

$$\begin{array}{ccccccc}
 (\mathbb{F}_{q^m})^n & \xrightarrow[\sim]{\tilde{\pi}_2} & (\mathbb{F}_q^m)^n & \xrightarrow{\tilde{L}_3} & (\mathbb{F}_q^m)^n & \xrightarrow[\sim]{} & \mathbb{F}_q^{mn} \\
 & & & & & \nearrow & \\
 & & & & & L_3 &
 \end{array}$$