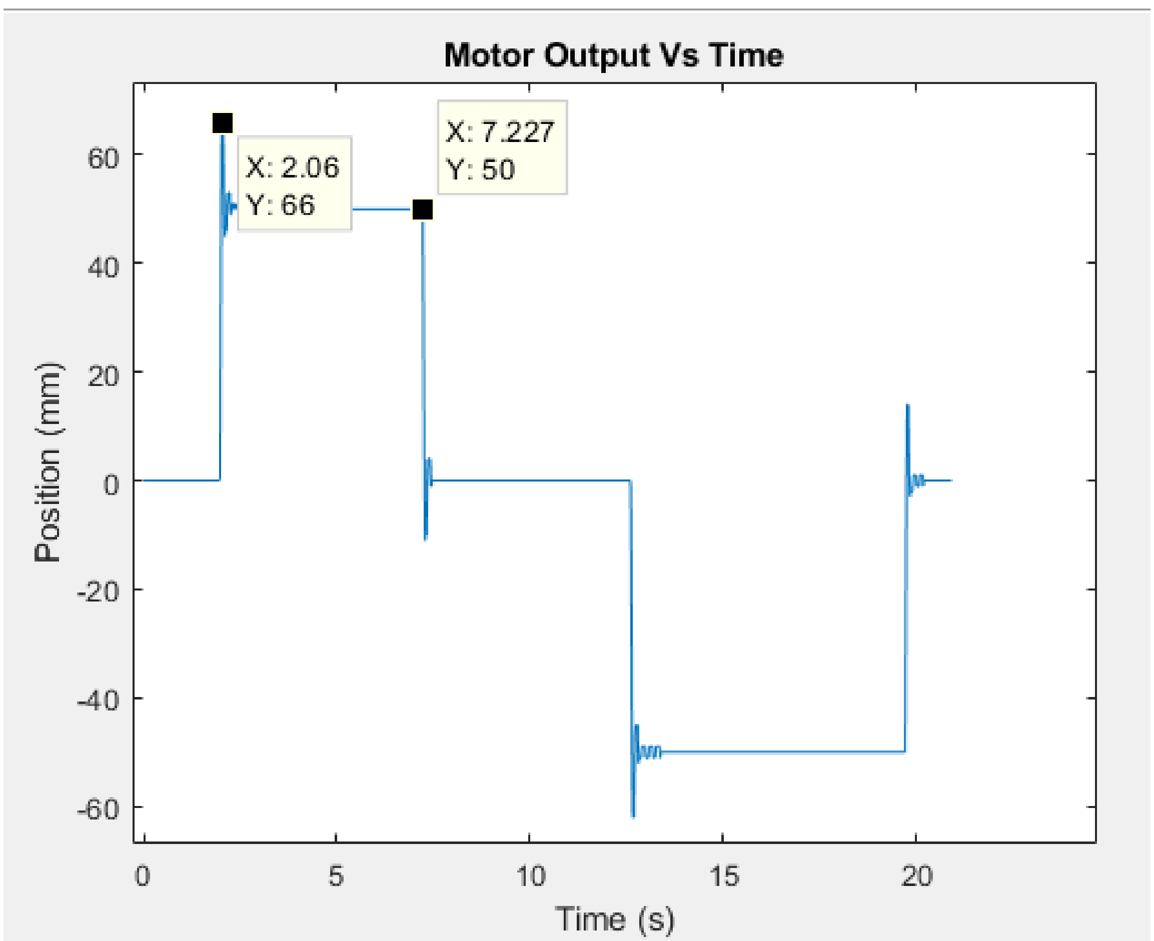

Figure 1 Motor Position and Reference signal vs Time graph for proportional gain, Kp = 7 (best data set)

2.2 Question Two

In lab 3, the percent overshoot, settling time, and peak time corresponding to the proportional gain of 7 were used to find the transfer function of the DC motor. Through observing the overshoot in figure 1, it was assumed that the system was a second order system. Therefore, the following calculations were based on standard form of a second order system.

The following equation was used to calculate the percent overshoot of the system

$$\%\,Overshoot = \left(\frac{ymax - yss}{yss}\right) x\ 100\ \%$$

From **Figure 2**, the percent overshoot of the system was determined to be 32%

$$\%\,Overshoot = \left(\frac{66\ - 50}{50}\right) x\ 100\ \%$$

$$= \quad 32\%$$

The calculated P.O. value is used to calculate the damping ratio using the following formula

$$\zeta = \sqrt{\frac{\left(ln\left(\frac{P.O}{100}\right)\right)^2}{\pi^2 - \left(ln\left(\frac{P.O}{100}\right)\right)^2} = \frac{\left(ln\left(\frac{32}{100}\right)\right)^2}{\pi^2 - \left(ln\left(\frac{32}{100}\right)\right)^2}}$$

ζ **= 0.3892**

The system is considered to be underdamped as the calculated damping ratio is between 0 and 1.

Furthermore, the natural frequency (ω_n) of the system can be found by first calculating the peak time, **Tp** using the selected data points from **Figure 2**, and then using the following formula.

Peak time (Tp) = 2.18 – 2.06s = 0.107s

Using the equation below and calculated values for Tp and damping ratio, $\boldsymbol{\omega_n} =$ **31.87**

$$\boxed{\omega_n = \frac{\pi}{T_p\sqrt{1 - \zeta^2}}}$$

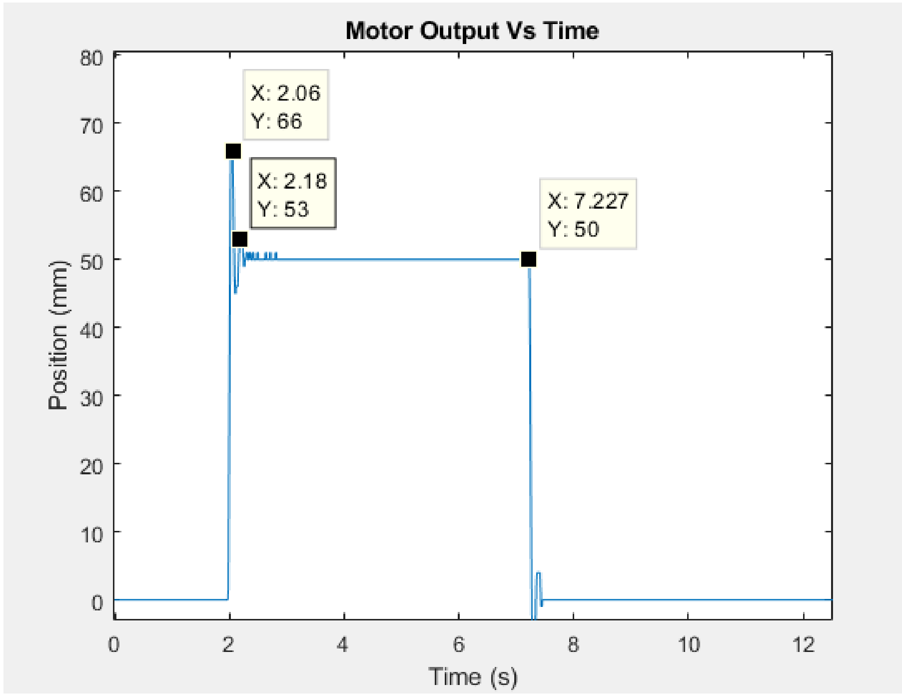

Figure 2 Motor output of the system with selected data points

Using the calculated values for damping ratio and natural frequency, time constant, τ, of the DC motor transfer function and the DC gain, k of the DC motor can be found. The following two equations were used to obtain these values.

$$\tau = \frac{1}{2\zeta\omega_n} \qquad k = \frac{\tau{\omega_n}^2}{k_p}$$

From the above equations, $\boldsymbol{\tau = 0.0403}$ **and** $\boldsymbol{k = 4.815}$

Using the calculated values of the time constant and DC gain of the system and the standard form of a second order transfer function shown below, the transfer function of the DC motor system can be generated

$$H(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Transfer function of the motor wit Kp =7

$$\boldsymbol{H(s) = \frac{1015}{s^2 + 25s + 1015}}$$

2.3 Question Three

The validation for the transfer function was done by comparing the transfer function response to the actual data response for a proportional gain of 5 and 7 (+/- 10%). The verification is done by overlaying the actual system response with the ideal response determined via MATLAB (shown in **Figure 3** and **4**). Similar calculations as shown in *Question 2*, were done for the other two data sets that were used for validation. The following were the results of the calculation.

	Proportional Gain	% Overshoot	Peak time (sec)
Data Set 1 (Main)	7	32%	0.12 s
Data Set 2	6	28%	0.09 s
Data Set 3	5	20%	0.06 s

The following figure plots the response of the response of the system with Kp = 6

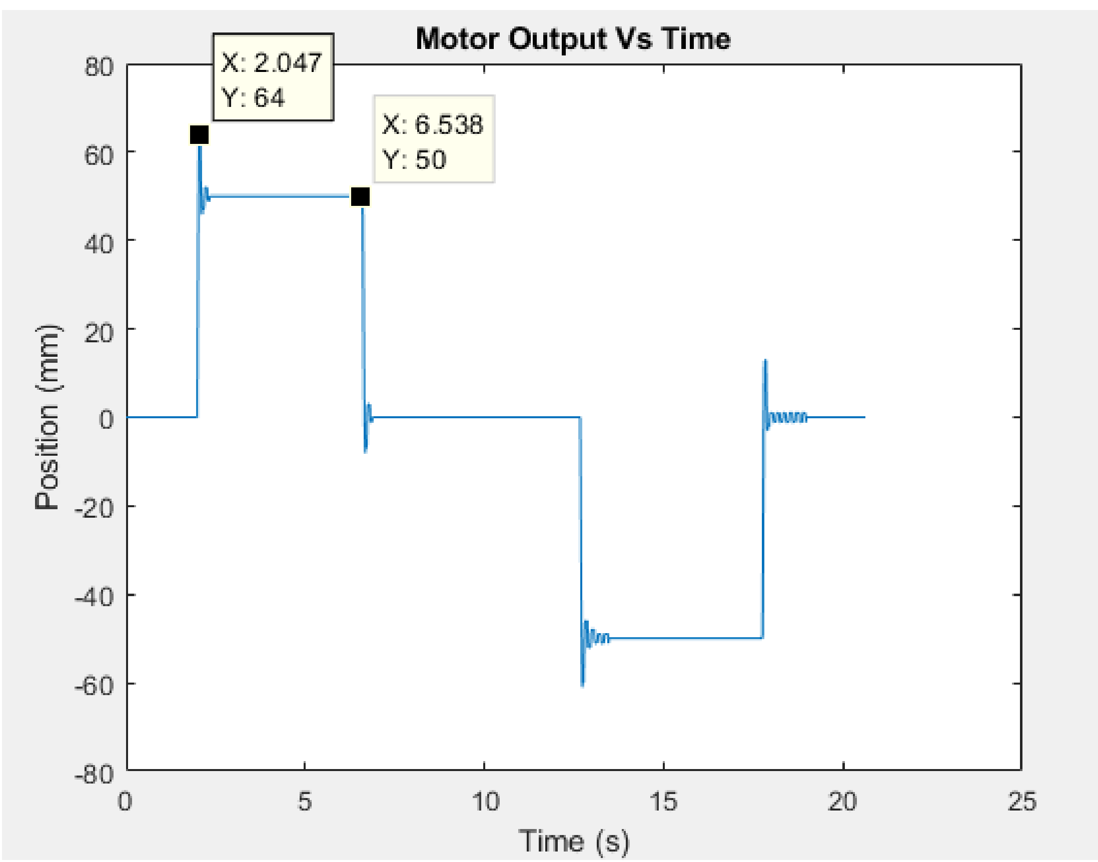

Figure 3 Transfer function validation with Kp =6

Data set 2 (Gain of 6):

$$H(s) = \frac{1516.33}{s^2 + 34.3s + 1248.90}$$

The following figure plots the response of the response of the system with Kp = 5

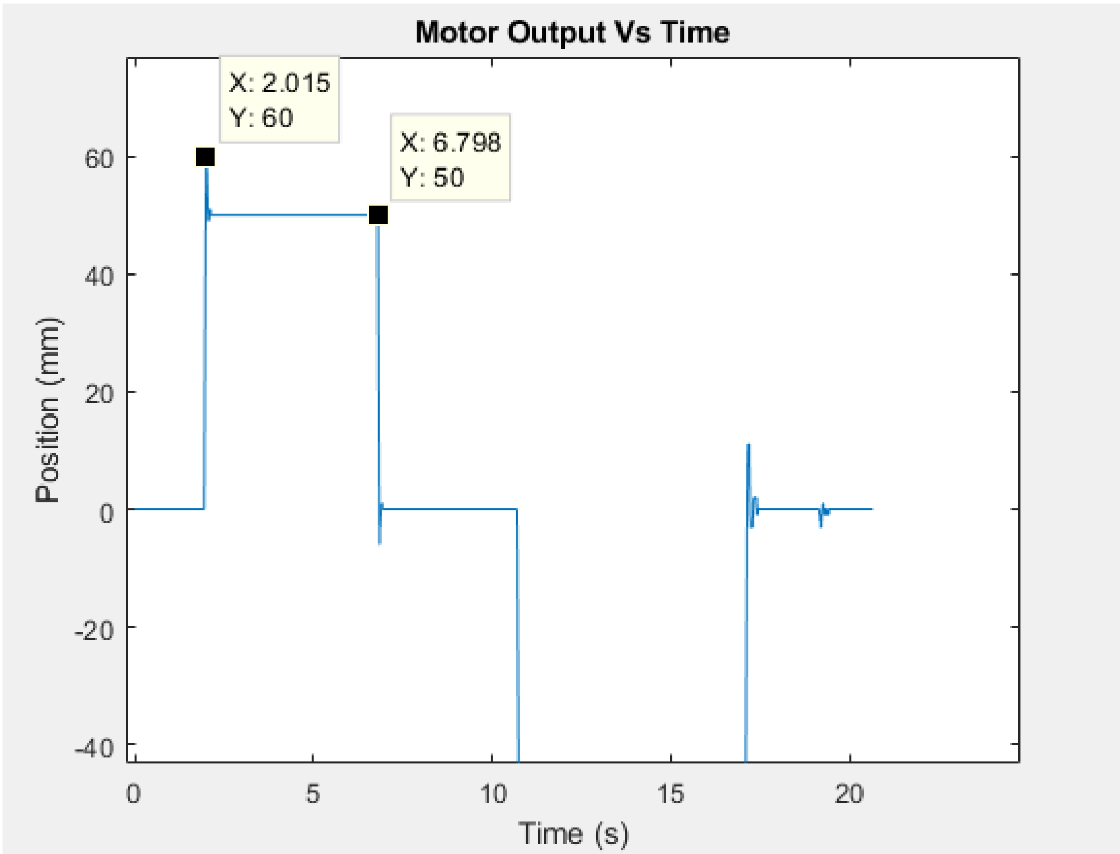

Figure 4 Transfer function validation with Kp =5

Data set 3 (Gain of 5):

$$H(s) = \frac{4256.04}{s^2 + 77s + 4256.04}$$

From the above figures, it can be observed that the motor response closely followed the ideal response behavior (Derived T.F.). Comparing the H(s) of the original equation vs the data set 3 equation, the peak time is 2 times lower. This causes the b coefficient to be much larger (77 compared to 25). Human error plays a part in which the results are not the same.

2.4 Question Four

In this lab, there are many sources of error that can affect the output of the system which results in a different transfer function. Applying input when system is not ready results in inaccurate results. This could have been the main reason for Data set 3. Disturbance in the signal is another source of error as it effects the output of the system. Parallax and human error could have contributed towards discrepancies during calculations for damping ratio, natural frequency, and tau. Such errors could cause inaccuracy when finding the transfer function.

3.0 Conclusions

In conclusion, it was observed that the derived functions followed a close relationship with the actual response of the DC motor. This was validated by testing it against different proportional gains and their transfer functions. Changing the gain of the system also affected the output of the system.