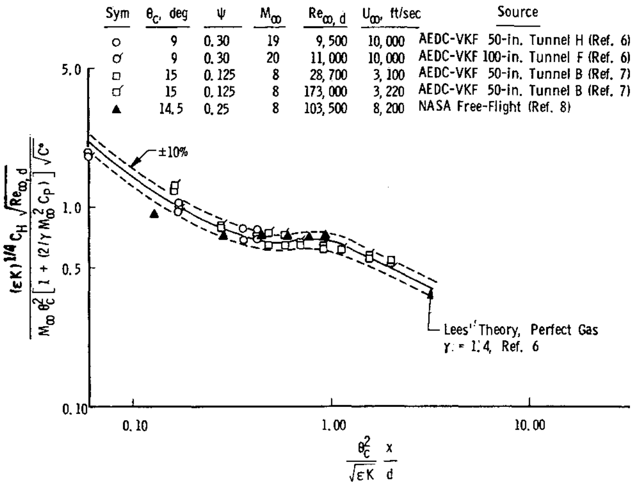

Fig. 4 Correlation of heat-transfer distribution to spherically blunted cones.

Figure 4 presents a correlation (following Griffith and Lewis[6]) of experimental and theoretical heat-transfer data over slender cones. The data presented represent data[7] from the AEDC-VKF 50-in. Mach 8 tunnel (*B*), data[6] from the AEDC-VKF 50- and 100-in. Mach 20 tunnels (*H* and *F*) and free-flight data[8] taken by NASA (NACA) personnel on the conical nose region of a spacecraft configuration. This correlation again indicates that support interference effects on the forebody are insignificant in the flight regime represented by these data.

In conclusion, the comparisons with free-flight data indicate that, for some situations of interest, real-gas effects are insignificant at velocities up to 18,000 fps and that support interference effects are negligible. Therefore, in this speed range it is possible to provide significant flight simulation in terms of Mach and Reynolds numbers alone, i.e., in existing test facilities that operate in the velocity regime of 10,000 fps.

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Reduction of Stiffness and Mass Matrices

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JUST as it is often necessary to reduce the size of the stiffness matrix in statical structural analysis, the simultaneous reduction of the nondiagonal mass matrix for natural mode analysis may also be required. The basis for one such reduction technique may follow the procedure used in Ref. 1 for the stiffness matrix, namely, the elimination of coordinates at which no forces are applied.

Arrange the structural equations $\{F\} = [K]\{x\}$ so that after partitioning in the form

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} A & B \\ B' & C \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

the forces F_2 are to be zero. The two resulting equations yield

$$F_1 = (A - BC^{-1}B')x_1$$

from which the reduced stiffness matrix is seen to be

$$K_1 = A - BC^{-1}B'$$

The foregoing amounts to a coordinate transformation $x = Tx_1$ or

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} I \\ -C^{-1}B' \end{bmatrix} \{x_1\}$$

If the structure energies are written $T = \frac{1}{2}\dot{x}'M\dot{x}$ and $V = \frac{1}{2}x'Kx$ and the foregoing transformation is employed, the result is

$$T = \tfrac{1}{2}\dot{x}_1'T'MT\dot{x}_1$$

$$V = \tfrac{1}{2}x_1'T'KTx_1$$

The reduced stiffness matrix is seen to be $K_1 = T'KT$ and the reduced mass matrix $M_1 = T'MT$. Then with

$$[M] = \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}' & \bar{C} \end{bmatrix}$$

the reduced mass matrix becomes

$$M_1 = \bar{A} - \bar{B}C^{-1}B' - (C^{-1}B')'(\bar{B}' - \bar{C}C^{-1}B')$$

In the case of the reduced stiffness matrix, none of the structural complexity is lost since all elements of the original stiffness matrix contribute. However, in the reduced mass matrix, combinations of stiffness and mass elements appear. The result is that the eigenvalue-eigenvector problem is closely but not exactly preserved. Some comparative results are reported in Ref. 2 for beam vibrations.

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