MAT 271E Probability and Statistics – 2015-2016 Spring - Midterm 1

Do all parts of all problems. Show your work for credit. Write your name on all submitted sheets. 100 minutes

1. Suppose we have three events A,B,C. Each event occurs with probability 0.4. Events A and C can not occur together. If we know that event B occurs, event A occurs with probability 0.3 and event C occurs with probability 0.7.

i) What is the probability that at least one of the events A, B or C occurs? (10 pts)

ii) What is the probability that event A occurs, if we know that B does not occur? (10 pts)

2. A marksman is able to hit a target 99 out of 100 times, on the average, when he fires his rifle. On a practice session, he fires 200 rounds. He is rated satisfactory if he hits the target more than 195 times.

i) What is the probability that he does not miss a shot in a practice session? (10 pts)

iii) What is the probability that he hit the target more than 198 times if he is rated satisfactory in a practice session? (15 pts)

3. Can two events be disjoint and independent at the same time? What does this imply about either one or both of these two events? (10 pts)

4. Using the six letters *a,b,c,d ,e,f,* we write sequences that are 4 letters long where each letter occurs in each sequence no more than once.

i) How many such sequences can you write? (5 pts)

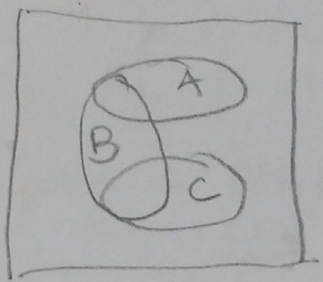
ii) If each sequence is equally likely as the others, what is the probability of observing a particular sequence? (5 pts)

iii)What is the probability of observing a sequence with *a* and *b* in it? (15 pts)

5. There are two light bulb brands, brand A and brand B. On the average, one out of 10^5 Brand A light bulbs are defective. On the average, two out of 10^5 Brand B light bulbs are defective. 40% of the defective light bulbs are of brand A. What is the market share percentage of brand A bulbs? (20 pts)

MAT 271E Midterm I Solutions 2015-2016

1.



$P(A) = P(B) = P(C) = 0.4$

$P(A|B) = 0.3$

$P(C|B) = 0.7$

i) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C)$

$= 1.2 - (0.3)(0.4) - (0.7)(0.4)$

$= 0.8$

ii) $P(A) = P(A \cap \bar{B}) + P(A \cap B)$

$0.4 = P(A|\bar{B})P(\bar{B}) + P(A|B)P(B)$

$0.4 = P(A|\bar{B})(0.6) + (0.3)(0.4)$

$P(A|\bar{B}) = \dfrac{0.28}{0.6} = 0.466$

2.

i) $\Pr\left\{ \begin{matrix} 0 \text{ misses in} \\ 200 \text{ trials} \end{matrix} \right\} = \binom{200}{200}(0.01)^0 (0.99)^{200}$

$n = 200$

$p = 0.01$

$a = np = 2$

$= \dfrac{e^{-2} a^0}{0!}$

$= e^{-2} = \dfrac{1}{(2.7)^2} \approx 0.135$

ii) $Pr\left\{ \begin{matrix}199 \text{ or } 200 \text{ hits} \\ \text{in } 200 \text{ rounds}\end{matrix} \middle| \begin{matrix}196, 197, 198, 199, 200 \text{ hits} \\ \text{in } 200 \text{ rounds}\end{matrix} \right\}$

$$= \frac{Pr\left\{ \begin{matrix}199 \text{ or } 200 \text{ hits} \\ \text{in } 200 \text{ rounds}\end{matrix} \right\}}{Pr\left\{ \begin{matrix}196, 197, 198, 199, 200 \text{ hits} \\ \text{in } 200 \text{ rounds}\end{matrix} \right\}}$$

$$= \frac{Pr\left\{ \bigcup_{k=0}^{1} \{k \text{ misses}\} \right\}}{Pr\left\{ \bigcup_{k=0}^{4} \{k \text{ misses}\} \right\}}$$

$$= \frac{e^{-2} + e^{-2} \cdot 2}{e^{-2} + e^{-2} 2 + e^{-2} \frac{2^2}{2!} + e^{-2} \frac{2^3}{3!} + e^{-2} \frac{2^4}{4!}}$$

$$\approx \frac{3}{7}$$

3. disjoint: $A \cap B = \emptyset \Rightarrow P(A \cap B) = \emptyset$.

independent: $P(A \cap B) = P(A)P(B)$

$P(A)P(B) - \emptyset$

implies

$P(A) = \emptyset$ and/or $P(B) = \emptyset$

4. a). # of paths in a 4-level tree where each node has one less sibling than its parent. Therefore $6.5.4.3 = \frac{6!}{\underbrace{2!}_{n-k}} = 360$

$n = 6$
$k = 4$

b) $Pr\left\{ \begin{matrix}\text{a particular} \\ \text{4 letter sequence}\end{matrix} \right\} = \frac{1}{360}$

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c) $\Pr\left\{\begin{matrix}\text{Observing a} \\ \text{sequence with "a"} \\ \text{and "b" in it}\end{matrix}\right\} = \dfrac{\text{\# sequences having "a" and "b"}}{\text{total \# sequences.}}$

To count the # in the numerator we first place "a" and "b". We can place "a" in 4 different ways. We can then place "b" in 3 different ways. Therefore we can place "a" and "b" in 12 different ways. In the remaining 2 places there are 4 possibilities for the first place followed by 3 possibilities for the second place. Therefore there are $12 \times 12 = 144$ sequences that have "a" and "b" in it.

$$\Pr\left\{\begin{matrix}\text{Observing a} \\ \text{sequence with} \\ \text{"a" and "b" in it}\end{matrix}\right\} = \frac{144}{360} = \frac{2}{5}$$

5. D: Defective

$P(D|A) = \dfrac{1}{10^5}$ $\quad P(D|B) = \dfrac{2}{10^5}$

$P(A|D) = 0.4.$

$P(A) = ?$

$P(B) = 1 - P(A)$

$$P(A|D) = \frac{P(D|A)\,P(A)}{P(D|A)\,P(A) + P(D|B)\,(1-P(A))}$$

$$0.4 = \frac{10^{-5}\,P(A)}{10^{-5}\,P(A) + 2\times10^{-5}\,(1-P(A))}$$

$0.8\times10^{-5} - 0.4\times10^{-5}\,P(A) = 10^{-5}\,P(A)$ $\quad P(A) = \dfrac{0.8}{1.4} \simeq 0.57$