Linear Model with higher order features

Our error analysis of the toy problem suggested that a straight line was perhaps not the best fit

- positive errors in the extremes
- negative errors in the center

Perhaps a "curve" would be a better hypothesis ? What if our data is not linear ?

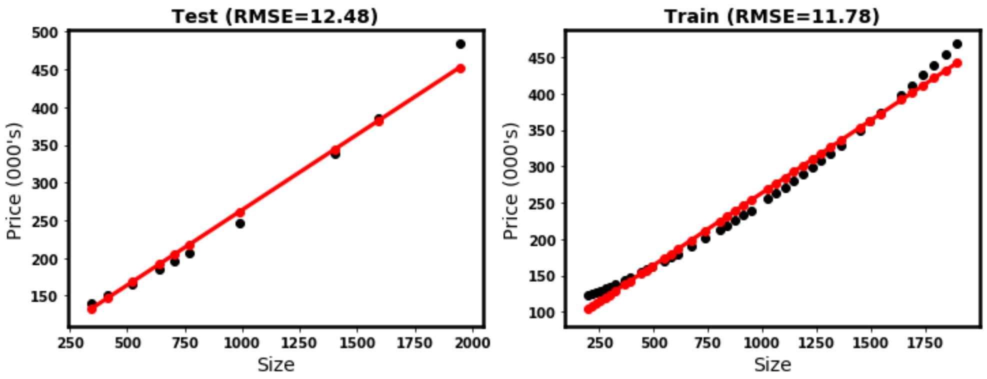
Here's what the dataset looked like

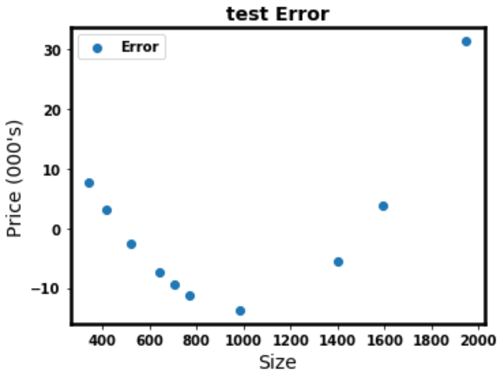
In [4]:

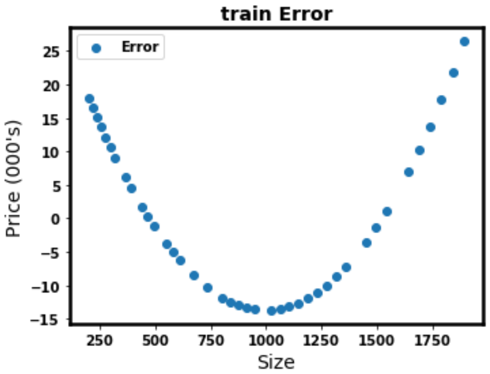
```
(xlabel, ylabel) = ("Size", "Price (000's)")

v1, a1 = 1, .005
v2, a2 = v1, a1*2
curv = recipe_helper.Recipe_Helper(v = v2, a = a2)
X_curve, y_curve = curv.gen_data(num=50)
# _= curv.gen_plot(X_curve,y_curve, xlabel, ylabel)
```

In [5]:
```
_ = curv.regress_with_error(X_curve, y_curve, xlabel=xlabel, ylabel=ylabel)
```

```
Coefficients:
 [64.04203363] [[0.1996918]]


R-squared (test): 0.99
Root Mean squared error (test): 12.48


R-squared (train): 0.99
Root Mean squared error (train): 11.78
```




test Error
Error
Price (000's)
Size
30
20
10
0
−10
400
600
800
1000
1200
1400
1600
1800
2000


train Error
Error
Price (000's)
Size
25
20
15
10
5
0
−5
−10
−15
250
500
750
1000
1250
1500
1750

The plot of the "fitted line" (our predictor) versus the training/test data doesn't look too bad.

- It's only by examining the errors that we see that our predictor has a systematic error.

But the bad news also suggests a solution

- change the model
- from a straight line
- to a curve

Fortunately, we can do this within the framework of a linear model.

Curvature in a linear model

Our (first-order) linear model was

$$\mathbf{y} = \Theta_0 + \Theta_1 \mathbf{x}$$

We can create a *second order* linear model by adding a feature x^2:

$$\mathbf{y} = \Theta_0 + \Theta_1 \mathbf{x} + \Theta_2 \mathbf{x}^2$$

$\mathbf{y}$ is a second order polynomial, whose plot is a curve

- but it is linear in features $\mathbf{x}, \mathbf{x}^2$

In other words, we are performing feature iteration

- in this case: adding the missing feature $\mathbf{x}^2$

Let's modify $\mathbf{x^{(i)}}$ from a vector of length 1:

$$\mathbf{x^{(i)}} = (\mathbf{x_1^{(i)}})$$

to a vector of length 2:

$$\mathbf{x^{(i)}} = (\mathbf{x_1^{(i)}}, \mathbf{x_1^{(i)}}^2)$$

by adding a squared term to the vector $\mathbf{x^{(i)}}$, for each i.

The modified $\mathbf{X}'$ becomes:

$$\mathbf{X} = \begin{pmatrix} 1 & \mathbf{x}_1^{(1)} & (\mathbf{x}_1^{(1)})^2 \\ 1 & \mathbf{x}_1^{(2)} & (\mathbf{x}_1^{(2)})^2 \\ \vdots & \vdots & \\ 1 & \mathbf{x}_1^{(m)} & (\mathbf{x}_1^{(m)})^2 \end{pmatrix}$$

Note that this modified $\mathbf{X}'$ fits perfectly within our Linear hypothesis

$$\hat{\mathbf{y}} = \mathbf{X}'\Theta$$

The requirement is that the model be linear in its *features*, **not** that the features be linear !

What we have done is added a second feature, that just so happens to be related to the first.

We can now run our linear model with the modified feature vectors

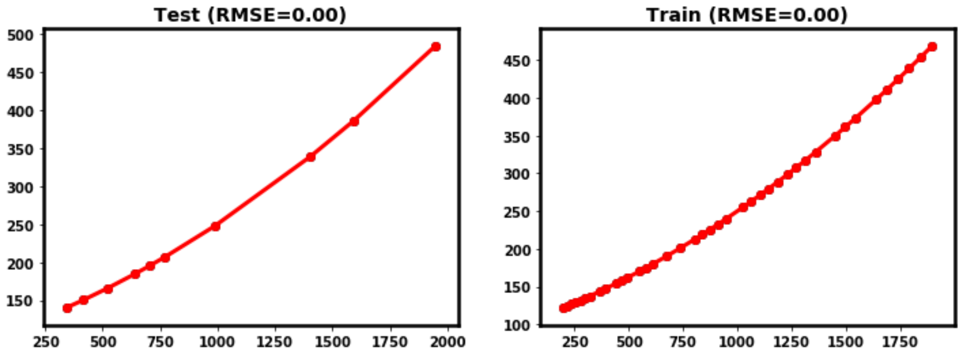
A word about our module

- we add the $\mathbf{x}^2$ column by setting optional parameter `run_transform` to `True`

In [6]:

```
_= curv.run_regress(X_curve, y_curve, run_transforms=True)
```

```
Coefficients:
 [100.] [[1.e-01 5.e-05]]


R-squared (test): 1.00
Root Mean squared error (test): 0.00


R-squared (train): 1.00
Root Mean squared error (train): 0.00
```



Perfect fit !

TIP

- Don't stop just because you scored 91%. And don't give up if the score was awful.
- Examining the errors (residuals) reveals a lot about how to improve your model.
    - Where was the fit good ? Where was it bad ?
    - Is there a pattern to the badly fit observations that points to a missing feature ?

One of the real arts of ML is diagnosing model deficiencies and knowing how to improve them.

We will have a separate module on this topic.

In [7]:

```
print("Done !")
```

```
Done !
```