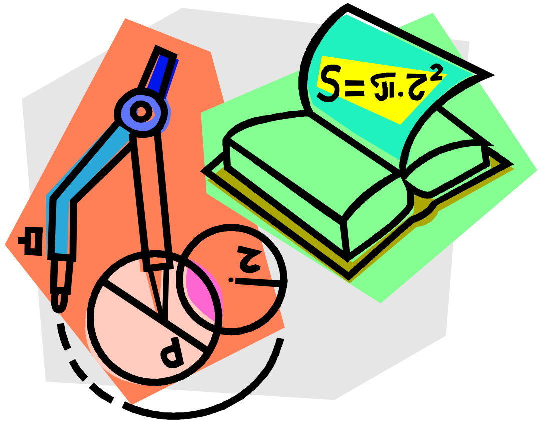
Linear Programming Concepts

- Decision Making Under Certainty
- Objectives
- Decision Variables
- Constraints
- Linear Relationships Between the Variables



Linear Programming Problem

- The company makes two toy products -trucks and cars. Each truck and each car has a contribution of $2 per unit. Each requires time in each of three different departments. The time required for the products in the three departments is shown in Table 1. Time available in the departments is also shown.
- The objective is to make the most money possible.

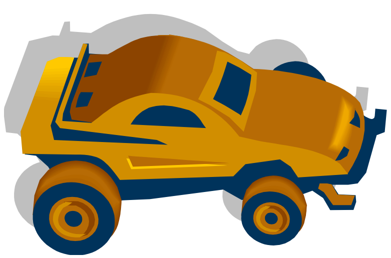
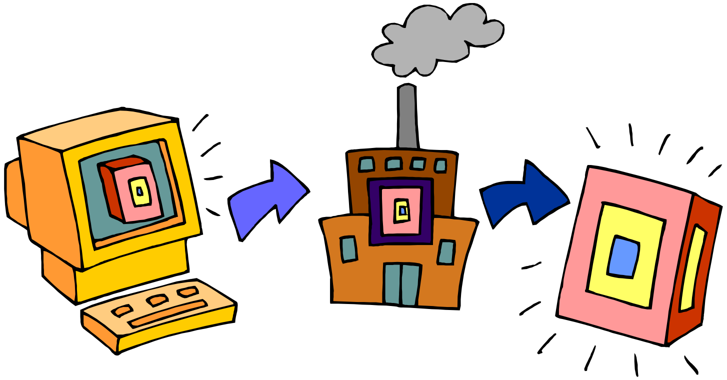
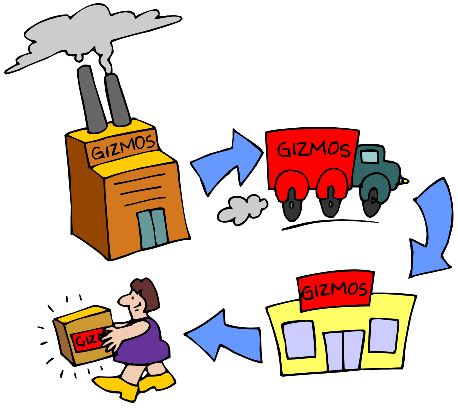


TABLE 1

Product	Mold	Paint	Pack
Car	2	1	1
Truck	1	2	1
Mins. avail.	400	600	500

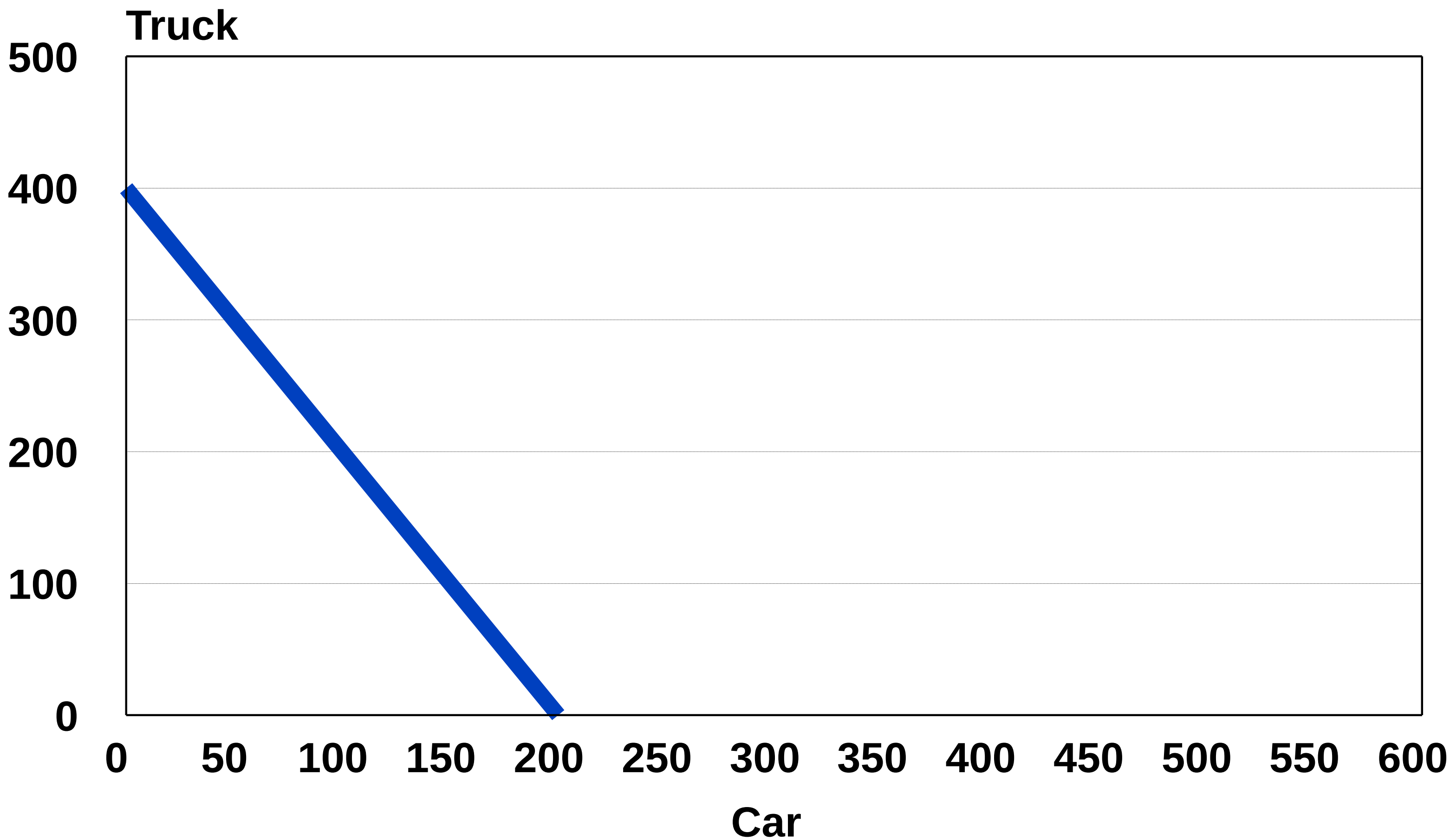


Linear Programming Formulation

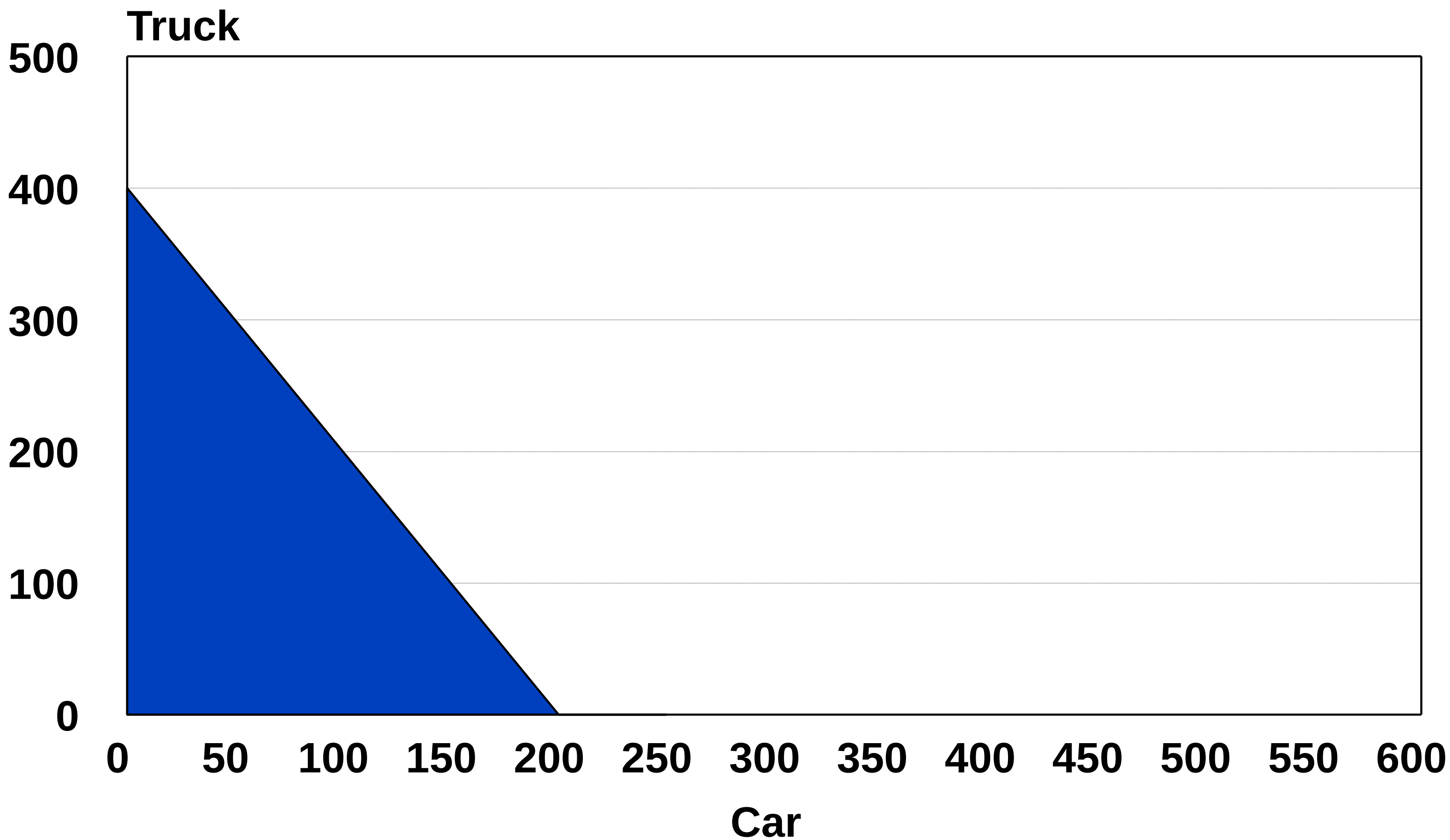


- **Objective Function**
- **Maximize: 2car+2truck**
- **Subject to:**
- **2car+1truck<=400 Mold Department**
- **1car+2truck<=600 Paint Department**
- **1car+1truck<=500 Pack Department**

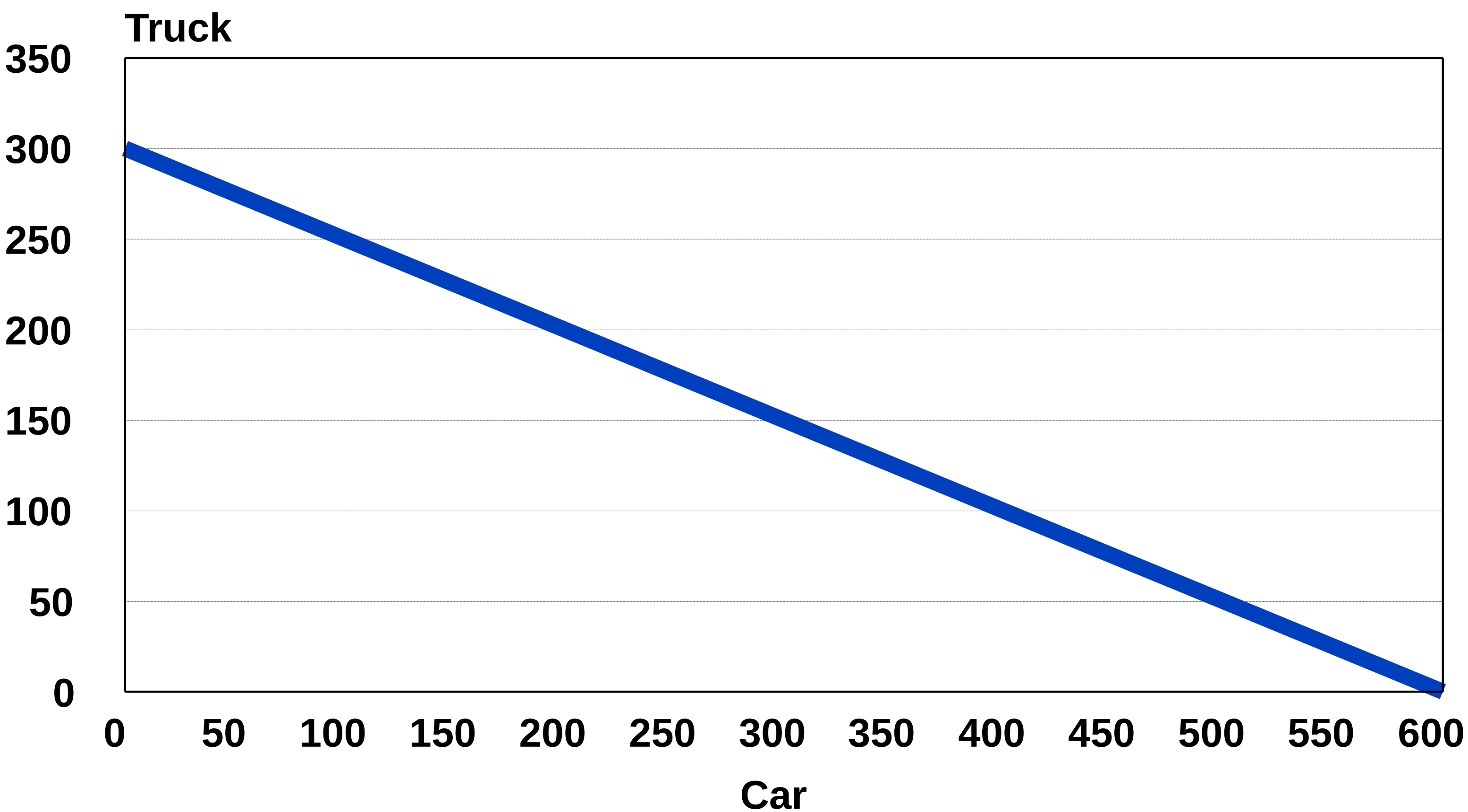
Mold Department



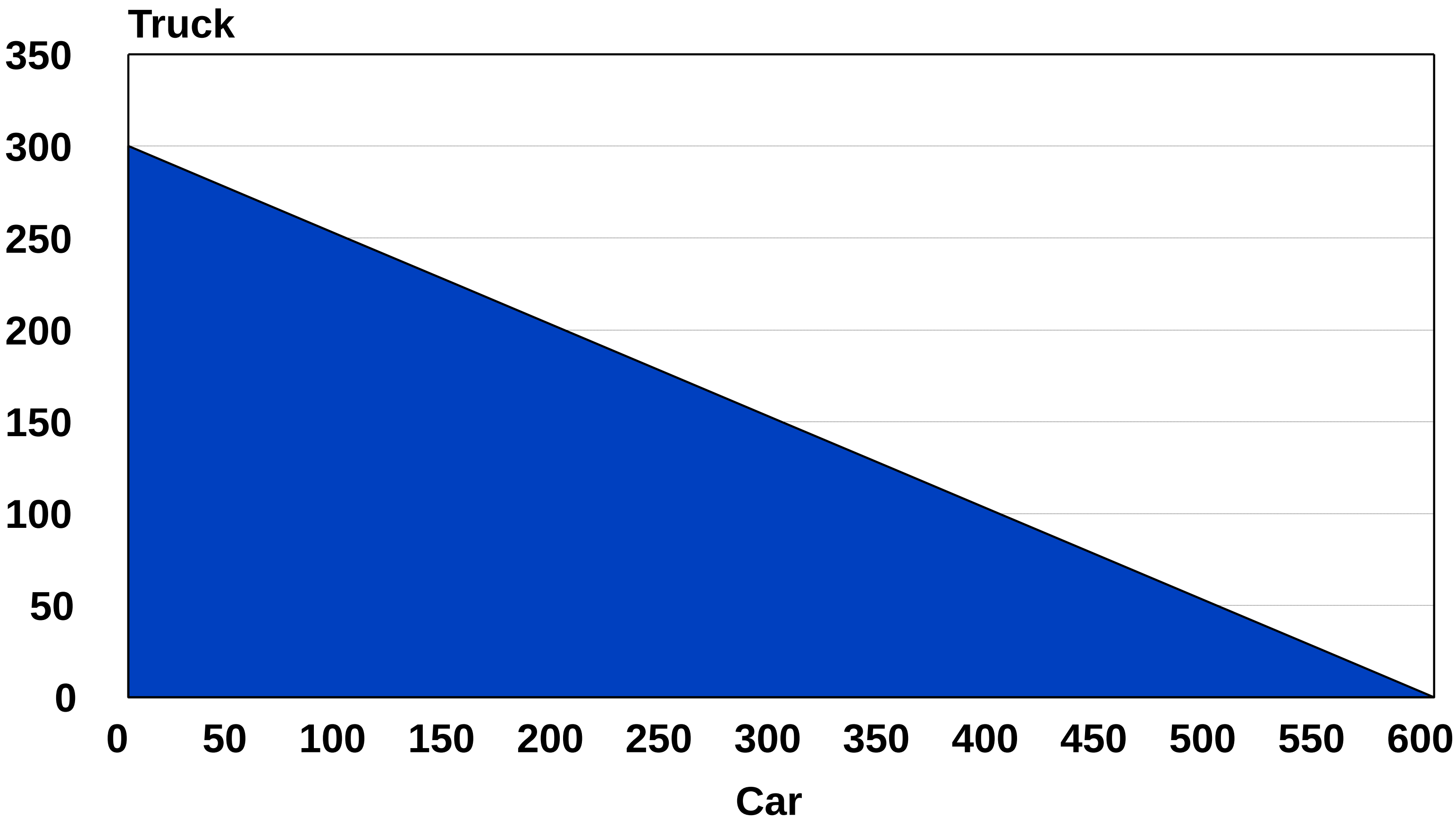
Mold Department



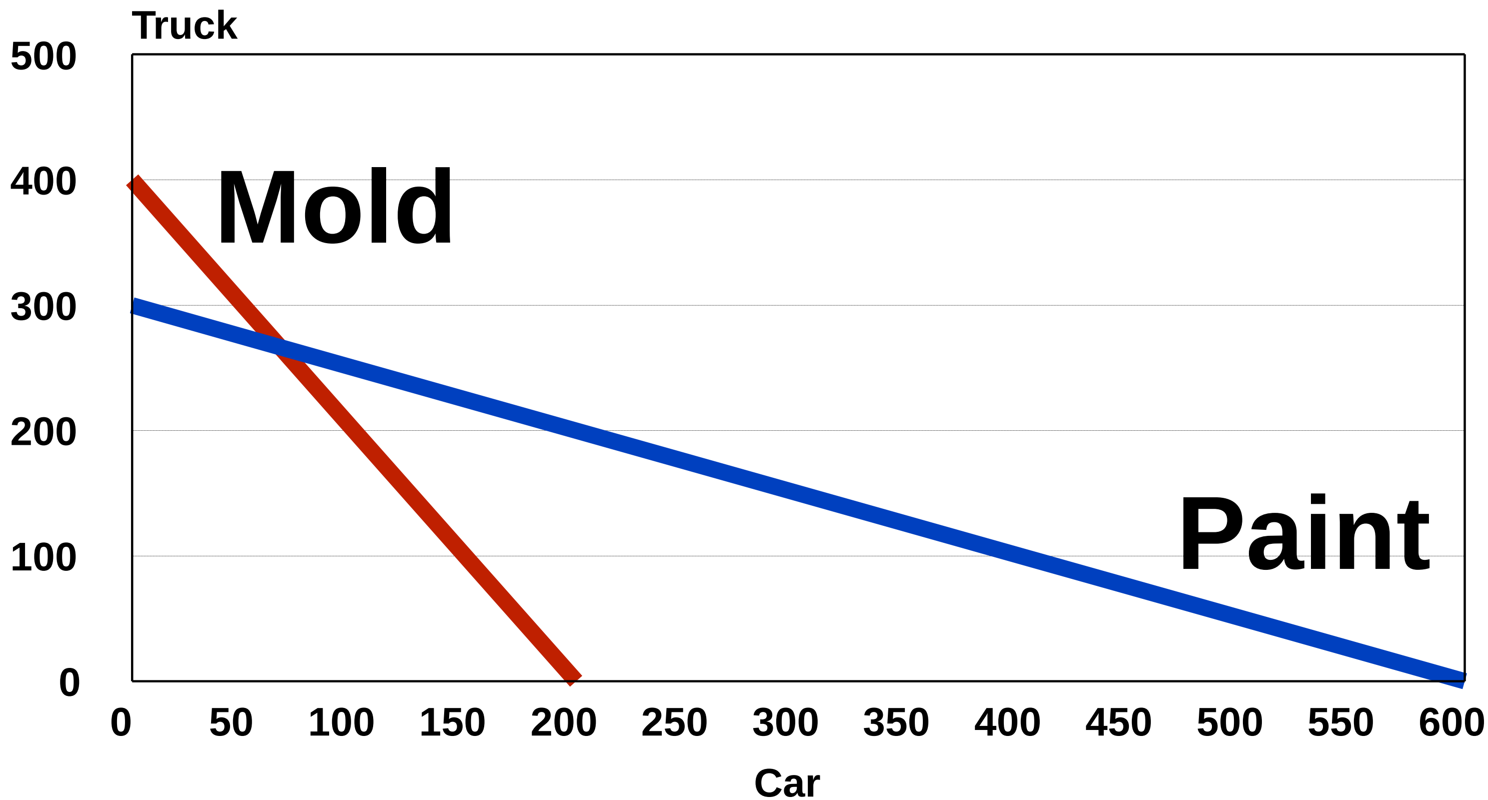
Paint Department



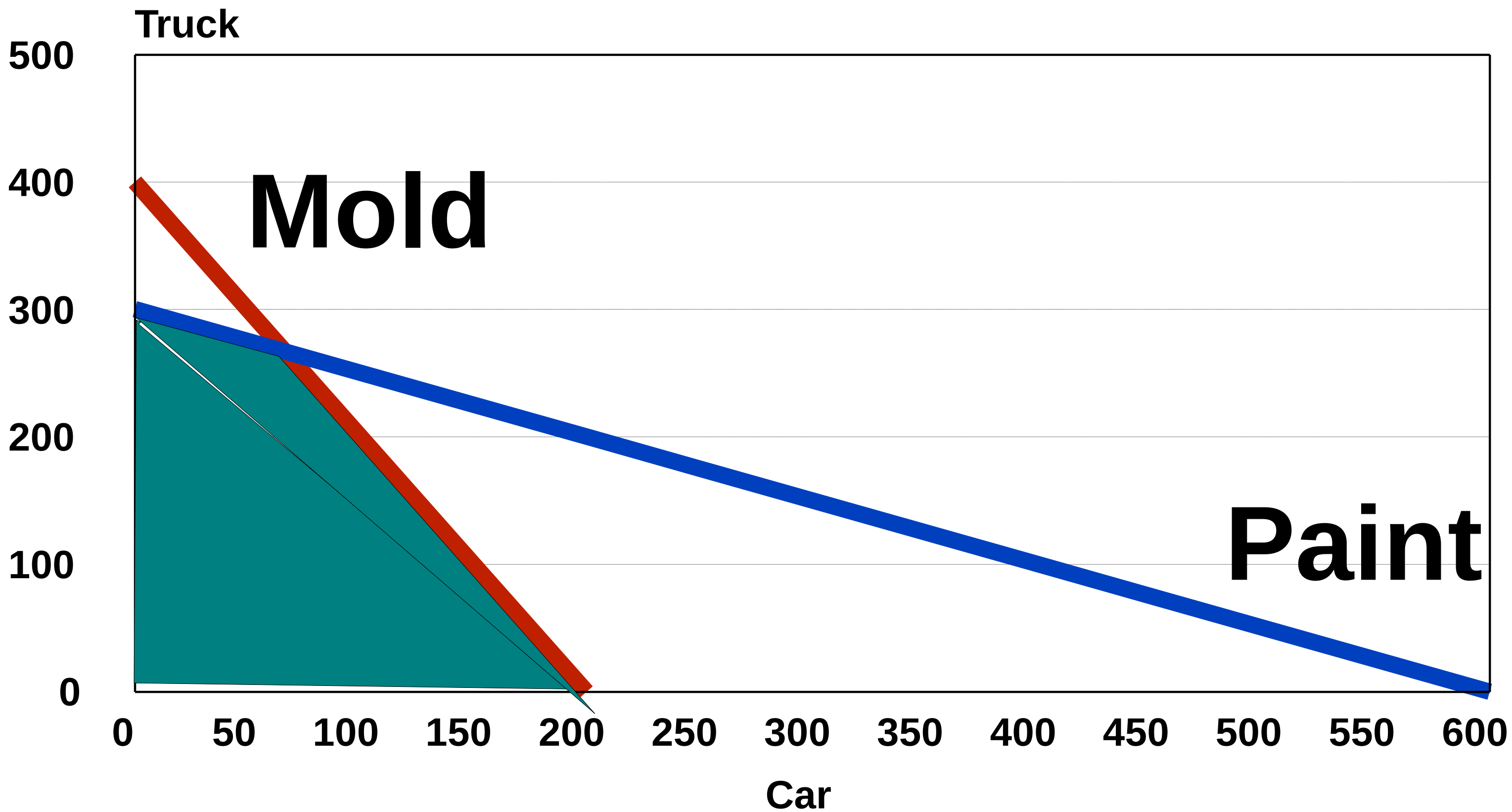
Paint Department



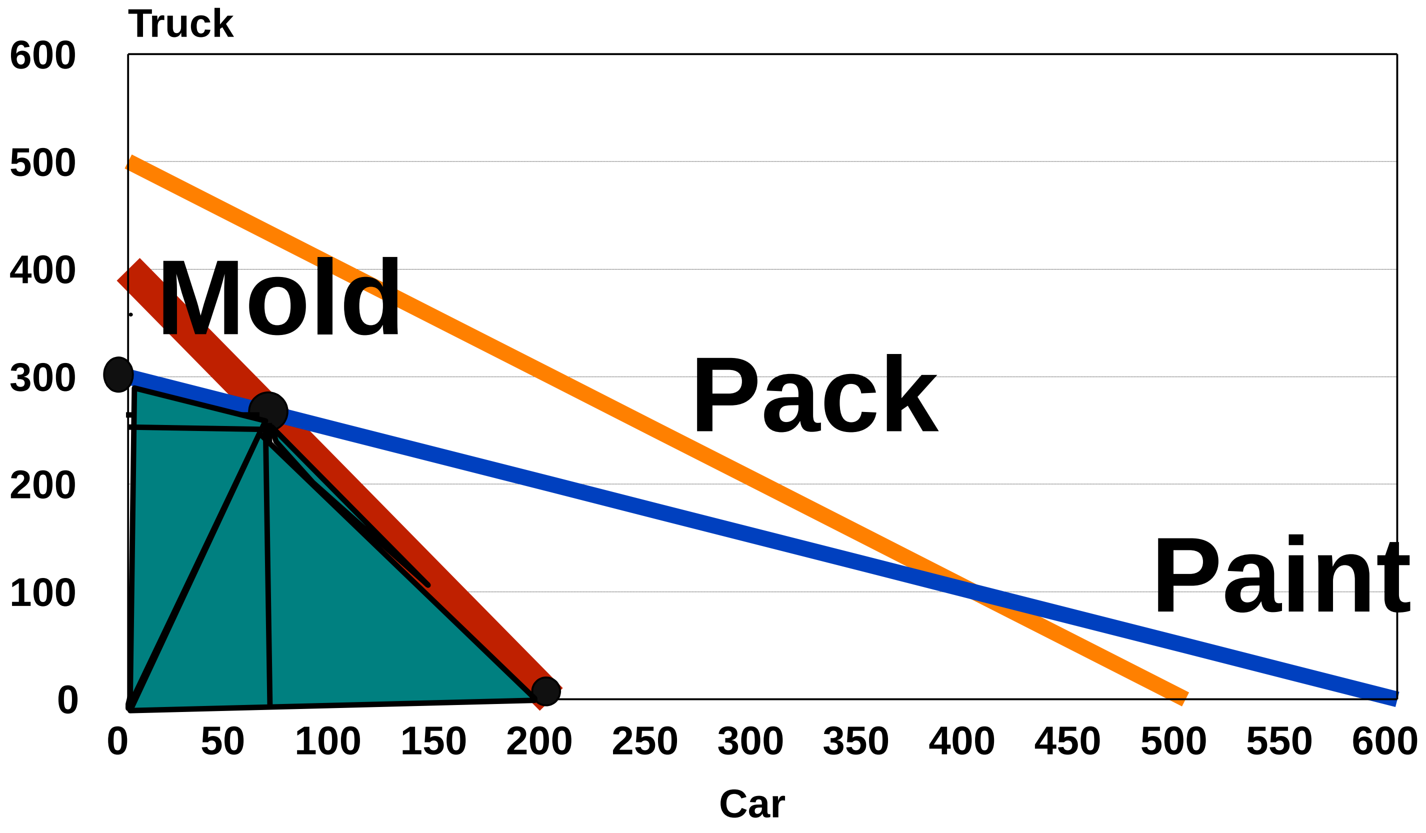
Mold and Paint



Mold and Paint



3 Departments



Linear Programming Solution

- The Best Solution Will Always Be Found
- At A Corner Point Of The Feasible Region
- Objective Function: Max: 2car+2truck

Cars	Trucks	Contribution
0	300	600
200	0	400
66.7	266.7	668

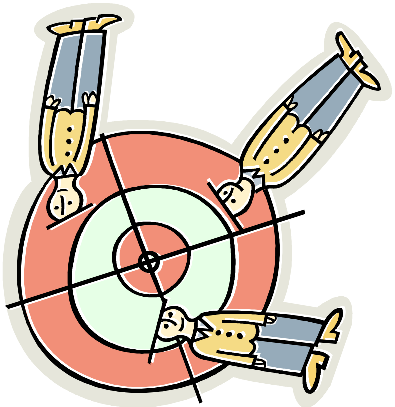
Linear Programming Problem

- A firm makes three similar products which all follow the same three-step process which consists of milling, inspection, and drilling. Product a requires 12 minutes of milling, 5 minutes for inspection and 10 minutes for drilling per unit; product b requires 10 minutes of milling, 4 minutes for inspection and 8 minutes of drilling per unit; and product c requires 8 minutes of milling, 4 minutes for inspection, and 16 minutes of drilling. The department has 20 hours available during the next period for milling, 15 hours for inspection, and 24 hours for drilling. Product a contributes $2.40 per unit to profit, b contributes $2.50 per unit, and c contributes $3.00 per unit. Use linear programming to determine the optimum mix of products in terms of maximizing contribution to profits for the period.

3 PRODUCT PROBLEM

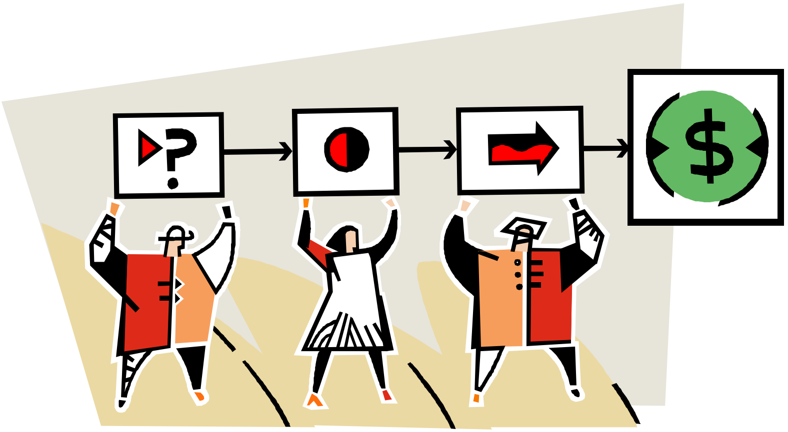
- MAXIMIZE 2.4 A + 2.5 B + 3.0C

- SUBJECT TO :
- 12A + 10B + 8C <= 1200 MILL
- 5A + 4B + 4C <= 900 INSPECT
- 10A + 8B + 16C <=1440 DRILL



L.P. Solution

- PAYOFF 350
- B 80
- C 50
- INSPECT 380



L.P. Problem from Textbook
15th Edition page 709 # 4

4 A diet is being prepared for the University of Arizona dorms. The objective is to feed the students at the least cost, but the diet must have between 1,800 and 3,600 calories. No more than 1,400 calories can be starch, and no fewer than 400 can be protein. The varied diet is to be made of two foods: *A* and *B*. Food *A* costs \$0.75 per pound and contains 600 calories, 400 of which are protein and 200 starch. No more than two pounds of food *A* can be used per resident. Food *B* costs \$0.15 per pound and contains 900 calories, of which 700 are starch, 100 are protein, and 100 are fat.

a. Write the equations representing this information.

b. Solve the problem graphically for the amounts of each food that should be used.

5 Do Problem 4 with the added constraint that not more than 150 calories shall be fat and that the price of food has escalated to \$1.75 per pound for food *A* and \$2.50 per pound for food *B*.

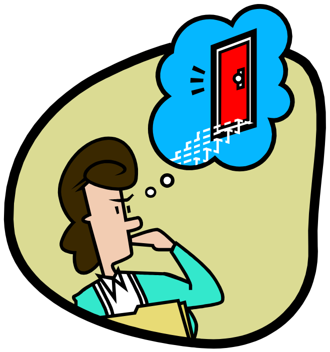
Operations Homework # 10

L.P. Problem from Textbook
13th Edition page 734 # 4

- Objective function : minimize .75A + .15B
- Subject to:
- 600A + 900B<= 3,600 upper calorie limit
- 200A + 700B<= 1,400 starch
- A <=2 limit on food A
- 600A + 900B >=1,800 lower calorie limit
- 400A + 100B >=400 protein

Operations Homework # 10

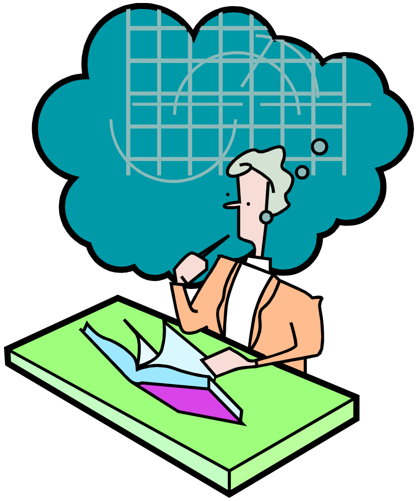
Solution

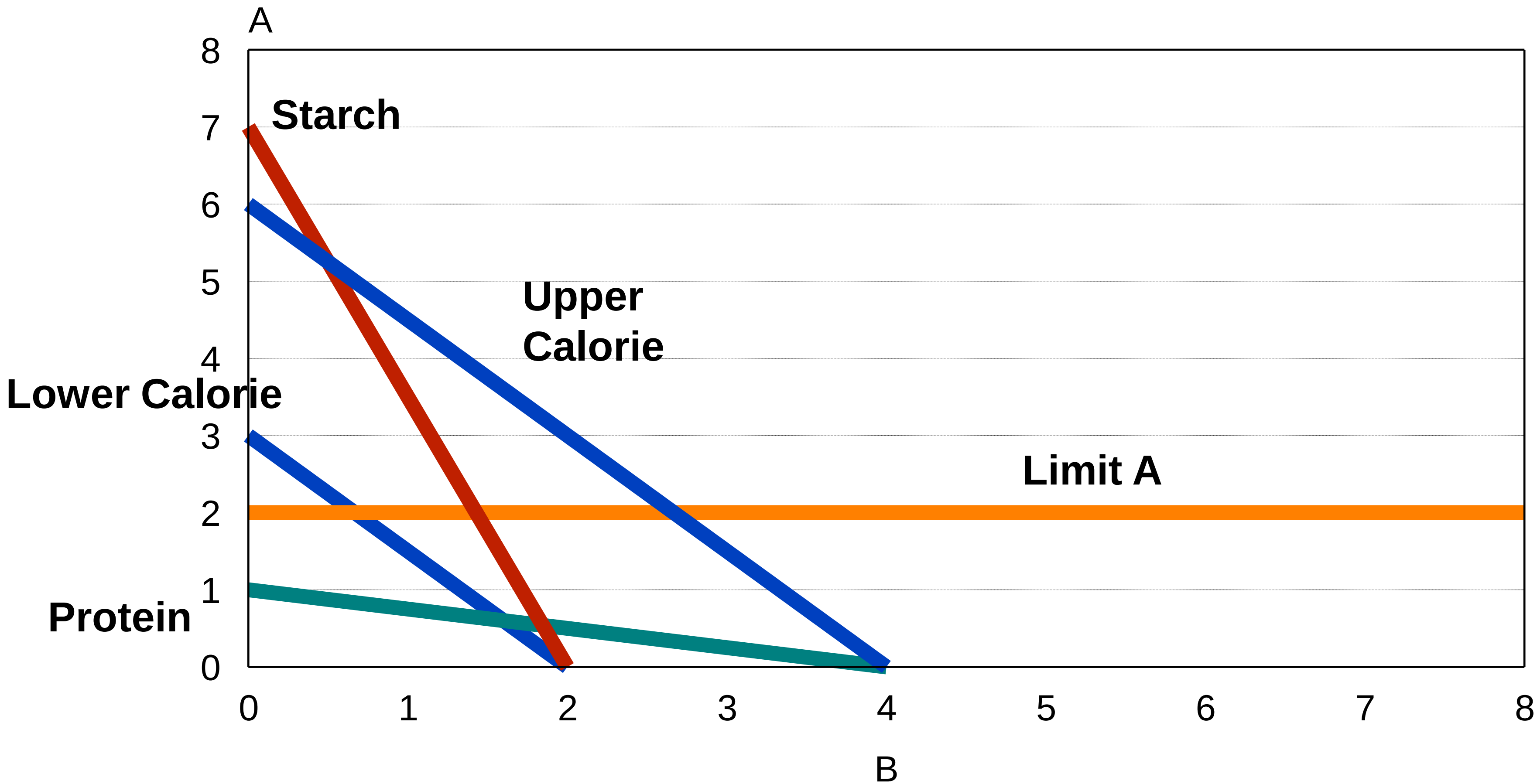


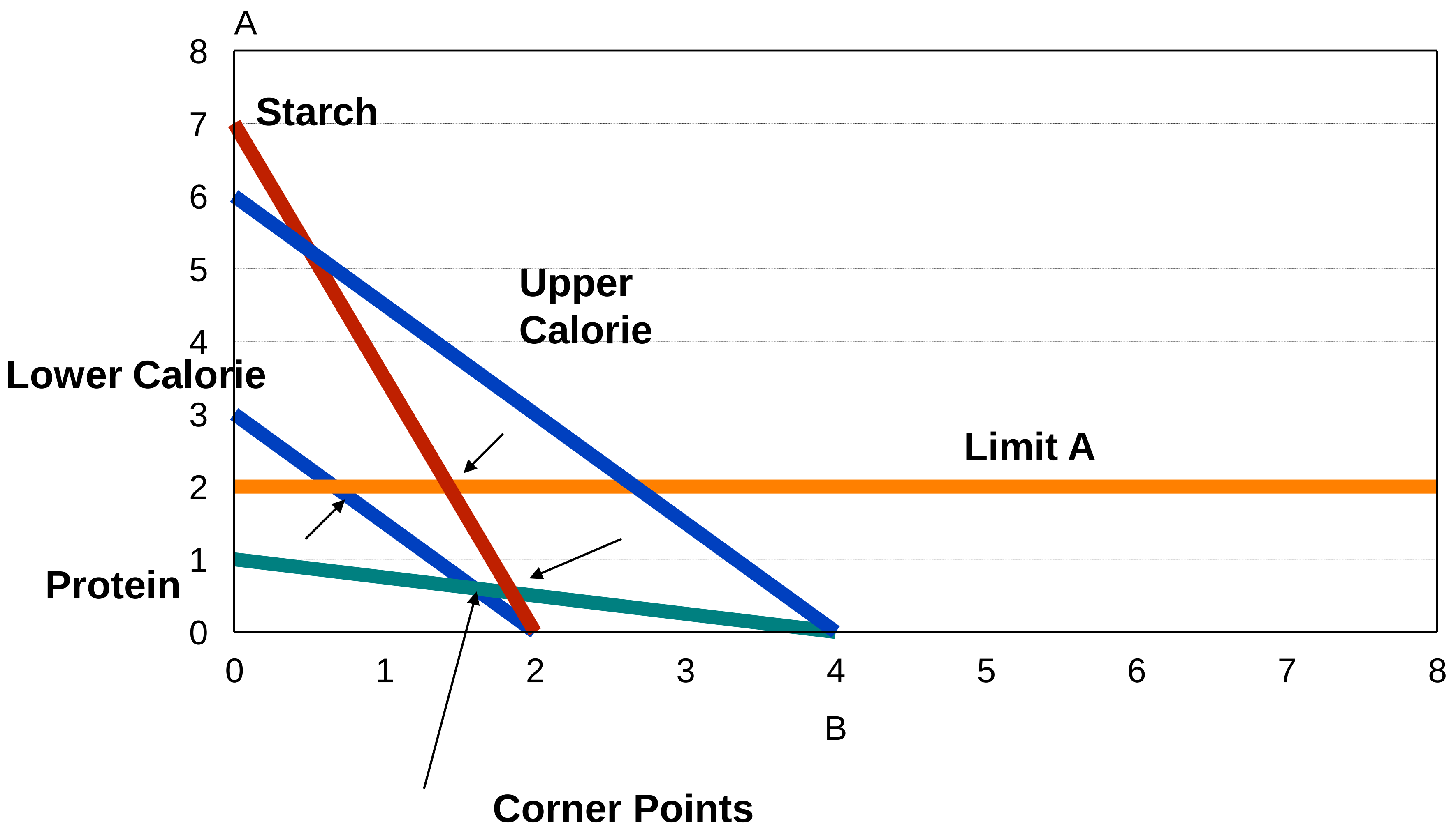
- PAYOFF .6807692
- B XXXXXX AMOUNT OF B
- A XXXXXX AMOUNT OF A
- UPPER CALORIE LIMIT 1615.385
- LOWER CALORIE LIMIT 184.6154
- LIMIT ON FOOD A 1.461539

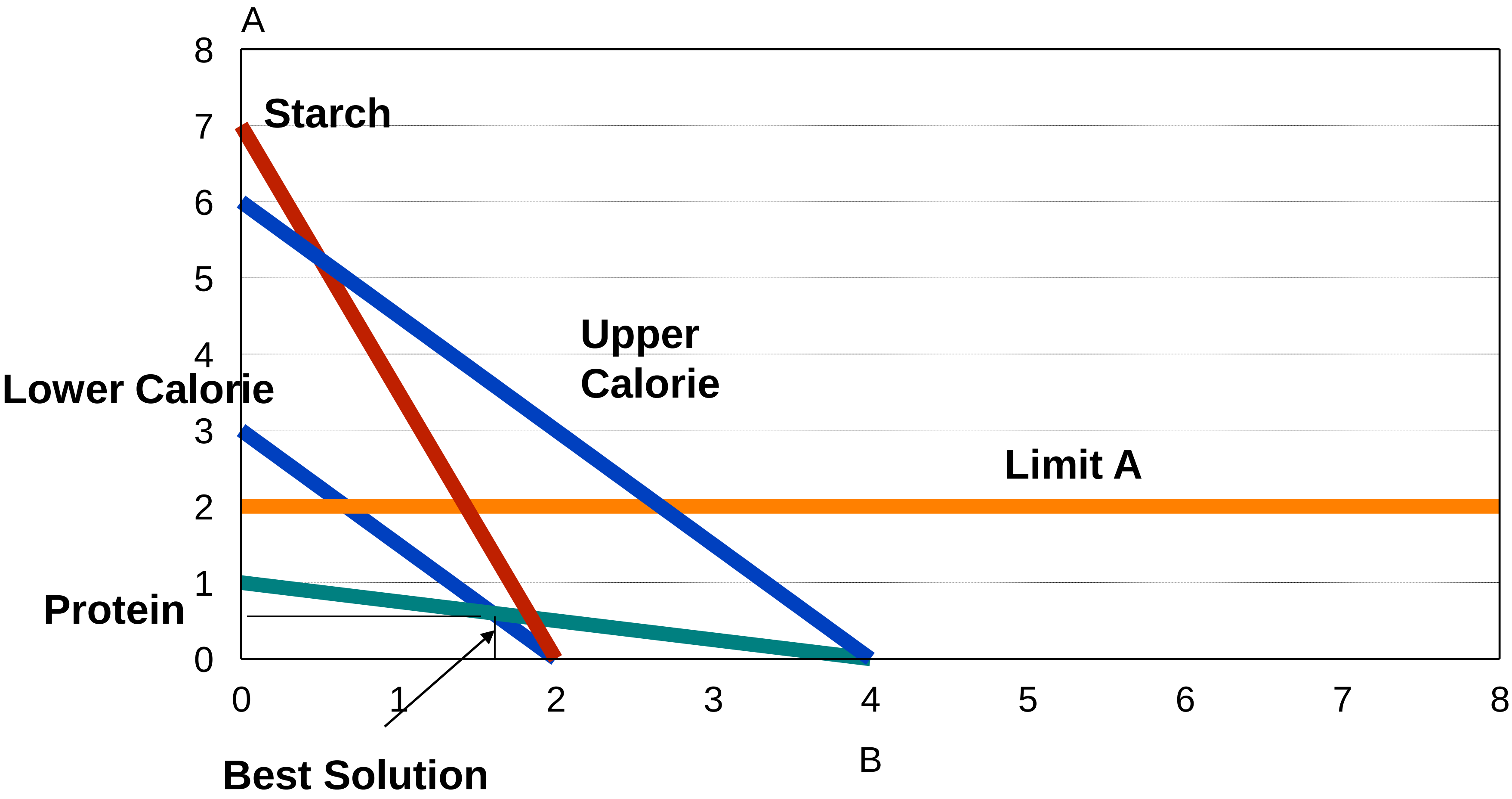
Revised Problem

- New objective function
- Minimize: 1.75A + 2.50B
- 100B <= 150 Limit on Fat
- Payoff 5.0625
- STARCH 200
- A .75
- LIMIT ON FOOD A 1.25
- B 1.5
- PROTEIN 50
- UPPER CALORIE 1800




A
8
7
6
5
4
3
2
1
0
Starch
Upper
Calorie
Lower Calorie
Limit A
Protein
0
1
2
3
4
5
6
7
8
B


A
8
7
6
5
4
3
2
1
0
Starch
Upper
Calorie
Lower Calorie
Limit A
Protein
0
1
2
3
4
5
6
7
8
B
Corner Points


A
8
7
6
5
4
3
2
1
0
Starch
Upper
Calorie
Lower Calorie
Limit A
Protein
0
1
2
3
4
5
6
7
8
B
Best Solution

Operations Homework # 11
Page 709 15th. ed. Problem 3

- A manufacturing firm has discontinued production of a certain unprofitable product line. Considerable excess production capacity was created as a result. Management is considering devoting this excess capacity to one or more of three products, X1, X2 and X3.
- Machine hours required per unit are as follows:

Machine type	X1	X2	X3
		Product	
Milling machine	8	2	3
Lathe	4	3	0
Grinder	2	0	1

Operations Homework # 11(cont.)

- The available time in machine hours per week are:
- Machine hours per week
- Milling 800
- Lathes 480
- Grinders 320
- The sales-people estimate that they can sell all the units of X1 and X2 that can be made. But the sales potential of X3 is 80 units per week maximum.
- Unit contribution for the three products are:
- Unit contribution
- X1 $20
- X2 6
- X3 8
- Formulate the Linear Programming problem to maximize the profit per week.

L.P.Problem

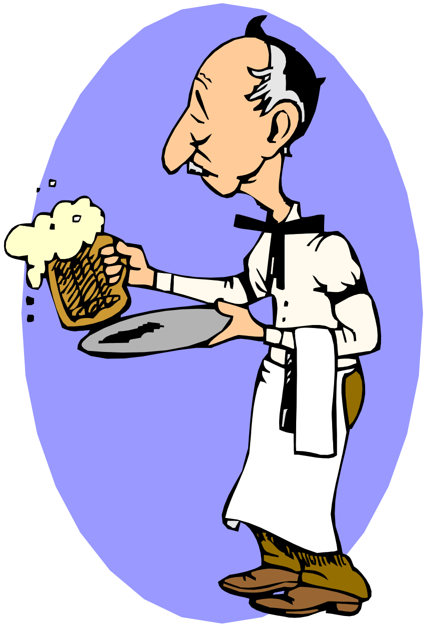
- Maximize: 20X1 + 6X2 + 8X3
- S.T. 8X1 + 2X2 + 3X3 <= 800 MILL
- 4X1 + 3X2 <= 480 LATHE
- 2X1 +1X3 <= 320 GRIND
- 1X3 <= 80 MKT-X3
  - payoff 2140
- X1 xx MILL 0
- X3 80 LATHE 0
- X2 xx
- GRIND 150

2ND SCREEN

• Lathe	.5
• Mill	2.25
• Grind	0
• MKT-X3	1.25

Operations Homework 12

- Billy Frank Haywood is the bartender at
- Oceanside Motel. When he checked the
- supply cabinet this afternoon,
- it contained these items:
-
- Gin 120 oz.
- Bourbon 108 oz.
- Vermouth 60 oz.
- Scotch 72 oz.
- Vodka 48 oz.



- The cabinet also contained cherries, orange slices, lemons, limes, onions and juices, as well as other garnishes which he might need. Billy offers a limited bar menu consisting of six mixed drinks, which he pre-mixes and places on trays. He then circulates through the crowd to sell them. Based on past experience, he knows that he cannot sell more than 60 martinis. His current bar menu is:

Scotch-on-the- rocks	2 oz. scotch
Martini	1.5 oz. gin
	.25 oz. vermouth
Atomic Bomb	1.5 oz. scotch
	1.5 oz. vodka
Snowdrift	2 oz. bourbon
Kentucky Colonel	2 oz. bourbon
	1 oz. vermouth
Steamroller	2 oz. gin
	1 oz. vodka
	.5 oz. scotch

- Each drink sells for $2.50, and Billy can sell as many drinks of each kind, except martinis, as he can pre-mix. What should he pre-mix this evening? Solve the problem using a computer

Billy Frank Haywood Problem

- MAXIMIZE:
- 2.5S + 2.5 M + 2.5A + 2.5D + 2.5K + 2.5R
- S.T. 1.5M + 2R <= 120 GIN
- 2D + 2K <= 108 BOURBON
- .25M + 1K <=60 VERMOUTH
- 2S + 1.5A + .5R <=72 SCOTCH
- 1.5A + 1R <=48 VODKA
- M <=60 MARTINI MAX
  - Payoff 416.875
- M=xx D=xx
- S=xx A=xx
- R=xx K=xx

Linear Programming Minimization Problem

- The research staff at The Braxton Chemical Company has developed a new type of wonder glue which is composed of possible ingredients
- A, B, C, and D. The amount of each ingredient
- can vary to some degree as long as the following specifications are met:
- a. The glue must contain at least 50%
- ingredient B by weight.
- b. There should be no more than
- 2 parts of A to 3 parts of C by weight.
- c. Ingredient D cannot exceed 25% by weight.

Linear Programming Minimization Problem (cont)

- You have checked and found that the cost of the four ingredients is as follows:

Ingredient	$ per ounce
A	.25
B	.33
C	.14
D	.09

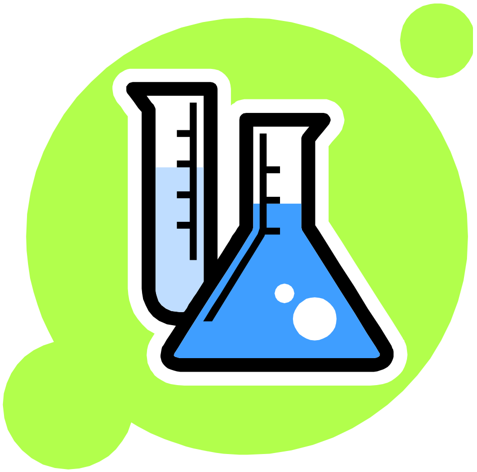
- You would-like to determine what mix of the four ingredients to blend together in order to determine the
- least cost mixture, but still remain in compliance with the specifications. Set up the problem so that it can be
- solved with linear programming and that the resulting objective function value will equal the cost of the ingredients used in a four-ounce bottle of the wonder glue.
-

Braxton Chemical Company

- MINIMIZE .25A + .33B + .14C + .09D
- SUBJECT TO:
- B >= 2 50% B
- 2A<=3C
- 2A-3C <= 0 2A/3C
- D <= 1 25% D
- A+B+C+D = 4 4 OZ. MIX

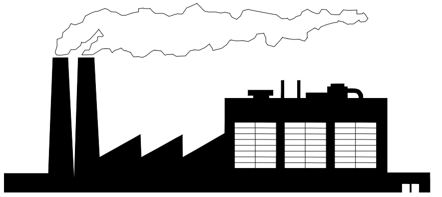
Solution

- PAYOFF .89
- D 1
- C 1
- B 2
- 2 A/3C 3



During the coming week a factory can manufacture combinations of the following products:

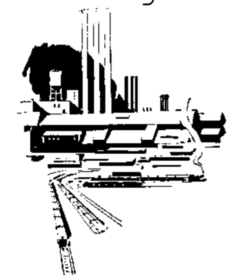
Product	Contribution per unit
X1	$3
X2	4
X3	5
X4	2
X5	6



The manufacturing facilities of the firm are divided into four departments through which the products may or may not have to pass, depending on individual manufacturing requirements. Individual requirements for each product in terms of hours and the total number of available hours in each department are given below:

	Hours Der unit			
Product	Dept. 1	Dept 2	Dept. 3	Dept. 4
X1	3	8	2	6
X2	4	3	1	0
X3	2	2	0	2
X4	2	1	3	4
X5	5	4	4	3
Total hours available	700	600	400	900

In addition to the above manufacturing restrictions on output, the following list represents the maximum sales anticipated for each of the five products during the coming week. No production is scheduled for inventory.



X1	100 units
X2	50 units
X3	90 units
X4	70 units
X5	30 units

Each of the five products is made from five raw materials, A, B, C, D and E. The following table illustrates the per-unit requirements in pounds for each product and the total availability of each raw material for the coming week:

			Pounds per unit		
Product	A	B	C	D	E
X1	4	2	0	1	3
X2	7	4	4	0	4
X3	6	2	5	7	0
X4	1	1	6	4	2
X5	3	0	2	3	4
pounds avail.	1,000	900	300	400	1,600

Production L.P. Problem

- Product Contribution
- X1 $3
- X2 4
- X3 5
- X4 2
- X5 6

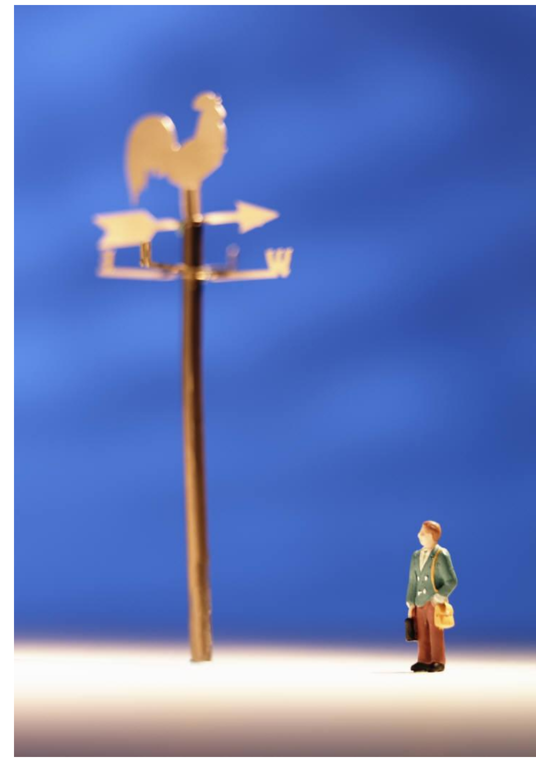
Routing and Capacity

– Departments

Product	1	2	3	4
X1	3	8	2	6
X2	4	3	1	0
X3	2	2	0	2
X4	2	1	3	4
X5	5	4	4	3
Hrs. Avail.	700	600	400	900

Market Constraints

- X1 100 units
- X2 50 units
- X3 90 units
- X4 70 units
- X5 30 units



Inventory

	Pounds per unit				
Product	A	B	C	D	E
X1	4	2	0	1	3
X2	7	4	4	0	4
X3	6	2	5	7	0
X4	1	1	6	4	2
X5	3	0	2	3	4

- Lbs. Avail. 1,000 900 300 400 1,600

L.P. Formulation

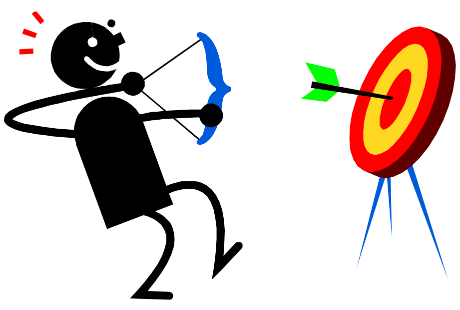
- **MAX: 3 X 1 + 4 X 2 + 5 X3 + 2 X 4 + 6 X 5**
- **Subject To:**

3 X1 + 4 X 2 + 2 X 3 + 2 X 4 + 5 X 5 <= 700 Dept 1

8 X 1 + 3 X 2 + 2 X 3 + 1 X 4 + 4 X 5 <=600 Dept 2

2 X 1 + 1 X 2 + 0 X 3 + 3 X 4 + 4 X 5 <= 400 Dept 3

6 X 1 + 0 X 2 + 2 X 3 + 4 X 4 + 3 X 5 <= 900 Dept 4

- 1X1+0X2+0X3+0X4+0X5<= 100 MKT. X1
- 1X 2 <= 50 MKT X2
- 1X3 <= 90 MKT X3
- 1 X 4 <= 70 MKT. X4
-  1 X 5 <= 30 MKT. X5

- $4X1+7X2+6X3+1X4+3X5 \le 1000$ INV. A
- $2X1+4X2+1X3+1X4+0X5 \le 900$ INV. B
- $0X1+4X2+5X3+6X4+2X5 \le 300$ INV. C
- $1X1+0X2+7X3+4X4+3X5 \le 400$ INV. D
- $3X1+4X2+0\,X3+2X4+4X5 \le 1600$ INV. E

- PAYOFF 557.2727
- X1 45.75758
- X5 30
- X2 12.81385
- X3 37.74892
- DEPT-1 285.9741
- DEPT-3 175.671
- DEPT-4 459.9567
- MKT-X1 54.24242
- MKT-X2 37.18615
- MKT-X3 52.25108
- MKT-X4 70

INV-A 410.77

INV-B 681.73

INV-E 1291.47

- X4 -3.090909
- DEPT-2 .3636364
- INV-C .7272728
- INV-D 9.090908E-02
- MKT-X5 2.818182
- DEP1-1 0 INV-A 0
- DEPT-3 0 INV-B 0
- DEPT-4 0 INVE 0
- MKT-X1 0
- MKT-X2 0
- MKT-X3 0
- MKT-X4 0