



Monsoon depression amplification by moist barotropic instability in a vertically sheared environment

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This study makes the case that monsoon depressions over South Asia can form from a variant of moist barotropic instability. Using an idealized numerical framework in which the atmosphere is partitioned into a basic state and a perturbation, we simulate vortices resembling monsoon depressions that draw energy from the meridional shear of the monsoon trough and amplify when they interact with precipitating ascent. The influence of the basic vertical shear on the vortex induces upward velocity which couples precipitation with a Rossby wave-like mode arising from dry barotropic growth, allowing the vortex to intensify. Sensitivity experiments reveal that both the sheared basic state and latent heating are necessary to achieve positive growth rates and that this process requires a sufficiently large initial perturbation. Trajectory analyses suggest that the combined flow of the vortex and the large-scale monsoon transport diabatically generated potential vorticity from southwest of the vortex into the vortex center, thus enabling growth. In contrast with tropical cyclones, this mechanism does not require a feedback between surface wind speed and surface heat and moisture fluxes, though their presence does ultimately result in a slightly stronger vortex.

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1. Introduction

Condensational heating from cumulus convection has long been recognized as the primary energy source for the growth of monsoon depressions in the Bay of Bengal (Krishnamurti *et al.* 1976; Shukla 1978). However, the mechanism that couples this convection to a depression-scale vortex and allows it to grow is less well understood. On its own, convection operates on a much smaller time and space scale compared to a monsoon depression. Thus, the challenge has been to find a mechanism that can organize convection onto a larger scale so that its energy can be used to foster the growth of a depression-scale vortex. A number of ideas have been discussed in past studies that warrant consideration.

One candidate is Conditional Instability of the Second Kind (CISK). There are two categories of CISK: one in which convection is organized by moisture convergence induced by boundary layer friction (Charney and Eliassen 1964) and another in which this moisture convergence is linked to an internal wave mode (Lindzen 1974). Though Shukla (1978) is perhaps the only study devoted to monsoon depressions that has applied this theory quantitatively, many others have employed the concept qualitatively to explain some aspect of monsoon depressions (Chen and Yoon 2000; Dash *et al.* 2004; Chen *et al.* 2005; Sørland *et al.* 2016) or have speculated it to be important based on their results (Keshavamurty *et al.* 1978; Goswami *et al.* 1980; Sørland and Sorteberg 2015). One nagging defect of CISK is that it predicts maximum growth rates at the smallest

resolvable scales (e.g. Crum and Dunkerton 1992) and this flaw persists when applied to monsoon depressions (Shukla 1978). Additionally, most wave-CISK theories are based on highly simplistic parameterizations of moist convection and classical wave-CISK modes do not appear in either observations or cloud-resolving numerical simulations (Bretherton 2003).

Another possibility is that monsoon depressions amplify by a mechanism similar to that of tropical cyclones. The most widely accepted theory for tropical cyclone intensification is Wind Induced Surface Heat Exchange (WISHE), a process by which convection is organized by wind-speed dependent surface heat and moisture fluxes (Emanuel 1986; Rotunno and Emanuel 1987). From a weather forecasting perspective, tropical cyclones are perhaps the closest analogue to monsoon depressions, with the Indian Meteorological Department (IMD) ranking weather systems in the Indian Ocean on a continuum which includes lows, depressions, and cyclonic storms. Storms within the last of those categories are equivalent to tropical cyclones and are given official names. Additionally, because of similarities between environmental conditions favoring monsoon depressions (Sikka 1977) and those favoring tropical cyclones, some studies have used the Genesis Potential Index (GPI) developed for tropical cyclones (Emanuel and Nolan 2004) to explain long-term trends in monsoon depression frequency (Prajesh *et al.* 2013; Vishnu *et al.* 2016, 2018). Parameters included in the GPI, such as vertical wind shear, sea surface temperature, absolute vorticity, and relative humidity, are also frequently discussed in IMD bulletins providing forecasts for monsoon depressions. Despite

these similarities, one curious discrepancy is that monsoon depressions often form in an environment of high vertical wind shear. In fact, because of this shear, the GPI predicts conditions over the Bay of Bengal to be less favorable for tropical cyclogenesis during the middle of summer (Camargo *et al.* 2007; Tippett *et al.* 2011), a time when monsoon depression frequency peaks (Hurley and Boos 2015; Ditchek *et al.* 2016). Whether they form because of this shear, or in spite of it, is not clear. Additionally, monsoon depressions occasionally form over land, where the GPI is undefined since it requires a value for sea surface temperature. Both of these observations suggest that monsoon depressions may be a phenomenon distinct from tropical cyclones.

Though generally detrimental to tropical cyclone intensification, vertical shear could promote other growth mechanisms. Unlike most of the maritime tropics, the South Asian monsoon features a strong time-mean meridional temperature gradient which is in approximate thermal wind balance with its strong vertical shear. This peculiarity has led many to argue that monsoon depressions are a manifestation of moist baroclinic instability (Moorthi and Arakawa 1985; Krishnakumar *et al.* 1992; Kasture *et al.* 1993; Aravequia *et al.* 1995; Krishnamurti *et al.* 2013). In this theory, precipitation is organized by a synoptic-scale baroclinic wave, whose warm advection drives ascent and condensation. When this condensation is biased into the warm part of the wave, instead of remaining strictly in quadrature, it enhances the growth of the original baroclinic wave. A major problem with this mechanism as an explanation for monsoon depressions is that it requires a growing disturbance to tilt upshear (Cohen and Boos 2016). Monsoon depressions by contrast generally exhibit no tilt or a slightly downshear tilt (Keshavamurty *et al.* 1978; Cohen and Boos 2016).

In addition to a strong meridional temperature gradient, the South Asian monsoon also exhibits a substantial meridional gradient of moisture. Adames and Ming (2018a) argue that this juxtaposition could lead to growing depression-scale disturbances through a process they have termed moisture-vortex instability, which is an extension of the moisture mode theory of Sobel *et al.* (2001). According to this idea, convection is organized by the advection of warm air and moisture in the northerly flow to the west of the vortex. This convection in turn amplifies the vortex through vorticity stretching. A necessary feature for growth in this mechanism is a time delay between the peak moistening rate and convection, which shifts the vortex stretching into phase with the positive vorticity. A problem with this mechanism in regards to monsoon depressions is that it requires the ascent driven by meridional temperature advection to be weak compared to the ascent driven by meridional moisture advection, a circumstance in disagreement with the presence of a strong temperature gradient and the results of Adames and Ming (2018b).

Another prominent feature of the South Asian monsoon is a strong meridional gradient of vorticity. Various studies have noted that this gradient can sometimes reverse sign, leading to the potential for barotropic instability, with the meridional shear serving as an energy source (Krishnamurti *et al.* 1976; Keshavamurty *et al.* 1978; Goswami *et al.* 1980; Subrahmanyam *et al.* 1981; Nitta and Masuda 1981; Lindzen *et al.* 1983; Mishra 2018). Recently, Diaz and Boos (2019) provided additional evidence to support this idea: a normal-mode linear instability analysis of a basic state representative of the environment preceding monsoon depression genesis over the Bay of Bengal yielded wave structures resembling monsoon depressions whose primary energy source was the basic-state meridional shear. However, they also found that, although zonally symmetric basic states were unstable, a basic state with realistic zonal variations was not. In this latter case, although wave structures were isolated that drew energy

from the meridional shear and resembled monsoon depressions, energy lost to dissipative processes led to the net growth rate being weakly negative. Nevertheless, they noted that these wave structures had regions of upward vertical velocity which they speculated could couple them to convection and lead to positive net growth. Building upon the results of Diaz and Boos (2019), the goal of this study is to test that speculation. In other words, are monsoon depressions a variant of moist barotropic instability?

This study is outlined as follows: In section 2, we introduce the numerical modeling framework. We present the structure and energetics of the disturbance for the control simulation in 3.1 and then discuss how it interacts with moist convection in section 3.2. In section 3.3, we present sensitivity experiments to gain further insight into which physical processes are important, including assessing to what extent the WISHE and frictional CISK mechanisms are applicable. Finally, in section 4, we summarize the results.

2. Methods

We use the same numerical modeling framework developed in Diaz and Boos (2019) as a starting point. In that study, we took the anelastic equations for atmospheric flow and derived a new equation set that is partitioned into a basic state and a perturbation. This partitioning provides a conceptually simpler interpretation of how a disturbance interacts with a specified background flow compared to solving the unpartitioned equation set. In the present study, we will augment this equation set to include representations of moist convection, turbulence, and surface fluxes.

2.1. Governing Equations

The momentum, heat, and pressure fields are governed by the following equations

$$\frac{\partial \mathbf{u}'}{\partial t} = -(\bar{\mathbf{U}} + \mathbf{u}') \cdot \nabla \mathbf{u}' - \mathbf{u}' \cdot \nabla \bar{\mathbf{U}} - \nabla \phi' + b' \hat{\mathbf{k}} - f \hat{\mathbf{k}} \times \mathbf{v}' + \mathbf{T}'_{\mathbf{u}} \quad (1a)$$

$$\frac{\partial \theta'}{\partial t} = -(\bar{\mathbf{U}} + \mathbf{u}') \cdot \nabla \theta' - \mathbf{u}' \cdot \nabla \bar{\theta} - w' \frac{d\theta_0}{dz} + M'_{\theta} + T'_{\theta} \quad (1b)$$

$$\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial}{\partial z} \left(\rho_0 \frac{\partial \phi'}{\partial z} \right) = \nabla \cdot (\rho_0 G \{ \mathbf{u}' \}) \quad (1c)$$

where $\mathbf{u} = \langle u, v, w \rangle$ represents the three-dimensional velocity field, $\mathbf{v} = \langle u, v, 0 \rangle$ the horizontal velocity field, θ is potential temperature, ρ_0 is the base state density, and ϕ is the pressure variable, defined as $\phi \equiv \frac{p}{\rho_0}$, with p being standard pressure. The primes denote perturbation values and bars basic state values. As is commonly done in anelastic systems, the basic state potential temperature is partitioned into a horizontally uniform part, θ_0 , and a horizontally varying part, $\bar{\theta}$. Microphysical processes, i.e. condensation and evaporation, are represented by M and tendencies from surface fluxes and subgrid-scale turbulence by T . The buoyancy term (b') is defined in terms of virtual potential temperature,

$$b' = g \left(\frac{\theta'}{\theta_0} + 0.61q' - c - r \right) \quad (2)$$

where q' is the perturbation water vapor mixing ratio, c is cloud water mixing ratio, and r is rain water mixing ratio. These moisture variables obey the following equations:

$$\frac{\partial q'}{\partial t} = -(\bar{\mathbf{U}} + \mathbf{u}') \cdot \nabla q' - \mathbf{u}' \cdot \nabla \bar{q} - w' \frac{dq_0}{dz} + M'_q + T'_q \quad (3a)$$

$$\frac{\partial c}{\partial t} = -(\bar{\mathbf{U}} + \mathbf{u}') \cdot \nabla c + M_c + T_c \quad (3b)$$

$$\frac{\partial r}{\partial t} = -(\bar{\mathbf{U}} + \mathbf{u}') \cdot \nabla r - w_r \frac{\partial r}{\partial z} + M_r + T_r \quad (3c)$$

where w_t is the terminal velocity of rain, and M and T represent sources and sinks of each moisture variable due to microphysical processes and subgrid-scale turbulence, respectively. The equation for water vapor (q) is derived in the same manner as that for θ . By contrast, cloud and rain water do not have a basic state. This formulation is equivalent to neglecting advection of the basic state cloud and rain field. The microphysics tendencies are calculated using a Kessler microphysics scheme (Kessler 1969), which includes only warm rain processes.

2.2. Surface Heat and Momentum Fluxes

Since it is possible that monsoon depressions draw energy through a WISHE-like or a CISK-like process, and because frictional dissipation can be a large sink of energy, it is important to have a representation of surface fluxes of heat, moisture, and momentum. With the existence of a time-independent basic state, special consideration must be given in their formulation. We assume that the basic state temperature, moisture, and wind distributions are in equilibrium with their respective basic state surface fluxes, such that, when added to Eq. 1, these fluxes preserve the time independence of the basic state. The perturbation flux for a generic scalar variable a can be defined as

$$F'_a = F_a - \bar{F}_a. \quad (4)$$

The total flux can be parameterized as

$$F_a = C_E |\mathbf{v}_{z_0}| (a_s - a_{z_0}) \quad (5)$$

where a_{z_0} is the value of a at the lowest model level, a_s is its surface value, $|\mathbf{v}_{z_0}|$ is the wind speed at the lowest model level, and C_E is a non-dimensional drag or exchange coefficient. Partitioning all variables into their basic state and perturbation values and substituting into Eqs. 5 and 4 yields the perturbation surface latent and sensible heat fluxes

$$F'_q = C_E \{ (|\mathbf{v}| - |\bar{\mathbf{v}}|) (q_s^* - \bar{q} - q_0) - |\mathbf{v}| q' \}_{z_0} \quad (6a)$$

$$F'_\theta = C_E \{ (|\mathbf{v}| - |\bar{\mathbf{v}}|) (T_s - \bar{T}) - |\mathbf{v}| \theta' \}_{z_0} \quad (6b)$$

where q_s^* is the saturation mixing ratio at the ocean surface, T_s the surface temperature of the ocean, and \bar{T} the basic state surface air temperature. All quantities except for q_s^* and T_s are evaluated at the lowest model level (z_0). One interpretation of Eq. 6a is that anomalous positive moisture fluxes occur in regions where the near-surface air is anomalously cool or dry or where the near-surface wind speed exceeds its basic state value. Following Rotunno and Emanuel (1987), C_E is set to

$$C_E = 1.1 \times 10^{-3} + 4 \times 10^{-5} |\mathbf{v}_{z_0}| \quad (7)$$

over the ocean. Over land, F'_q and F'_θ are set to zero. Note that although these perturbation fluxes are zero over land, there are still basic state heat (\bar{F}_θ) and moisture (\bar{F}_q) fluxes implicit in the formulation of the equations. Analogous to the preceding derivation, the perturbation surface momentum stresses can be defined as

$$\tau'_{xz} = C_D \{ |\mathbf{v}| u' + (|\mathbf{v}| - |\bar{\mathbf{v}}|) \bar{U} \}_{z_0} \quad (8a)$$

$$\tau'_{yz} = C_D \{ |\mathbf{v}| v' + (|\mathbf{v}| - |\bar{\mathbf{v}}|) \bar{V} \}_{z_0} \quad (8b)$$

where C_D is the drag coefficient. Over water, $C_D = C_E$, and over land, $C_D = 3 \times 10^{-3}$.

2.3. Subgrid-scale Turbulence Closure

We base our subgrid-scale turbulence parameterization on the first-order turbulence closure discussed in Rotunno and Emanuel (1987) and Skamarock *et al.* (2008), with special considerations for our inclusion of a basic state. For compactness, we use tensor notation, with the wind components defined as $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and the coordinate system as $\mathbf{r} = \langle x_1, x_2, x_3 \rangle$. The tendencies of the perturbation wind due to subgrid-scale turbulence can be represented as

$$T'_{u_i} = \frac{\partial \tau'_{ij}}{\partial x_j}. \quad (9)$$

The stress tensor τ'_{ij} is defined as

$$\tau'_{ij} = K(\mathbf{r}, t) d'_{ij} \quad (10)$$

where K is a mixing coefficient and d'_{ij} is the rate of deformation tensor, defined as

$$d'_{ij} = \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}. \quad (11)$$

The tendencies for a generic perturbation scalar quantity a due to subgrid-scale turbulence can be represented as

$$T'_a = -\frac{\partial F'_j}{\partial x_j} \quad (12)$$

where $F'_j = -K(\mathbf{r}, t) \partial a' / \partial x_j$.

We parameterize the mixing coefficient K using

$$K(\mathbf{r}, t) = C_s^2 l_{h,v}^2 \left(D^2 - P_r^{-1} N^2 \right)^{1/2} \quad (13)$$

where $C_s = 0.25$, $P_r^{-1} = 3$, N^2 is the Brunt-Väisälä frequency, and D^2 is a function of the deformation of the flow. If the quantity $D^2 - P_r^{-1} N^2$ is less than zero, K is set to zero. As in Skamarock *et al.* (2008), we set the vertical mixing length l_v equal to the vertical grid spacing, which is a function of height. The horizontal mixing length l_h is set to 2000 m. This value is less than that of Skamarock *et al.* (2008), who suggests using a value equal to the horizontal grid spacing (i.e. 6000 m) for anisotropic turbulence, but is closer to the 3000 m used in Rotunno and Emanuel (1987). Upon testing, we found that the higher values used by Skamarock *et al.* (2008) and Rotunno and Emanuel (1987) appeared to be overly diffusive and thus opted for a smaller value. We also noted that the decreasing amount of energy dissipation accompanying decreases in l_h tend to be compensated by increases in energy dissipated through numerical diffusion, thus decreasing overall sensitivity to changes in l_h .

The total rate of deformation is defined as

$$D_{ij} = d'_{ij} + \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \quad (14)$$

and D^2 is defined as

$$D^2 = \frac{1}{2} \left[D_{11}^2 + D_{22}^2 + D_{33}^2 \right] + D_{12}^2 + D_{13}^2 + D_{23}^2. \quad (15)$$

The Brunt-Väisälä frequency is defined as

$$N^2 = g \left[A \frac{\partial \theta_e}{\partial z} - \frac{\partial (q + c + r)}{\partial z} \right], \text{ if saturated} \quad (16a)$$

$$N^2 = g \left[\frac{1}{\theta} \frac{\partial \theta}{\partial z} + 0.61 \frac{\partial q}{\partial z} - \frac{\partial c}{\partial z} - \frac{\partial r}{\partial z} \right], \text{ if unsaturated} \quad (16b)$$

and

$$A \equiv \frac{1}{\theta} \frac{1 + \frac{Lq}{R_d T}}{1 + \frac{0.622 L^2 q}{c_p R_d T^2}} \quad (17)$$

where θ_e is equivalent potential temperature, L is the latent heat of vaporization, R_d the dry gas constant, and c_p the specific heat of air at constant pressure. All values pertaining to atmospheric fields in Eq. 16 and 17 are calculated from the full fields, i.e. basic state plus perturbation.

2.4. Basic State, Initialization, and Model Grid Parameters

The basic state (i.e. \bar{U} , \bar{V} , \bar{W} , $\bar{\theta}$, and \bar{q}) used in this study is identical to that of [Diaz and Boos \(2019\)](#), who performed an extensive analysis of its instability characteristics for a dry atmosphere. It is obtained from a composite of atmospheric states preceding monsoon depression genesis derived from the ERA-Interim reanalysis ([Dee et al. 2011](#)). As noted previously, a key feature of this basic state is that its slowest-decaying normal mode has a structure similar to that of observed monsoon depressions. Figure 1 shows the basic state wind at 3.36 km and moisture field at 1.16 km of the full model domain. An important feature of the background environment for monsoon depressions is the west-to-east elongated monsoon trough, along whose axis the zonal wind shifts direction. Monsoon depressions generally form within this trough and track westward along its axis. Low-level moisture is abundant within the monsoon trough, with slowly decreasing values to the south and rapidly decreasing values to the west of India in the desert regions. Although this basic state was chosen to be representative of that in which monsoon depressions typically form, the large-scale weather pattern in this region can vary considerably from day to day. Thus, an important caveat to keep in mind throughout this study is that different growth mechanisms could be operating for monsoon depressions which form in basic states substantially different from the one explored here.

Choosing a suitable perturbation field (i.e. u' , θ' , and q') to initialize the simulation is complicated by lack of knowledge of the initial disturbances from which monsoon depressions typically arise. For lack of a better option, we use the least stable normal mode of this basic state obtained in the dry simulation of [Diaz and Boos \(2019\)](#). This method has some precedent, with a number of previous idealized instability analyses using a most unstable linear normal mode to initialize nonlinear dry and moist experiments ([Thorncroft and Hoskins 1994](#); [Lambaerts et al. 2011, 2012](#)). This choice also has the benefit of making our results more comparable with those of [Diaz and Boos \(2019\)](#). Since the dry normal mode has no perturbation moisture field, the initial q' is set to zero.

All of our experiments use 6 km grid spacing. Although this resolution is coarser than what is typically recommended for convection-resolving simulations (e.g. [Weisman et al. 1997](#)), we found little difference between a 6 km and a 4 km simulation and decided using finer resolution was not worth the additional computing resources. Interestingly, we were able to get reasonable simulations up to about 25 km grid spacing, perhaps because synoptic-scale dynamics organize the moist convection on larger scales. For monsoon depressions themselves, this outcome is consistent with the results of [Hunt and Turner \(2017a\)](#), who found only marginal improvements in the structure of monsoon depressions simulated in the Met Office Unified Model as resolution was refined beyond about a 39 km grid spacing. However, since their simulation included a convective parameterization, it may not be directly comparable. Nevertheless, our results do not seem to be overly sensitive to resolution, at

least within the above-stated ranges. With 6 km grid spacing, our domain, which stretches from 58.0°E to 115°E and 3.5°N to 36°N, requires 1060 by 620 grid points. The domain extends to 20 km in the vertical and consists of 42 vertical levels. The vertical grid spacing is stretched from 50 m at the lowest level to 1000 m at the uppermost using the technique of [Anthes \(1970\)](#).

3. Results

Most of our analysis will be dedicated to a control simulation that we deem to best represent the development of a monsoon depression in the Bay of Bengal. We then analyze a series of sensitivity experiments designed to elucidate some of the physical processes important for the growth of the simulated disturbance.

3.1. Disturbance Structure and Energetics

Figure 2 shows the time evolution of the perturbation pressure (p') and wind (\mathbf{v}') overlaid with the basic state zonal wind (\bar{U}) at 3.36 km above sea level. At Day 2 of the simulation, a weak area of low pressure is approaching the Bay of Bengal with an area of higher pressure to its west. This low pressure amplifies as it moves over the ocean (Day 4) and makes landfall over northeastern India (Day 6). As it moves over land, it weakens and a region of high pressure forms to its east (Days 8 and 10). By Day 12, the entire wave train is weakening as it continues moving westward. The evolution of this disturbance is similar to that of observed monsoon depressions, though its lifespan lies at the higher end of typical values ([Krishnan et al. 2011](#)). A possible explanation for this longer lifespan is that the basic state, which was chosen to favor monsoon depression formation, does not change with time.

Figure 3 shows a cross section of the perturbation wind and potential temperature (θ') through the center of the low pressure at 6.3 d into the simulation, a time shortly after landfall. The section is drawn to be roughly parallel to the axis of the monsoon trough such that it passes directly through the wind maxima on either side of the low pressure center. The wind values shown are the components orthogonal to the plane of the cross section. To highlight the larger-scale structure of the disturbance, both fields are averaged 240 km on either side of the cross section. Similar to observed monsoon depressions (e.g. [Hurley and Boos 2015](#); [Hunt et al. 2016](#)), this disturbance has a warm-over-cold core structure with a lower-to-middle tropospheric maximum in perturbation wind. Although the warm core is centered near 9 km at this time, the altitude of this center varies throughout the evolution of the vortex from near 5 km during its early phase to near 10 km at later times.

To better understand the physical processes that lead to the growth of this disturbance, we examine the time evolution of its perturbation kinetic energy, defined as $K_p \equiv 0.5(u'^2 + v'^2)$. For the model equation set, the rate of change of K_p can be expressed as

$$\begin{aligned} & \frac{\partial K_p}{\partial t} + (\bar{\mathbf{U}} + \mathbf{u}') \cdot \nabla K_p + \\ & u' v' \partial_y \bar{U} + u' v' \partial_x \bar{V} + u'^2 \partial_x \bar{U} + v'^2 \partial_y \bar{V} + u' w' \partial_z \bar{U} + v' w' \partial_z \bar{V} \\ & = -\mathbf{v}' \cdot \left(\frac{\partial \phi'}{\partial x} \hat{\mathbf{i}} + \frac{\partial \phi'}{\partial y} \hat{\mathbf{j}} \right) + \mathbf{v}' \cdot (T_u' \hat{\mathbf{i}} + T_v' \hat{\mathbf{j}}) + \mathbf{v}' \cdot (n_u' \hat{\mathbf{i}} + n_v' \hat{\mathbf{j}}). \end{aligned} \quad (18)$$

On the left-hand side, the first term represents the local change of K_p , the second term represents advection of K_p , and the remaining terms represent barotropic conversion. On the right-hand side, the first term represents work done by the pressure gradient force, the second term represents tendencies from friction and subgrid-scale turbulence, and the final term is the diffusion implicit in the numerical method used to calculate

advection, with \mathbf{n}' being the velocity tendency vector due to this process. For succinctness, we combine the barotropic terms into a horizontal shear term ($u'v'\partial_y\bar{U} + u'v'\partial_x\bar{V}$), a horizontal convergence term ($u'^2\partial_x\bar{U} + v'^2\partial_y\bar{V}$), and a vertical shear term ($u'w'\partial_z\bar{U} + v'w'\partial_z\bar{V}$). Figure 4 shows the time evolution of each term averaged over the entire model domain, with Fig. 4a and Fig. 4b showing them expressed as the rate of energy generation and Fig. 4c and Fig. 4d as growth rates. To highlight the correspondance between precipitation and pressure work, Fig. 4a also shows the rain water mixing ratio averaged within a $15^\circ \times 15^\circ$ box centered on the minimum pressure.

During most of the first two days, K_p is decaying. The dominant energy source is from horizontal shear with the other terms remaining small or negative. Growth rates increase rapidly beyond day 2 as the pressure work term becomes dominant. This sudden increase in pressure work coincides with the increase in rain water (Fig. 4a). Thus, we attribute this energy to latent heat release from convection decreasing hydrostatic pressure which in turn leads to an acceleration of the perturbation wind. From day 4 through 8, growth waxes and wanes with positive contributions mainly from pressure work and horizontal shear and losses from friction and numerical diffusion. The growth rates of about 0.25 d^{-1} to 0.3 d^{-1} from horizontal shear throughout the simulation are similar to that of the most unstable normal mode of this basic state (Fig. 11 in Diaz and Boos (2019)). By day 9, the total growth rate becomes permanently negative as the disturbance decays. The gradual loss of pressure work as a dominant energy source corresponds with a decrease in precipitation. However, meridional shear continues to be a large energy source, slowing down its overall decay as it moves inland.

In comparison with the results of Diaz and Boos (2019), the disturbance in this simulation exhibits many of the features of the least stable dry normal mode of this basic state, including its overall size, track, vertical structure, and the fact that it gains energy from the meridional shear of the basic state. A major difference is that, with the addition of moist convection, this disturbance amplifies with a large source of energy coming from pressure work. The following section will detail how this convection leads to its amplification

3.2. Role of Convection in Vortex Amplification

The interaction with latent heat release is essential for the disturbance presented in section 3.1 to achieve net growth. This necessity will be demonstrated explicitly in section 3.3. Here we explore how the disturbance organizes convection and how this convection leads to its amplification.

3.2.1. Organization of the Rain Field

One distinguishing feature of monsoon depressions is their asymmetric precipitation field. Whereas precipitation in well organized tropical cyclones is roughly symmetric about their center, precipitation in monsoon depressions is biased toward the southwestern quadrant of their circulation (Godbole 1977; Sanders 1984). To summarize the precipitation field in our simulation, we examine a mass-weighted average of rain water mixing ratio (r) from 0.4 km to 15.0 km (Fig. 5). For reference, this quantity is overlaid with the perturbation pressure (p') and wind field (\mathbf{v}'), both at 3.36 km and smoothed with a Gaussian filter to emphasize the larger-scale circulation. Figure 5a shows these fields at 3.7 d (88 h) into the simulation, a time roughly coinciding with the peak growth rate (Fig. 4). At this time, rain is most extensive and intense south and west of the circulation center, with the highest values located about 300 km west of center. Rain in the vicinity of the center is much weaker and more scattered. As noted above, this pattern is typical of

monsoon depressions. By 4.8 d (114 h), a time near the peak vortex intensity, the heaviest rain has become concentrated into a band located south and west of the circulation center, with the central region becoming devoid of precipitation (Fig. 5b). By 7.6 d (182 h), the rain has become less intense but is still widespread (Fig. 5c). This decreasing precipitation aligns well with the decreasing growth rates from pressure work (Fig. 4).

The precipitation distribution within monsoon depressions has generally been linked to their interaction with the background wind shear and temperature gradient associated with the large-scale monsoon (e.g. Rao and Rajamani 1970; Rajamani and Rao 1981; Sanders 1984; Boos et al. 2015; Adames and Ming 2018b). This interaction has been most frequently expressed using the quasi-geostrophic (QG) omega equation. Following this approach, we use the Q-vector form of the QG-omega equation. Starting with the height coordinate version (Hoskins et al. 1978; Hoskins 1997) and retaining the separation between the basic state and perturbation, the two components of the Q-vector become

$$Q_1 = -\frac{g}{\theta_0} \left(\frac{\partial \bar{\mathbf{V}}}{\partial x} \cdot \nabla_h \theta' + \frac{\partial \mathbf{v}'_\psi}{\partial x} \cdot \nabla_h \bar{\theta} + \frac{\partial \mathbf{v}'_\psi}{\partial x} \cdot \nabla_h \theta' \right) \quad (19a)$$

$$Q_2 = -\frac{g}{\theta_0} \left(\frac{\partial \bar{\mathbf{V}}}{\partial y} \cdot \nabla_h \theta' + \frac{\partial \mathbf{v}'_\psi}{\partial y} \cdot \nabla_h \bar{\theta} + \frac{\partial \mathbf{v}'_\psi}{\partial y} \cdot \nabla_h \theta' \right) \quad (19b)$$

Following previous applications of the QG-omega equation in the tropics (Kiladis et al. 2006; Boos et al. 2015), we have substituted the rotational component of the wind (\mathbf{v}'_ψ) for the geostrophic wind. For simplicity, we have used the full basic state wind instead of its rotational component. The QG-omega equation can be expressed as

$$\left(N^2 \nabla_h^2 + f_o^2 \frac{\partial^2}{\partial z^2} \right) w'_q = 2 \nabla_h \cdot \mathbf{Q} + \beta \frac{g}{\theta_0} \frac{\partial \theta'}{\partial x} \quad (20)$$

where β is the meridional gradient of the Coriolis parameter and w'_q is the perturbation QG vertical velocity. The quantity w'_q can then be determined by inverting the operator on the left-hand side of Eq. 20. At all boundaries, we assume that w'_q goes to zero. This matches the boundary condition for w' in the model equation set. The resulting w'_q field corresponding to times in Fig. 5 is shown in Fig. 6, along with the 3.36 km perturbation stream function (ψ'). To provide a more integrated perspective of QG vertical motion, the w'_q field is averaged from 1.45 km to 5.36 km above sea-level. Because of the simulation's high resolution, considerable spatial variability occurs in the vicinity of the convection. Since QG dynamics are presumably less applicable at this fine scale, the following discussion focuses on the broader scale w'_q structure.

At 3.7 d, QG ascent lies south and west of the circulation center, with QG descent to the northeast (Fig. 6a). This arrangement corresponds well with the precipitation field, which has extensive rain to the southwest and little rain to the northeast (Fig. 5a). Observed monsoon depressions exhibit a similar dipole of QG vertical motion (e.g. Fig. 4 in Boos et al. (2015)). The positive w'_q values of 1 cm s^{-1} to 2 cm s^{-1} west of the circulation center are of similar magnitude to the actual vertical velocity (w') of the dry simulations of Diaz and Boos (2019) (their Fig. 15). The pattern of QG ascent remains similar at 4.8 d with ascent to the south and west and descent to the north and east (Fig. 6b). At this time, precipitation occurs within the southern portion of the QG ascent region (Fig. 5b). This offset could result from the delay between when an air parcel begins its ascent and when it becomes saturated with water vapor, during which time it is advected southward. The pattern of QG ascent changes by 7.6 d, with a large area of QG ascent west of the circulation center and a small area of QG descent to the east (Fig. 6c). Though precipitation remains within areas of positive w'_q (Fig. 5c), a large region of QG ascent

unaccompanied by precipitation occurs north of the vortex. Again, we speculate that this offset may be attributed to a delay between the start of ascent and the onset of saturation.

3.2.2. Potential Vorticity Analysis

We now work toward understanding how latent heat release amplifies the circulation. Previous studies showed that the dominant source of rotation in monsoon depressions comes from vorticity stretching, presumably from the vertical motion coupled to convection (Daggupaty and Sikka 1977; Boos *et al.* 2015). To make this link more explicit, we use a potential vorticity budget, in which the contribution from latent heating can be isolated. Recognizing the partition between perturbation and basic state, the perturbation potential vorticity (P') is expressed as

$$P' = \frac{1}{\rho_0} [\zeta' \cdot \nabla(\bar{\theta} + \theta_0) + (\bar{\zeta}_a + \zeta') \cdot \nabla\theta'] \quad (21)$$

where ζ and ζ_a are the relative and absolute vorticity vectors, respectively, and the prime and bar notation follows the same logic as the original governing equation set. The distribution of P' generated from diabatic heating over the time interval T_1 to T_2 can be expressed as

$$P'_d = \frac{1}{\rho_0} \int_{T_1}^{T_2} (\bar{\zeta}_a + \zeta') \cdot \nabla M'_\theta dt. \quad (22)$$

The potential vorticity framework is useful because, through the invertibility principle (Hoskins *et al.* 1985), one can conceive of the P' generated by latent heat release as contributing to the larger-scale circulation of the monsoon depression. Additionally, since P' is conserved following the flow in the absence of P' sources and sinks, one can envision P' being generated by convection and then passively advected to regions away from the convection, as will be elaborated upon in Sec. 3.2.3. The time evolution of P' and P'_d averaged from 0.4 km to 5.13 km are shown in Fig. 7 with perturbation winds (\mathbf{v}') at 3.36 km overlaid. Values of P'_d in each plot are integrated over the two hours previous to the time shown and then converted to an hourly generation rate. To make the fields more presentable, they are smoothed with a Gaussian filter and plotted on a \log_2 scale for P' and a \log_3 scale for P'_d .

At 3.7 d, a broad region of higher P' is located within the circulation center (Fig. 7a). However, the largest P' and P'_d are positioned well to the west of the circulation center in the region of maximum precipitation (Fig. 5a). Values of P'_d near the circulation center, though non-zero, are about one to two orders of magnitude smaller than those far to the west. This arrangement of P' and P'_d resembles the vorticity budgets examined in previous studies, where the maximum vorticity production through vorticity stretching occurs southwest of the vortex center in the region of highest precipitation (Daggupaty and Sikka 1977; Krishnan *et al.* 2011; Boos *et al.* 2015). By 4.8 d, the maximum P'_d has become concentrated within the precipitation band on the southern side of the vortex (Fig. 7b). A broad area of high P' remains located within the vortex center, though it is separated from the region of P'_d . By 7.6 d, the central region of P' has become fragmented with an east-to-west oriented region of P'_d (Fig. 7c). This transformation is consistent with the weakening rain field (Fig. 5c) and a decreasing amount of energy generated from pressure work (Fig. 4).

3.2.3. Trajectory Analysis

As seen in Sec. 3.2.2, the region of highest P'_d associated with convection is offset south and west of the circulation center.

However, large values of P' have also accumulated within the circulation center, away from the convection. A reasonable explanation is that this P' was created within the precipitating region then passively advected downstream. This process can readily be seen in animations of potential vorticity (not shown). Here, we test this supposition by calculating trajectories forward in time emanating from the region of maximum P'_d . We focus on the interval from 80 h to 116 h, near the period of peak growth (Fig. 4). Results are shown in Fig. 8. The dashed-line box, which is drawn to roughly enclose the region of strongest convection (shading) and thus highest P'_d , delineates the region from which the trajectories are specified to originate. The vertical profile of P' and P'_d averaged over this box is shown in Fig. 9. With P' generation at low levels and P' destruction at high levels, this profile is more characteristic of convective heating than stratiform. We note that profiles farther south are more characteristic of stratiform heating, with P' generation at upper levels and P' destruction at lower levels (not shown), consistent with Krishnan *et al.* (2011) and Boos *et al.* (2017), who noted that heating profiles in monsoon depressions tend to be more stratiform. Since P'_d within the box is largest at low levels, we limit our trajectories to those which begin between 0.41 km and 2.28 km above the surface. The lines in Fig. 8 depict individual trajectories colored by altitude from 80 h to 116 h originating from the above-described volume.

Figure 8 suggests two general paths taken by air parcels leaving the convection. Some parcels ascend high enough within the convection to be rapidly advected westward in the strong flow aloft. Others remain low enough to be advected eastward and then swept cyclonically around the circulation center. It is these eastward-moving low-level air parcels which are capable of transporting high potential vorticity generated at low-levels by diabatic heating toward the circulation center where they can accumulate and contribute to the depression-scale rotation. We argue that this process links convection south and west of the vortex center to the amplification of the depression-scale circulation. The integrated effect of this process may explain the large values of P' in regions devoid of convection seen in Fig. 7.

This intensification mechanism, incorporating both the potential vorticity and trajectory analysis, is summarized in Fig. 10. The path of the P' center of mass from 56 h to 116 h is marked with \times symbols and the path of the P'_d center of mass is marked by $+$ symbols (Fig. 10a). Both are calculated using positive values averaged from 0.41 km to 5.13 km above the surface. The arrowed lines show two median trajectory paths emanating from the region of maximum P'_d over the 80 h to 116 h time period. The dotted line includes trajectories which, after 10 hours, lie east of the box depicted in Fig. 8 and are below 2 km. The dashed line is the same, except for those lying between 2 km and 5 km. The former contains 1135 of the original 4842 trajectories and the latter 1577. Their median altitude as a function of time is shown in Fig. 10b. Over the 80 h to 116 h timeframe, the P'_d center of mass lies west of the P' center of mass at earlier times, with their orientation gradually shifting to southwest-to-northeast at later times. Low-level air parcels originating from the P'_d region are located within the fast westerly flow and are thus quickly advected toward the east. In fact, within a period of only four hours, the median trajectory is already positioned east of the P' center of mass. The two median trajectories then orbit the P' center of mass where their vorticity is incorporated into the broader-scale circulation.

Some studies have argued that persistent vortex stretching induced by convection to the west of a circulation center causes monsoon depressions to propagate westward (e.g. Chen *et al.* 2005). This explanation may sound reasonable from an Eulerian perspective. However, the Lagrangian perspective presented above offers a different viewpoint. Being embedded within fast westerly

flow, these high vorticity air parcels reach the longitude of the circulation center within a mere 2 to 4 hours (Fig. 10a). Thus, vorticity advection overwhelms the local vorticity tendency caused by vortex stretching. A similar cancellation between vorticity stretching and advection for monsoon depressions was shown by Boos *et al.* (2015). This observation, along with the fact that the disturbance translates westward at a similar speed in the dry simulation (Diaz and Boos 2019), suggests that the convection is not causing the disturbance to propagate westward.

3.3. Sensitivity Experiments

The previous sections focused exclusively on our control simulation, in which a rapidly growing disturbance resembling a monsoon depression drew energy from the meridional shear of the basic state and from its interaction with latent heat release. In this section, we present a series of sensitivity experiments to better understand the processes influencing this growth. These experiments will be grouped into four categories, namely Exp. A, B, C, and D, with sub-experiments under each category meant to be compared with each other and the control. For each experiment, we plot the minimum perturbation pressure 0.74 km above sea level every two hours, smoothed with a three-point moving average (Fig. 11). To facilitate comparison, the control simulation is duplicated in each panel. However, it should be noted that there is some uncertainty in comparing individual experiments since stochastic processes associated with convection can lead to rapid short term fluctuations in pressure. With this in mind, our analysis focuses more on the day-to-day differences and less on the hourly ones. For each experiment, we also show the total accumulated perturbation kinetic energy associated with each term in Eq. 18 (Fig. 12). These values are calculated by integrating the domain-averaged energy generation rate for each term from hour 4 to hour 220 of the simulation. For an additional point of comparison, the average growth rates over this time period are shown in Fig. 13.

3.3.1. Energy Sources (Exp. A)

Exp. A focuses on the energy sources leading to growth. As previously discussed, latent heating is essential for amplification of this disturbance. To illustrate this, we run a simulation in which we deactivate the microphysics parameterization. As expected, the disturbance fails to amplify, with the minimum perturbation pressure remaining near -1 hPa (Fig. 11a). Nevertheless, it still draws energy from the basic state meridional shear (Fig. 12a), with the growth rate due to this process only slightly reduced compared to the control (Fig. 13a). This result is consistent with Diaz and Boos (2019), who found that this basic state can support waves which draw energy from the meridional shear, but that this energy source is insufficient to produce net growth once dissipative processes are included.

In the second part of Exp. A, we assess the role of the basic state meridional shear by eliminating all terms in Eqs. 1 and 3 with overbars (i.e. \bar{U} , $\bar{\theta}$, and \bar{q}). The result is a disturbance initialized in a horizontally homogeneous environment with basic state temperature and moisture profiles taken from the midpoint of the domain, which lies in the northernmost Bay of Bengal. This approach is imperfect since other features of the basic state, such as vertical and zonal gradients of wind, could also be important for the disturbance evolution. Unfortunately, the various gradients present in this basic state cannot be independently modified while maintaining realistic balance constraints. Nevertheless, this type of experiment can verify whether some aspect of the basic state is necessary for disturbance amplification. As shown in Fig. 11a, without the basic state, the disturbance fails to amplify, though it does maintain a pressure near or slightly below -1 hPa for the duration of the simulation. Isolated convection still forms within

the circulation (not shown), presumably influenced by some combination of frictional convergence and surface heat fluxes, and the resulting latent heat release leads to limited growth through pressure work (Fig. 12a and Fig. 13a). Though this contribution is small, the proportion of energy lost through turbulence and numerical diffusion is also much smaller relative to the control since the disturbance does not have to contend with the basic state shear (Fig. 13a).

For the final part of Exp. A, instead of removing the basic state gradients at the beginning of the simulation, we remove them midway through the period of rapid intensification, namely at hour 68. This scenario places an approximately -3 hPa low pressure area over a warm ocean with no wind shear. In this experiment, after a brief period of weakening immediately after removing the basic state, the vortex intensifies rapidly (Fig. 11a). Though not shown on the plot, the perturbation pressure reaches -58 hPa on day 6, after which it makes landfall and weakens. In this case, the disturbance clearly undergoes tropical cyclogenesis and intensifies through the WISHE mechanism. This experiment illustrates that, though the basic state may contribute to disturbance growth initially, it is ultimately detrimental to continued amplification. This is presumably due to its strong vertical wind shear and its tendency to advect the disturbance westward into the adjacent landmass. Interestingly, although the pressure deficit is an order of magnitude larger in this experiment relative to the control (Fig. 11a), the total energy extracted is similar when both pressure work and horizontal shear are added together (Fig. 12a); the energy of the tropical cyclone is concentrated into a much smaller area compared to the monsoon depression.

3.3.2. Surface Heat Fluxes (Exp. B)

Surface sensible and latent heat fluxes are necessary for sustained tropical cyclone amplification, and they could also be important for monsoon depressions, despite the much weaker surface winds of those storms. In Exp. B we modify the surface flux parameterization (Eq. 6) to assess the role of surface heat fluxes in the amplification of the simulated disturbance. For the first part of Exp. B, we turn off the perturbation sensible and latent heat fluxes completely by setting F'_q and F'_θ in Eq. 6 to zero. It is important to distinguish this scenario from turning off the total surface flux, as one might in a traditional modeling experiment: implicit basic state surface fluxes \bar{F}_q and \bar{F}_θ remain to maintain the basic state moisture and temperature fields (Eq. 4). Considering the strength of the low-level southwesterly flow in this region associated with the monsoon, the basic state fluxes presumably contribute substantially to maintaining the basic state moisture field. Setting F'_q and F'_θ to zero will, however, inhibit the WISHE mechanism by preventing a feedback between surface wind and surface fluxes.

As seen in Fig. 11b, a surface flux feedback plays little to no role in the initial amplification of the disturbance since the minimum pressure is nearly the same as the control for the first five days. Afterward, however, the minimum pressure is about 1 hPa to 2 hPa higher than that of the control, indicating that surface fluxes play some role in maintaining the strength of the disturbance after it has reached its maximum intensity. This is also reflected in slightly less energy gained from pressure work relative to the control (Fig. 12b). Interestingly, the largest differences in intensity occur primarily after landfall (Fig. 2), when one might expect surface fluxes from the ocean to be less important. The reason is that the perturbation southwesterly flow east of the circulation center remains over water and continues to contribute to the anomalous sensible and latent heat flux even after the center makes landfall. Although none of our simulations includes anomalous surface heat and moisture fluxes over land, some

studies have argued that surface moisture fluxes are important for monsoon depressions to maintain their intensity over land (Hunt and Turner 2017b).

In the second part of Exp. B, we enhance the surface sensible and latent heat fluxes by increasing the sea surface temperature from 28.0 °C to 29.0 °C. This is accomplished by increasing the saturation mixing ratio q_s^* and surface temperature T_s in Eq. 4. This results in a slightly higher intensity throughout most of the period when the disturbance is strongest (i.e. days 4–8), with negligible changes at other times (Fig. 11b). From Exp. B, we conclude that monsoon depressions do not require WISHE for their formation, but that WISHE can contribute to higher intensity, especially once a monsoon depression reaches larger amplitude.

3.3.3. Initial Amplitude (Exp. C)

In the next set of experiments (Exp. C), we vary the initial amplitude of the disturbance. Although theories of normal mode baroclinic and barotropic instability allow disturbances to develop spontaneously from an infinitesimally small initial perturbation or even random noise, other theories, such as those involving growth of tropical cyclones by WISHE, require some threshold amplitude. To determine which case is more applicable here, we conduct two experiments: one with the initial amplitude reduced to 75 % of its value in the control and another where it is reduced to 50 %. In the former case, although the rapid amplification is delayed by about one day, it still achieves a similar, albeit slightly higher, minimum pressure (Fig. 11c). However, for the latter case, the disturbance does not strengthen at all. We speculate that, when the initial amplitude falls below a certain threshold, the vertical velocity is too weak to generate the convection necessary to initiate the growth process seen in the control simulation. Interestingly, growth rates from horizontal shear are not substantially different among the three experiments while the growth rate from pressure work falls by a factor of two as the initial amplitude is halved (Fig. 13c). This suggests that the synoptic-scale wave structure is acting independently from the convection. The results of Exp. C demonstrate that the growth process simulated here requires an initial disturbance of sufficient amplitude. In the real atmosphere, this points to the importance of a predecessor disturbance from upstream (e.g. Chen and Weng 1999) or some initial convectively generated vortex to start the growth process.

3.3.4. Friction (Exp. D)

In our final set of experiments (Exp. D), we examine the sensitivity to friction. As noted in the introduction, a number of studies have cited CISK as an important growth process for monsoon depressions. We test its importance in our simulation by deactivating surface friction, i.e. setting C_E in Eq. 8 to zero. This modification tests only the version of CISK which requires frictional convergence in the boundary layer to force ascent and convection and not the version which links this convergence to a wave mode (i.e. wave-CISK). Also recall that, similar to the experiment that altered surface heat and moisture fluxes, we are altering only the perturbation friction. Results are shown in Fig. 11d. Judging by the –18 hPa pressure perturbation achieved by the disturbance without friction, convergence induced by friction does not appear to be an essential part of the growth mechanism. Interestingly, energy lost through numerical diffusion increases substantially in this experiment, with the turbulence parameterization reduced to a secondary energy sink (Fig. 12d). This indicates that a larger portion of the perturbation kinetic energy is concentrated near the grid scale, where numerical diffusion acts most strongly. A possible explanation is that, without friction to reduce surface winds, substantial grid-scale

noise develops. Nevertheless, there is some evidence that the lack of frictional convergence at the surface does lead to proportionately less disturbance growth through latent heat release: despite the minimum pressure being substantially lower than in the control, the growth rate from pressure work is smaller (Fig. 13d). The fact that the total energy gained from pressure work is nearly the same between the two experiments (Fig. 12d) is more a reflection of the much larger amplitude of the disturbance in the frictionless case. Cross-referencing plots of the rain field (not shown), the disturbance is somewhat less convectively active, especially near the circulation center, where surface convergence forced by boundary layer friction should be largest.

In the final part of Exp. D, we double the friction coefficient in Eq. 8 over land in order to test the extent to which increased friction over land leads to the disturbance's demise. Note that, even in the control simulation, friction over land is specified to be higher than it is over water. As seen in Fig. 11d, this modification reduces the amplitude of the low pressure slightly after day 6, which is near landfall. This suggests that, at least in this simulation, the increase in friction over land has a small impact on disturbance decay. This is further supported by the fact that, even without friction, the disturbance begins to weaken at approximately the same time.

4. Summary and Discussion

The experiments presented here suggest that monsoon depressions are a type of moist barotropic instability. For a dry atmosphere, a basic state constructed from observations of the South Asian monsoon supports stable Rossby wave-like disturbances that draw energy from the meridional shear of the basic state and that resemble monsoon depressions (Diaz and Boos 2019). Being stable, these waves cannot spontaneously appear but rather require some preexisting disturbance of sufficient amplitude. With the addition of moisture, the pattern of vertical motion induced by their interaction with basic state gradients of wind, temperature, and vorticity couples them with condensation. This coupling amplifies the otherwise decaying disturbance. Although the condensation is offset from the circulation center, the combined flow of the vortex and large-scale monsoon enables potential vorticity generated by this condensation to remain colocated with the larger-scale vortex as it spirals around the circulation center. This contrasts with theories of moist baroclinic instability, which require disturbances to tilt upshear in the latitude-height plane in order to bring the diabatic potential vorticity (PV) generation in phase with positive PV anomalies. It also contrasts with the moisture-vortex instability of Adames and Ming (2018a), which relies on a convective time delay to bring the PV generation in phase with the PV anomalies. In summary, the least stable normal mode of the basic state explains the structure and synoptic-scale size of the monsoon depression, while coupling with moist convection takes this system from weakly stable to unstable.

Although we refer to disturbance growth in our simulations as moist barotropic instability, a distinction must be made between this mechanism and that explored by Lambaerts *et al.* (2011), which is arguably a more pure form of moist barotropic instability. In their case, precipitation is generated by the ageostrophic response to the barotropic instability itself. In our case, we argue that strong vertical shear and gradients of temperature and vorticity interact with the vortex to generate the ageostrophic flow that causes precipitation. Thus, even though the environmental baroclinicity does not seem to contribute to growth through a classic baroclinic instability, it may be essential for producing the distribution of vertical velocity necessary to couple this disturbance to convection.

Though theoretical work on moist barotropic instability is limited (Kuo 1978; Lambaerts *et al.* 2011), the mechanism

presented above bears some resemblance to explanations for the amplification of easterly waves which, similar to monsoon depressions, occasionally evolve into tropical cyclones. For example, African easterly waves draw energy from the meridional shear of the background flow and are coupled to convection (Kiladis *et al.* 2006). This coupling has been explained using the QG-omega equation (Kiladis *et al.* 2006) and some have argued that it is essential for their growth (Berry and Thorncroft 2012). Rydbeck and Maloney (2015) have made similar arguments for easterly waves in the eastern Pacific, with the additional proposal that convection forces the wave axis to elongate from southwest to northeast, a configuration needed to draw energy from the meridional shear. These views of moist barotropic instability contrast with that of Schubert *et al.* (1991), which focuses on how convection alters the basic state itself, though it is possible that this process is also relevant to monsoon depressions.

Our explanation of how convection interacts with the monsoon depression vortex has parallels with the “marsupial paradigm” of tropical cyclogenesis introduced by Dunkerton *et al.* (2009). According to that theory, the region where an easterly wave trough intersects its critical latitude favors tropical cyclogenesis due to the presence of a metaphorical “pouch,” a region of strong rotation, higher inertial stability, and flow stagnation. For our basic state, since the slowest decaying normal-mode moves slowly westward, it would have a critical latitude (defined as the location where wave-relative flow goes to zero) just north of the zero-wind line. In our simulations, convection occurs close enough to this region that its generated vorticity can be recirculated rather than differentially advected by the strong basic state meridional and vertical shear. Thus, this particular theory of tropical cyclogenesis could be relevant to monsoon depression genesis. As suggested by the IMD ranking system, perhaps monsoon depressions represent an early phase of tropical cyclogenesis that is ultimately inhibited by vertical wind shear. In fact, when we abruptly remove the basic state fields in our simulation while the disturbance is amplifying, it becomes much stronger as it undergoes tropical cyclogenesis and the WISHE mechanism begins to operate. Thus, once the monsoon depression reaches a certain amplitude, the basic state which nurtured its growth may become detrimental, presumably from the high vertical wind shear.

Our results suggest that the WISHE mechanism is not necessary for the formation of monsoon depressions: even with anomalous surface heat and moisture fluxes eliminated, the simulated disturbance amplifies at nearly the same rate and reaches a similar minimum pressure. Nevertheless, without these fluxes, the disturbance weakens faster after reaching its maximum intensity, suggesting that surface heat and moisture fluxes do become an important energy source once it reaches sufficient amplitude. This result should not be interpreted as suggesting that warm water is not favorable for monsoon depression genesis. The formulation of our equations requires an implicit surface moisture flux to maintain the basic state moisture field, and this moisture ultimately originated from the ocean. Indeed, statistical analysis showed that monsoon depressions form more frequently where the climatological surface moist energy content is elevated relative to that of the upper troposphere (Ditchek *et al.* 2016), extending ideas of the importance of “relative SST” (Vecchi and Soden 2007) and tropical cyclone potential intensity (Emanuel 1988) to monsoon depressions. Our results only suggest that a WISHE-like feedback between the surface wind speed of the amplifying vortex and the surface heat and moisture flux is not essential. It should also be noted that some have argued that WISHE may not always be important in the early phase of tropical cyclogenesis and that a moderate amount of wind shear at this early stage could be beneficial by enhancing convection (Molinari *et al.* 2004).

Additionally, our results do not support frictionally driven CISK being an essential growth process; turning off surface

friction resulted in a substantially stronger vortex in our experiments, presumably because friction is an energy sink. However, considering that the growth rate from pressure work was smaller in the no-friction simulation compared to the control, frictional convergence likely makes some contribution to enhancing convection, even if it is not the leading factor. Although we argue that this disturbance has Rossby wave-like properties, the classical wave-CISK paradigm also does not seem applicable since it is usually framed in terms of equatorial wave modes (Lindzen 1974).

An important caveat is that we did not explore the sensitivity of our results to the basic state. The South Asian monsoon environment varies substantially with time and our simulations used only one possible state, albeit one that we aimed to make as relevant as possible to monsoon depressions. Other basic states might support different ratios of the aforementioned growth mechanisms or different ones entirely. Storm systems in this region might exist on a continuum from more monsoon depression like, as in the one we have simulated, to more tropical cyclone like.

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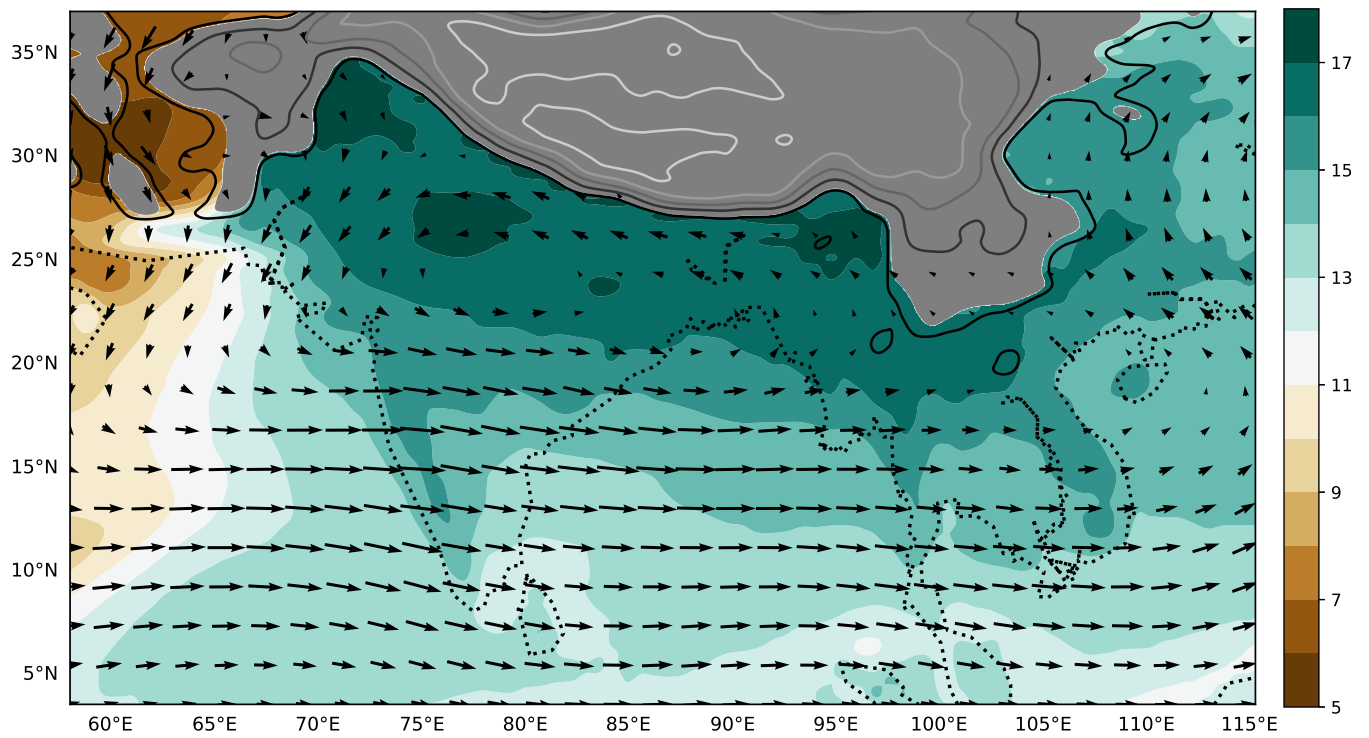


Figure 1. Basic state water vapor mixing ratio at 1.16 km (shading, $\bar{q} + q_0$, g kg^{-1}), basic state wind at 3.36 km (vectors, $\bar{\mathbf{V}}$), and terrain height (contoured every 1.0 km starting at 1.0 km). Areas below the terrain surface are shaded in gray and wind vectors with magnitudes less than 2 m s^{-1} are excluded.

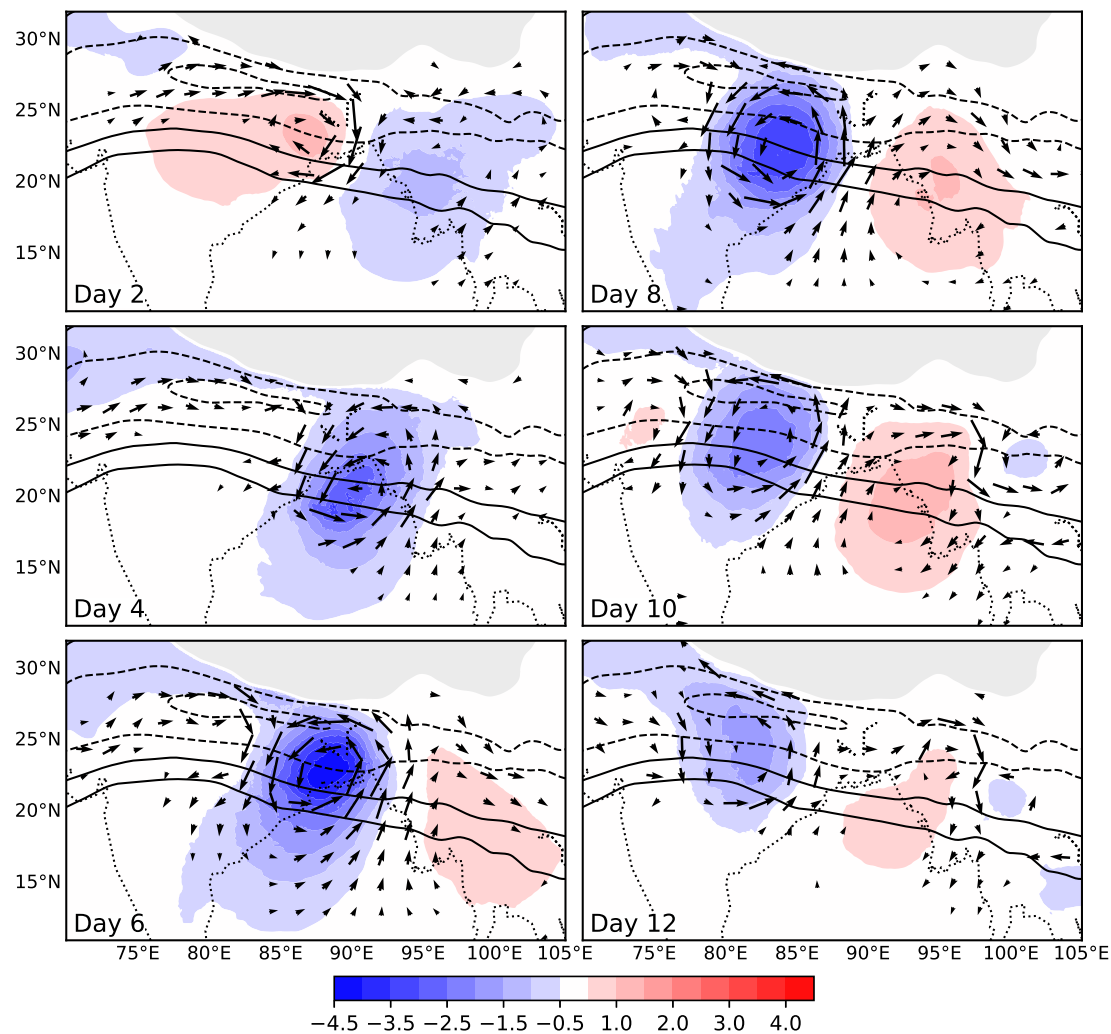


Figure 2. For the control simulation, time evolution of perturbation pressure at 3.36 km (p' , shading, hPa), perturbation winds at 3.36 km (\mathbf{v}' , vectors), and basic state zonal wind at 3.36 km (\bar{U} , contoured every 4 m s^{-1} between -6 m s^{-1} and 6 m s^{-1}). Areas below the terrain are shaded in gray and wind vectors less than 2 m s^{-1} are excluded.

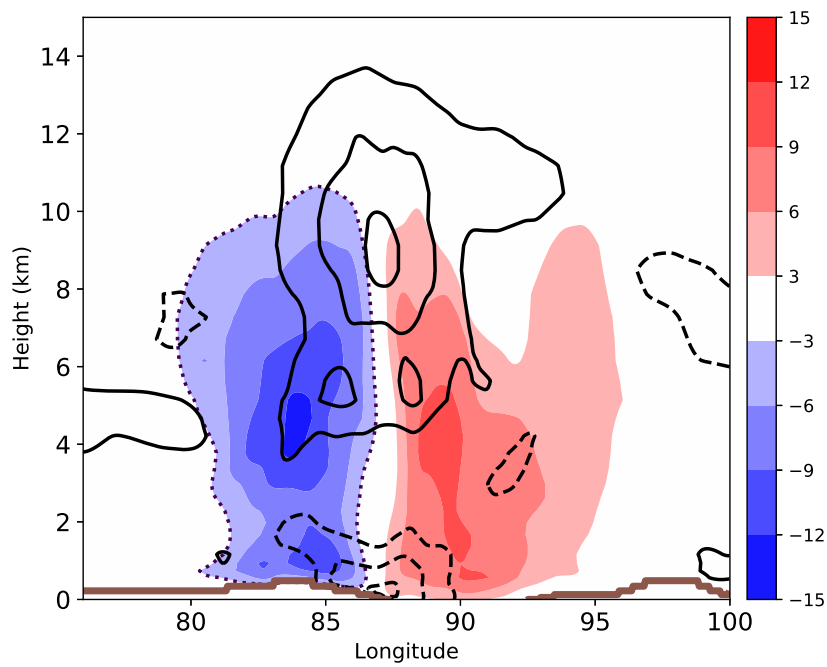


Figure 3. Cross section from $[26.5^{\circ}\text{N}, 75^{\circ}\text{E}]$ to $[18^{\circ}\text{N}, 100^{\circ}\text{E}]$ at 6.3 d. Fields shown are perturbation wind components orthogonal to the plane of the cross section (shading, m s^{-1}) and perturbation potential temperature (contoured every 0.8 K , θ'). The brown line demarcates the terrain surface and negative values of wind are enclosed within the dotted line.

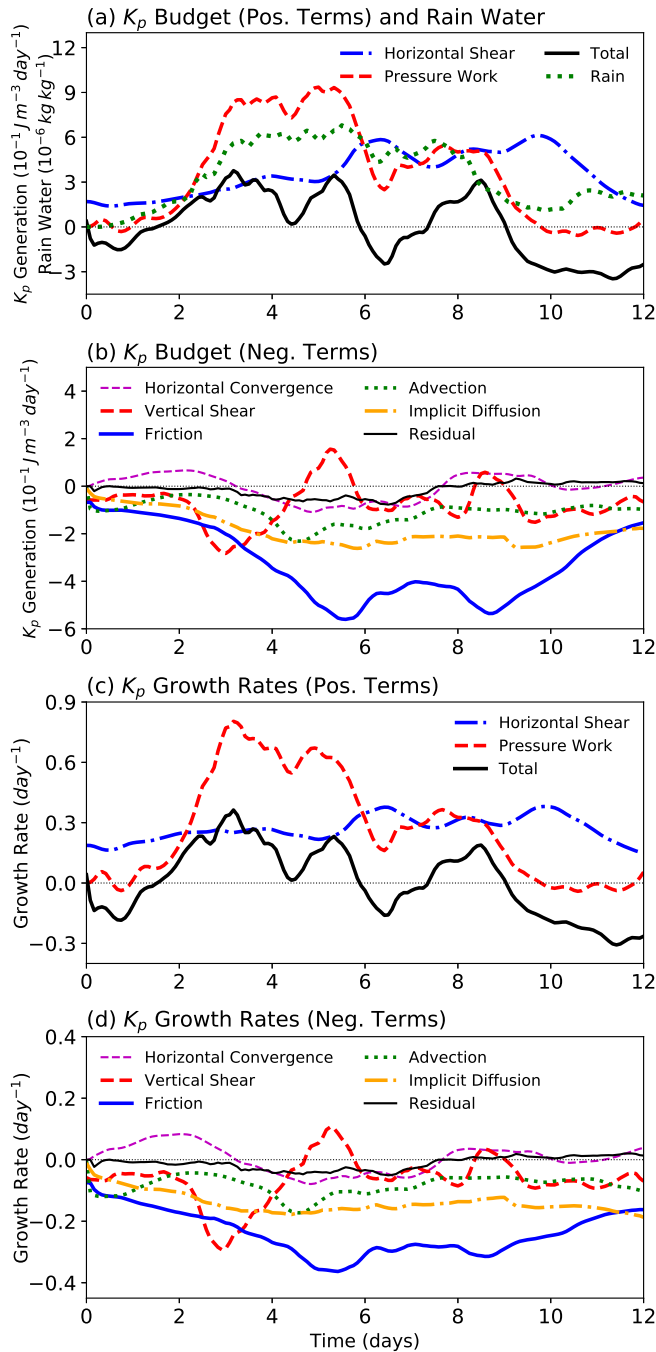


Figure 4. Domain averaged terms in the K_p budget (Eq. 18). (a) shows K_p generation rates for the predominately positive terms along with the rain water mixing ratio averaged within a $15^\circ \times 15^\circ$ box centered on the minimum pressure, (b) shows the generation rates for the predominately negative terms. (c) shows the corresponding growth rates for the terms shown in (a), and (d) for the terms shown in (b). For all plots a six-hour moving average is applied to smooth out the higher frequency noise.

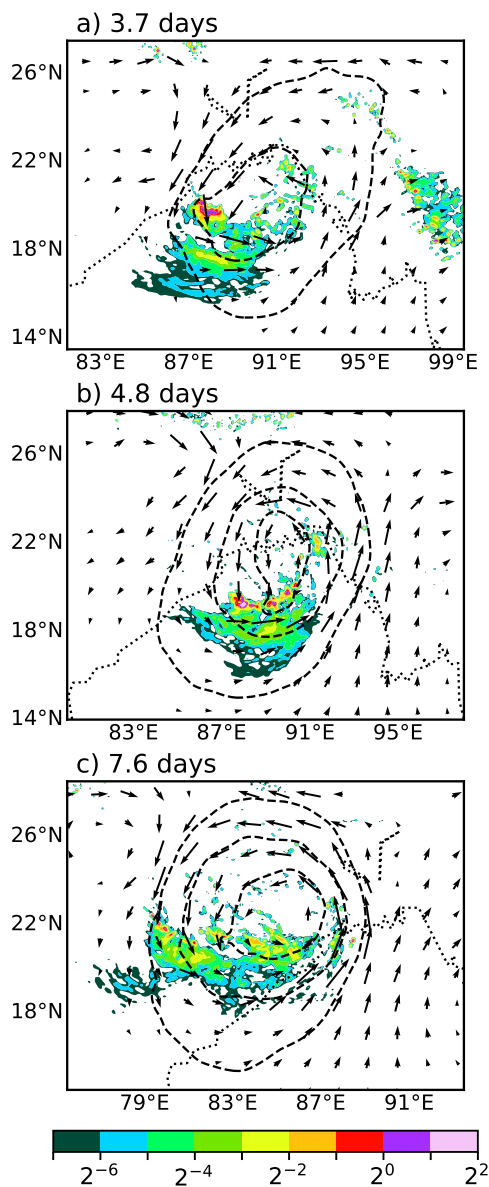


Figure 5. Time evolution of rain water mixing ratio (r , shading, g kg^{-1} , 0.08 km to 15 km mass-weighted average), perturbation pressure (p' , contours, 3.36 km), and perturbation winds (v' , vectors, 3.36 km) for the control simulation at (a) 3.7 d (88 h), (b) 4.8 d (114 h), and (c) 7.6 d (182 h). To emphasize the broader-scale pattern, the pressure field has been smoothed with a Gaussian filter.

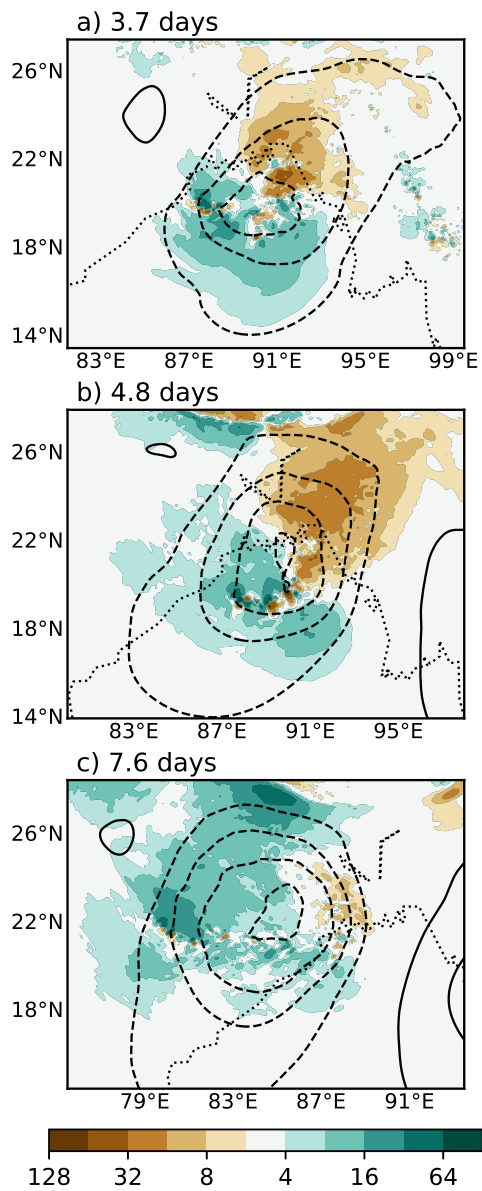


Figure 6. Time evolution of vertical velocity (w'_q) from the QG-omega equation (Eq. 20) (shading, mm s^{-1} , 0.04 km to 5.13 km average) and perturbation stream function (ψ') at 3.36 km (contours) for the control simulation at (a) 3.7 d, (b) 4.8 d, and (c) 7.6 d.

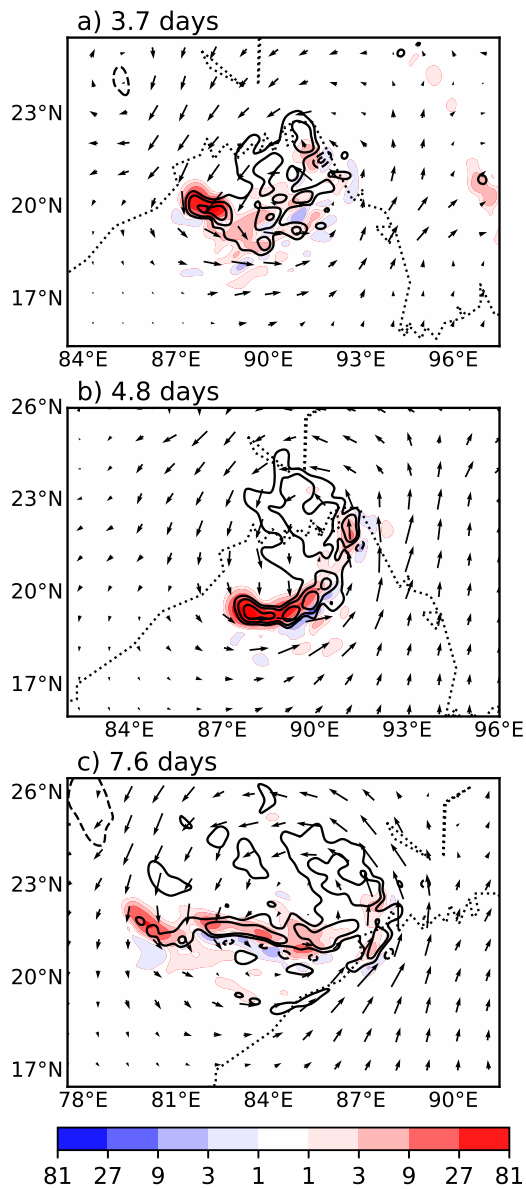


Figure 7. Time evolution of diabatic generation of perturbation PV (P'_d , shading, 0.7 PVU h^{-1} , 0.08 km to 15 km mass-weighted average), perturbation winds at 3.36 km (\mathbf{v}' , vectors), and perturbation PV (P' , contoured on a \log_2 scale beginning at 0.3 PVU) for the control simulation at (a) 3.7 d , (b) 4.8 h , and (c) 7.6 d . Values of P'_d in each plot are integrated over the two hours previous to the time shown and then converted to an hourly generation rate. To emphasize the broader-scale pattern, the PV and PV generation field have been smoothed with a Gaussian filter.

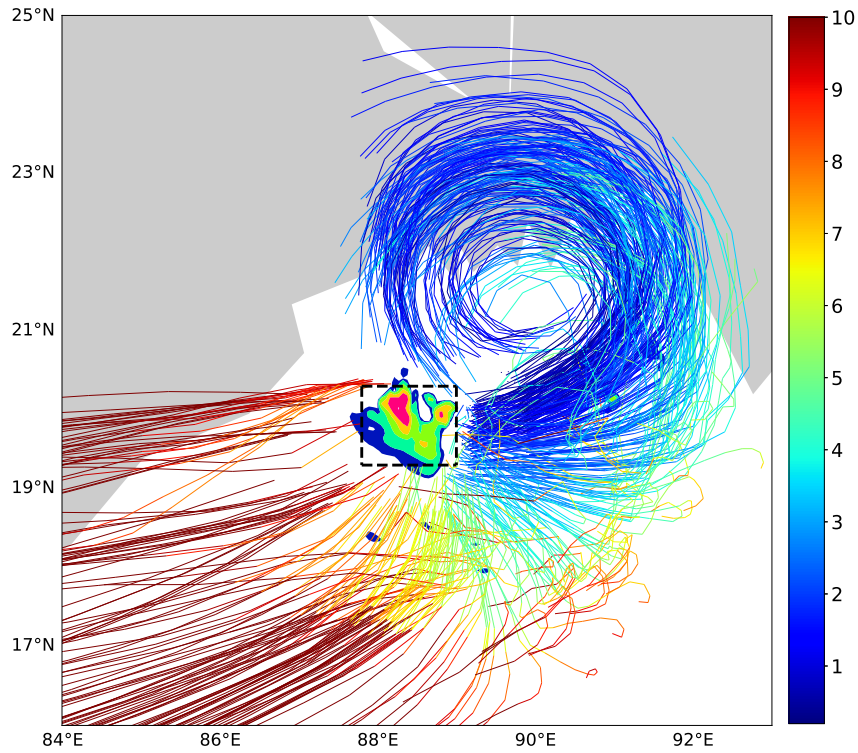


Figure 8. Trajectory paths colored by altitude (km) from 80 h to 116 h. The dashed-line box shows the starting points of the trajectories and the shading shows rain water mixing ratio at 80 h. The color bar corresponds to the trajectory heights.

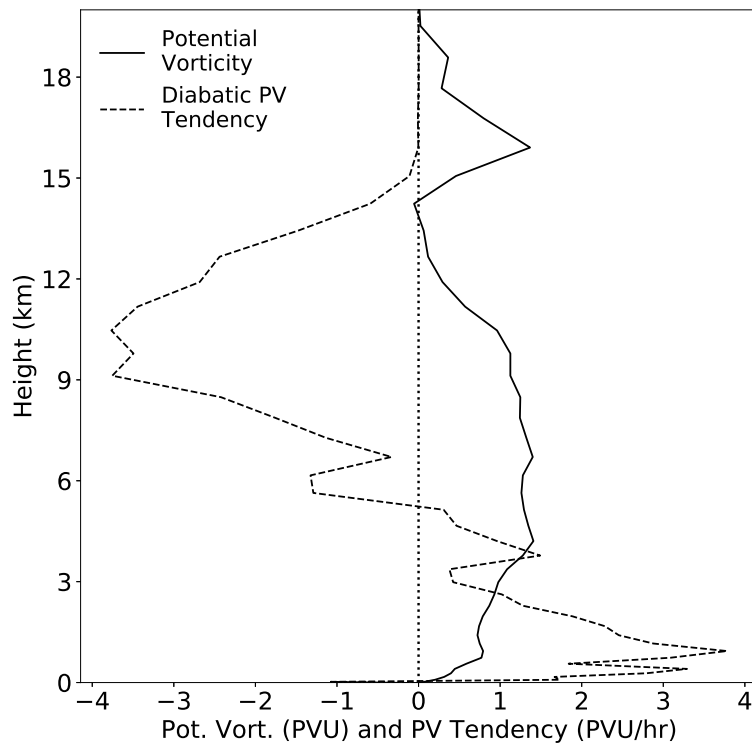


Figure 9. Vertical profile of P' and P'_d averaged over the box shown in Fig. 8 at 80 h. As in Fig. 7, the values of P'_d are integrated over two hours previous to this time and converted to an hourly rate.

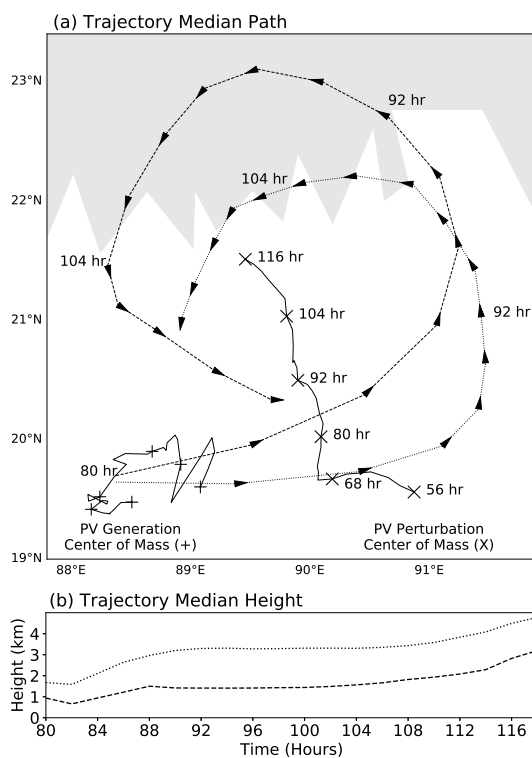


Figure 10. (a) The relationship between air parcel median trajectories (lines with directional arrows), the P' center of mass (X's), and the P'_d center of mass (+s). The arrows on the trajectories are drawn every two hours from hour 80 to hour 112 and labeled with the simulation time every 12 hours. The P' center of mass is labeled every 12 hours from hour 56 to hour 116. For the P'_d center of mass, a symbol (+) is drawn every 12 hours from hour 56 to hour 116, but for clarity is not labeled. The easternmost symbol on its track is at hour 56. (b) the height of the two median trajectories as a function of time with the line style chosen to match those in (a).

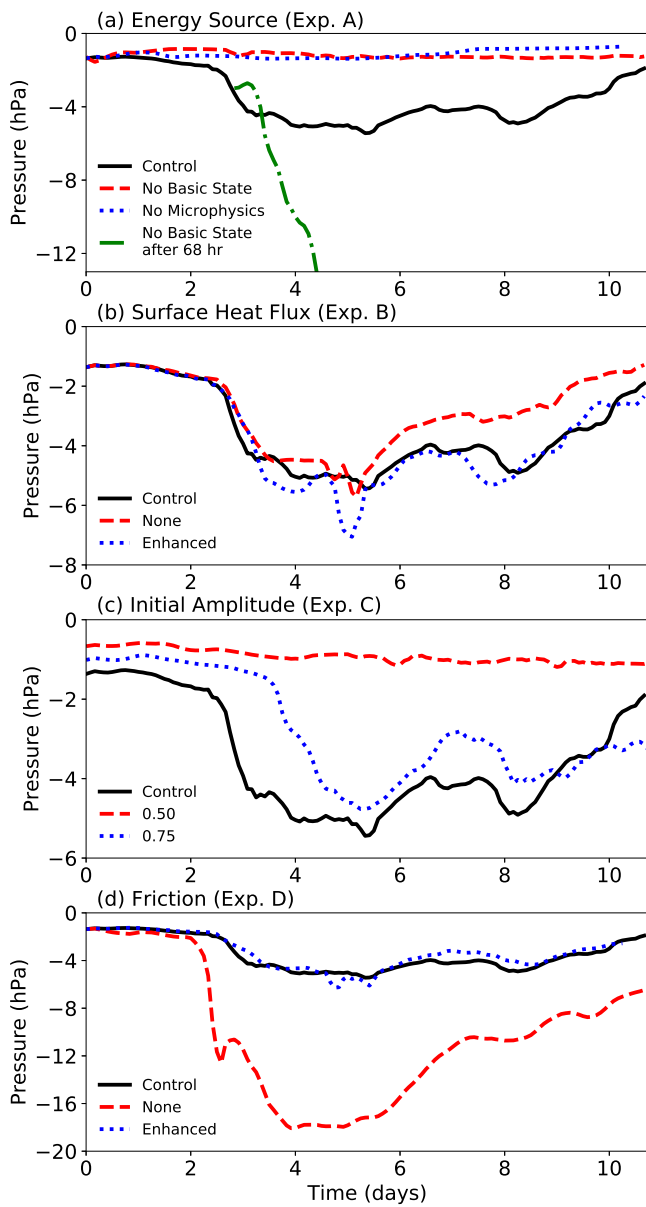


Figure 11. Minimum perturbation pressure (hPa) at 0.74 km for sensitivity experiments. (a) shows the results for Exp. A, (b) results for Exp. B, (c) results for Exp. C, and (d) results for Exp. D.

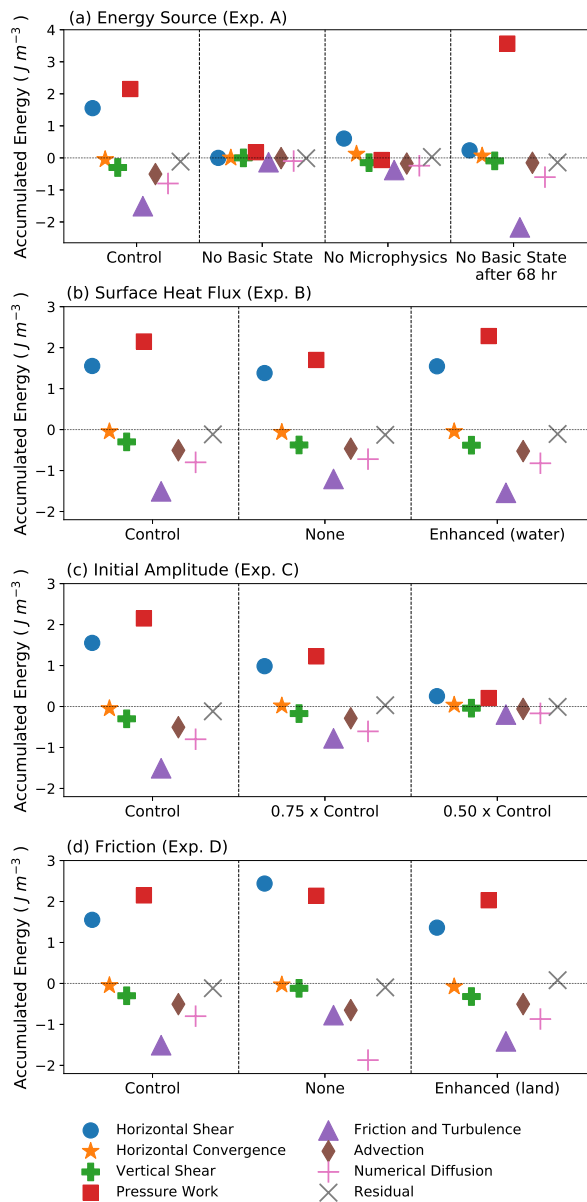


Figure 12. Mass-weighted domain average accumulated kinetic energy for sensitivity experiments from hour 4 to hour 220 of each simulation. (a) shows the results for Exp. A, (b) results for Exp. B, (c) results for Exp. C, and (d) results for Exp. D.

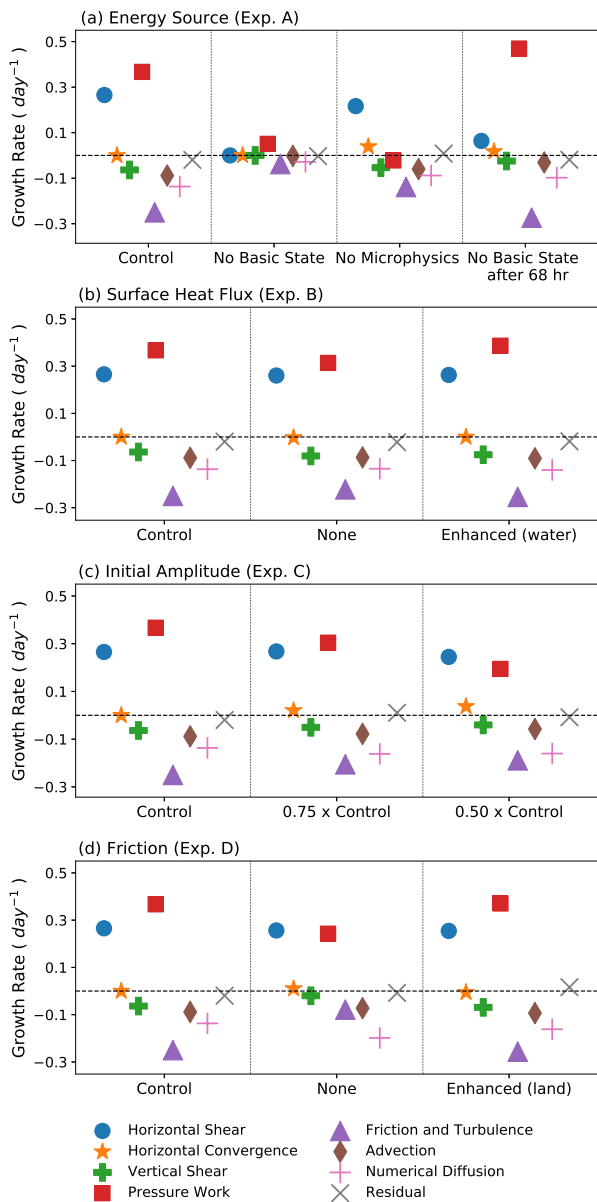


Figure 13. Same as in Fig. 12, but for the average growth rates.