Question4：

a)

Logically:

We have three literals, $G_A = true$, $G_B = true$, $G_C = true$, which denotes that the goat is in location A, B or C and $S_A = true$, $S_B = true$, $S_C = true$ denotes I select A, B or C and goat is in the location I select, otherwise false.

So, we have a logical formula, goat in x and I select y.

$$\varphi(x, y) = G_x \wedge S_y$$

and it is easily to know that l_x and S_x have the same assignment, so the formula can be written as, x is the goat location, and y is my selected location, if x is not y, I lose to find the goat, so I can transform $\varphi(x, y)$ to the following formula, $l_x = true$ for goat in x, x is A, B or C and $l_y = false$ for y in union(A, B, C) except x. So, I use a XNOR gate $(x \odot y) = (x \wedge y) \vee (\bar{x} \wedge \bar{y})$, and denote the bool variable $l_A = A \wedge \bar{B} \wedge \bar{C}$ $l_B = B \wedge \bar{A} \wedge \bar{C}$ $l_C = C \wedge \bar{A} \wedge \bar{B}$ for all A=B=C=true. Function is:

$$\phi(x, y) = x \wedge (x \odot y)$$

Means that x is the goat, y is the selected location, if x=y that is selected is where the goat is, we get a true to win the goat. We have the truth table,

Goat	I select	Win or lose
A	A	1
A	B	0
A	C	0
B	A	0
B	B	1
B	C	0
C	A	0
C	B	0
C	C	1

Probabilistically:

We denote the probability of goat in A, B, C as P_A, P_B, P_C.

So, $P_A = P_B = P_C = 1/3$.

b)

For all y in A, B, C, $S_y = true$. So, for x in A, B or C, we will have ϕs have a value of one true and two false.

If goat is in location x and I select x then the result of ϕ would be true and the other two would be false, and ϕ get an assignment. If ϕ is true means we have found goat, otherwise false.

There is no obvious choice of best action because regardless what location we choose, there would be 9 possible outcomes and 3 of them is true, and for select A, B or C either have one true outcome and two false outcomes. So, we can't know whether the location we choose would definitely find a goat so that the formula is true.

c)

Under probability, I can tell difference between them just compare their probability. But as we know, $P_A = P_B = P_C = 1/3$, so all three choices will have the same chance to get the goat. So, there is no obvious choice of best action.

d)

Let literal $R_Z = false$ be that CBMH opens door z to show the empty (reveal z).
So, we have a new,

$$\phi'(\mathrm{x},\mathrm{y},\mathrm{z}) = \mathrm{x} \wedge (x \odot \mathrm{y}) \wedge (x \oplus \mathrm{z}) \wedge \bar{\mathrm{z}}$$

So, truth table is: (just the location not the value)

Goat	I select	CBMH	Win or lose
A	A	B	1
A	A	C	1
A	B	C	0
A	C	B	0
B	A	C	0
B	B	A	1
B	B	C	1
B	C	A	0
C	A	B	0
C	B	A	0
C	C	A	1
C	C	B	1

e)

In order to use Bayes' Theorem, we need to first assign an event to H_A and R_B. Let event H_A be that the goat is behind door number A (Hide behind A). Let event R_B be that CBMH opens door B to show the empty (reveal B). Here's the Bayes' solution,

$$\mathrm{P}(H_A|R_B) = \frac{\mathrm{P}(R_B|H_A) \times \mathrm{P}(H_A)}{P(R_B)} = \frac{1/2 \times 1/3}{1/3 \times 1/2 + 1/3 \times 0 + 1/3 \times 1} = 1/3$$

$\mathrm{P}(H_A)$ (or P_A) is simple to figure out. There is a 1/3 chance that the goat is behind door A. There are two doors left, and each has a 1/2 chance of being chosen — which gives us $\mathrm{P}(R_B|H_A)$, or the probability of event R_B, given H_A.
$\mathrm{P}(R_B)$, in the denominator, is a little trickier to figure out. Consider that:
You choose door A. CBMH shows you an empty behind door B. If the goat is behind door A, CBMH will not choose it. He'll open door B and show an empty 1/2 of the time. If the goat is behind door B, CBMH will always open the door C, as he never reveals the goat. If the goat is behind door C, CBMH will open door B 100% of the time. As CBMH has opened door B, you know the goat is either behind door A (your choice) or door C. The probability of the goat being behind door A is 1/3. This means that the probability of the goat being behind door C is 1 – (1/3) = 2/3. That is $\mathrm{P}(H_C|R_B) = 2/3$

f)

As in d), I have this table, so I can know re-select A which has 1 win and 1 lose is equal to re-select C which has 1 win and 1 lose and re-select B will lose in probability 1.

Goat	I select	CBMH	Re-select	Win or lose
A	A	B	A	1
A	A	B	B	0
A	A	B	C	0
C	A	B	A	0
C	A	B	B	0
C	A	B	C	1

So, best is to choose A or C but not B.

g)

In e), I get the probability of re-select A, B, C to win is 1/3, 0, 2/3. So, obviously the best action is re-select C.

h)

From b), d), f), we can know that the winning rate for re-select A and C are the same, and before the CBMH's help, A, B, C are also the same, so it doesn't do much help to me. Because A and C are still at equal position for next choosing step, that is change or doesn't change will have the same outcome. So, I can either stick with A or change to C.

i)

From c), e), g), we can easily know that the winning rate for re-select A and C are different, and C would be prior to A in the re-selection with a probability 2/3 greater than the 1/3. So, the CBMH does give me help. I must change my selection to location C.

j)

The probabilityGDBot is more successful in the problem.

Because in this problem, we cannot think the probability of choose a goat is the same before and after the CBMH's help, if we don't have an initial choice and the CBMH helps me, then I choose one out of the remain two locations, it is the regular problem.

But in this problem, actually we can transfer it to a problem:

I choose goat location, CBMH choose one of the empty, switch will lose.

I choose empty, CBMH choose the other empty, switch will win.

So, the probability for re-selection actually equal to the probability of our first choice whether I select the goat in the beginning.

$$\mathrm{P}(\text{switch to win}) = \mathrm{P}(stick\ to\ lose) = \mathrm{P}(\text{choose goat location in the beginning}) = 2/3$$

$$\mathrm{P}(\text{switch to lose}) = \mathrm{P}(stick\ to\ win) = \mathrm{P}(\text{choose empty location in the beginning}) = 1/3$$

So, in reality, switch will always give us a bigger chance to win. So, the probabilityGDBot describes the actual situation of this problem rather than logicalGDBot and it would make us closer to win.

Bonus)

I would like to draw a sheet to denote the calculate results.

Form of utility for stick (P is the probability of having the goat in location X)

Goat	I select	CBMH	Re-select	Win or lose
A P=1.0	A P=0.33	C P=1-p	A P=1.0	WIN 0.33-0.33p
		B P=p	A P=1.0	WIN 0.33p
	B P=0.33	C P=1.0	B P=1.0	LOSE 0.33
	C P=0.33	B P=1.0	C P=1.0	LOSE 0.33

Form of utility for switch

Goat	I select	CBMH	Re-select	Win or lose
A P=1.0	A P=0.33	C P=1-p	B P=1.0	LOSE P=0.33-0.33p
		B P=p	C P=1.0	LOSE P=0.33p
	B P=0.33	C P=1.0	A P=1.0	WIN P=0.33
	C P=0.33	B P=1.0	A P=1.0	WIN P=0.33

So, the utility of sticking with initial selection is 0.33, the U of switching is 0.33, so it is independent to p.