Where am I? The symmedian point!

Bill Lionheart

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What if the GPS goes off?

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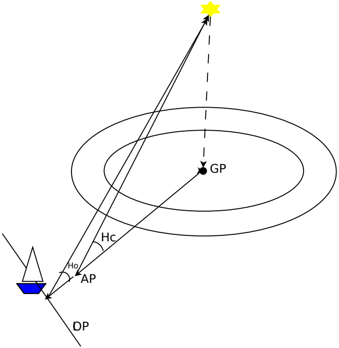
Use the old fashioned method of Celestial Navigation. Measuring the position of the stars and planets and an accurate clock to find our position.

How to do it

1. Identify the stars or planets you can use.



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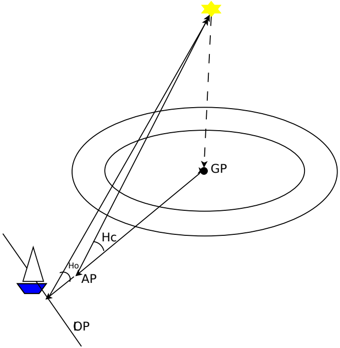
2. Measure the angle it makes to horizon H_o and note time.



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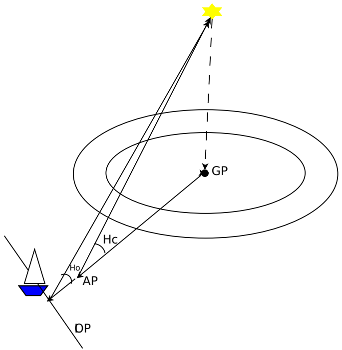

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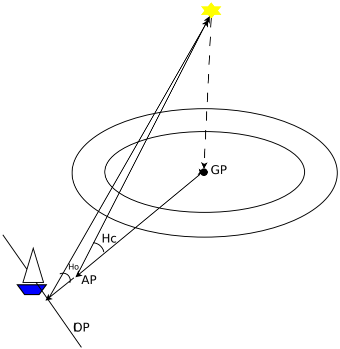

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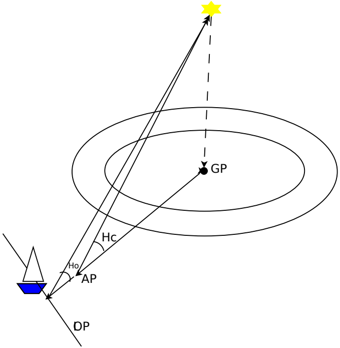
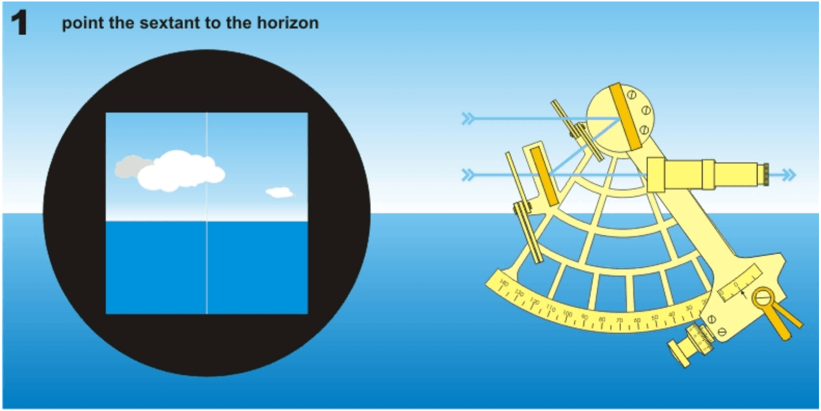
5. Difference between measured H_o and calculated H_c angle and the bearing gives you a position line (LOP).



Sextant animation



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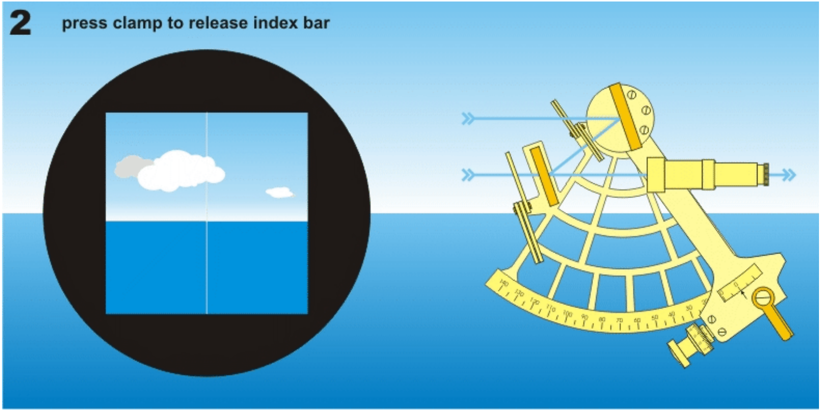

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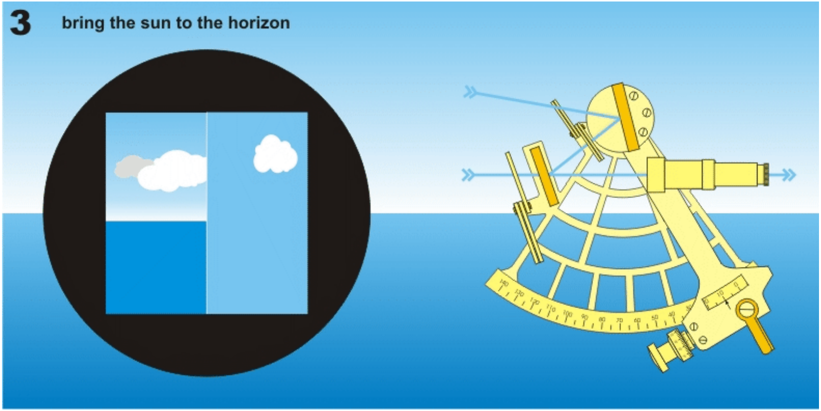

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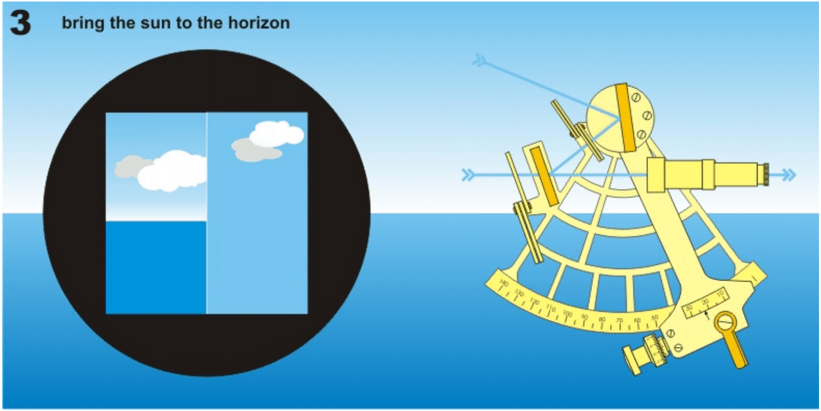

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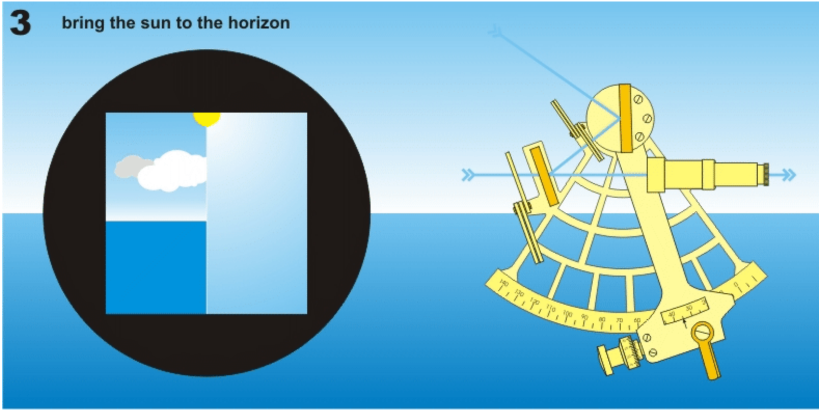

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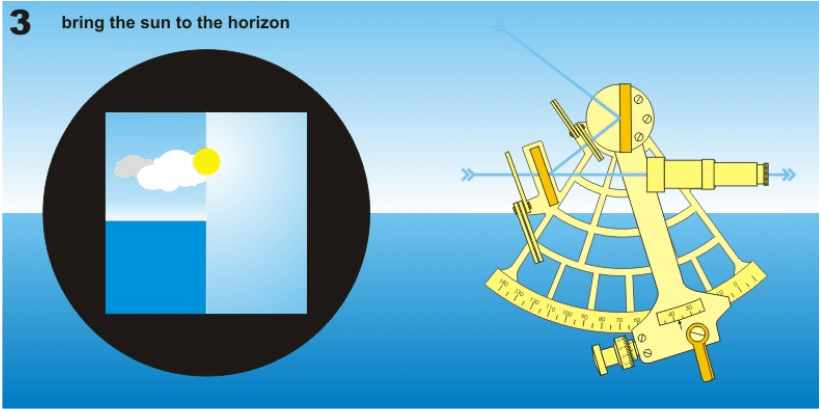

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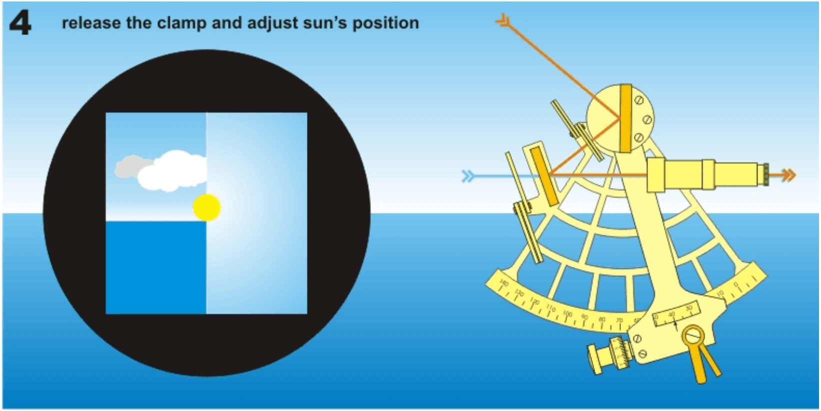

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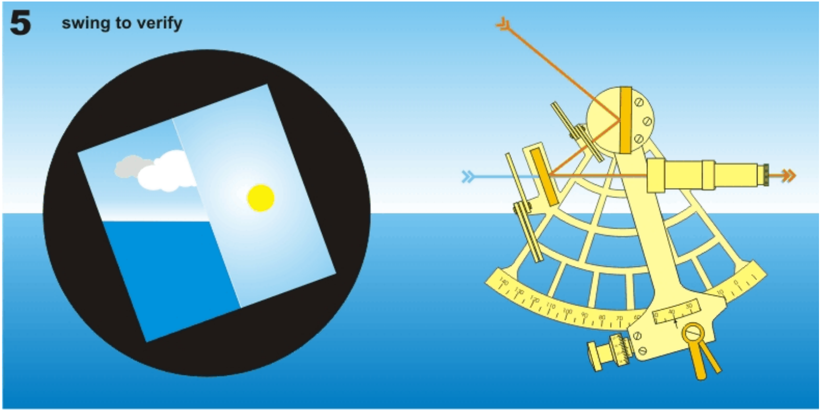

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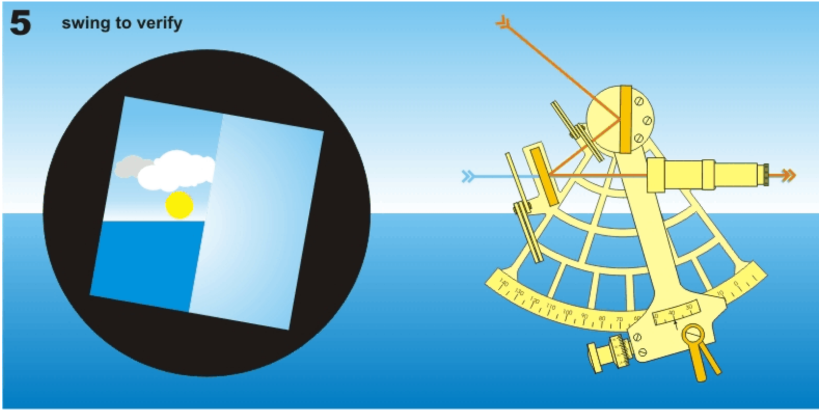

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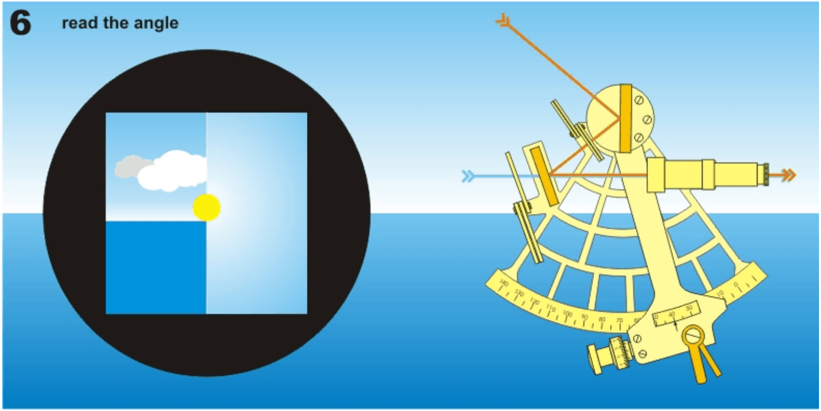
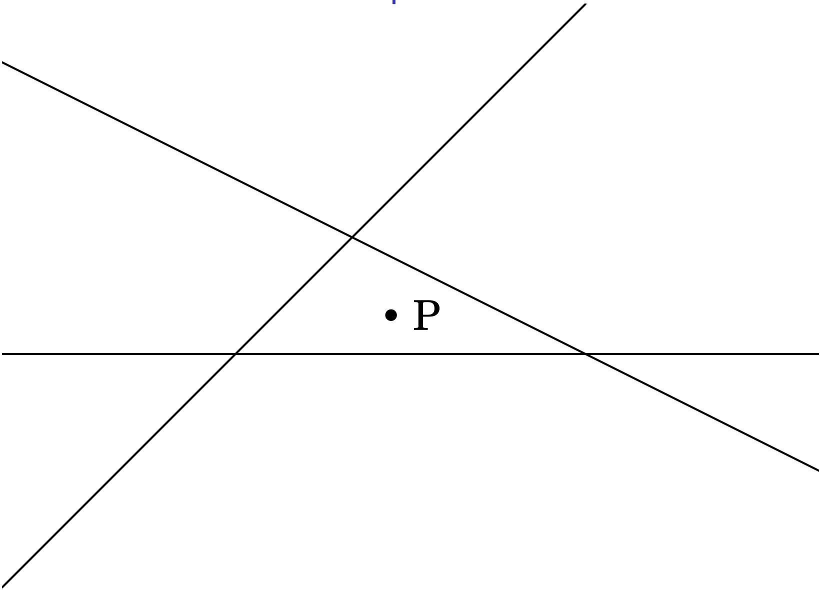


Animation from Wikipedia/Sextant(Marine) Joaquim Alves Gaspar CC BY-SA 2.5

Three lines don't meet at a point!



Solve three equations in two variables

$$\begin{aligned}a_{11}x_1 + a_{12}x_2 &= b_1\\a_{21}x_1 + a_{22}x_2 &= b_2\\a_{31}x_1 + a_{32}x_2 &= b_3\end{aligned}$$

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$$
Ax = b
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Least squares solution

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\begin{aligned}
d_1 &= a_{11}x_1 + a_{12}x_2 - b_1 \\
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Minimize

$$d_1^2 + d_2^2 + d_3^2 = ||d||^2$$

Least squares solution

$$||d||^2 \geq 0$$

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$||d||^2 \geq 0$ Differentiate to find minimum point

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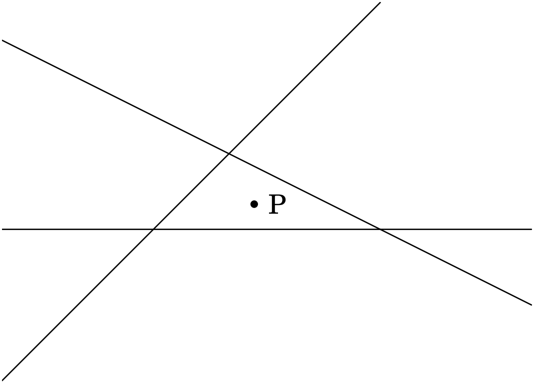
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Where is the centre of a triangle?



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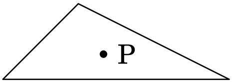

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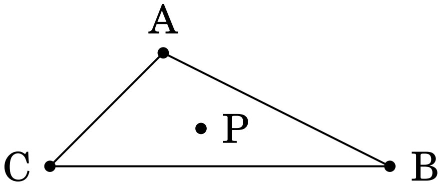

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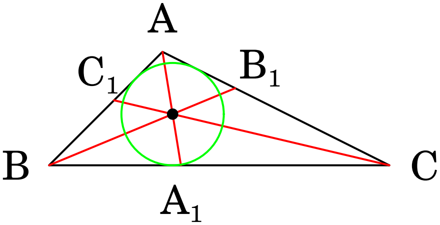
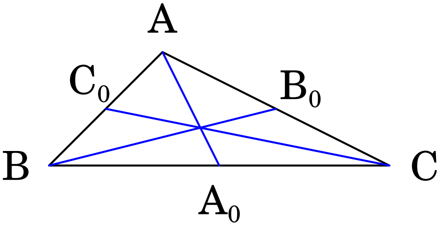


The bisectors of the angles meet at the centre of the inscribed circle - the *incentre*.

Where is the centre of a triangle?



The medians are lines joining vertices to midpoints of the opposite sides. They meet at the *centroid*, the centre of gravity

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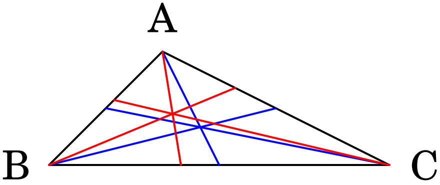

These centres are typically different

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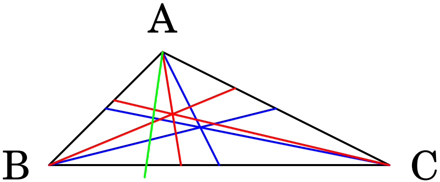

Now reflect the median line (blue) in the bisector (red). The green line is a *symmedian* line

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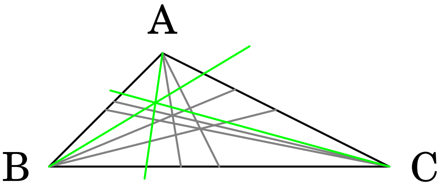

The green lines meet at the *symmedian* point. This is the point that minimizes the sum of squared distances from the sides. Note it is closer to the shorter side.

So how many ‘centres of a triangle’ are there?

There is a list: Clark Kimberling’s *Encyclopedia of Triangle Centers*.

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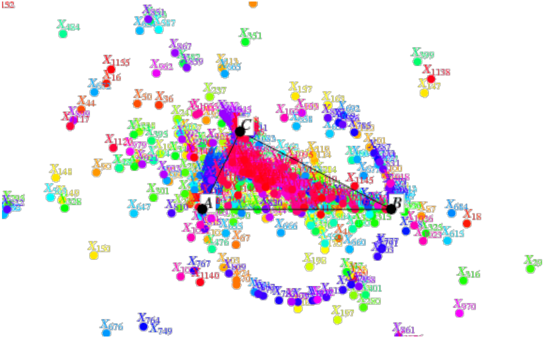

From:Weisstein, Eric W. "Kimberling Cent er." MathWorld–A Wolfram Web Resource.

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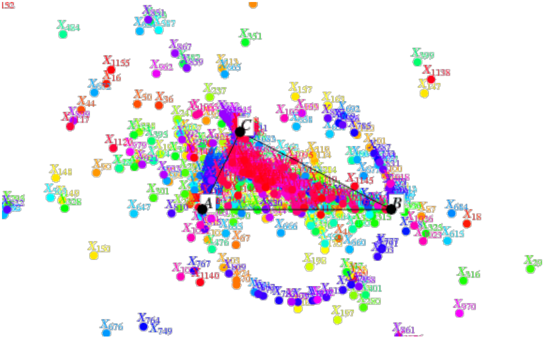

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There are currently 11809

And GPS?

To find your position (including height) using GPS you need the distance to four satellites. The intersection of four planes in three dimensional space.

I will leave it to you to see if you can extend what we have done today to that case!

Notes

- If you want a quick overview of Celestial Navigation I recommend Blue Planet Cruising School's "Celestial Navigation the missing introduction" `http://www.blueplanetcruisingschool.com/celestial-navigation-the-missing-introduction/` Diagrams are better than mine!
- If you are interested in the question I raised about GPS and the symmedian of a tetrahedron, []lease have a go yourself. When you have done that you might consult Jawad Sadek, Majid Bani-Yaghoub, and Noah H. Rhee, Forum Geometricorum, Isogonal Conjugates in a Tetrahedron, Forum Geo,Volume 16 (2016) 4350.