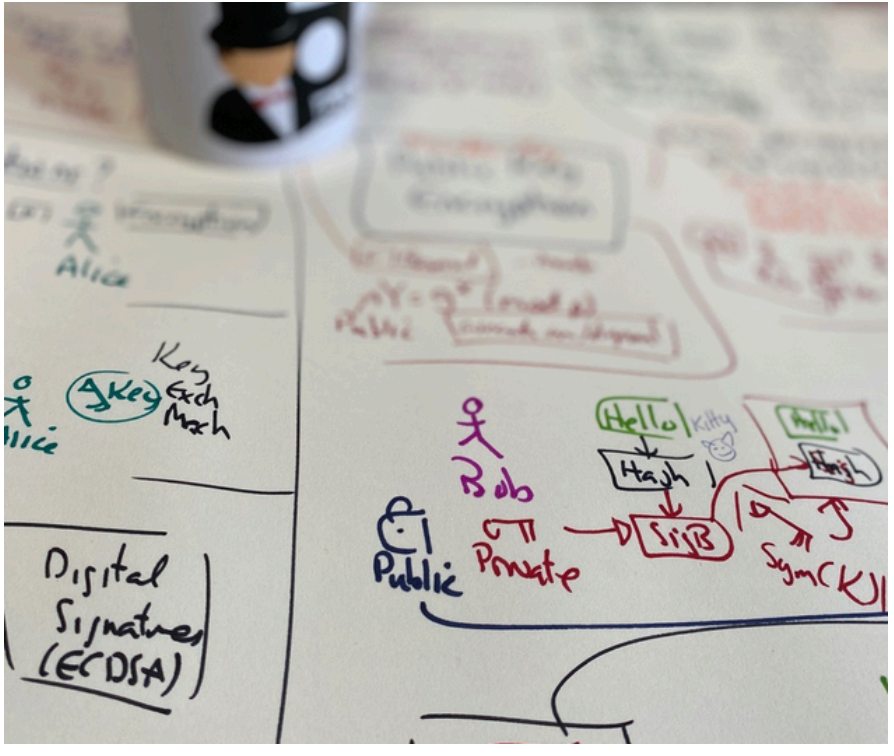
Unit 4: Public Key

Basics
RSA
Elliptic Curve
ElGamal
KMS



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https://asecuritysite.com/ecc
https://asecuritysite.com/elgaml

Unit 4: Public Key

Basics

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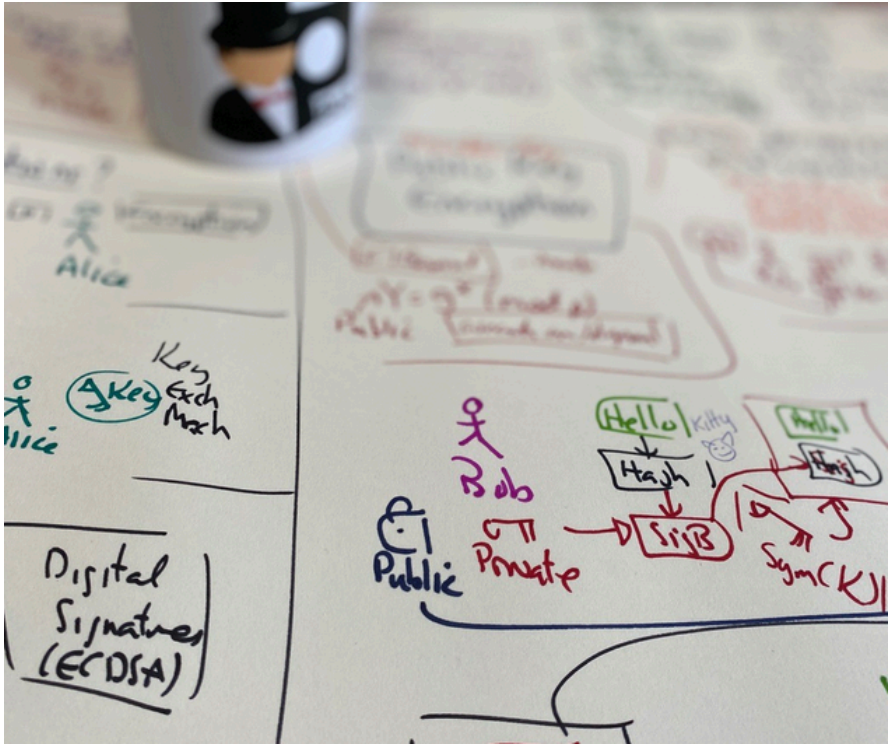
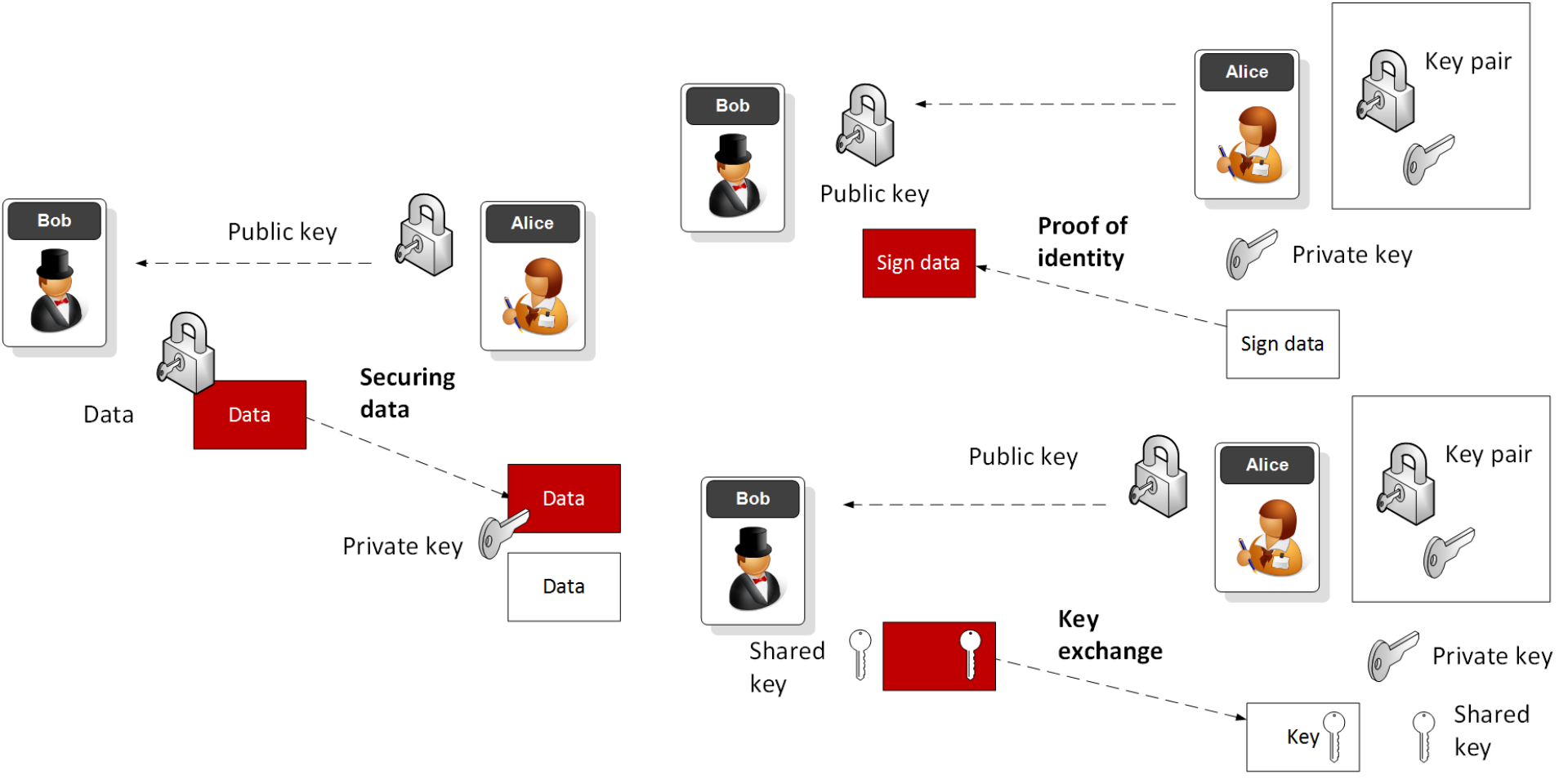
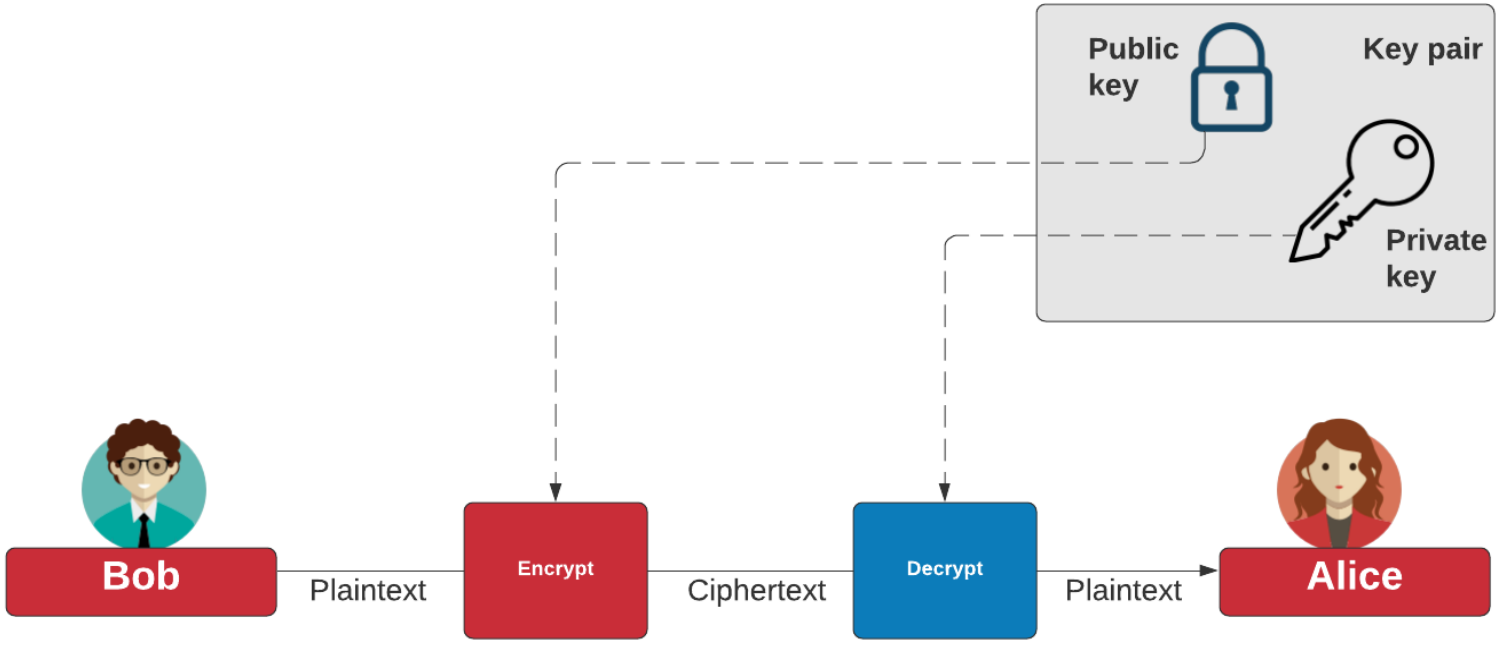


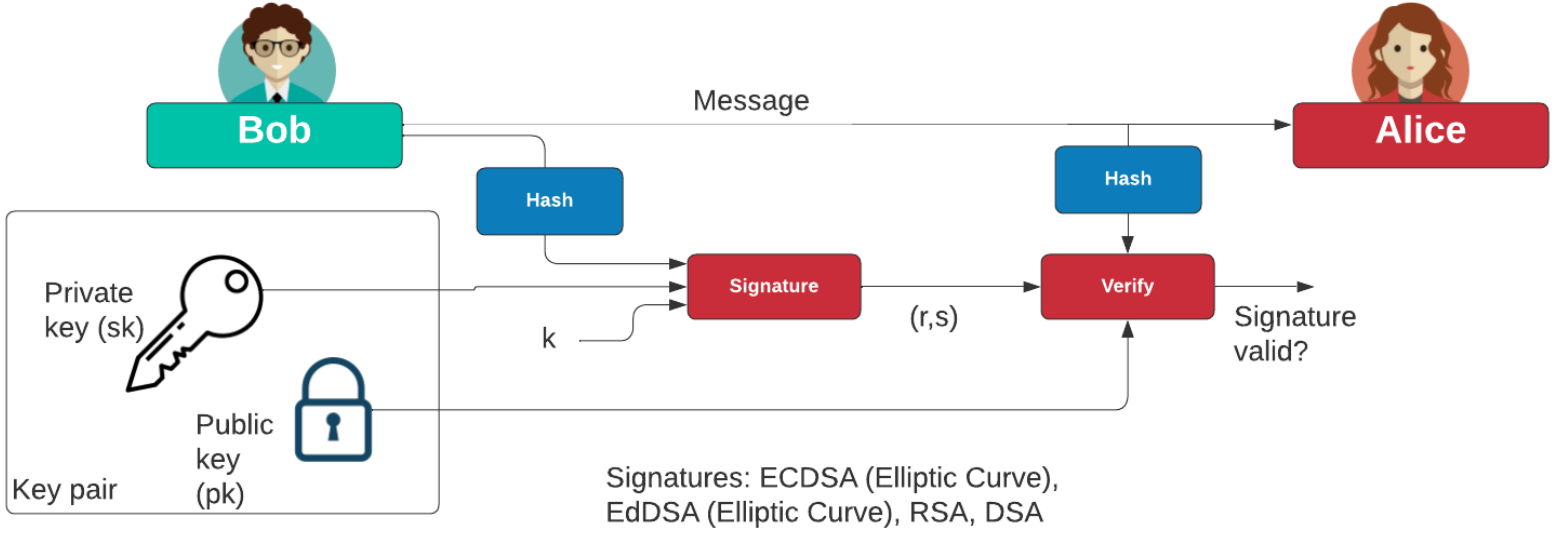
Public Key Methods



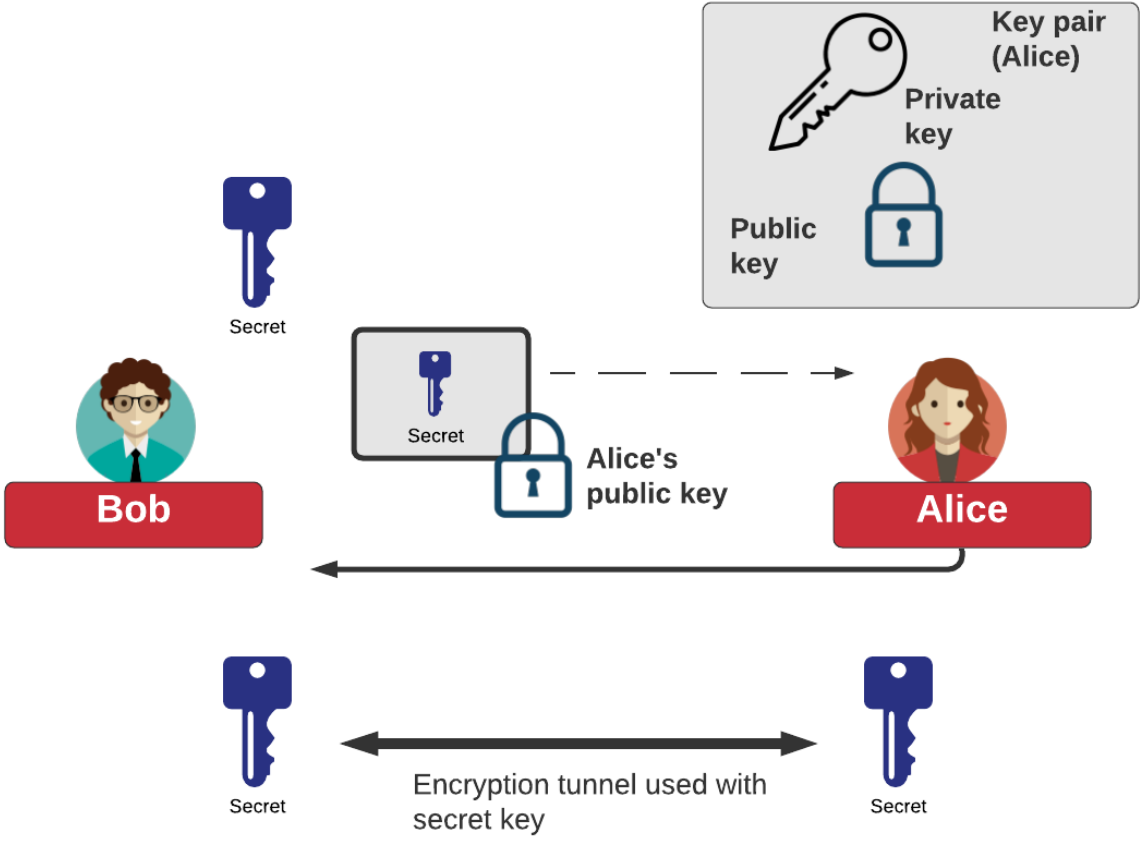
Public Key Encryption



Digital Signatures



Key Exchange with Public Key



Public Key Methods

- **Integer Factorization**. Using prime numbers. Example: RSA. Digital Certs/SSL.
- **Discrete Logarithms**. $Y = G^x \bmod P$. Example: ElGamal.
- **Elliptic Curve Relationships**. Example: Elliptic Curve. Smart Cards, IoT, Tor, Bitcoin.

Public Key Methods

- **Integer Factorization**. Using prime numbers. Example: RSA. Digital Certs/SSL.
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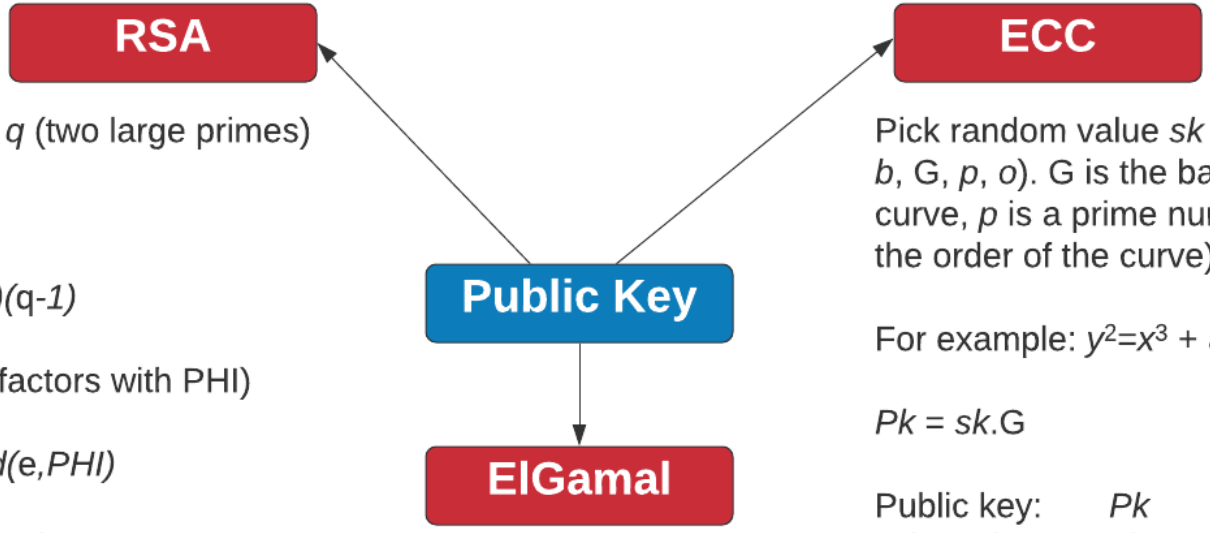
- **Elliptic Curve Relationships**. Example: Elliptic Curve. Smart Cards, IoT, Tor, Bitcoin.

Public Key Methods

- **Integer Factorization**. Using prime numbers.
  Ex
- **Di**
  El
- **El**
  Cu

security level	volume of water to bring to a boil	symmetric key	cryptographic hash	RSA modulus
teaspoon security	0.0025 liter	35	70	242
shower security	80 liter	50	100	453
pool security	2 500 000 liter	65	130	745
rain security	0.082 km^3	80	160	1130
lake security	89 km^3	90	180	1440
sea security	3 750 000 km^3	105	210	1990
global security	1 400 000 000 km^3	114	228	2380
solar security	-	140	280	3730

Public Key Encryption Methods



Unit 4: Public Key

RSA

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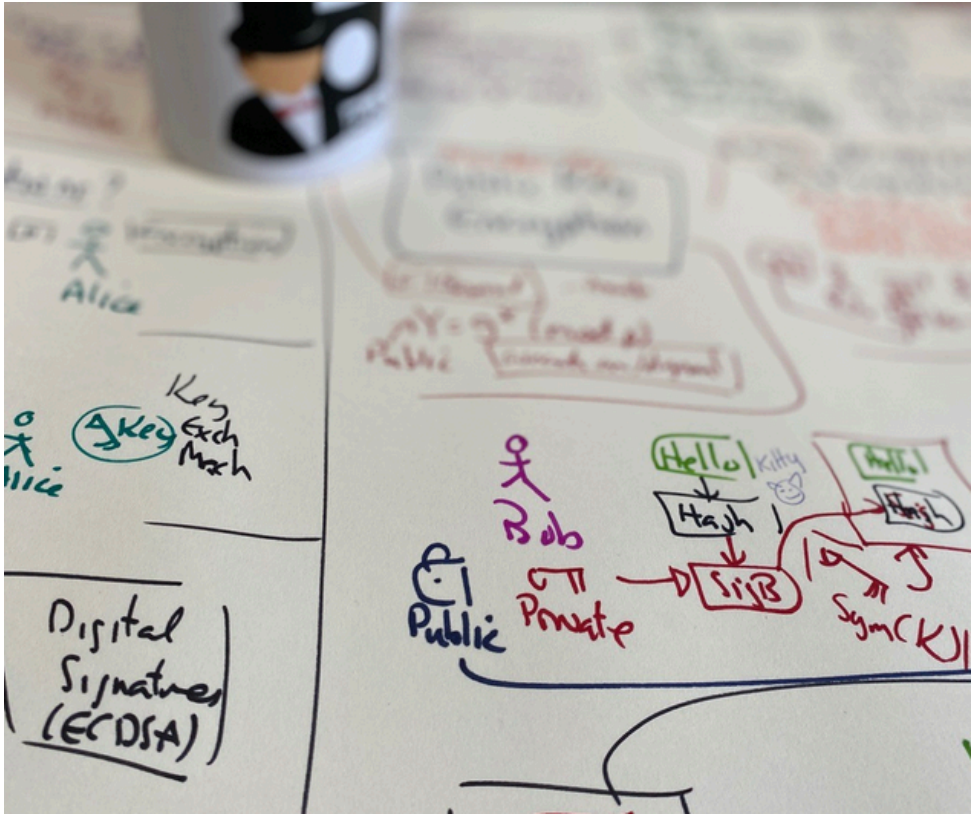
https://asecuritysite.com/elgaml



With Diffie-Hellman we need the other side to be active before we send data. Can we generate a special one-way function which allows is to distribute an encryption key, while we have the decryption key?

Solved in 1977, By Ron Rivest, Adi Shamir, and Len Aldeman created the RSA algorithm for public-key encryption.

RSA

- Two primes p, q.
- Calculate N (modulus) as p x q eg 3 and 11. n=33.
- Calculate PHI as (p-1)x(q-1). PHI=20
- Select e for no common factor with PHI. e=3.
- Encryption key [e,n] or [3,33].
- (d x e) mod 20 = 1
- (d x 3) mod 20 = 1
- d= 7
- Decryption key [d,n] or [7,33]

RSA

Calc

Example

- Encryption key [e,n] or [3,33].
- Decryption key [d,n] or [7,33]
- Cipher = M^e mod N

eg M=5.

- Cipher = 5^3 mod 33 = 26
- Decipher = C^d mod N
- Decipher = $(26)^7$ mod 33 = 5

RSA

openssl genrsa -out private.pem 1024

$C = M^e \bmod N$

$D = C^d \bmod N$

$N = p\ q$

cat private.pem

```
-----BEGIN RSA PRIVATE KEY-----
MIICXQIBAAKBgQC3qXK4kCxn3BNk87vJUMwIznU8pTjr10Kma9+Jkj4zEy/fiZtY
xvdmn1rKNq/8fEUDCcRVC8hQBpevqxFiJ3dbA7ZM6VjUAmztOfRfxSezgvkjswVS
F1/cgBM32AB4nx1dkCV/Wgedn3KFIFU+b8LH1ZLoyRMyLnwWmAkT/mBC/QIDAQAB
AoGAE8Yao+Rh44y+SdA0F6irTwdrd+wSBNJYSrKyjo1ARR97uAWIxDYnzNS7Yaoh
qH14sKsMiFuMZZFQI4m3hWnaX7OFjhJvxKjP6+BdXKsnwWxpwec7RS6n9ptA7q1E
aIFfVARyiWjG+q+8Bg8CTaHjGgtYPnfLzJM0Vef6gKg5vgECQQDZSKGxtdbpXwXw
VAC78SyfOOYmWKL1HiZs0nyTOnZmhMSkE4+S38zhDTjITh0cuKTksTFeUku/sRij
4T4Y9iz5AkEA2GMpeeRT3IQntmzQgTc7Rgez73Y/UWFynuErg++9gzI758TO3AoV
lFs4NOUAqhZ5fdwizs6sa0bjYm+BC1mbJQJBAMQVts4QItVSSqK6vDrfh/xctd4v
KUh5oAWe4otfPBCCio7jlDLgwxzp+K9TRxRvUWeMvNe4/uEMKgdiss6GAskCQQCf
MpVZMDriifgNppDgABqDszcWfhCnduI1McQqFT+APn0ETy9Bg8nMlDAN+k061b4c
ctDJBhSj+EtiKFbwWsRhAkAnEPn+6m3djTwJMw82DxK1q2fcIjTR0ng8pyrF2iIR
P7oBP8I4hGix/FOrV8M8virK6iCsslEcZBo39FkEqc0N
-----END RSA PRIVATE KEY-----
```

openssl rsa -in private.pem -text

```
Private-Key: (1024 bit)
modulus:
    00:b7:a9:72:b8:90:2c:67:dc:13:64:f3:bb:c9:50:
    cc:08:ce:75:3c:a5:38:eb:d7:42:a6:6b:df:89:92:
    3e:33:13:2f:df:89:9b:58:c6:f7:66:9f:5a:ca:36:
    af:fc:7c:45:03:09:c4:55:0b:c8:50:06:97:af:ab:
    11:62:27:77:5b:03:b6:4c:e9:58:d4:02:6c:ed:39:
    f4:5f:c5:27:b3:82:f9:23:b3:05:52:17:5f:dc:80:
    13:37:d8:00:78:9f:1d:5d:90:25:7f:5a:07:9d:9f:
    72:85:20:55:3e:6f:c2:c7:d5:92:e8:c9:13:32:2e:
    7c:16:98:09:13:fe:60:42:fd
publicExponent: 65537 (0x10001)
privateExponent:
    13:c6:1a:a3:e4:61:e3:8c:be:49:d0:34:17:a8:ab:
    4f:07:6b:77:ec:12:04:d2:58:4a:b2:b2:8e:8d:40:
    45:1f:7b:b8:05:88:c4:36:27:cc:d4:bb:61:aa:21:
    a8:7d:78:b0:ab:0c:88:5b:8c:65:91:50:23:89:b7:
    85:69:da:5f:b3:85:8e:12:6f:c4:a8:cf:eb:e0:5d:
    5c:ab:27:c1:6c:69:c1:e7:3b:45:2e:a7:f6:9b:40:
    ee:a9:44:68:81:5f:54:04:72:89:68:c6:fa:af:bc:
    06:0f:02:4d:a1:e3:1a:0b:58:3e:77:cb:cc:93:34:
    55:e7:fa:80:a8:39:be:01
prime1:
    00:d9:48:a1:b1:b5:d6:e9:5f:05:f0:54:00:bb:f1:
    2c:9f:38:e6:26:58:a2:f5:1e:26:6c:d2:7c:93:3a:
    76:66:84:c4:a4:13:8f:92:df:cc:e1:0d:38:c8:4e:
    1d:1c:b8:a4:e4:b1:31:5e:52:4b:bf:b1:18:a3:e1:
    3e:18:f6:2c:f9
prime2:
    00:d8:63:29:79:e4:53:dc:84:27:b6:6c:d0:81:37:
    3b:46:07:b3:ef:76:3f:51:61:72:9e:e1:2b:83:ef:
    bd:83:32:3b:e7:c4:ce:dc:0a:15:94:5b:38:34:e5:
    00:aa:16:79:7d:dc:22:ce:ce:ac:6b:46:e3:62:6f:
    81:0b:59:9b:25
```

RSA

openssl genrsa -out private.pem 1024

$$C = M^e \bmod N$$

openssl rsa -in private.pem -text

```
Private-Key: (1024 bit)
modulus:
    00:b7:a9:72:b8:90:2c:67:dc:13:64:f3:bb:c9:50:
    cc:08:ce:75:3c:a5:38:eb:d7:42:a6:6b:df:89:92:
    3e:33:13:2f:df:89:9b:58:c6:f7:66:9f:5a:ca:36:
```

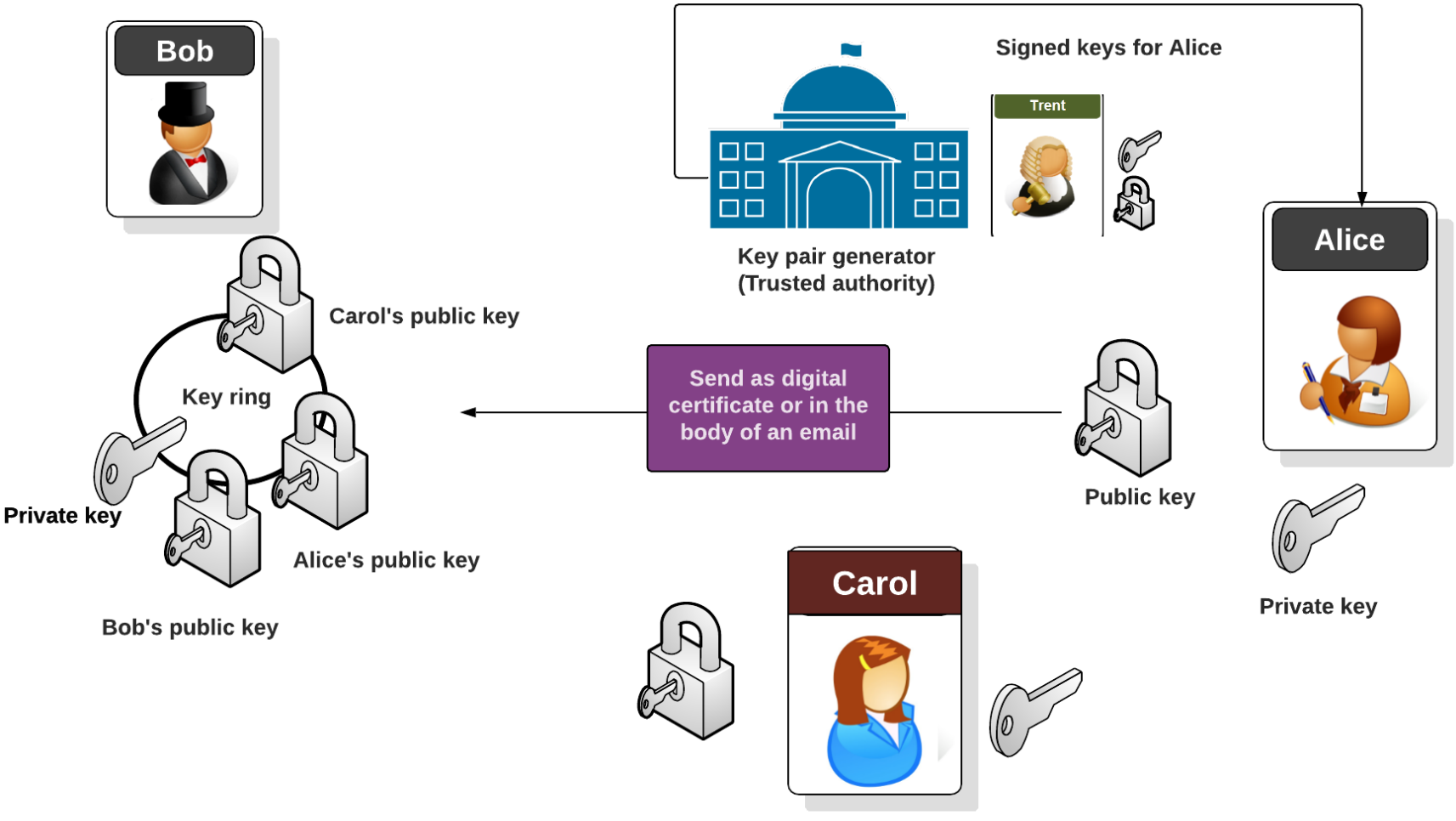
```
-----BEGIN PGP MESSAGE-----
Version: GnuPG v1

hQEMA8anVEMIIe/JAQf/cUmIvTbhQQhr70vPY817xRld7NNUrfIqWoz0S7BfpXDi
kvNbw/tIR5yS8gIbm25QFl5kUCukZh3zBq1vZ4pSq35e0ReH4RZQRmDe6Wtn244D
OPJ6WOe4c4y+87shZdJhAwgpLZl5gqZ3YnySoX7kH2CbqDYJUr+4giq/TWGYGb+F
7ztIBwnTZEyijFpWrYhtBVz2DM1HfMDgH3wNWLH0LbE+s7XwqBP/3FHp4Holaqrt
BMU9+MZlM5rqq/AnGXW80/VR8eELJs500qRZmHcI8DO6p8sgNBTeuchadSkKZLgP
iO+l/m2/9nOFg++JSCRpul+JVQU+IngP9pgG13NvktJUAa2/McEaBRYeIr1X4v6g
Syr5jcHBqCR3zyMVO6rg2+r0VK3Z0a9DV4QGOyKewJhPEwPDfLo4SoWZa5n9zwNP
JWm6iiSYz2wLiYd5Pg6Zr/DpGbDF
=5Ost
-----END PGP MESSAGE-----
```

```
MpVZMDriifgNppDgABqDszcWfhCnduI1McQqFT+APn0ETy9Bg8nMlDAN+k061b4c
ctDJBhSj+EtiKFbwWsRhAkAnEPn+6m3djTwJMw82DxK1q2fcIjTR0ng8pyrF2iIR
P7oBP8I4hGix/FOrV8M8virK6iCsslEcZBo39FkEqc0N
-----END RSA PRIVATE KEY-----
```

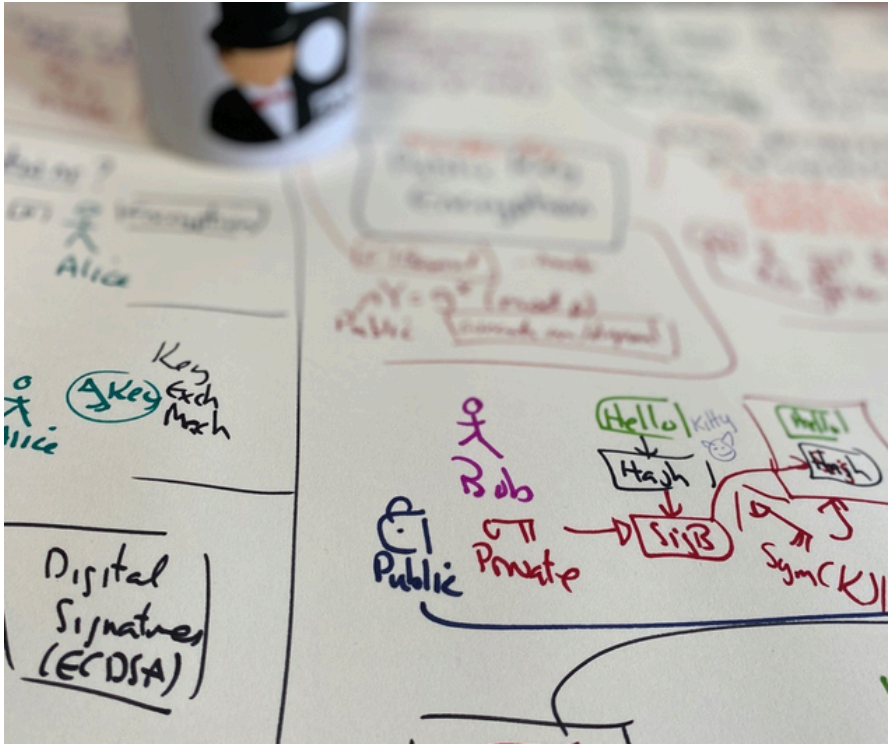
```
    3e:18:f6:2c:f9
prime2:
    00:d8:63:29:79:e4:53:dc:84:27:b6:6c:d0:81:37:
    3b:46:07:b3:ef:76:3f:51:61:72:9e:e1:2b:83:ef:
    bd:83:32:3b:e7:c4:ce:dc:0a:15:94:5b:38:34:e5:
    00:aa:16:79:7d:dc:22:ce:ce:ac:6b:46:e3:62:6f:
    81:0b:59:9b:25
```

Key ring


Bob
Signed keys for Alice
Trent
Key pair generator
(Trusted authority)
Alice
Carol's public key
Key ring
Send as digital certificate or in the body of an email
Public key
Private key
Private key
Alice's public key
Bob's public key
Carol

Unit 4: Public Key

Elliptic Curve



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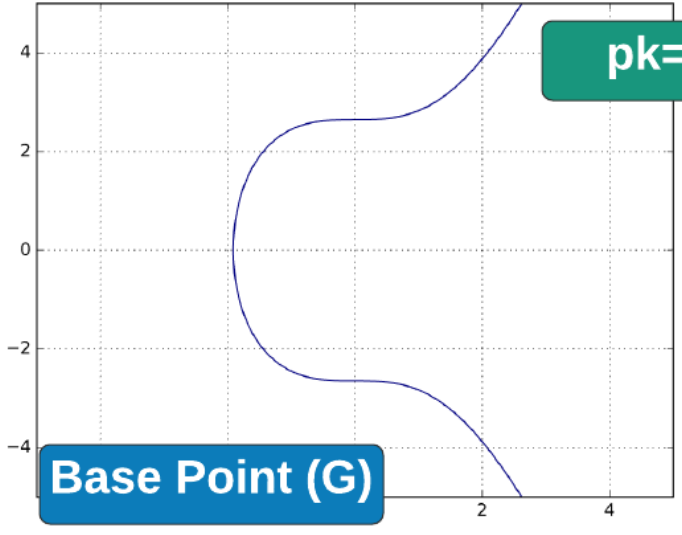
Elliptic Curve Methods

$y^2=x^3+ax+b \pmod p$



Base Point (G)

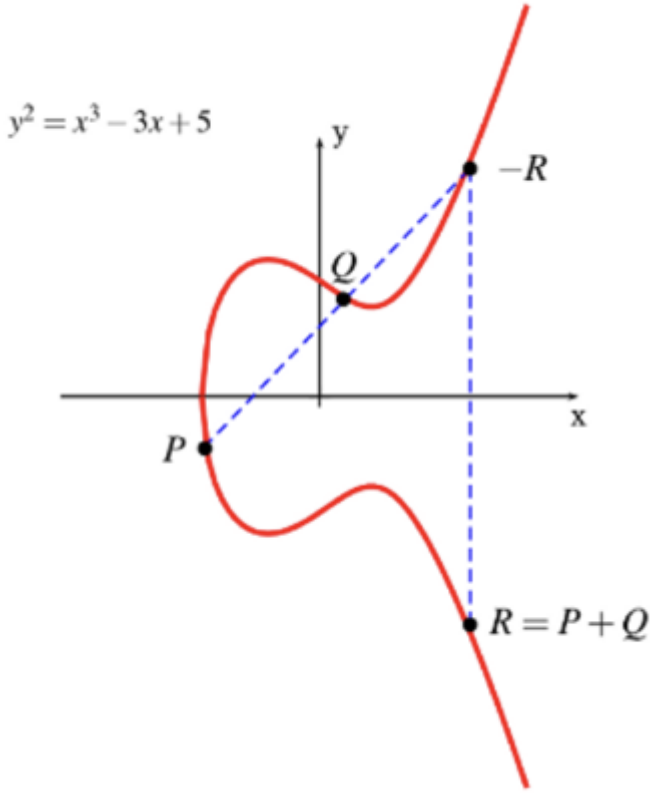
pk=sk.G

Bob

sk (private key)

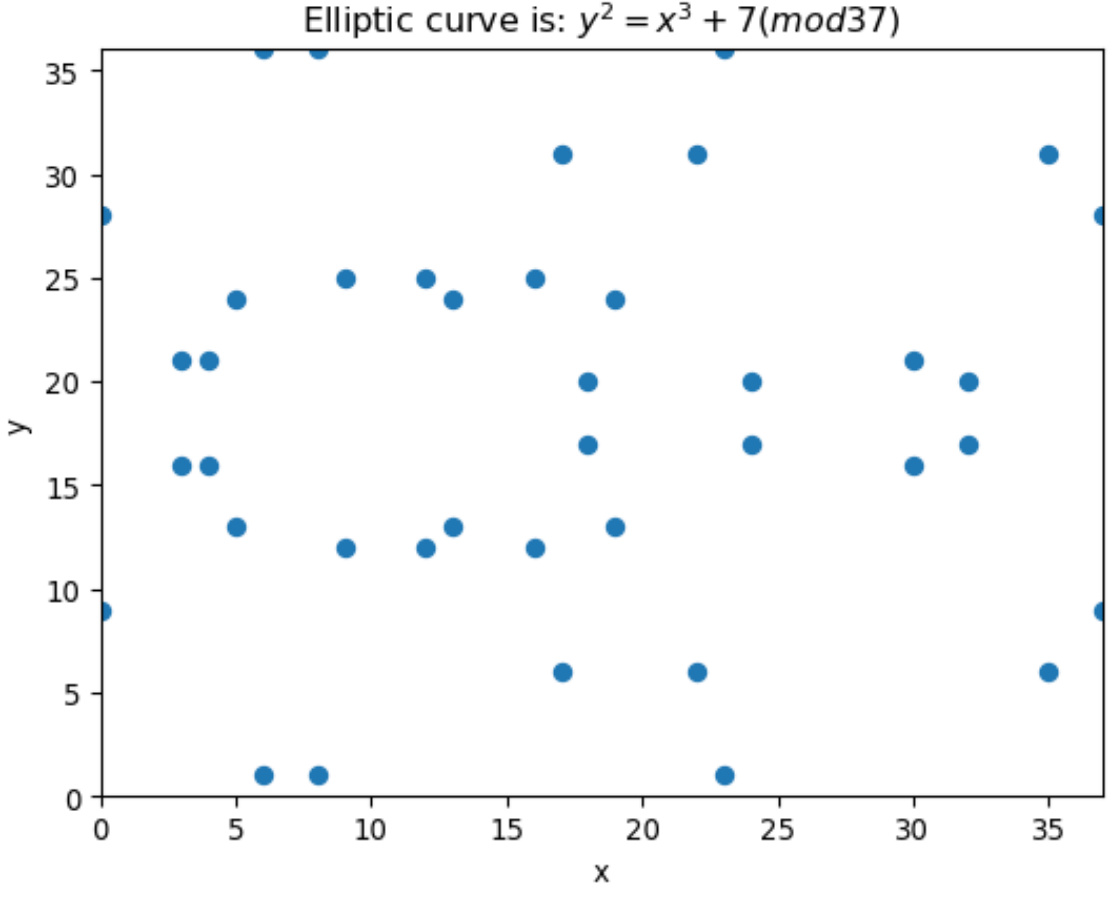
Basic operations:
- Point add.
- Point double.

Elliptic Curve (EC)

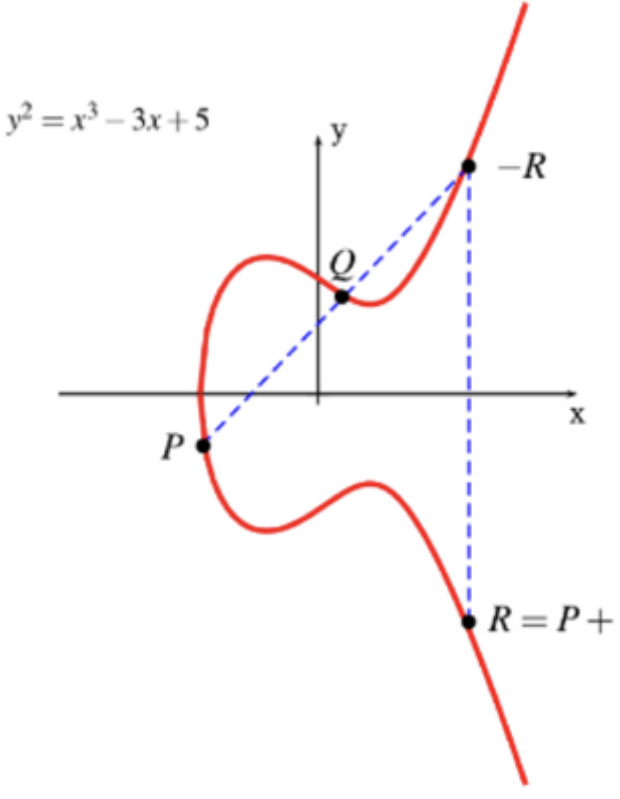


- Pick a point on the elliptic curve (G).
- Generate a random number (n) – this will be the private key.
- Public key is P = n x G (mod p), where p is a prime number (eg 256-bit prime for Curve 25519).
- n is a scalar value which multiples with G to give P (public key)
- Bitcoin uses secp256k1 and Tor uses Curve 25519 [here].

Elliptic Curve (EC) - (mod p)



Bitcoin Key Generation



Private key

904081f45a6890ae6070c4ca0c6231720546c3c6ad53f26c6b6c2e61f5892141

Base58

WiF private key

5JupJddAVbKB8Fa8T1SP999kzePLiQFYG39Kk3XjZgWf8VKDTcr

Elliptic Curve DSA

04f94454315341f0dc0700329d7c7cca833bf29a722df04c1ce7f88d85c88a80b49777d30372ec71e44dc6ffb7059c4e2f29ad30a69021f576594c170eabc4d314

512-bit public key with prefix

SHA-256 RIPEM-160

Public key hash

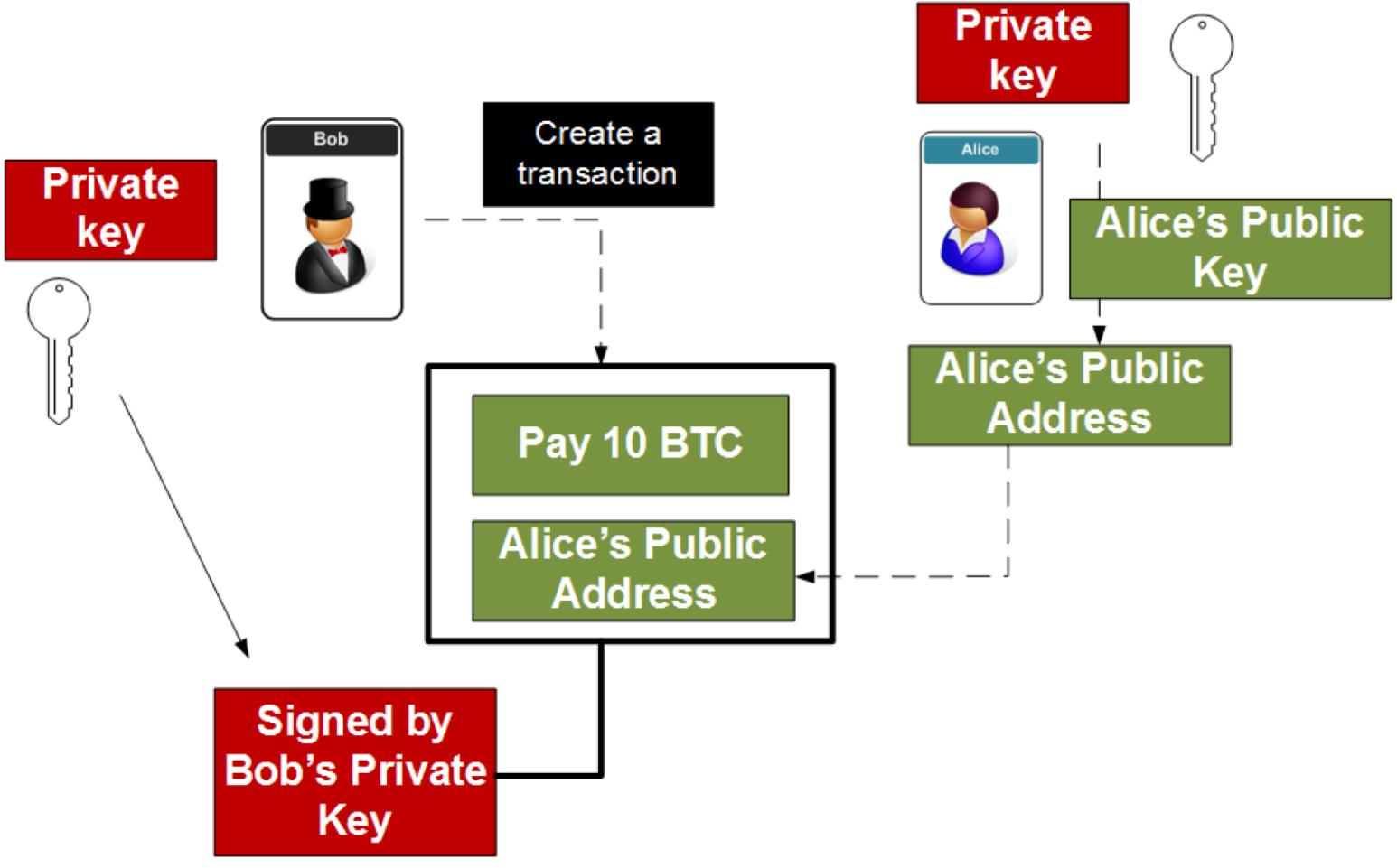
66276E465A08B20D9A51CBC43A5A56F489304BED

Base58

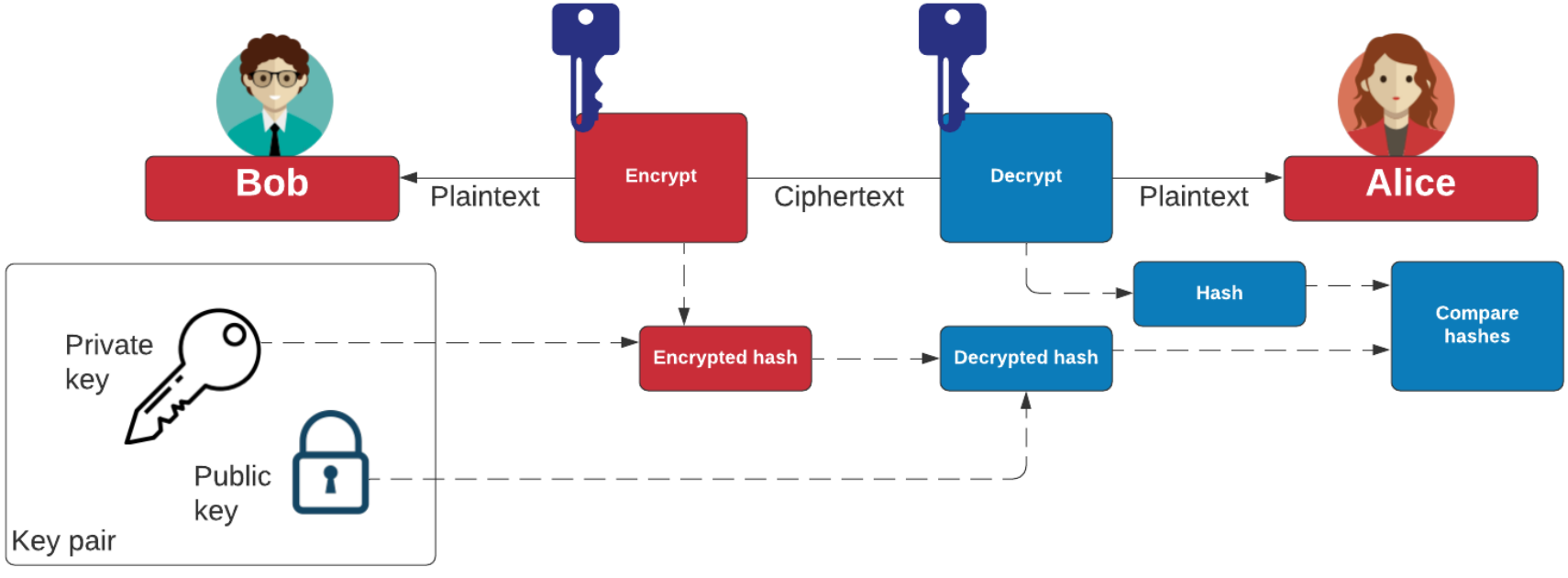
1EsPD1GVW3eyshZG8bJ1dydWPc7Abic4V2

Address

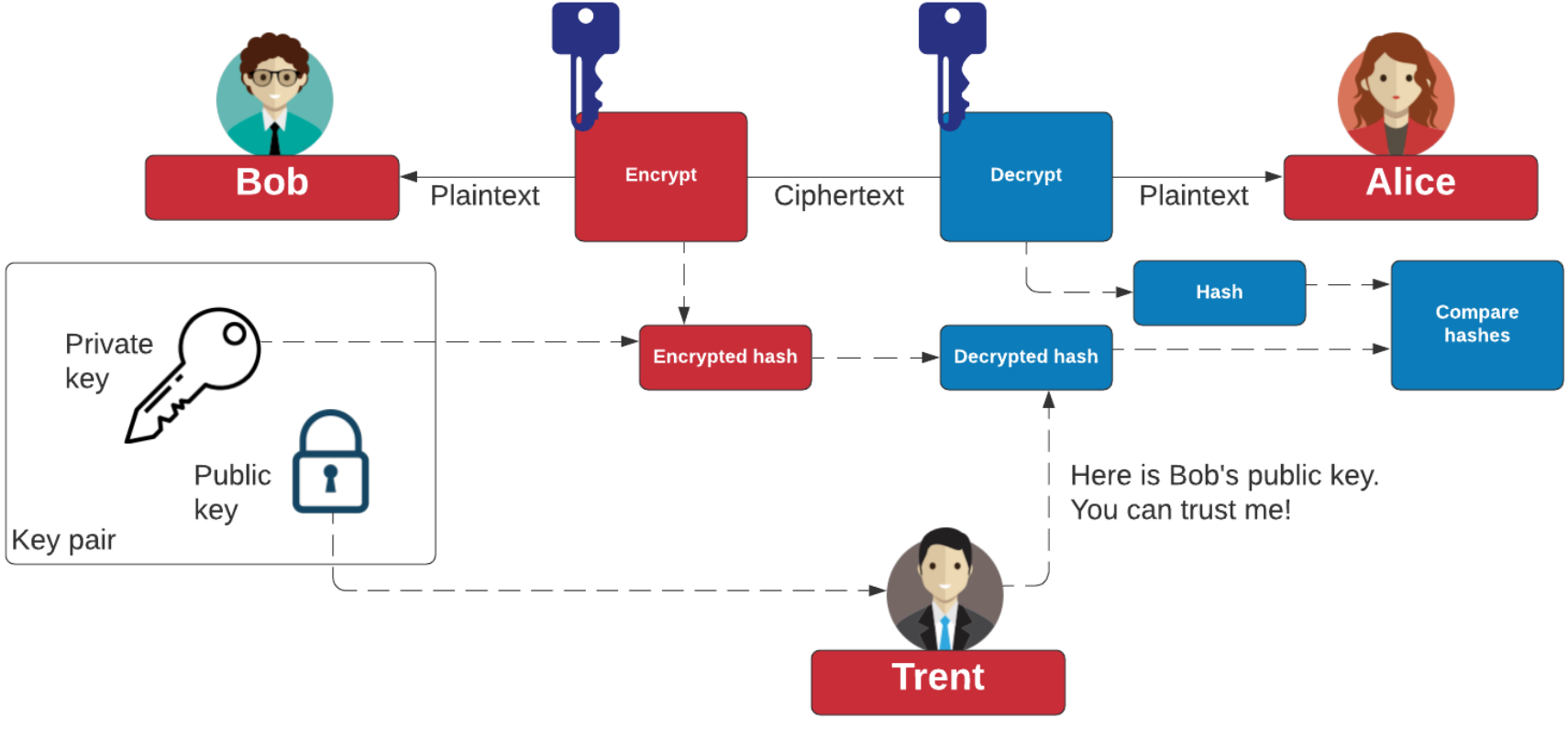
Bitcoin Transaction



Signing


Bob
Plaintext
Encrypt
Ciphertext
Decrypt
Plaintext
Alice
Hash
Compare hashes
Private key
Encrypted hash
Decrypted hash
Public key
Key pair

Signing


Bob
Plaintext
Encrypt
Ciphertext
Decrypt
Plaintext
Alice
Private key
Public key
Key pair
Encrypted hash
Decrypted hash
Hash
Compare hashes
Here is Bob's public key.
You can trust me!
Trent

Example

Elliptic Curve (EC)

```
C \ > openssl ecparam -name secp256k1 -genkey -out priv.pem

C \ > type ec-priv.pem
-----BEGIN EC PARAMETERS-----
BgUrgQQACg==
-----END EC PARAMETERS-----
-----BEGIN EC PRIVATE KEY-----
MHQCAQEEIEa56GG2PTUJyIt4FydaMNItYsjNj6ZIbd7jXvDY4ElfoAcGBSuBBAAK
oUQDQgAEJQDn8/vd8oQpA/VE3ch0lM6VAprOTiV9VLp38rwfOog3qUYcTxxX/sxJ
l1M4HncqEopYIKkkovoFFi62Yph6nw==
-----END EC PRIVATE KEY-----
```

Example

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-----BEGIN EC PRIVATE KEY-----
MHQCAQEEIEa56GG2PTUJyIt4FydaMNItYsjNj6ZIbd7jXvDY4ElfoAcGBSuBBAAK
oUQDQgAEJQDn8/vd8oQpA/VE3ch0lM6VAprOTiV9VLp38rwfOog3qUYcTxxX/sxJ
l1M4HncqEopYIKkkovoFFi62Yph6nw==
-----END EC PRIVATE KEY-----
```

Example

Elliptic Curve (EC)

```
C \> openssl ecparam -name secp256k1 -genkey -out priv.pem

C \>
-----B
BgUr
-----E
-----B
MHC
oUQ
l1M4
-----E
```

```
C \> openssl ec -in priv.pem -text -noout
read EC key
Private-Key  (256 bit)
priv
    46 b9 e8 61 b6 3d 35 09 c8 8b 78 17 27 5a 30
    d2 2d 62 c8 cd 8f a6 48 6d de e3 5e f0 d8 e0
    49 5f
pub
    04 25 00 e7 f3 fb dd f2 84 29 03 f5 44 dd c8
    74 94 ce 95 02 9a ce 4e 25 7d 54 ba 77 f2 bc
    1f 3a 88 37 a9 46 1c 4f 1c 57 fe cc 49 97 53
    38 1e 77 2a 12 8a 58 20 a9 24 a2 fa 05 16 2e
    b6 62 98 7a 9f
ASN1 OID  secp256k1
```

Example

Elliptic Curve (EC)

```
C \ > openssl ecparam -name secp256k1 -genkey -out priv.pem

C \ > type ec-priv.pem
-----BEGIN EC PARAMETERS-----
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-----END EC PARAMETERS-----
-----BEGIN EC PRIVATE KEY-----
MHQCAQEEIEa56GG2PTUJyIt4FydaMNItYsjNj6ZIbd7jXvDY4ElfoAcGBSuBBAAK
oUQDQgAEJQDn8/vd8oQpA/VE3ch0lM6VAprOTiV9VLp38rwfOog3qUYcTxxX/sxJ
l1M4HncqEopYIKkkovoFFi62Yph6nw==
-----END EC PRIVATE KEY-----
```

Example

Elliptic Curve (EC)

```
C \> openssl ecparam -name secp256k1 -genkey -out priv.pem

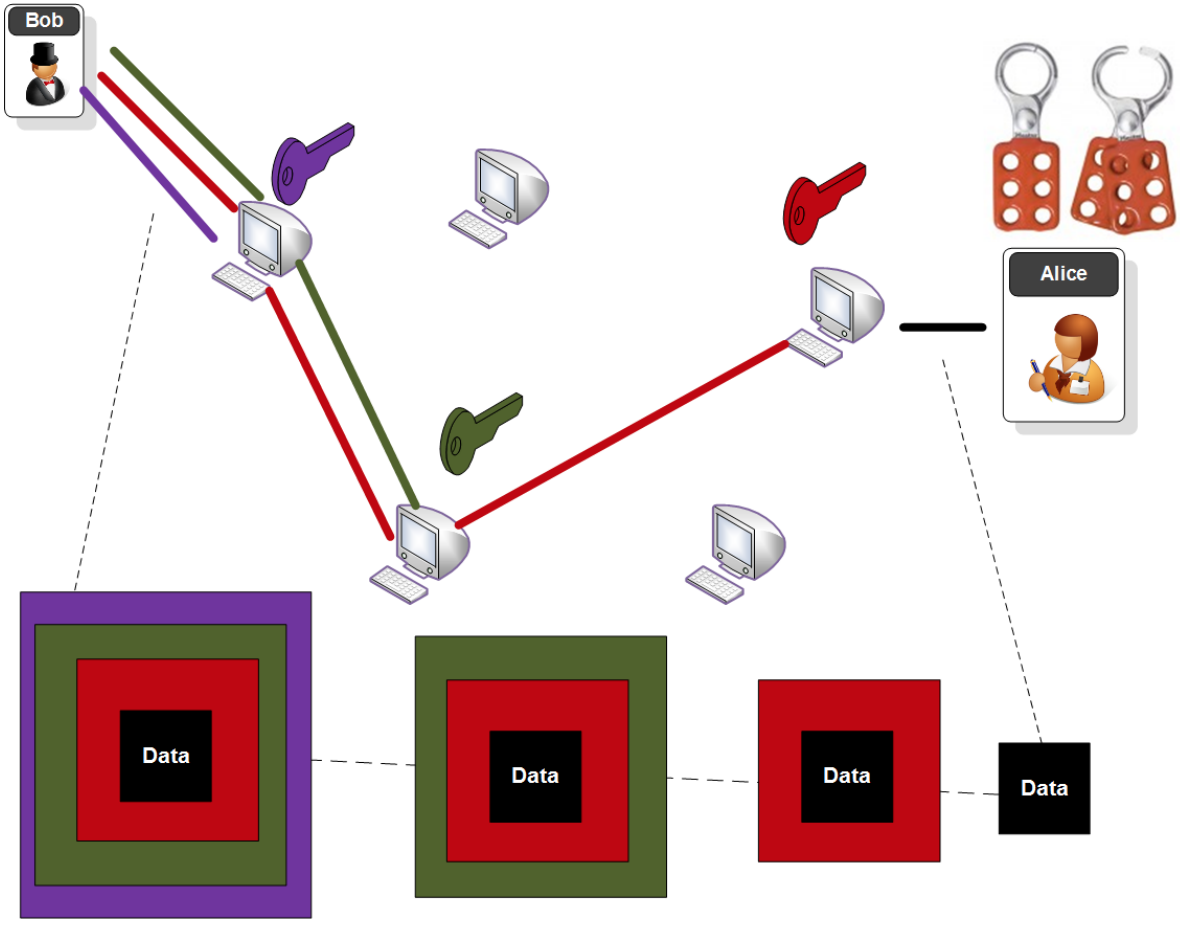
C \>
-----B
BgUr
-----E
-----B
MHC
oUQ
l1M4
-----E
```

```
C \> openssl ec -in priv.pem -text -noout
read EC key
Private-Key  (256 bit)
priv
    46 b9 e8 61 b6 3d 35 09 c8 8b 78 17 27 5a 30
    d2 2d 62 c8 cd 8f a6 48 6d de e3 5e f0 d8 e0
    49 5f
pub
    04 25 00 e7 f3 fb dd f2 84 29 03 f5 44 dd c8
    74 94 ce 95 02 9a ce 4e 25 7d 54 ba 77 f2 bc
    1f 3a 88 37 a9 46 1c 4f 1c 57 fe cc 49 97 53
    38 1e 77 2a 12 8a 58 20 a9 24 a2 fa 05 16 2e
    b6 62 98 7a 9f
ASN1 OID  secp256k1
```

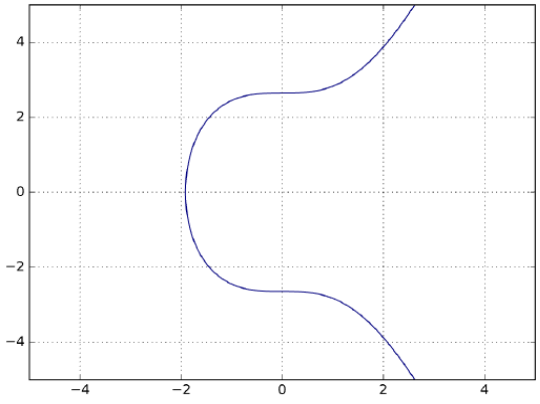
Example

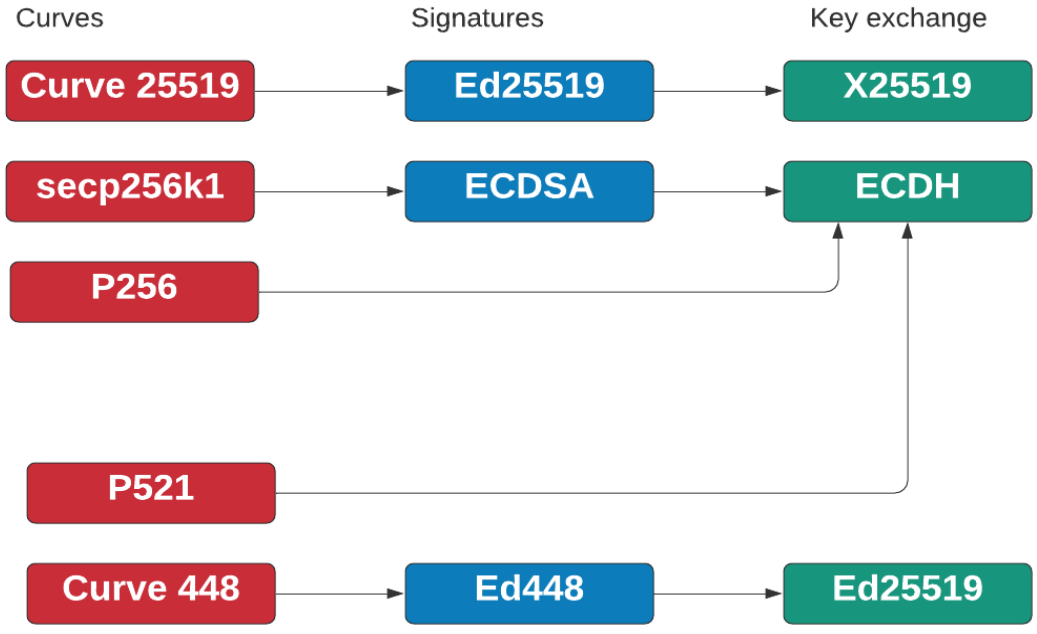
```
C:> openssl ecparam -in priv.pem -text -param_enc explicit -noout
Field Type: prime-field
Prime:
    00:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:
    ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:fe:ff:
    ff:fc:2f
A:    0
B:    7 (0x7)
Generator (uncompressed):
    04:79:be:66:7e:f9:dc:bb:ac:55:a0:62:95:ce:87:
    0b:07:02:9b:fc:db:2d:ce:28:d9:59:f2:81:5b:16:
    f8:17:98:48:3a:da:77:26:a3:c4:65:5d:a4:fb:fc:
    0e:11:08:a8:fd:17:b4:48:a6:85:54:19:9c:47:d0:
    8f:fb:10:d4:b8
Order:
    00:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:ff:
    ff:fe:ba:ae:dc:e6:af:48:a0:3b:bf:d2:5e:8c:d0:
    36:41:41
Cofactor:  1 (0x1)
```

Elliptic Curve Diffie Hellman (ECDH)

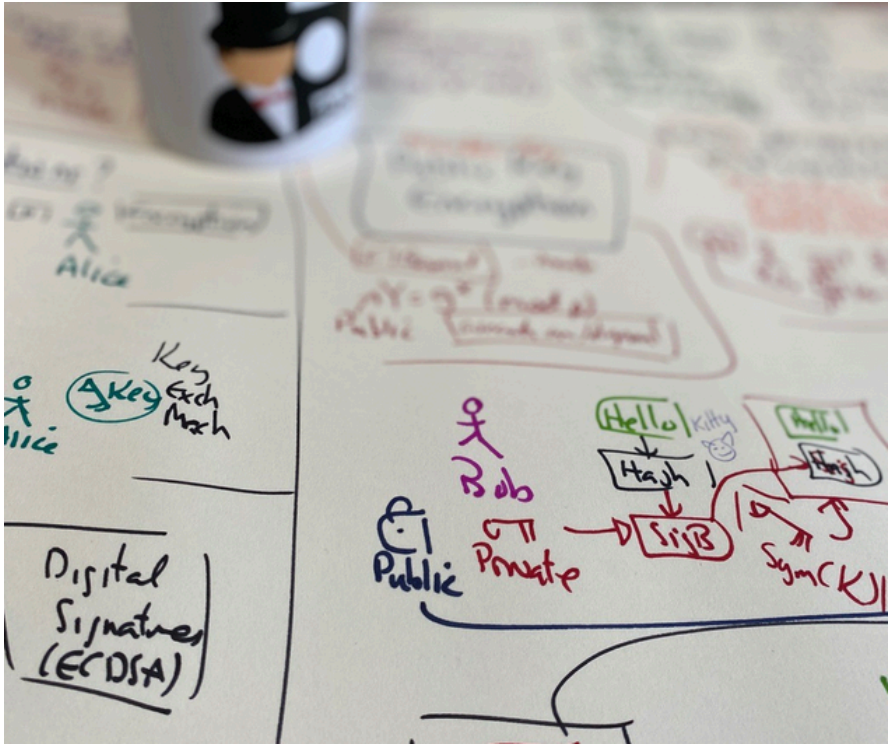


ECC Methods


4
2
0
−2
−4
−4
−2
0
2
4


Curves
Signatures
Key exchange
Curve 25519
Ed25519
X25519
secp256k1
ECDSA
ECDH
P256
P521
Curve 448
Ed448
Ed25519

Unit 4: Public Key



ElGamal

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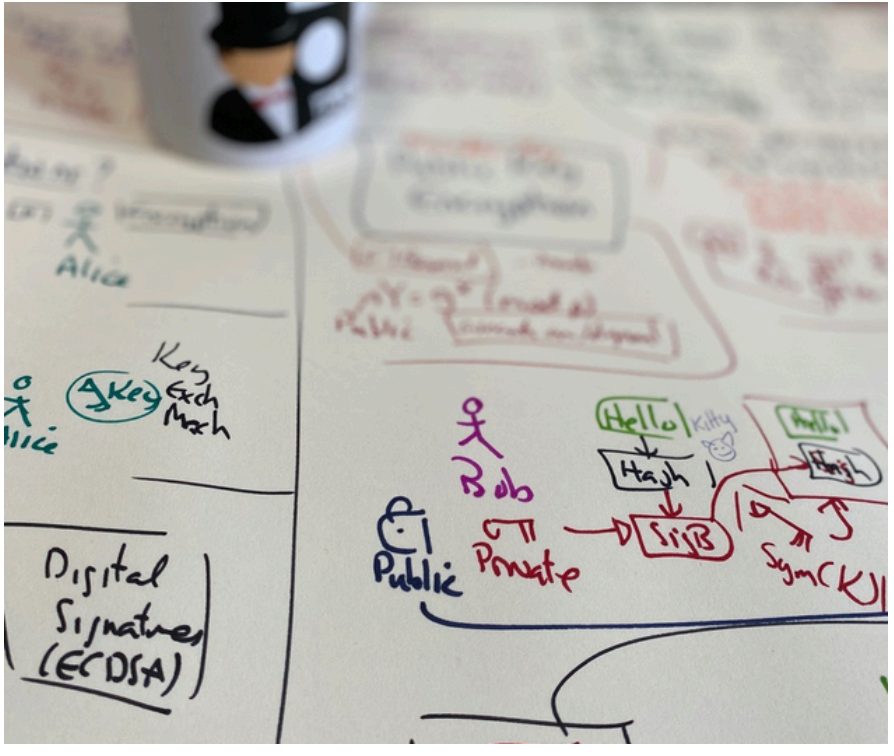
https://asecuritysite.com/ecc
https://asecuritysite.com/elgamal

ElGamal

- $Y = g^x \bmod p$
- g is picked from cyclic group (Explained in Key Handshaking section). Here.
- p is a prime number.
- Example here.

Unit 4: Public Key



PGP/Box

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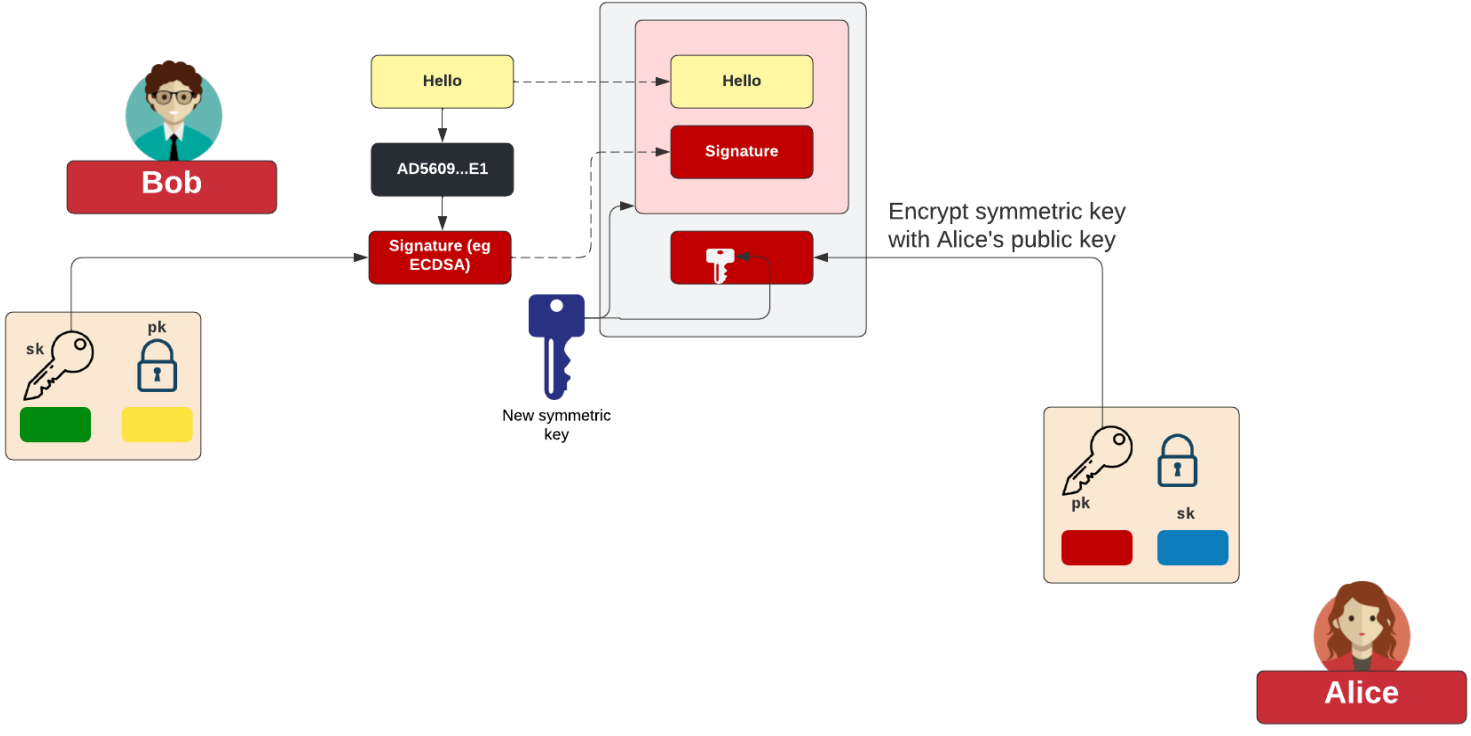
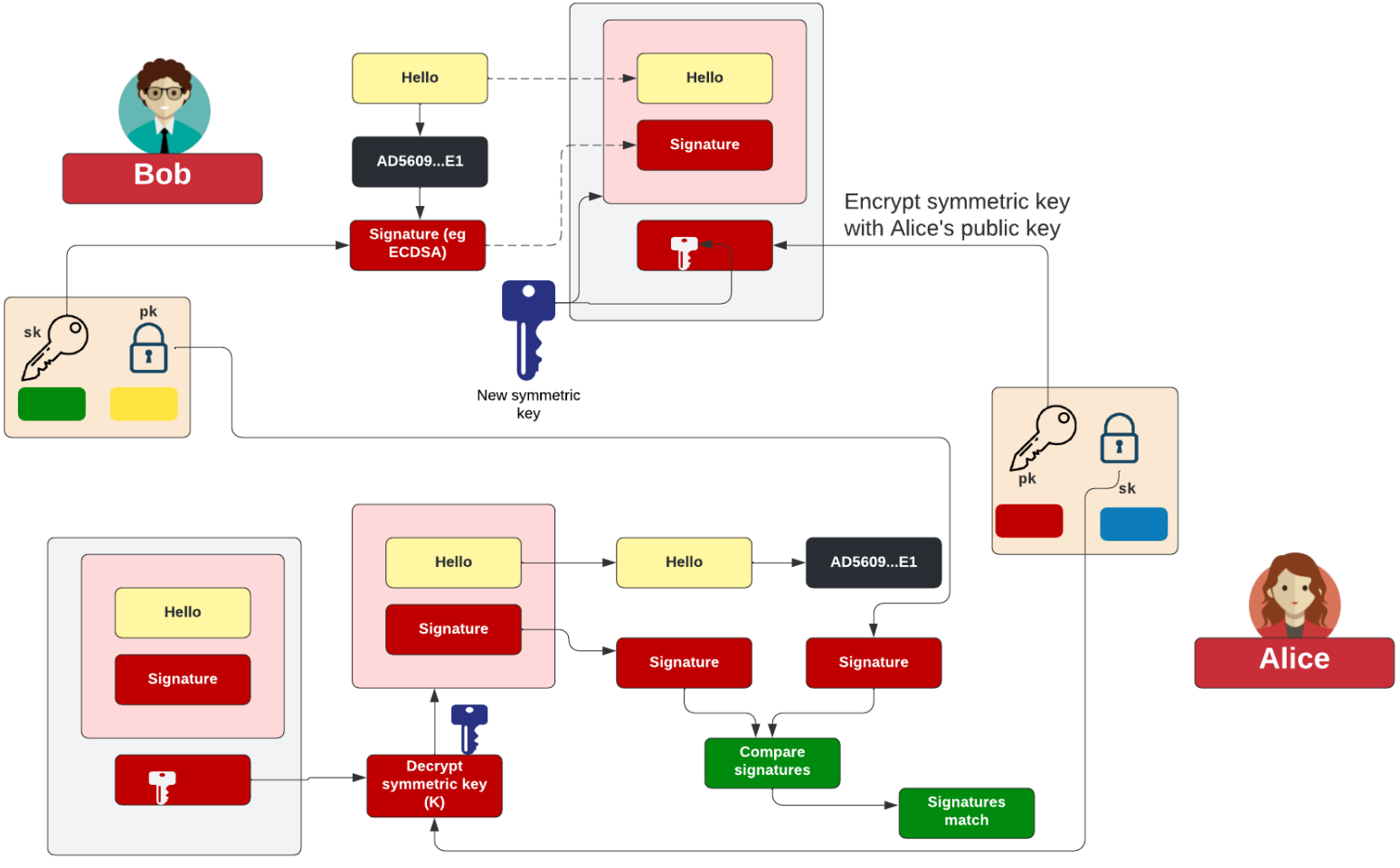
PGP

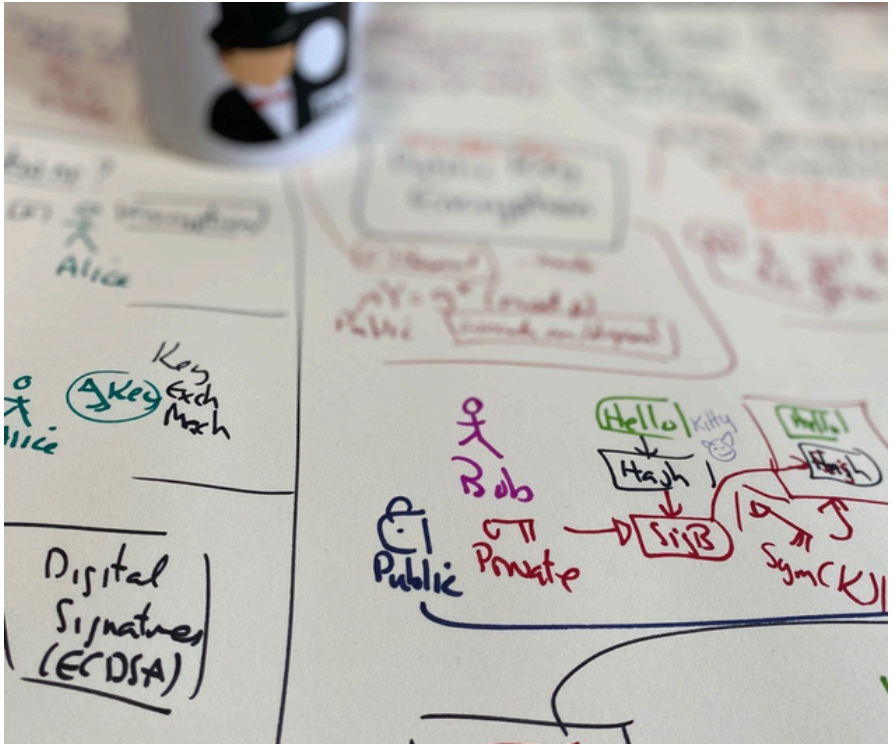

Bob
Hello
AD5609...E1
Signature (eg ECDSA)
Hello
Signature
Encrypt symmetric key with Alice's public key
sk
pk
New symmetric key
pk
sk
Alice

PGP


Bob
Hello
AD5609...E1
Signature (eg ECDSA)
Hello
Signature
Encrypt symmetric key with Alice's public key
sk
pk
New symmetric key
pk
sk
Alice
Hello
Signature
Hello
Signature
Hello
Hello
AD5609...E1
Signature
Signature
Compare signatures
Signatures match
Decrypt symmetric key (K)

Unit 4: Public Key

AWS Key Management System



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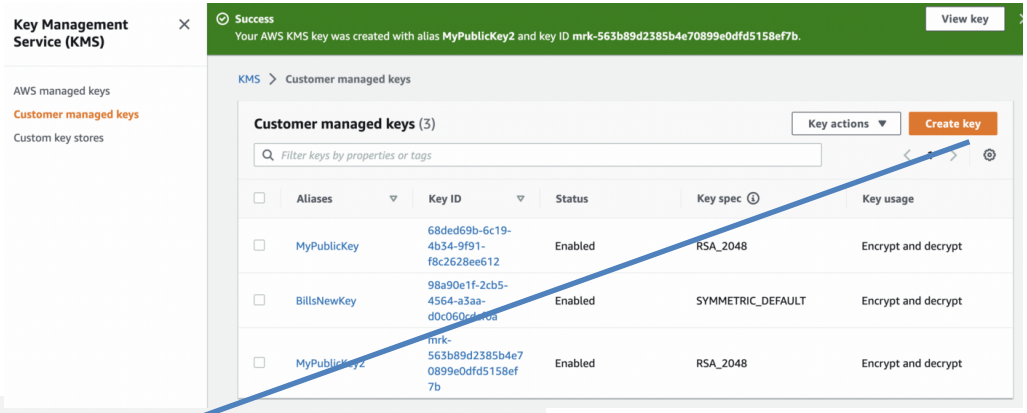
https://asecuritysite.com/elgamal

KMS



Demo

Configure key

Key type Help me choose

- ○ Symmetric
  A single key used for encrypting and decrypting data or generating and verifying HMAC codes.
- ◉ Asymmetric
  A public and private key pair used for encrypting and decrypting data or signing and verifying messages.

Key usage Help me choose

- ◉ Encrypt and decrypt
  Use the key only to encrypt and decrypt data.
- ○ Sign and verify
  Key pairs for digital signing
  Uses the private key for signing and the public key for verification.

Key spec Help me choose

- ◉ RSA_2048
- ○ RSA_3072
- ○ RSA_4096

Viewing public key

Key policy | Cryptographic configuration | Tags | Public key | Aliases

Public key Copy Download

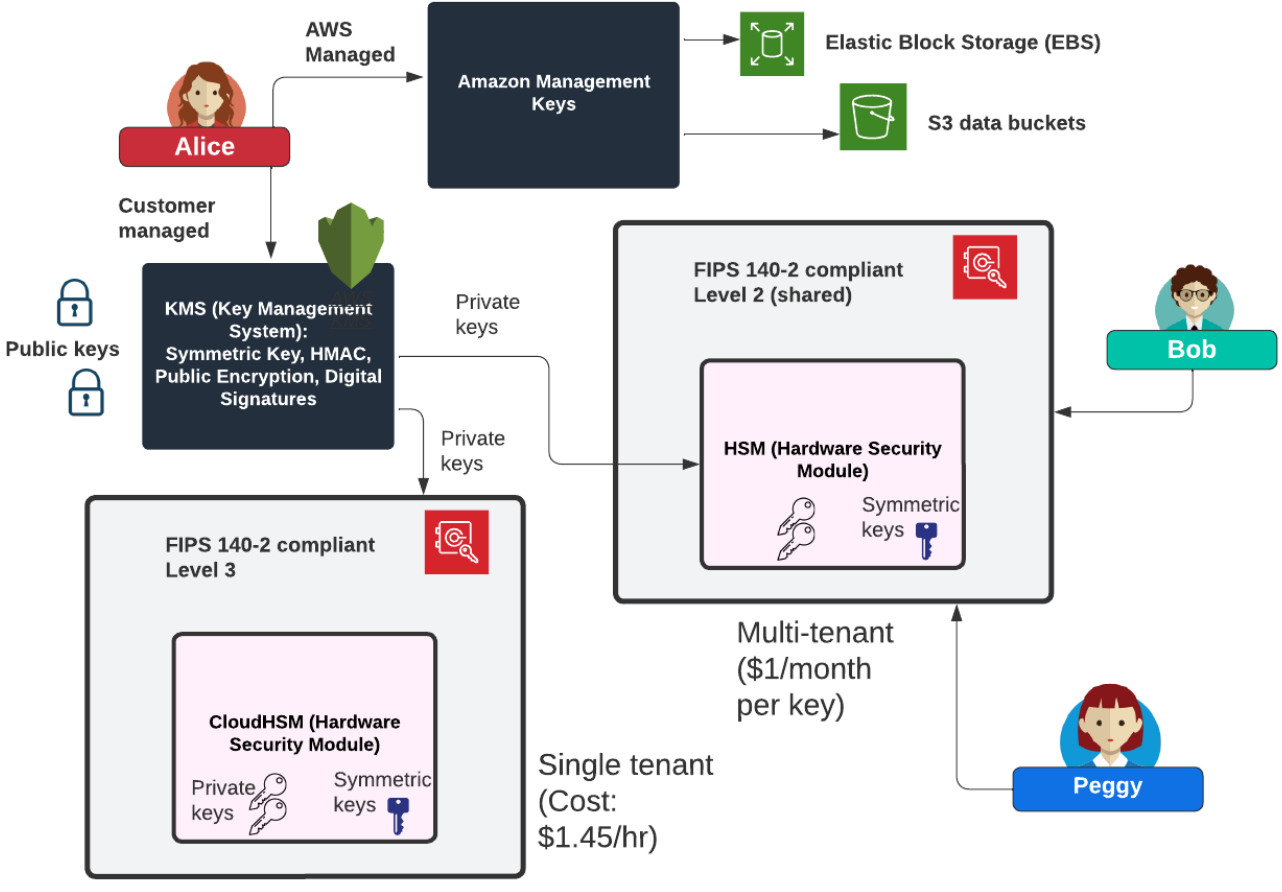
Public key for encryption operations only. Learn more

```
-----BEGIN PUBLIC KEY-----
MIIBIjANBgkqhkiG9w0BAQEFAAOCAQ8AMIIBCgKCAQEAsXDtHOdCeteObzugPf6E
Njeft6CDGjbaR9t40++q4jqtSd5JsdYel1Rn3mYL+oXqKQJz9o+aoXdCcMFkhu6w
qqDVbIOPT2nsXIuO3p+0G7uUS93g3cc5RodEAn3jb2yBjHjvfs9OBSBM7bh6Kw21
YuN/omU1GaL/d4o7+NYu0mDEAmb0Nh+1Q6lrpf+bu1YZ31gVpbLd78xGlv1dz2nq
y8G8VaZW90fr05jDjcpDnWm1O9QXl0pEhwNGcvcsxcHodslAZrlzKUre/nZ5MTNL
3uigw8w5l2uQLRFiIBpLlHKpcNBaxZu3Za5Mk2Dvj+1+L2PejLydAPfqQB5N8ds0
AQIDAQAB
-----END PUBLIC KEY-----
```

Download public key

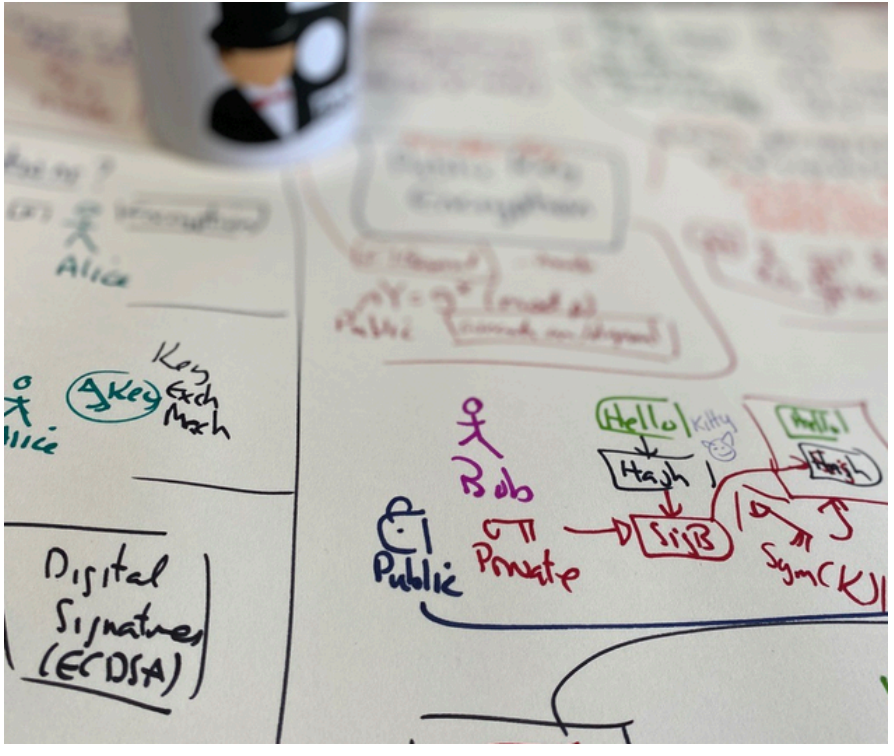
```
% aws kms get-public-key --key-id alias/PublicKeyForDemo
{
    "KeyId": "arn:aws:kms:us-east-1:103269750866:key/de30e8e6-c753-4a2c-881a-53c761242644",
    "PublicKey": "MIIBIjANBgkqhkiG9w0BAQEFAAOCAQ8AMIIBCgKCAQEAsXDtHOdCeteObzugPf6ENjeft6CDGjbaR9t40++q4jqtSd5JsdYel1Rn3
mYL+oXqKQJz9o+aoXdCcMFkhu6wqqDVbIOPT2nsXIuO3p+0G7uUS93g3cc5RodEAn3jb2yBjHjvfs9OBSBM7bh6Kw21YuN/omU1GaL/d4o7+NYu0mDEAmb0
Nh+1Q6lrpf+bu1YZ31gVpbLd78xGlv1dz2nqyBG8VaZW90fr05jDjcpDnWm1O9QXl0pEhwNGcvcsxcHodslAZrlzKUre/nZ5MTNL3uigw8w5l2uQLRFiIBp
LlHKpcNBaxZu3Za5Mk2Dvj+1+L2PejLydAPfqQB5N8dsOAQIDAQAB",
    "CustomerMasterKeySpec": "RSA_2048",
    "KeySpec": "RSA_2048",
    "KeyUsage": "ENCRYPT_DECRYPT",
    "EncryptionAlgorithms": [
        "RSAES_OAEP_SHA_1",
        "RSAES_OAEP_SHA_256"
    ]
}
```

HSM or CloudHSM



Unit 4: Public Key

Basics
RSA
Elliptic Curve
ElGamal
KMS



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