



COMPILATION OF A TYPED R-LIKE LANGUAGE TO WEBASSEMBLY

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Bachelor's thesis
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December 28, 2025

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Citation of this thesis: Baljinnyam Bilguudei. *Compilation of a typed R-like language to WebAssembly*. Bachelor's thesis. Czech Technical University in Prague, Faculty of Information Technology, 2026.

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Declaration

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In Prague on December 28, 2025

Abstract

R is a widely-used dynamic language for statistical computing, but its dynamic nature prevents efficient ahead-of-time compilation. We present the design and implementation of a compiler for a statically-typed subset of R that targets WebAssembly. The defined language retains R’s core characteristics, including first-class vector operations and lexical scoping, while introducing static typing to enable compilation. The compiler leverages the WebAssembly Garbage Collection proposal, which simplifies memory management and enables efficient representation of high-level language constructs. Source programs are transformed through parsing, type checking, and intermediate representation lowering to generate WebAssembly bytecode, supported by a runtime system providing vector operations and host environment interoperability. Evaluation on representative statistical programs demonstrates the viability of compiling R-like languages to WebAssembly using modern proposals.

Keywords WebAssembly, WASM GC, typed R, WASM bytecode, pass-based Compiler

Abstrakt

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Klíčová slova enter, comma, separated, list, of, keywords, in, CZECH

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List of abbreviations

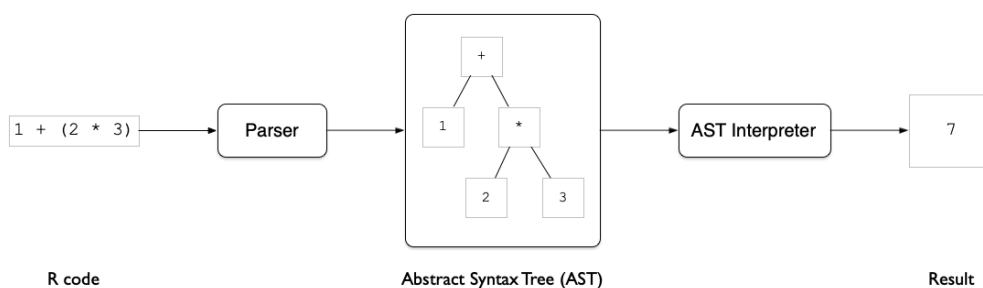
WASM	Web Assembly
WAT	Web Assembly Text
GC	Garbage Collector
IR	Intermediate Representation

Chapter 1

Introduction

The R programming language is ubiquitous for statistical computing and data analysis, offering powerful abstraction for data manipulation and visualization. Inspired by S-Language, the R language has been an important part for industries working with statistics and been a pioneer for introducing widespread tools currently used by the community such as DataFrame.

Traditionally, R is an interpreted language that runs on most major operating systems. By interpreted language, we mean that the language source code is not translated to certain binary to be executed by a processor like compiled languages. Instead, an interpreter is a program, usually written in low-level languages such as C, C++ or Rust, which takes the source code, does lexing and parsing to get AST and directly executes the program from its AST representation by evaluating from the top node[1]. Figure 1.1 shows the pipeline of R code being interpreted.



■ **Figure 1.1** Example of R program being interpreted

Nowadays, the industry has been expanding the environments where we can run our programs. The interpreter program we talked about, is traditionally compiled to bytecode, and then binary that the processor can execute. However, with the invention of WebAssembly, we can take this interpreter and compile it to so-called WASM bytecode, and run it on the web. This is exactly what webR(cite webR) did. This way, R code we write, can seamlessly run on

the web environment on most browsers.

Compiling the interpreter to WASM introduces certain challenges. Since the interpreter consists of a substantial amount of code, every R program executed in the browser must include the entire interpreter compiled to WASM. This increases the overall memory footprint and can negatively impact performance, particularly in resource-constrained environments.

Thus, this thesis explores direct compilation of R-like language to WebAssembly. I create subset of R with type annotations, in order to make compilation possible to WASM, as it needs static typing. Then explore the challenges of mapping a dynamic high-level language to WASM, and finally evaluate the performance and correctness of my compiler.

Background

2.1 The R Programming Language

R is a domain-specific language designed for statistical computing and graphics, originally developed by Ross Ihaka and Robert Gentleman in the early 1990s as an open-source implementation of the **S** language. With its CRAN package manager providing more than 20000 packages, it's a widely programming language used by industries where experimenting with data is involved, such as data analytics, data mining and bio-informatics.

2.1.1 Key Characteristics

- **Dynamic typing:** Variables have no declared types; types are determined at runtime. This enables rapid prototyping but prevents static verification and certain compiler optimizations.
- **Vectorization:** Operations apply element-wise to vectors, matrices, and arrays. For example,

$$c(1,2,3) + c(4,5,6) \Rightarrow c(5,7,9)$$

without the need for explicit loops.

- **Lexical scoping:** R uses lexical (static) scoping with closures. Functions capture their defining environment, enabling functional programming patterns and higher-order abstractions.
- **Lazy evaluation:** Function arguments are evaluated lazily (call-by-need), allowing non-standard evaluation mechanisms that support domain-specific sublanguages.
- **Copy-on-modify semantics:** R employs implicit copying to preserve referential transparency. While this simplifies reasoning about programs, it may introduce performance overhead in memory-intensive workloads.

2.1.2 Assignment Operators

R provides multiple assignment operators with distinct scoping behavior:

- `<-`: Regular assignment in the current environment.
- `<<-`: Superaassignment, which modifies the nearest existing binding in an enclosing environment.

2.1.3 Type System

R employs a dynamic type system with several foundational types, all of which are first-class objects. Unlike statically-typed languages, type information is associated with values at runtime rather than with variable declarations. Atomic Types R provides six atomic vector types:

- **Logical:** Boolean values `TRUE`, `FALSE`, and `NA` (missing).

```
x <- TRUE
typeof(x)      # returns "logical"
```

- **Integer:** Whole numbers, denoted with an `L` suffix.

```
x <- 42L
typeof(x)      # returns "integer"
```

- **Double:** Floating-point numbers (default numeric type).

```
x <- 3.14
typeof(x)      # returns "double"
```

- **Character:** Strings of text.

```
x <- "hello"
typeof(x)      # returns "character"
```

- **Complex:** Complex numbers with real and imaginary parts.

```
x <- 2 + 3i
typeof(x)      # returns "complex"
```

- **Raw:** Raw bytes (rarely used).

```
x <- charToRaw("A")
typeof(x)      # returns "raw"
```

Composite Types Beyond atomic vectors, R supports several composite data structures:

- **List:** Heterogeneous collections that can hold elements of different types.

```
x <- list(42, "text", TRUE)
typeof(x)      # returns "list"
```

- **Function:** Functions are first-class objects.

```
f <- function(x) x + 1
typeof(f)      # returns "closure"
```

This means functions are treated as normal variables. It can be nested definition, passed as parameter and returned from a function.

- **Environment:** Hash-like structures for variable scoping.

```
e <- new.env()
typeof(e)      # returns "environment"
```

Type Coercion R performs implicit type coercion following a hierarchy: logical \rightarrow integer \rightarrow double \rightarrow character. When combining types, R automatically converts to the most general type:

```
c(TRUE, 1L, 2.5, "text") # returns character vector:
# "TRUE" "1" "2.5" "text"
```

This automatic coercion simplifies interactive use but can lead to unexpected behavior if types are not carefully managed.

2.1.4 Peculiarities

As every programming language comes with their nuances and uniqueness, R is no short of those. Below, the ones mention worthy examples

- Vectors and lists are indexed by 1. For example,

```
v <- c(10,20,30) // initializes a vector of 10,20,30
v[1]             // returns 10
v[2]             // returns 20
```

- Even a scalar is represented as a vector of 1 element.

```
x <- 5           // assign 5
is.vector(x)     // return TRUE
```


The reason for it is to recycle the vectors easily. The language is designed for vector operations

```
x <- 5           // assign 5
v <- c(10,20,30) // initializes a vector of 10,20,30
x + v           // returns a vector of 15,25,35
```

2.2 WebAssembly

WebAssembly is a binary instruction format designed as a portable compilation target for high-level languages. In simpler terms, it's a bytecode format made to be a common-ground between web environment and different programming languages. For web environment, JavaScript is the only language that can be executed. However, with WebAssembly, commonly known as WASM, we can execute languages such as C, Rust in web environment after compiling them to WASM bytecode format. WASM main characteristics include:

- Stack-based virtual machine: WASM uses a structured stack machine with explicit control flow constructs (block, loop, if) rather than arbitrary jumps.
- Linear memory: A contiguous, resizable array of bytes for heap allocation, isolated from the host environment for security.
- Type safety: WASM enforces type safety at validation time, preventing type confusion and memory safety violations.

Language Design

3.1 Design Goals

To design typed R, I tried to be as close as possible to R semantics to easily compile available R codes with wasmR. Although there are obviously inherent limitations compiling dynamic language to statically typed bytecode. Issues include mutability, typing, reflections. For example, how do we deal with

```
var <- 21 // assign integer
print(var)
var <- c(1,2,3) // assign vector
```

This is a perfectly fine code for R but compiling it to WASM brings difficulties. Therefore, like other static languages, we will add checking against mutability of different types.

- R compatibility: Preserve R's syntax and core semantics where possible
- Type safety: Static type checking with sound type system
- Ahead-of-time compilation
- A single executable WASM file, no linking required
- First-class functions: Support functional programming with closures
- Practicality: Provide essential functions for data processing (builtins)

3.2 Typed R-like Language

This section presents the design of a statically-typed programming language inspired by R's syntax, which we refer to as the *Typed R-like Language*. The language maintains R's characteristic features such as the left-assignment operator

(`<-`), first-class functions, and vector-oriented programming, while introducing a static type system to enable ahead-of-time compilation to WebAssembly.

The language supports:

- **Static typing** with type inference and explicit type annotations
- **First-class functions** with closures and lexical scoping
- **Vector operations** as a fundamental data structure
- **Control flow** constructs including conditionals and loops
- **Higher-order functions** enabling functional programming patterns
- **Lexical scoping** with support for nested functions and variable capture
- **Named arguments** enabling option to have positional and optional arguments

The design philosophy emphasizes a familiar R-like syntax while ensuring type safety and efficient compilation to WebAssembly. Unlike dynamically-typed R, all type information is resolved at compile time, enabling optimized code generation and early error detection.

3.3 Syntax

The syntax of the Typed R-like Language closely follows R conventions with extensions for explicit type annotations. This section describes the core syntactic constructs.

3.3.1 Lexical Elements

The language uses the following lexical tokens:

- **Keywords:** `function`, `if`, `else`, `for`, `in`, `while`, `return`
- **Type keywords:** `int`, `double`, `string`, `char`, `void`, `logical`, `any`, `vector`, `list`
- **Operators:**
 - Assignment: `<-` (assignment), `<<-` (super-assignment)
 - Arithmetic: `+`, `-`, `*`, `/`, `%%` (modulo)
 - Comparison: `==`, `!=`, `<`, `<=`, `>`, `>=`
 - Logical: `&` (and), `|` (or), `!` (not)
 - Range: `:` (sequence generation)

- Type annotation: `:` (in declaration context)
- Function arrow: `->` (in type signatures)
- **Delimiters:** `(,)`, `{, }`, `[,]`, `,`
- **Literals:** Numeric literals, string literals (double-quoted), logical literals (`TRUE`, `FALSE`)
- **Identifiers:** Alphanumeric sequences starting with a letter or underscore
- **Special:** `...` (varargs placeholder)

3.3.2 Variable Declaration and Assignment

Variables are declared using the left-assignment operator with optional type annotations:

■ **Code listing 3.1** Variable assignment examples

```
# Simple assignment with type inference
x <- 10

# Assignment with explicit type annotation
y: int <- 20

# String assignment
name: string <- "Hello"

# Vector assignment
vec <- c(1, 2, 3)
```

The super-assignment operator `<<-` modifies variables in enclosing function scopes:

■ **Code listing 3.2** Super-assignment example

```
outer <- function() {
  x <- 0
  inner <- function() {
    x <<- 10 # Modifies x in outer scope
  }
  inner()
  return(x) # Returns 10
}
```

3.3.3 Function Definitions

Functions are first-class values defined using the `function` keyword:

Code listing 3.3 Function definition examples

```
# Simple function with type annotations
add <- function(a: int, b: int): int {
    return(a + b)
}

# Function returning a vector
create_vector <- function(): vector<double> {
    return(c(1.0, 2.0, 3.0))
}

# Higher-order function
apply_twice <- function(f: int -> int, x: int): int {
    return(f(f(x)))
}

# Function with default parameters
greet <- function(name: string, prefix: string = "Hello"): string {
    return(prefix) # Simplified example
}
```

Function types use arrow notation: `param_types -> return_type`. Multiple parameters are comma-separated, and parentheses group complex function types when used as parameters.

3.3.4 Control Flow

3.3.4.1 Conditional Statements

The language supports `if-else` statements and expressions:

Code listing 3.4 Conditional examples

```
# If statement
if (x > 0) {
    print(x)
}

# If-else statement
if (x > 0) {
    y <- 1
} else {
    y <- -1
}

# If expression (returns value)
result <- if (x > 0) { 1 } else { -1 }
```

3.3.4.2 Loops

Two loop constructs are provided: `for` and `while`.

■ **Code listing 3.5** Loop examples

```
# For loop iterating over range
for (i in 1:10) {
  print(i)
}

# For loop iterating over vector
vec <- c(1, 2, 3, 4, 5)
for (x in vec) {
  print(x)
}

# While loop
i <- 1
sum <- 0
while (i <= 5) {
  sum <- sum + i
  i <- i + 1
}
```

3.3.5 Expressions

The language supports various expression forms:

- **Literals:** 42L, 3 3.14, TRUE, FALSE
- **Identifiers:** Variable references
- **Binary operations:** Arithmetic, comparison, logical, range (1:10)
- **Unary operations:** Negation (`-x`), logical not (`!x`)
- **Function calls:** `f(arg1, arg2)` with positional or named arguments
- **Vector indexing:** `vec[i]`
- **Vector construction:** `c(1, 2, 3)`
- **Function definitions:** Anonymous functions as expressions
- **If expressions:** Conditional expressions returning values

3.3.6 Blocks and Tail Expressions

Blocks consist of zero or more statements followed by an optional tail expression. The tail expression (final expression without semicolon) determines the block's value:

■ **Code listing 3.6** Block with tail expression

```
f <- function(x: int): int {  
  y <- x * 2  
  z <- y + 1  
  z # Tail expression - returned automatically  
}
```

3.4 Type System

The type system ensures type safety through static analysis while supporting type inference to maintain concise syntax. Though it's important to note that there are many programs where type inference will fail to identify the type and our compiler won't work due to lack of type information. (reference needed as to why type inference is hard)

3.4.1 Primitive Types

The language provides the following primitive types:

- **int**: 32-bit signed integers (maps to WebAssembly i32)
- **double**: 64-bit floating-point numbers (maps to WebAssembly f64)
- **logical**: Boolean values (TRUE or FALSE)
- **void**: Absence of value (used for functions with no return)

3.4.2 Composite Types

3.4.2.1 Vector Types

Vectors are homogeneous arrays parameterized by element type:

■ **Code listing 3.7** Vector type examples

```
# Vector of integers  
v1: vector<int> <- c(1, 2, 3)  
  
# Vector of doubles  
v2: vector<double> <- c(1.5, 2.5, 3.5)  
  
# Vector operations (component-wise)
```

```
v3: vector<double> <- v2 + c(1.0, 2.0, 3.0)
```

Vectors support component-wise arithmetic operations when both operands have compatible vector types.

3.4.2.2 Function Types

Function types represent callable values with parameter and return types:

```
<type> ::= <function_type>

<function_type> ::= <param_list> "->" <type>
                  | <primary_type>

<param_list> ::= <primary_type>
                | <primary_type> "," <param_list>

<primary_type> ::= <builtin_type>
                  | <generic_type>
                  | "(" <function_type> ")"
                  | "function"

<builtin_type> ::= "int" | "double" | "string" | "char"
                 | "void" | "logical" | "any"

<generic_type> ::= "vector" "<" <type> ">"
                 | "list" "<" <type> ">"
```

Examples of function types:

- `int -> int`: Function taking an integer, returning an integer
- `int, int -> double`: Function taking two integers, returning a double
- `(int -> int) -> int`: Higher-order function taking a function as parameter
- `int, (int, int -> int) -> int`: Function taking an integer and a function

3.4.3 Type Inference

The type checker performs bidirectional type inference:

- **Bottom-up inference:** Expression types are inferred from literal values and operator signatures

- **Top-down checking:** Function return types and variable annotations provide expected types
- **Unification:** Type constraints are solved to determine concrete types

Type annotations are required in the following contexts:

- Function parameters
- Function return types (when not inferrable from return statements)
- Ambiguous variable declarations

3.4.4 Typing Rules

Selected typing rules are presented below using inference rule notation.

3.4.4.1 Variable Reference

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T}$$

A variable reference has the type bound in the environment Γ .

3.4.4.2 Function Application

$$\frac{\Gamma \vdash e_1 : T_1, \dots, T_n \rightarrow T \quad \Gamma \vdash e_2 : T_1 \quad \dots \quad \Gamma \vdash e_{n+1} : T_n}{\Gamma \vdash e_1(e_2, \dots, e_{n+1}) : T}$$

Function application checks that argument types match parameter types and produces the return type.

3.4.4.3 Function Definition

$$\frac{\Gamma, x_1 : T_1, \dots, x_n : T_n \vdash e : T_{ret}}{\Gamma \vdash \text{function}(x_1 : T_1, \dots, x_n : T_n) : T_{ret}\{e\} : (T_1, \dots, T_n \rightarrow T_{ret})}$$

A function definition has a function type where parameters and return type match the declared signature.

3.4.4.4 Binary Operations

$$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

$$\frac{\Gamma \vdash e_1 : \text{double} \quad \Gamma \vdash e_2 : \text{double}}{\Gamma \vdash e_1 + e_2 : \text{double}}$$

Arithmetic operators are overloaded for numeric types. Comparison operators produce `logical` values.

3.4.4.5 Vector Operations

$$\frac{\Gamma \vdash e_1 : \mathbf{vector} < T > \quad \Gamma \vdash e_2 : \mathbf{vector} < T >}{\Gamma \vdash e_1 + e_2 : \mathbf{vector} < T >}$$

Component-wise vector operations require matching element types.

3.4.4.6 Conditionals

$$\frac{\Gamma \vdash e_{cond} : \mathbf{logical} \quad \Gamma \vdash e_{then} : T \quad \Gamma \vdash e_{else} : T}{\Gamma \vdash \mathbf{if} (e_{cond}) \{e_{then}\} \mathbf{else} \{e_{else}\} : T}$$

If-expressions require a logical condition and both branches must have the same type.

3.4.5 Scoping and Closures

The language uses lexical scoping where:

- Only functions create new scopes (blocks, if-statements, and loops do not)
- Functions can capture variables from enclosing function scopes
- Captured variables are implemented using closure environments in WebAssembly
- Super-assignment (`<<-`) searches enclosing function scopes to modify variables

Example demonstrating closure:

■ Code listing 3.8 Closure example

```
make_counter <- function(start: int): () -> int {
  count <- start
  function(): int {
    count <<- count + 1
    return(count)
  }
}

counter <- make_counter(0)
print(counter()) # Prints 1
print(counter()) # Prints 2
```

3.5 Semantics

This section describes the operational semantics of key language constructs.

3.5.1 Expression Evaluation

Expression evaluation follows a small-step operational semantics with evaluation contexts.

3.5.1.1 Literal Evaluation

Literals evaluate to themselves:

$$\begin{aligned} n &\Downarrow n && \text{(numeric literal)} \\ \text{"str"} &\Downarrow \text{"str"} && \text{(string literal)} \\ \text{TRUE} &\Downarrow \text{TRUE} && \text{(logical literal)} \end{aligned}$$

3.5.1.2 Variable Lookup

Variable references evaluate by environment lookup:

$$\frac{\rho(x) = v}{\langle x, \rho \rangle \Downarrow v}$$

where ρ is the runtime environment mapping identifiers to values.

3.5.1.3 Binary Operations

Binary operations evaluate operands left-to-right, then apply the operation:

$$\frac{\langle e_1, \rho \rangle \Downarrow v_1 \quad \langle e_2, \rho \rangle \Downarrow v_2 \quad v_1 \oplus v_2 = v}{\langle e_1 \oplus e_2, \rho \rangle \Downarrow v}$$

where \oplus represents any binary operator.

Arithmetic Operators (+, -, *, /, %) on same types:

$$\begin{aligned} n_1 + n_2 &= n_1 + n_2 && (\text{int} + \text{int} \rightarrow \text{int}) \\ d_1 + d_2 &= d_1 + d_2 && (\text{double} + \text{double} \rightarrow \text{double}) \\ n_1 - n_2 &= n_1 - n_2 \\ n_1 * n_2 &= n_1 \times n_2 \\ n_1 / n_2 &= \lfloor n_1 \div n_2 \rfloor && (\text{integer division}) \\ d_1 / d_2 &= d_1 \div d_2 && (\text{floating-point division}) \\ n_1 \bmod n_2 &= n_1 \bmod n_2 \end{aligned}$$

Type Promotion: When operands have different numeric types, implicit casting promotes to the wider type:

$$\begin{aligned} \text{int} \oplus \text{double} &\rightarrow \text{double} \\ \text{logical} \oplus \text{int} &\rightarrow \text{int} \\ \text{logical} \oplus \text{double} &\rightarrow \text{double} \end{aligned}$$

Example: $3 + 2.5$ evaluates as $3.0 + 2.5 = 5.5$ after promoting 3 to double.

Comparison Operators ($=$, $!=$, $<$, $<=$, $>$, $>=$) produce logical values:

$$\begin{aligned} n_1 < n_2 &= \text{TRUE if } n_1 < n_2, \text{ else FALSE} \\ n_1 == n_2 &= \text{TRUE if } n_1 = n_2, \text{ else FALSE} \end{aligned}$$

Type promotion applies: $3 < 2.5$ compares as $3.0 < 2.5$.

Logical Operators ($\&$, $|$) on boolean values:

$$\begin{aligned} b_1 \ \& \ b_2 &= b_1 \wedge b_2 \\ b_1 \ | \ b_2 &= b_1 \vee b_2 \end{aligned}$$

Vector Operations are applied component-wise after ensuring compatible types:

$$\text{vector}[v_1, \dots, v_n] \oplus \text{vector}[w_1, \dots, w_n] = \text{vector}[v_1 \oplus w_1, \dots, v_n \oplus w_n]$$

For mixed vector types, element types are promoted (e.g., `vector<int> + vector<double>` produces `vector<double>`).

Vector-Scalar Operations: Scalars are broadcast to match vector length:

$$\text{vector}[v_1, \dots, v_n] \oplus s = \text{vector}[v_1 \oplus s, \dots, v_n \oplus s]$$

3.5.1.4 Function Application

Function application evaluates the callee to obtain a closure, evaluates arguments, extends the closure environment, and evaluates the body:

$$\frac{\begin{array}{l} \langle e_0, \rho \rangle \Downarrow \langle \lambda(x_1, \dots, x_n).e, \rho' \rangle \\ \langle e_1, \rho \rangle \Downarrow v_1 \quad \dots \quad \langle e_n, \rho \rangle \Downarrow v_n \\ \langle e, \rho' [x_1 \mapsto v_1, \dots, x_n \mapsto v_n] \rangle \Downarrow v \end{array}}{\langle e_0(e_1, \dots, e_n), \rho \rangle \Downarrow v}$$

3.5.2 Statement Execution

Statements modify the environment and may alter control flow.

3.5.2.1 Assignment

Assignment evaluates the right-hand side and binds the result to the identifier:

$$\frac{\langle e, \rho \rangle \Downarrow v}{\langle x \leftarrow e, \rho \rangle \rightarrow \rho[x \mapsto v]}$$

Super-assignment searches parent scopes:

$$\frac{\langle e, \rho \rangle \Downarrow v \quad x \in \text{dom}(\rho_{\text{parent}})}{\langle x \leftarrow\leftarrow e, \rho \rangle \rightarrow \rho[\rho_{\text{parent}}[x \mapsto v]]}$$

3.5.2.2 Conditional Execution

If-statements evaluate the condition and execute the appropriate branch:

$$\frac{\langle e_{cond}, \rho \rangle \Downarrow \text{TRUE} \quad \langle s_{then}, \rho \rangle \rightarrow \rho'}{\langle \text{if } (e_{cond}) \{s_{then}\}, \rho \rangle \rightarrow \rho'}$$

$$\frac{\langle e_{cond}, \rho \rangle \Downarrow \text{FALSE} \quad \langle s_{else}, \rho \rangle \rightarrow \rho'}{\langle \text{if } (e_{cond}) \{s_{then}\} \text{ else } \{s_{else}\}, \rho \rangle \rightarrow \rho'}$$

3.5.2.3 While Loops

While loops repeatedly execute the body while the condition is true:

$$\frac{\langle e_{cond}, \rho \rangle \Downarrow \text{FALSE}}{\langle \text{while } (e_{cond}) \{s\}, \rho \rangle \rightarrow \rho}$$

$$\frac{\langle e_{cond}, \rho \rangle \Downarrow \text{TRUE} \quad \langle s, \rho \rangle \rightarrow \rho' \quad \langle \text{while } (e_{cond}) \{s\}, \rho' \rangle \rightarrow \rho''}{\langle \text{while } (e_{cond}) \{s\}, \rho \rangle \rightarrow \rho''}$$

3.5.2.4 For Loops

For loops iterate over vectors or ranges:

$$\langle e_{vec}, \rho \rangle \Downarrow \text{vector}[v_1, \dots, v_n]$$

$$\langle s, \rho[x \mapsto v_1] \rangle \rightarrow \rho_1$$

$$\dots$$

$$\frac{\langle s, \rho_{n-1}[x \mapsto v_n] \rangle \rightarrow \rho_n}{\langle \text{for } (x \text{ in } e_{vec}) \{s\}, \rho \rangle \rightarrow \rho_n}$$

3.5.3 Function Calls and Return

Function calls push a new activation frame onto the call stack. The **return** statement terminates function execution and yields a value:

$$\frac{\langle e, \rho \rangle \Downarrow v}{\langle \text{return}(e), \rho \rangle \Rightarrow v}$$

The special \Rightarrow arrow indicates early exit from the function body.

3.5.4 Built-in Functions

The language provides built-in functions with special semantics:

- `c(e1, ..., en)`: Creates a vector from arguments. All arguments must have the same type.
- `print(e)`: Outputs the value of expression `e` to standard output.
- `length(v)`: Returns the number of elements in vector `v`.
- `sum(v)`: Returns the sum of all elements in numeric vector `v`.
- `gen_seq(start, end)`: Generates a sequence from `start` to `end` (equivalent to `start:end`).

Compiler and Runtime

4.1 Compiler Architecture

Frontend, type checking, intermediate representation, and backend.

4.2 WebAssembly Code Generation


Translation of language constructs and types to WebAssembly.

4.3 Runtime System

Vector representation, memory management, and built-in operations.

4.4 Implementation Details

Key implementation choices and limitations.



Chapter 5

Evaluation

5.1 Correctness

Testing methodology and comparison with R behavior.

5.2 Performance

Execution time, code size, and runtime overhead.

5.3 Discussion

Analysis of results and trade-offs.

Chapter 6

Conclusion

Summary of contributions and future work.

Appendix A

Some content

An example below to show how the bytecode for generic functions would look like in arbitrary typed bytecode. How would we compile untyped code for?

1. The high level code

```
c = a + b
```

2. Every value is a boxed value.

```
Value {  
  tag : TypeTag  
  payload : union {  
    int64  
    float64  
    pointer  
    object_ref  
  }  
}
```

```
TypeTags ::= INT | FLOAT | STRING | OBJECT | ...
```

3. Load variable and call the generic function (simplified instruction)

```
LOAD_LOCAL  a  
LOAD_LOCAL  b  
ADD_GENERIC  
STORE_LOCAL c
```

4. What ADD_GENERIC have to do?

```
b = pop()  
a = pop()
```

```
if a.tag == INT and b.tag == INT:
```

```
        push(int_add(a, b))
    elif a.tag == FLOAT and b.tag == FLOAT:
        push(float_add(a, b))
    elif a.tag == STRING and b.tag == STRING:
        push(string_concat(a, b))
    elif a.tag == OBJECT:
        call a.__add__(b)
    else:
        runtime_type_error()
```

Remember, this is optimistic scenario. What happens when we have on LHS(left-hand side) an INT and on RHS(right-hand side) FLOAT? Moreover what if one of them is composite types? As one can see this dispatcher function for every operation combined with every type will be a huge overhead.

Bibliography

1. NYSTROM, Robert. *Crafting Interpreters*. Genever Benning, 2021. ISBN 978-0990582939. Available also from: <https://craftinginterpreters.com/>.

Contents of the attachment

```
/
├── readme.txt ..... stručný popis obsahu média
├── exe ..... adresář se spustitelnou formou implementace
├── src
│   ├── impl ..... zdrojové kódy implementace
│   └── thesis ..... zdrojová forma práce ve formátu LATEX
├── text ..... text práce
└── thesis.pdf ..... text práce ve formátu PDF
```