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第一題手寫題

以"e"為底，即以下之 log 為"ln"

① $\frac{\partial \left(\log P(x_i ; \mu, \alpha^2) \right)}{\partial \mu} = 0$

$\Rightarrow \frac{\partial \left(-\frac{1}{2\alpha^2} \sum_{i=1}^{n} (x_i - \mu)^2 - \frac{N}{2} \log \alpha^2 - \frac{N}{2} \log(2\pi) \right)}{\partial \mu} = 0$

對 μ 常數　　對"μ 常數"

$\Rightarrow \frac{\partial \left(-\frac{1}{2\alpha^2} \sum_{i=1} \cancel{x_i^2} - 2\mu x_i + \mu^2 - \cancel{\frac{N}{2} \log \alpha} - \cancel{\frac{N}{2} \log(2\pi)} \right)}{\partial \mu} = 0$

$0 \quad 0 \quad 0$

$\Rightarrow \frac{\partial \left(-\frac{1}{2\alpha^2} \sum_{i=1}^{N} - 2\mu x_i + \mu^2 \right)}{\partial \mu} = 0$

$\Rightarrow \frac{\partial \left(-\frac{1}{2\alpha^2} \left(\sum_{i=1}^{N} - 2\mu x_i + \sum_{i=1}^{N} \mu^2 \right) \right)}{\partial \mu} = 0$

$\Rightarrow \frac{\partial \left(-\frac{1}{2\alpha^2} \cdot N \mu^2 + \frac{\mu}{\alpha^2} \sum_{i=1}^{N} x_i \right)}{\partial \mu} = 0$

$\Rightarrow -\frac{N}{\alpha^2} \mu + \frac{1}{\alpha^2} \sum_{i=1}^{N} x_i = 0$　$\boxed{P.1}$

$$\Rightarrow \frac{N}{\cancel{\alpha^2}} u = \frac{1}{\cancel{\alpha^2}} \sum_{i=1}^{N} x_i$$

$$\Rightarrow u = \frac{1}{N} \sum_{i=1}^{N} x_i \#$$

$$\therefore u_{ML} = \underset{u}{\operatorname{argmin}} \left(\sum_{i=1}^{N} \log P(x_i; u, \alpha^2) \right)$$

$$= \frac{1}{N} \sum_{i=1}^{N} x_i \#$$

② 因為我們 log 以 e 為底，= 取 ln 对 α 常數

$$\frac{\partial \left(-\frac{1}{2\alpha^2} \sum_{i=1}^{N} (x_i - u)^2 - \frac{N}{2} \log \alpha^2 - \cancel{\frac{N}{2} \log (2\pi)} \right)}{\partial \alpha} = 0$$

$$\Rightarrow \frac{\partial \left(-\frac{1}{2\alpha^2} \sum_{i=1}^{N} (x_i - u)^2 - \frac{N}{2} \log \alpha^2 \right)}{\partial \alpha} = 0$$

$$\Rightarrow \frac{\partial \left(-\frac{1}{2} \left(\sum_{i=1}^{N} (x_i - u)^2 \right) \alpha^{-2} - N \cdot \log \alpha \right)}{\partial \alpha} = 0$$

$$\Rightarrow -\frac{-2}{2} \sum_{i=1}^{N} (x_i - u)^2 \alpha^{-3} - N \cdot \frac{1}{\alpha} = 0$$

$$\Rightarrow \sum_{i=1}^{N} (x_i - u)^2 \alpha^{-2} - N = 0$$

$$\Rightarrow N = \sum_{i=1}^{N} (x_i - u)^2 \alpha^{-2}$$

承 $\boxed{P3}$

$$\Rightarrow N\sigma^2 = \sum_{i=1}^{N}(x_i-\mu)^2$$

$$\Rightarrow \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2$$

$$\Rightarrow \sigma = +\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2} \#$$

(不為負數)

$$\therefore \sigma_{ML} = \arg\max_{\sigma} P(x;\mu,\sigma^2)$$

$$= \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\mu)^2} \#$$