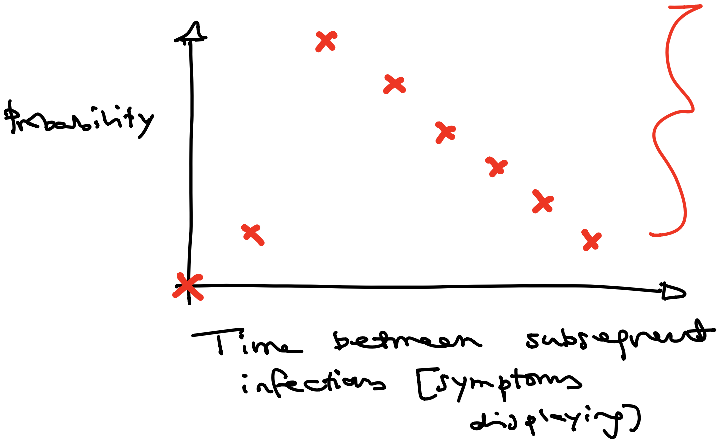
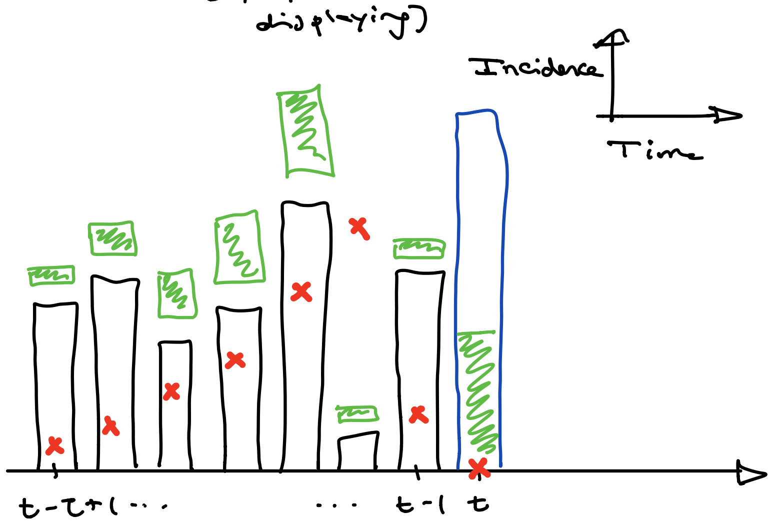



"Calculate what we expect the # local infections to be [if $R_t$ is 1] & then take ratio of actual incidence to get $R_t$ estimate"




Serial interval has been flipped

Key Idea: $$R_t = \frac{I_t^{local}}{\Lambda_t(w_s)}$$

Is this a 'Renewal Process'?

$$I_t^{local} = R_t \Lambda_t(w_s)$$

Using this, we can generate a stochastic model, i.e if

$\mathbb{E}[I_t^{local} | R_t, w_s, I_{t-1}] = R_t \Lambda_t(w_s)$ & $I_t^{local}$ is Poisson distributed,

must have $w_0 = 0$

$$I_t^{local} | R_t, w_s, I_{t-1} \sim \text{Poi}(R_t \Lambda_t(w_s))$$

$* \; I_{t,\tau}$

$$\Rightarrow P(I_t^{local} = i | R_t, w_s, I_{t-1}) = \frac{e^{-R_t \Lambda_t} (R_t \Lambda_t)^i}{i!}$$

We can go further & think about the likelihood of a stochastic process.

Poisson likelihood

i.e $p\left(I_t^{\text{local}}, I_{t-1}^{\text{local}}, \ldots, I_{t-\tau+1}^{\text{local}} \mid \underline{\underline{R_t}}, w_s, \underline{I}_{t-\tau}\right)$

really this should be $R_t, R_{t-1}, \ldots R_{t-\tau+1}$ but we assume $R_t$ is constant here.

$$= \prod_{k=t-\tau+1}^{t} \frac{e^{-R_{k(t)}\Lambda_k}\left(R_{k(t)}\Lambda_k\right)^{i_k}}{i_k!}$$

can replace $R_k$ with $R_t$.

We can now use Bayes' Th'm to infer $R_t$.

Recall: $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

$P(B|A) = \dfrac{P(A\cap B)}{P(A)}$

$$\Rightarrow P(B|A) = \frac{P(A\cap B)}{P(B)}\frac{P(B)}{P(A)} = P(A|B)\frac{P(B)}{P(A)}.$$

i.e $P(B|A) \propto P(A|B)\,P(B)$

$$\Rightarrow P\left(I_t^{\text{local}}, \ldots, I_{t-\tau+1}^{\text{local}} \mid R_t, w_s, \underline{I}_{t-\tau}\right)$$

$$\propto P\left(R_t \mid I_t^{\text{local}}, \ldots, I_{t-\tau+1}^{\text{local}}, I_{t-\tau}^{\text{local}}, w_s\right) P(R_t) \;*$$

Two problems with the model:

1) we are estimating instantaneous $R_t$.

2) we assume $R_t$ is constant from $[t, t-\tau+1]$, this certainly is not true during an control intervention for example.

$$P(R_t) = \frac{R_t^{a-1} e^{-R_t/b}}{\Gamma(a)\, b^a}$$

If we assume a Gamma prior for $R_t$, then we know this will be a conjugate prior to the Poisson likelihood. i.e

$$\underbrace{P(\theta \mid \text{data}, m)}_{\text{posterior}} = \frac{\overbrace{P(\text{data} \mid \theta, m)}^{\text{likelihood}} \times \overbrace{P(\theta \mid m)}^{\text{prior}}}{P(\text{data} \mid m)}$$

$$= \prod_{k=t-\tau+1}^{t} \frac{(R_t \Lambda_k(w_s))^{i_k} \exp(-R_t \Lambda_k(w_s))}{i_k!}$$

$$\times \frac{R_t^{a-1} e^{-\frac{R_t}{b}}}{\Gamma(a) b^a}$$

$$\propto R_t^{-1+a+\sum_{k=t-\tau+1}^{t} i_k} \exp\left\{-R_t\left[\sum_{k=t-\tau+1}^{t} \Lambda_k(w_s) + \frac{1}{b}\right]\right\}$$

$$\times \prod \frac{\Lambda_k(w_s)^{i_k}}{i_k!}$$

[Note that $i_k$ refers to $I_k^{local}$.]

Therefore $a' = a + \sum_{k=t-\tau+1}^{t} i_k$ &

$$b' = \frac{1}{R_t\left(\sum_{k=t-\tau+1}^{t} \Lambda_k(w_s) + \frac{1}{b}\right)}.$$