

$$\begin{array}{ccccccc}
\mathbb{F}_q^{nm} & \xrightarrow{\sim} & (\mathbb{F}_q^n)^m & \xrightarrow{\tilde{L}_1} & (\mathbb{F}_q^n)^m & \xrightarrow[\sim]{\tilde{\pi}_1} & (\mathbb{F}_{q^n})^m \\
& & & & & \nearrow & \\
& & & & & L_1 & 
\end{array}$$