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Calculating Cross-Orthogonality with Expanded Test Data

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Introduction

VARIOUS means exist for assessing correlation between math model predictions and modal test data. One relatively simple way is the so-called cross-orthogonality (c/o) computation. The pre- and post-multiplication by test, ϕ_{test}, and predicted, ϕ_{anal}, modal vectors is generally performed on the mass matrix M as indicated in Eq. (1).

$$[\text{c/o}] = [\phi_{\text{test}}]^T [M] [\phi_{\text{anal}}] \tag{1}$$

The c/o matrix is then examined for its orthogonality properties. While the concept is simple, cognizance of certain ramifications enhances the assessment of such data.

Compatibility

In general, math model dynamic degrees-of-freedom (DOF) will not be uniquely one-for-one with accelerometers placed upon the prototypes. Consequently, reduction of the math model will be required. If all accelerometers do not lie directly at math model DOF, this reduction will require more than one step. Following the standard force-free reduction, a second calculation must kinematically transform to juxtaposition. Once developed, of course, the double transformation may be combined into one.

This total transformation, applied to the math model mass matrix in a Guyan reduction,[1] satisfies compatibility requirements. The purpose of this Note is primarily to point out that more detailed information is retained if the mass matrix reduced in the above manner is *not* employed. Rather, the transformation (applied to the test modes) essentially provides for their expansion to math model size. Then the c/o computations will employ as independent quantities all DOF in the analytical modal vectors.

Normalization

Analytical modes used in this manner will already be correctly normalized. Test modes in their expanded form must be normalized in the same way, and supplemented by a null matrix.

Equation (2) shows the conventional transformations used to reduce the mass matrix.

$$\underset{N\times N}{\begin{bmatrix} \text{c/o} \end{bmatrix}} = \underset{N\times N}{\begin{bmatrix} \diagdown & & \\ & D & \\ & & \diagdown \end{bmatrix}}^{-1/2} \underset{N\times(A+O)}{\begin{bmatrix} \phi^T_{\text{test}} & \vdots & 0 \end{bmatrix}} \underset{(A+O)\times A}{\begin{bmatrix} T_2 \end{bmatrix}}^T$$

$$\times \underset{A\times(A+O)}{\begin{bmatrix} T_1 \end{bmatrix}}^T \underset{(A+O)\times(A+O)}{\left[\begin{array}{c:c} M_{AA} & M_{AO} \\ \hdashline M_{OA} & M_{OO} \end{array}\right]} \underset{(A+O)\times A}{\begin{bmatrix} T_1 \end{bmatrix}}$$

$$\times \underset{A\times(A+O)}{\begin{bmatrix} T_2 \end{bmatrix}} \underset{(A+O)\times N}{\left[\begin{array}{c} \phi_{\text{anal}_A} \\ \hdashline \phi_{\text{anal}_O} \end{array}\right]} \tag{2}$$

where

A = the kept set, compatible with the test set
O = the omit set, or reduced set
N = the number of modes

$T_1 = \begin{bmatrix} I \\ \hline G_{OA} \end{bmatrix}$ and $G_{OA} = -K_{OO}{}^{-1} K_{OA}$ (Ref. 1)

T_2 = kinematic transformation
= $[I \,\vdots\, 0]$ for coincident test/analysis points
D = test mode normalization factors

Equation (3) shows the proposed method of expanding the test modes to math model size through the transformation $T_2^T T_1^T$.

$$\begin{bmatrix} \text{c/o} \end{bmatrix} = \begin{bmatrix} \diagdown & & \\ & D & \\ & & \diagdown \end{bmatrix}^{-1/2} \begin{bmatrix} \phi_{\text{test}} & \vdots & 0 \end{bmatrix}^T$$

$$\times \begin{bmatrix} T_2 \end{bmatrix}^T \begin{bmatrix} T_1 \end{bmatrix}^T \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} \phi_{\text{anal}} \end{bmatrix} \tag{3}$$

Since dynamically the T_1 matrix will tend to introduce error through the static reduction term G_{OA} $(\phi_O \cong G_{OA}\phi_A)$, the elimination of one T_1 term by Eq. (3) provides a more accurate and simplified solution.

Reference

[1] Guyan, R. J., "Reduction of Stiffness and Mass Matrices," *AIAA Journal*, Vol. 3, Feb. 1965, p. 380.

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Amplification of Discontinuities in a Radiating Gas

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IN our earlier paper,[1] using the optically thin approximation to the radiative transfer equation, we discussed the effects of radiative flux on the amplification or attenuation of finite amplitude waves propagating into a region assumed to be uniform and at rest. In a radiating gas, the validity of this assumption is questionable, because the tendency of a thermal precursor, which propagates with a velocity comparable to that of light, is, in general, to disturb the flow ahead of a modified gasdynamic wave, see, e.g., Lick,[2] Prasad,[3] and Helliwell.[4] Thus, in the study of wave propagation in a radiating gas, one should, in general, take into account the unsteady behavior of the flow ahead of the wave.

The purpose of the present Note is to reanalyze the problem considered in Ref. 1, keeping in view the unsteady behavior of the flow ahead of the wave surface. It is shown that along the bicharacteristic curves of the governing differential equations, the growth equation for a weak discontinuity reduces to a Bernoulli type equation, the solution of which has been completely analyzed in one of our papers.[5] Using the general

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