Sampling from High Dimensional Distributions

A Gradient Based Approach

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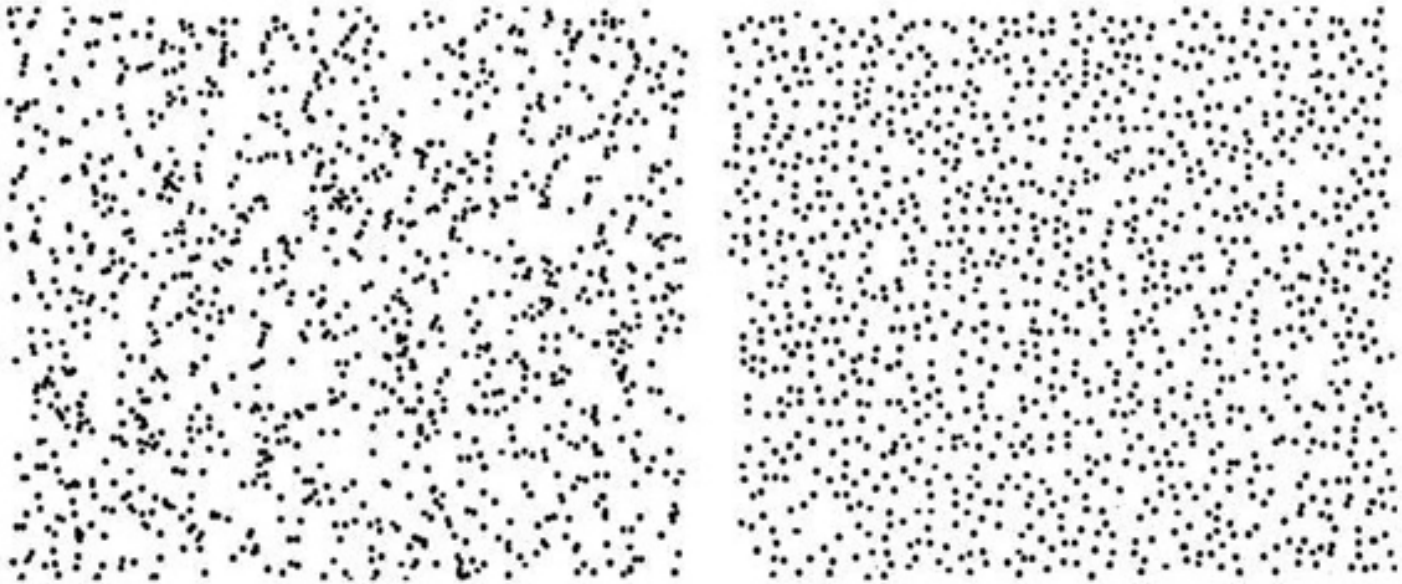
What is random?



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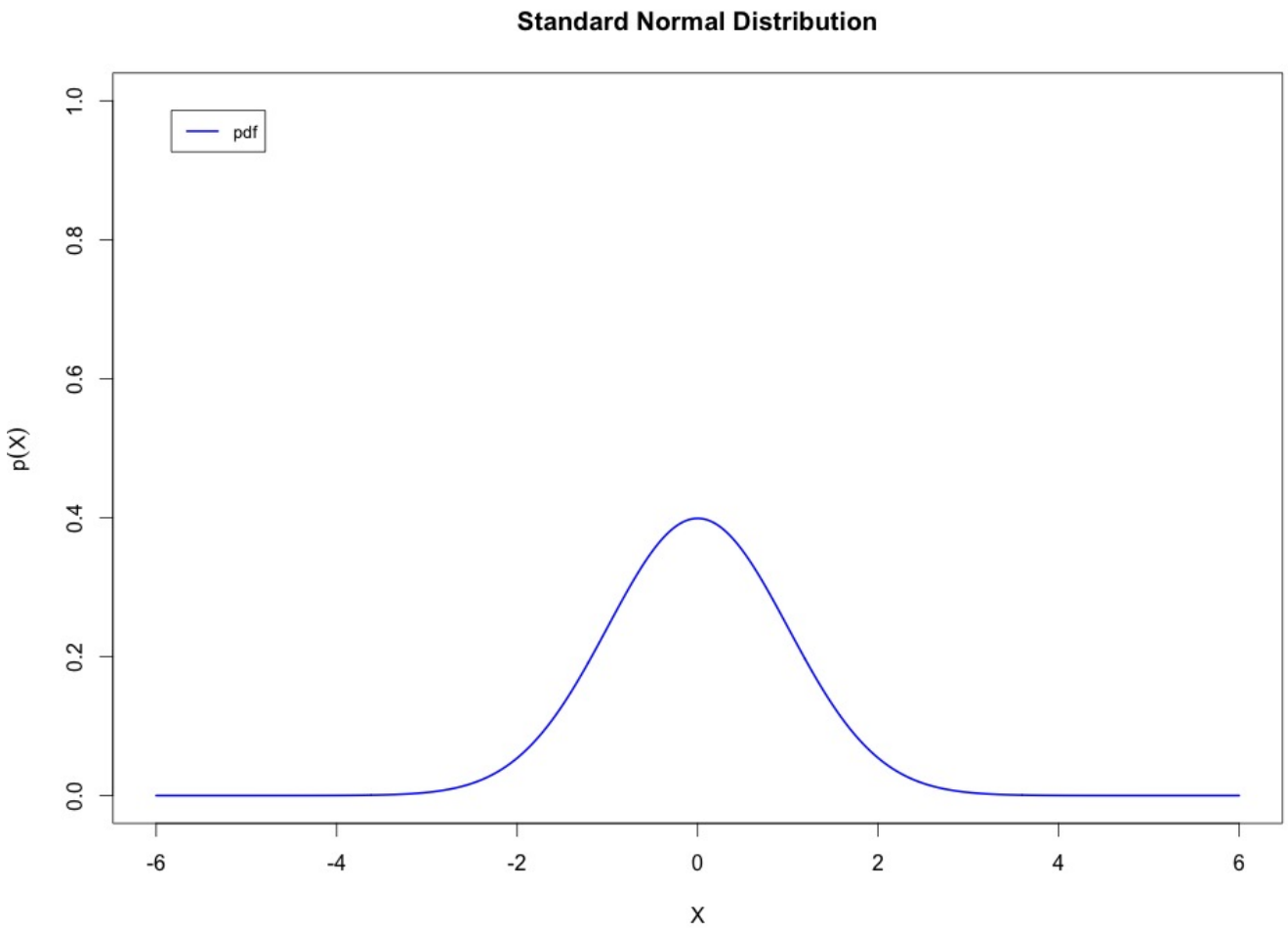
- Idea: transform the uniform random variables using the inverse of the indefinite integral of the desired distribution.

$$x = h^{-1}(z) = h^{-1}\left(\int_{-\infty}^{x} p(\hat{x})\, d\hat{x} \right)$$

Inverse Transform Example



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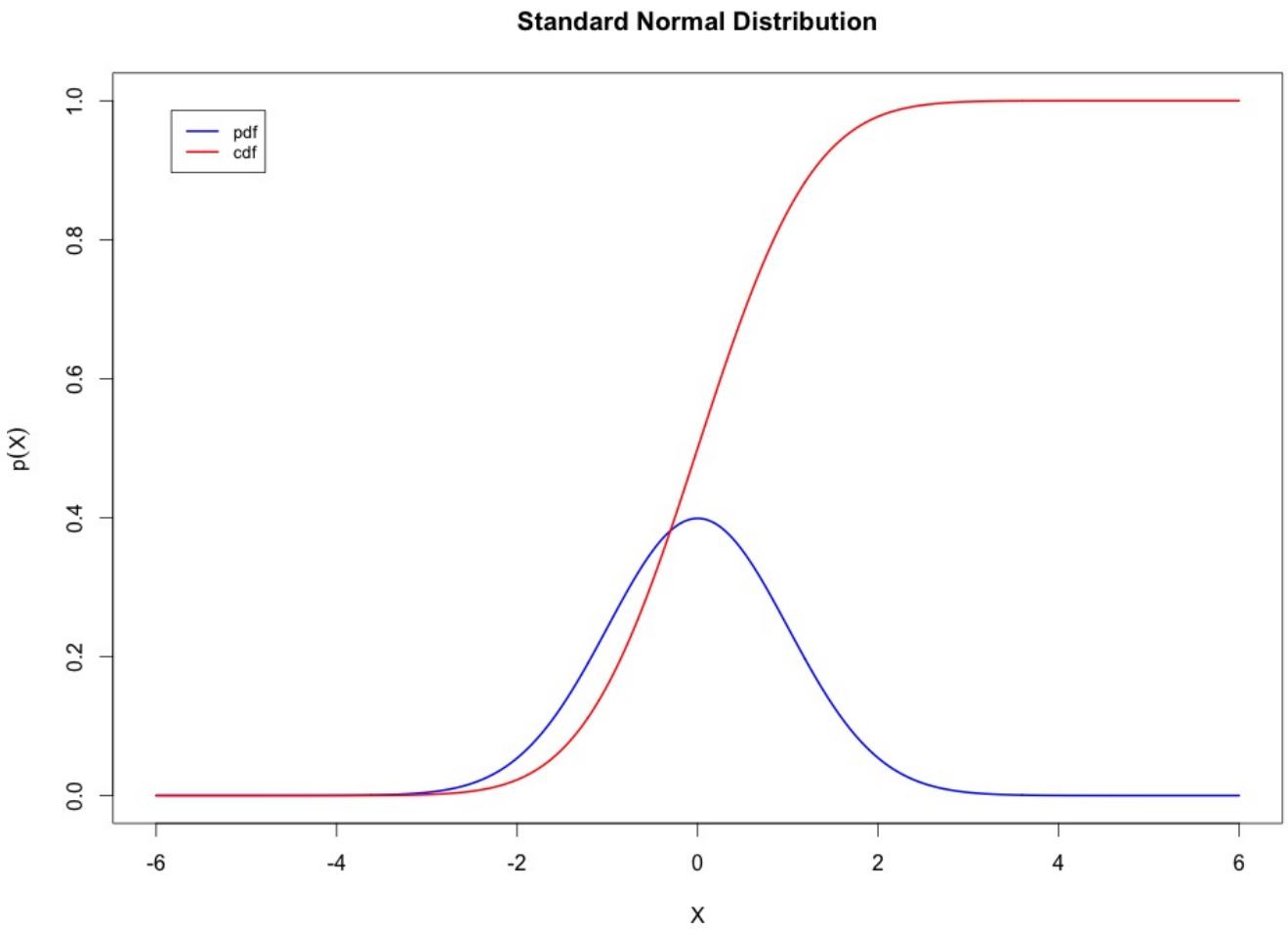

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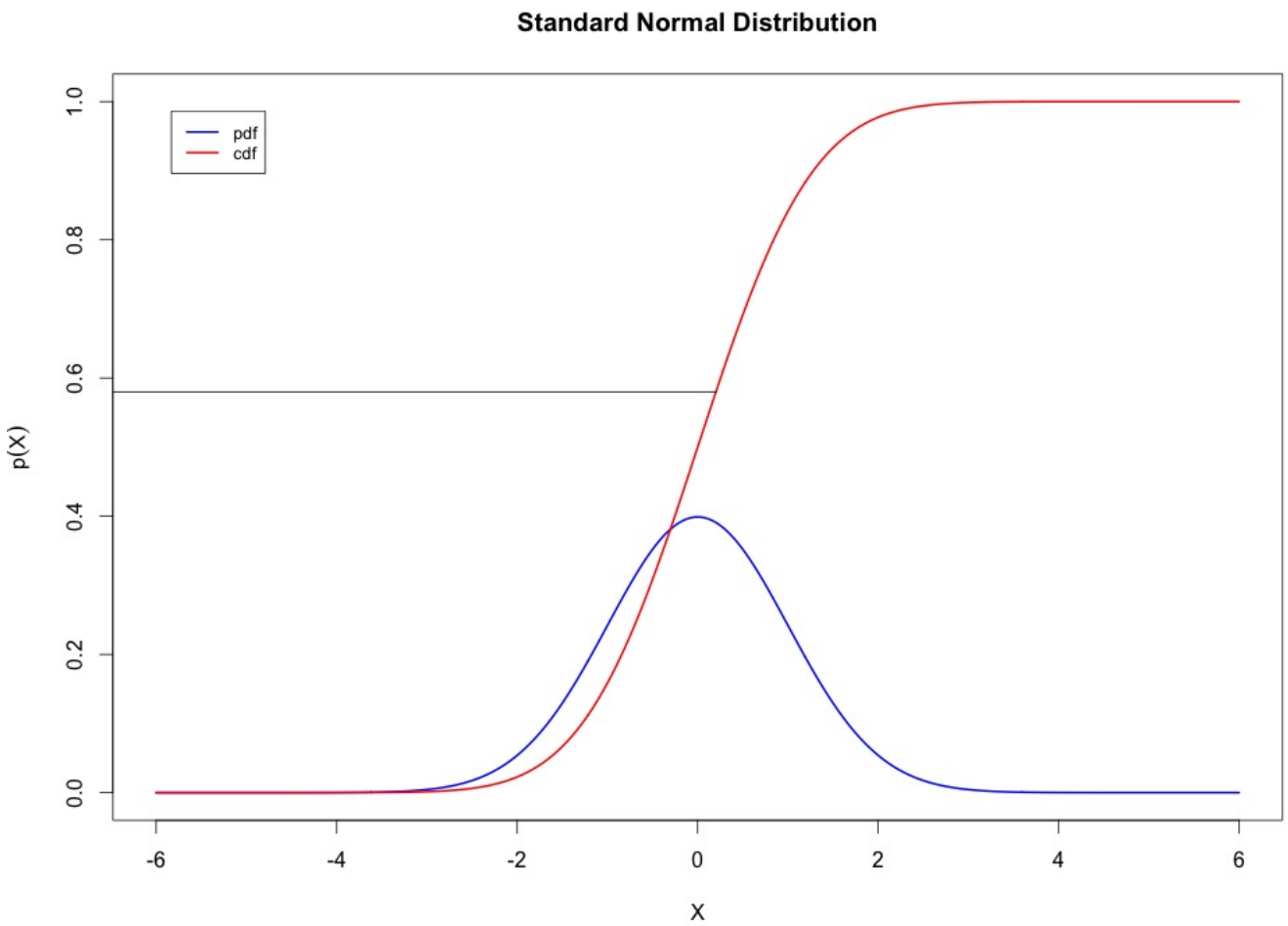

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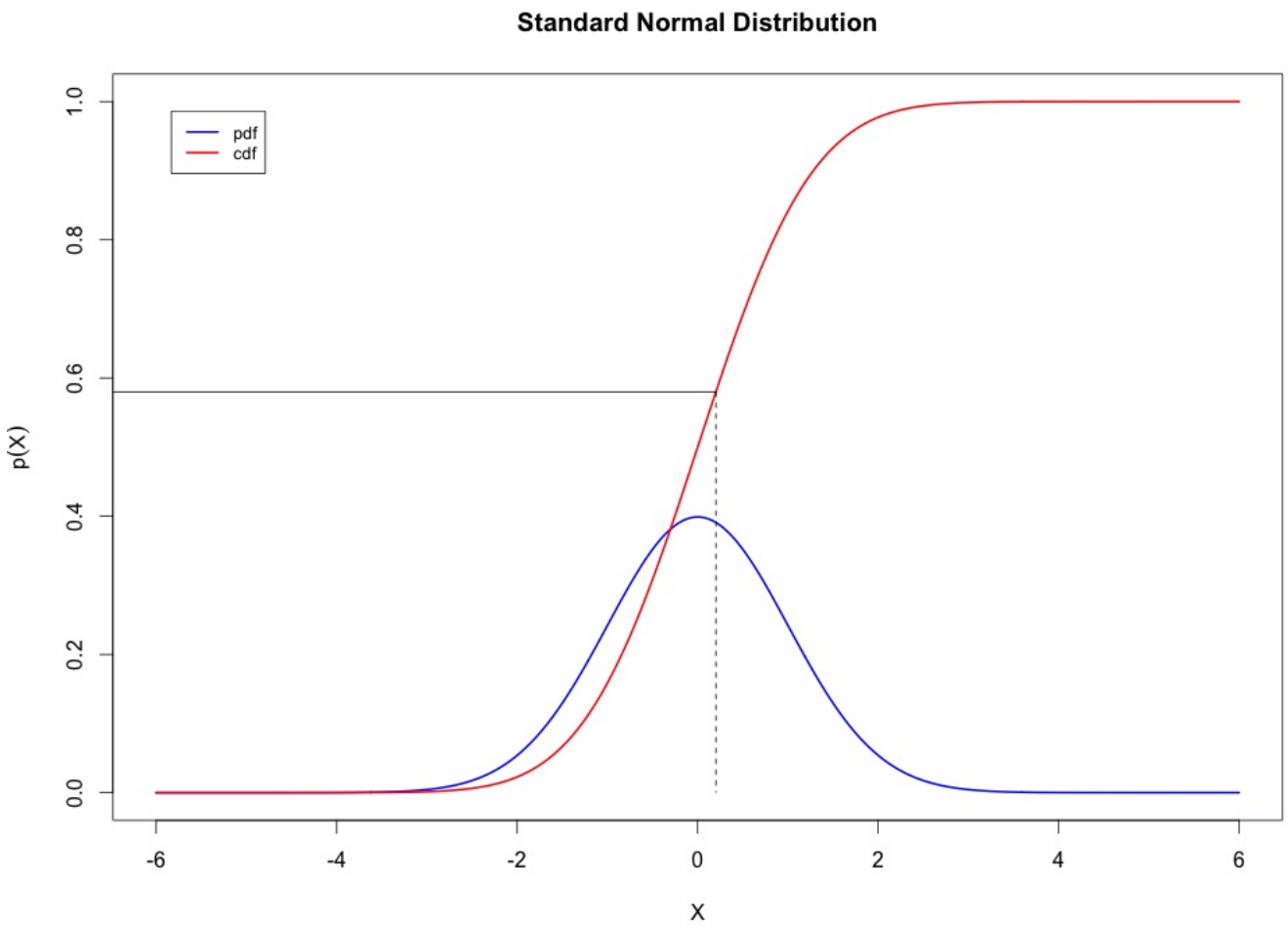

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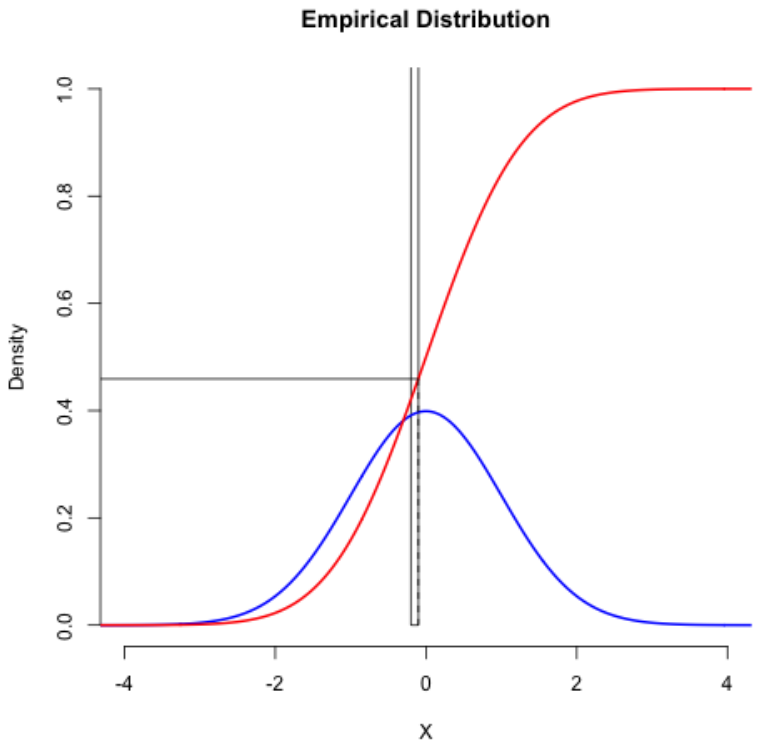

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  - Hamiltonian Monte Carlo