CS 536 : Homework Zero

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1) Let $X_1, X_2, X_3, \ldots, X_n$ be i.i.d. random variables uniformly distributed on $[0, L]$. What is the density of

$$Y = \mathbf{max}\,[X_1, X_2, \ldots, X_n]? \tag{1}$$

2) Let $X_1, X_2, X_3, \ldots, X_n$ be i.i.d. random variables with a normal distribution $N(\mu, \sigma^2)$.

   - What is the joint density of these random variables?
   - Define the sample mean and sample variance as

$$\begin{aligned} \bar{X}_n &= \frac{1}{n}\sum_{i=1}^{n} X_i \\ S_n^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2. \end{aligned} \tag{2}$$

   Show that $\bar{X}_n \sim N(\mu, \sigma^2/n)$, that $(n-1)S_n^n/\sigma^2 \sim \chi^2_{n-1}$, and $\bar{X}_n$ and S_n^2 are independent.

3) Let (X_i, Y_i) for $i = 1, \ldots n$ be a set of points in $\mathbb{R}^2$. Find w^*, b^* to minimize

$$\sum_{i=1}^{n} \left(wX_i + b - Y_i\right)^2. \tag{3}$$

4) Continuing on the previous problem, suppose that for each (X_i, Y_i), we had that the X_i value is given, and $Y_i = wX_i + b + Z_i$ where $Z_i \sim N(0, \sigma^2)$.

   - What is the expected value of w^* in this case? What is the expected value of b^*?
   - What is the variance of w^*? What is the variance of b^*?

   Simplify as much as possible.

5) Let (A, B), (C, D) be two points in $\mathbb{R}^2$ chosen such that A, B, C, D are all i.i.d. standard normal random variables. Consider the line drawn from (A, B) to (C, D). What is the distribution of its length, and what is the expected value of the length?