TAYLOR SPECTRUM FOR MODULE OVER LIE ALGEBRA

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1. Introduction

2. Preliminaries

Let $\mathfrak{g}$ be an arbitary finite dimensional Lie algebra. We will denote by $\mathfrak{g}$**-mod** and **mod-**$\mathfrak{g}$ the categories of left and right $\mathfrak{g}$-modules respectively. We define functors $\square^*\colon\ \mathfrak{g}\textbf{-mod}^{op} \to \textbf{mod-}\mathfrak{g}$ and $\square^\circ\colon\ \mathfrak{g}\textbf{-mod} \to \textbf{mod-}\mathfrak{g}$ as follows. The first $\square^*$, called duality functor, sends $\mathfrak{g}$-module V to it's dual vector space, on which the right action of $\mathfrak{g}$ defined as

$$(f \cdot g)(v) = f(g \cdot v), \quad \text{for all } f \in V^*,\ v \in V,\ g \in \mathfrak{g}.$$

The second $\square^\circ$, called antipode functor, sends V to itself as vector space with right action

$$v \cdot g = -g \cdot v, \quad \text{for all } v \in V,\ g \in \mathfrak{g}.$$

These two functors define equivallence of categories $\mathfrak{g}$**-mod**, **mod-**$\mathfrak{g}$, $\mathfrak{g}\textbf{-mod}^{op}$ and $\textbf{mod-}\mathfrak{g}^{op}$.

3. Taylor spectrum of $\mathfrak{g}$-module

Let $\mathfrak{g}$ be an arbitary Lie algebra and E be a left $\mathfrak{g}$-module. We will denote by $\hat{\mathfrak{g}}$ the set of isomorphism classes of simple finite dimensional $\mathfrak{g}$-modules.

Definition 1. *The Taylor spectrum of E is the set, defined as*

$$\sigma(E) = \{V \in \hat{\mathfrak{g}} \mid \exists k\colon \operatorname{Tor}_k^{U\mathfrak{g}}(V^*, E) \neq 0\}.$$

From it follows, that the definition above coincides with the original Taylor's definition in case of abelian $\mathfrak{g}$. Add ref

4. Case of semisimple Lie algebra

5. Spectrum of one-dimensional extensions

6. Case of solvable Lie algebra

7. Case of nilpotent Lie algebra

8. Case of borel subalgebra of semisimple Lie algebra