

Sets:

De Morgan Laws: $\overline{A \cup B} = \bar{A} \cap \bar{B}$, $\overline{A \cap B} = \bar{A} \cup \bar{B}$ Subtraction: $A - B = A / B = A \cap \bar{B}$

Distribution properties: $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$, $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Probability:

Axioms: **1)** $P(A) \geq 0$, **2)** $P(S) = 1$, **3)** $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \phi$

In general $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq P(A) + P(B)$

Classical definition: $P(A) = \frac{n_A}{n}$, n : no. equally likely outcomes

Frequency definition: $P(A) = \lim_{k \rightarrow \infty} \frac{k_A}{k}$, k : trials

Conditional probability: $P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{n_{AB}}{n_B}$

Partition: If B_i 's partition S then $\bigcup_i B_i = S$ and $B_i \cap B_j = \phi$

Total probability: $P(A) = \sum_i P(A \cup B_i) = \sum_i P(A | B_i) P(B_i)$

Independence: A ind. of B ($A \cap B \neq \phi$) $\Rightarrow P(A \cap B) = P(A)P(B)$, $P(A|B) = P(A)$, $P(B|A) = P(B)$

De Moivre Laplace approximation for binomial formula: Remember $\binom{N}{k} = \frac{N!}{(N-k)!k!}$

$\binom{N}{k} p^k (1-p)^{N-k} \approx \frac{1}{\sqrt{2\pi Np(1-p)}} \exp\left(-\frac{1}{2} \frac{(k - Np)^2}{Np(1-p)}\right)$ good for $N, k, N-k$ large, $|k - Np| < \sqrt{Np(1-p)}$

$\sum_{k=k_1}^{k_2} \binom{N}{k} p^k (1-p)^{N-k} \approx F\left(\frac{k_2 - Np}{\sqrt{Np(1-p)}}\right) - F\left(\frac{k_1 - Np}{\sqrt{Np(1-p)}}\right) = \text{erf}\left(\frac{k_2 - Np}{\sqrt{Np(1-p)}}\right) - \text{erf}\left(\frac{k_1 - Np}{\sqrt{Np(1-p)}}\right)$

x	F(x)	erf(x)	x	F(x)	erf(x)	x	F(x)	erf(x)
0	0.5	0	0.6	0.7257	0.2257	1.05	0.8531	0.3531
0.1	0.5398	0.0398	0.7	0.7580	0.2580	1.1	0.8643	0.3643
0.2	0.5793	0.0793	0.8	0.7881	0.2881	1.2	0.8849	0.3849
0.3	0.6179	0.1179	0.9	0.8159	0.3159	1.3	0.9032	0.4032
0.4	0.6554	0.1554	0.95	0.8289	0.3289	1.4	0.9192	0.4192
0.5	0.6915	0.1915	1.0	0.8413	0.3413	2.0	0.9773	0.4772

$F(-x) = 1 - F(x)$

$\text{erf}(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$

Poisson approximation for binomial formula: $\binom{N}{k} p^k (1-p)^{N-k} \approx \frac{e^{-Np} (Np)^k}{k!}$ for $N \gg 1$, $p \ll 1$, $Np = a$